2.1 a. $f_x(x) = 42x^5(1-x)$, $0 < x < 1$; $y = x^3 = g(x)$, monotone, and $\mathcal{Y} = (0,1)$. Use Theorem 2.1.5.

$$\begin{aligned} f_Y(y) &= f_x(g^{-1}(y))\left|\frac{d}{dy}g^{-1}(y)\right| = f_x(y^{1/3})\frac{d}{dy}(y^{1/3}) = 42y^{5/3}(1-y^{1/3})(\frac{1}{3}y^{-2/3}) \\ &= 14y(1-y^{1/3}) = 14y - 14y^{4/3}, \quad 0 < y < 1. \end{aligned}$$

To check the integral,

$$\int_0^1 (14y - 14y^{4/3})dy = 7y^2 - 14\frac{y^{7/3}}{7/3}\bigg|_0^1 = 7y^2 - 6y^{7/3}\bigg|_0^1 = 1 - 0 = 1.$$

2.12 We have $\tan x = y/d$, therefore $\tan^{-1}(y/d) = x$ and $\frac{d}{dy}\tan^{-1}(y/d) = \frac{1}{1+(y/d)^2}\frac{1}{d}dy = dx$. Thus,

$$f_Y(y) = \frac{2}{\pi d}\,\frac{1}{1+(y/d)^2}, \qquad 0 < y < \infty.$$

This is the Cauchy distribution restricted to $(0, \infty)$, and the mean is infinite.

2.31 Since the mgf is defined as $M_X(t) = \mathrm{E}e^{tX}$, we necessarily have $M_X(0) = \mathrm{E}e^0 = 1$. But $t/(1-t)$ is 0 at $t = 0$, therefore it cannot be an mgf.