1.13 If A and B are disjoint, $P(A \cup B) = P(A) + P(B) = \frac{1}{3} + \frac{3}{4} = \frac{13}{12}$, which is impossible. More generally, if A and B are disjoint, then $A \subset B^c$ and $P(A) \leq P(B^c)$. But here $P(A) > P(B^c)$, so A and B cannot be disjoint.

1.33 Using Bayes rule

$$P(M|CB) = \frac{P(CB|M)P(M)}{P(CB|M)P(M) + P(CB|F)P(F)} = \frac{.05 \times \frac{1}{2}}{.05 \times \frac{1}{2} + .0025 \times \frac{1}{2}} = .9524.$$

1.38 a. $P(A) = P(A \cap B) + P(A \cap B^c)$ from Theorem 1.2.11a. But $(A \cap B^c) \subset B^c$ and $P(B^c) = 1 - P(B) = 0$. So $P(A \cap B^c) = 0$, and $P(A) = P(A \cap B)$. Thus,

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)}{1} = P(A)$$

.

b. $A \subset B$ implies $A \cap B = A$. Thus,

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{P(A)}{P(A)} = 1.$$

And also,

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)}{P(B)}.$$

c. If A and B are mutually exclusive, then $P(A \cup B) = P(A) + P(B)$ and $A \cap (A \cup B) = A$. Thus,

$$P(A|A \cup B) = \frac{P(A \cap (A \cup B))}{P(A \cup B)} = \frac{P(A)}{P(A) + P(B)}.$$

d. $P(A \cap B \cap C) = P(A \cap (B \cap C)) = P(A|B \cap C)P(B \cap C) = P(A|B \cap C)P(B|C)P(C)$.

1.52 The function $g(\cdot)$ is clearly positive. Also,

$$\int_{x_0}^{\infty} g(x)dx = \int_{x_0}^{\infty} \frac{f(x)}{1-F(x_0)}dx = \frac{1-F(x_0)}{1-F(x_0)} = 1.$$