

1. A sample $X = (X_1, \dots, X_n)$ is taken from a Poisson distribution with parameter θ . Suppose that $\hat{\theta}_0$ is the Bayes (or GMLE) estimator of θ based on the sample X and on the improper natural non-informative prior, and $\hat{\theta}_\lambda$ is the Bayes (GMLE) estimator based on the sample X and on the proper exponential prior $p(\theta) = \lambda e^{-\lambda\theta}$. Prove that

$$\hat{\theta}_\lambda \rightarrow \hat{\theta}_0$$

in probability, as $\lambda \rightarrow 0$ (n is fixed).