**1.**

$$X_1, ..., X_n \sim N(\theta, 1)$$

$$\Theta = \{0, 1, 2\}, \Theta_0 = \{0\}, \Theta_1 = \{1, 2\}$$

So, $H_0 : \theta = 0$, $H_1 : \theta = 1$ or $\theta = 2$.

Prior $\pi(0) = 1/2$, $\pi(1) = p$ $(0 \leq p \leq 1/2)$, $\pi(2) = 1/2 - p$.

Which prior is the worst for $H_0$, i. e. for which $p$ probability $P(H_0|X)$ is minimal if

a) $n = 1000$, $\sum X_i = 1100$,

b) $n = 2000$, $\sum X_i = 3100$?

Discuss the result.

**2.** There is one observation $X$ from an exponential distribution with parameter $\theta$:

$$f_X(x|\theta) = \theta e^{-\theta x}, \quad x \geq 0, \quad \theta > 0.$$

One tests the hypothesis $H_0 : \theta \geq \theta_0$ versus $H_1 : \theta < \theta_0$ using Jeffreys' non-informative prior. Find the posterior density (precise expression). Prove that $H_0$ is accepted iff

$$X \leq \frac{\ln 2}{\theta_0}.$$