

1. A sample  $X = (X_1, \dots, X_n)$  is taken from a Poisson distribution with parameter  $\theta$ . Suppose that  $\hat{\theta}_0$  is the Bayes (or GMLE) estimator of  $\theta$  based on the sample  $X$  and on the improper natural non-informative prior, and  $\hat{\theta}_\lambda$  is the Bayes (GMLE) estimator based on the sample  $X$  and on the proper exponential prior  $\pi(\theta) = \lambda e^{-\lambda\theta}$ . Prove that

$$\hat{\theta}_\lambda \rightarrow \hat{\theta}_0$$

in probability, as  $\lambda \rightarrow 0$  ( $n$  is fixed).