1. The sample

$$1, 2, 0, 4$$

is taken from the binomial distribution with parameters $(10, \theta)$. Find the Bayesian estimate for θ if the prior for θ is the beta distribution with parameters (3,5).

2. Let $X_1, ..., X_n$ be a sample from the exponential distribution with unknown parameter θ. Find GMLE of θ if the prior distribution of θ is exponential with parameter λ.

3. A sample $X_1, ..., X_n$ is taken from an exponential distribution with parameter θ. The prior distribution (density) has the form $\pi(\theta) = \pi_1(\theta)\pi_2(\theta)$, where $\pi_1(\theta)$ is the density of gamma distribution with parameters (α, β) and $\pi_2(\theta)$ is uniform distribution over the interval $[a, b]$. Find GMLE of θ.

4. $X_1, ..., X_n \sim Pois(\theta)$, $\theta \sim U[0, 1]$. Find GMLE if $n = 1000$ and the sum of X_i is equal to a) 300, b) 3000.