Lecture 13

1) $f_x(x|\theta) = \theta^x (1-\theta)^{1-x} \quad x = 0, 1$

$\pi(\theta) \propto \theta^2$

$\pi(\theta|x) \propto \theta^{\sum x_i} (1-\theta)^{n - \sum x_i} \theta^2 =$

$= \theta^{2 + \sum x_i} (1-\theta)^{n - \sum x_i}$

$2 + \sum x_i = 10 - \sum x_i$

$\sum x_i = 4 \qquad n - \sum x_i = 6$

2) ~~$f(x|\theta)$~~ $\propto e^{-\theta n} \theta^{\sum x_i}$

$\pi(\theta) \propto \theta^{\alpha_0 - 1} e^{-\frac{\theta}{\beta_0}}$

$\pi(\theta|x) \propto \theta^{\alpha_0 + \sum x_i - 1} e^{-\theta\left(n + \frac{1}{\beta_0}\right)}$

$\alpha_0 + \sum x_i = \alpha_1 \qquad \sum x_i = \alpha_1 - \alpha_0$

$\frac{\beta_0}{1 + n\beta_0} = \beta_1 \qquad n = \frac{\beta_0 - \beta_1}{\beta_0 \beta_1}$

Lecture 16

1) $\pi(\theta) \propto \theta^2(1-\theta)^4$

$f(X|\theta) \propto \theta^{\sum X_i}(1-\theta)^{10n-\sum X_i} = \theta^7(1-\theta)^{33}$

$\pi(\theta|X) \propto \theta^9(1-\theta)^{37} = \theta^{10-1}(1-\theta)^{38-1}$

beta(10, 38)

$\hat{\theta}_B = \frac{10}{10+38} = \frac{10}{48}$

2) $X_1, \dots, X_n \sim \theta e^{-\theta x}$

$\pi(\theta) = \lambda e^{-\lambda\theta}$

$f(X|\theta) \propto \theta^n e^{-\theta\sum X_i}$

$\pi(\theta) \propto \theta^n e^{-\theta(\sum X_i + \lambda)}$

$\hat{\theta}_{GMLE} = \frac{n}{\lambda + \sum X_i}$

3) $X_1, \dots, X_n \sim \theta e^{-\theta x}$

$f(X|\theta) \propto \theta^n e^{-\theta\sum X_i}$

$\pi(\theta|X) \propto \theta^{n+\alpha-1} e^{-\theta/\frac{\beta}{1+\beta\sum X_i}} I_{[a,b]}(\theta)$

$$\hat{\theta}_{GMLE} = \begin{cases} \frac{(n+\alpha-1)\beta}{1+\beta\sum X_i} & \text{if } a \le \frac{(n+\alpha-1)\beta}{1+\beta\sum X_i} \le b \\ a & \text{if } \frac{(n+\alpha-1)\beta}{1+\beta\sum X_i} < a \\ b & \text{if } \frac{(n+\alpha-1)\beta}{1+\beta\sum X_i} > b \end{cases}$$

4) $X_1, \dots, X_n \sim \text{Pois}(\theta)$, $\theta \sim U[0,1]$

$$f(X|\theta) \propto e^{-n\theta} \theta^{\sum X_i}$$

$$\pi(\theta|X) \propto \theta^{\sum X_i} e^{-n\theta} I_{[0,1]}(\theta)$$

$$\hat{\theta}_{GMLE} = \begin{cases} \bar{X} & \text{if } \bar{X} \leq 1 \\ 1 & \text{if } \bar{X} > 1 \end{cases}$$

a) $\hat{\theta} = 0.3$

b) $\hat{\theta} = 1$