2.17 a. $\int_0^m 3x^2dx = m^3 \stackrel{set}{=} \frac{1}{2} \quad \Rightarrow \quad m = \left(\frac{1}{2}\right)^{1/3} = .794.$

b. The function is symmetric about zero, therefore $m = 0$ as long as the integral is finite.

$$\frac{1}{\pi}\int_{-\infty}^{\infty} \frac{1}{1+x^2}dx = \frac{1}{\pi}\tan^{-1}(x)\Big|_{-\infty}^{\infty} = \frac{1}{\pi}\left(\frac{\pi}{2}+\frac{\pi}{2}\right) = 1.$$

This is the Cauchy pdf.

2.18 $\mathrm{E}|X-a| = \int_{-\infty}^{\infty} |x-a|f(x)dx = \int_{-\infty}^{a} -(x-a)f(x)dx + \int_a^{\infty}(x-a)f(x)dx$. Then,

$$\frac{d}{da}\mathrm{E}|X-a| = \int_{-\infty}^{a} f(x)dx - \int_a^{\infty} f(x)dx \stackrel{\text{set}}{=} 0.$$

The solution to this equation is $a =$ median. This is a minimum since $d^2/da^2\mathrm{E}|X-a| = 2f(a) > 0$.

2.19

$$\begin{aligned}\frac{d}{da}\mathrm{E}(X-a)^2 &= \frac{d}{da}\int_{-\infty}^{\infty}(x-a)^2 f_X(x)dx = \int_{-\infty}^{\infty}\frac{d}{da}(x-a)^2 f_X(x)dx \\ &= \int_{-\infty}^{\infty} -2(x-a)f_X(x)dx = -2\left[\int_{-\infty}^{\infty} xf_X(x)dx - a\int_{-\infty}^{\infty} f_X(x)dx\right] \\ &= -2[\mathrm{E}X-a].\end{aligned}$$

Therefore if $\frac{d}{da}\mathrm{E}(X-a)^2 = 0$ then $-2[\mathrm{E}X-a] = 0$ which implies that $\mathrm{E}X = a$. If $\mathrm{E}X = a$ then $\frac{d}{da}\mathrm{E}(X-a)^2 = -2[\mathrm{E}X-a] = -2[a-a] = 0$. $\mathrm{E}X = a$ is a minimum since $d^2/da^2\mathrm{E}(X-a)^2 = 2 > 0$. The assumptions that are needed are the ones listed in Theorem 2.4.3.