

1.47 All of the functions are continuous, hence right-continuous. Thus we only need to check the limit, and that they are nondecreasing

a. $\lim_{x \rightarrow -\infty} \frac{1}{2} + \frac{1}{\pi} \tan^{-1}(x) = \frac{1}{2} + \frac{1}{\pi} \left(\frac{-\pi}{2} \right) = 0$, $\lim_{x \rightarrow \infty} \frac{1}{2} + \frac{1}{\pi} \tan^{-1}(x) = \frac{1}{2} + \frac{1}{\pi} \left(\frac{\pi}{2} \right) = 1$, and $\frac{d}{dx} \left(\frac{1}{2} + \frac{1}{\pi} \tan^{-1}(x) \right) = \frac{1}{1+x^2} > 0$, so $F(x)$ is increasing.

b. See Example 1.5.5.

c. $\lim_{x \rightarrow -\infty} e^{-e^{-x}} = 0$, $\lim_{x \rightarrow \infty} e^{-e^{-x}} = 1$, $\frac{d}{dx} e^{-e^{-x}} = e^{-x} e^{-e^{-x}} > 0$.

d. $\lim_{x \rightarrow -\infty} (1 - e^{-x}) = 0$, $\lim_{x \rightarrow \infty} (1 - e^{-x}) = 1$, $\frac{d}{dx} (1 - e^{-x}) = e^{-x} > 0$.

e. $\lim_{y \rightarrow -\infty} \frac{1-\epsilon}{1+e^{-y}} = 0$, $\lim_{y \rightarrow \infty} \epsilon + \frac{1-\epsilon}{1+e^{-y}} = 1$, $\frac{d}{dx} \left(\frac{1-\epsilon}{1+e^{-y}} \right) = \frac{(1-\epsilon)e^{-y}}{(1+e^{-y})^2} > 0$ and $\frac{d}{dx} \left(\epsilon + \frac{1-\epsilon}{1+e^{-y}} \right) > 0$, $F_Y(y)$ is continuous except on $y = 0$ where $\lim_{y \downarrow 0} \left(\epsilon + \frac{1-\epsilon}{1+e^{-y}} \right) = F(0)$. Thus is $F_Y(y)$ right continuous.