1. Let $X_1, ..., X_n$ be a sample from a distribution with the density

$$f(x|\theta) = \begin{cases} 2x/\theta^2 & \text{for } 0 \le x \le \theta, \\ 0 & \text{otherwise.} \end{cases}$$

The prior for θ is a Pareto distribution $Pa(\alpha, \beta)$, $\alpha > 0$, $\beta > 0$, with the density

$$\pi(\theta) = \begin{cases} \alpha\beta^\alpha/\theta^{\alpha+1} & \text{for } \theta \ge \beta, \\ 0 & \text{for } \theta < \beta. \end{cases}$$

a) Show that the posterior is a Pareto distribution and find its parameters (for convenience, use the following short notations: $\mu = \max\{X_1, ..., X_n\}$, $\gamma = \max\{\mu, \beta\} = \max\{X_1, ..., X_n, \beta\}$).

b) Find the GMLE of θ.

c) Find $(1 - \delta)$ HPD credible interval for θ $(0 < \delta < 1)$.