1.39 a. Suppose A and B are mutually exclusive. Then $A \cap B = \emptyset$ and $P(A \cap B) = 0$. If A and B are independent, then $0 = P(A \cap B) = P(A)P(B)$. But this cannot be since $P(A) > 0$ and $P(B) > 0$. Thus A and B cannot be independent.

b. If A and B are independent and both have positive probability, then

$$0 < P(A)P(B) = P(A \cap B).$$

This implies $A \cap B \neq \emptyset$, that is, A and B are not mutually exclusive.

1.49 For every t, $F_X(t) \le F_Y(t)$. Thus we have

$$P(X > t) = 1 - P(X \le t) = 1 - F_X(t) \ge 1 - F_Y(t) = 1 - P(Y \le t) = P(Y > t).$$

And for some t^*, $F_X(t^*) < F_Y(t^*)$. Then we have that

$$P(X > t^*) = 1 - P(X \le t^*) = 1 - F_X(t^*) > 1 - F_Y(t^*) = 1 - P(Y \le t^*) = P(Y > t^*).$$

2.20 From Example 1.5.4, if X = number of children until the first daughter, then

$$P(X = k) = (1-p)^{k-1}p,$$

where p = probability of a daughter. Thus X is a geometric random variable, and

$$\begin{aligned} \mathrm{E}X &= \sum_{k=1}^{\infty} k(1-p)^{k-1}p = p - \sum_{k=1}^{\infty} \frac{d}{dp}(1-p)^k = -p\frac{d}{dp}\left[\sum_{k=0}^{\infty}(1-p)^k - 1\right] \\ &= -p\frac{d}{dp}\left[\frac{1}{p} - 1\right] = \frac{1}{p}. \end{aligned}$$

Therefore, if p = $\frac{1}{2}$,the expected number of children is two.