Chapter 9

• Estimator (a random variable i.e. a function) and estimate (a realization of the estimator i.e. a number (or a vector)).

• Let $\hat{\theta}$ be an estimator of parameter θ. $\hat{\theta}$ is said to be unbiased if $E\hat{\theta} = \theta$. Examples: $\bar{X}$ is an unbiased estimator of μ, S^2 is an unbiased estimator of σ^2.

• If $\hat{\theta}_1$ and $\hat{\theta}_2$ are two unbiased estimators of θ, then $\hat{\theta}_1$ is said to be more efficient than $\hat{\theta}_2$ if $\mathrm{Var}(\hat{\theta}_1) \leq \mathrm{Var}(\hat{\theta}_2)$ for all values of θ and $\mathrm{Var}(\hat{\theta}_1) < \mathrm{Var}(\hat{\theta}_2)$ for at least one value.

• $(1-\alpha)$ confidence interval $[\hat{\theta}_L, \hat{\theta}_U]$:

$$P([\hat{\theta}_L \leq \theta \leq \hat{\theta}_U]) = 1 - \alpha.$$

• X-s are normally distributed or n is large enough, σ is known. The interval

$$\left[\bar{X} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}}, \bar{X} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right]$$

is a $(1-\alpha)$ confidence interval for μ.

Sample size for $P(|\bar{X} - \mu| \leq \varepsilon) \geq 1 - \alpha$ is

$$n \geq \left(z_{\alpha/2}\frac{\sigma}{\varepsilon}\right)^2.$$

One-sided confidence intervals:

$$\left[\bar{X} - z_{\alpha}\frac{\sigma}{\sqrt{n}}, \infty\right), \quad \left(-\infty, \bar{X} + z_{\alpha}\frac{\sigma}{\sqrt{n}}\right]$$

• X-s are normally distributed, σ is unknown. $(1-\alpha)$ confidence interval for μ is

$$\left[\bar{X} - t_{\alpha/2,n-1}\frac{S}{\sqrt{n}}, \bar{X} + t_{\alpha/2,n-1}\frac{S}{\sqrt{n}}\right].$$

It can be also used when X-s are not normally distributed but n is large.