1.1 a. Each sample point describes the result of the toss (H or T) for each of the four tosses. So, for example THTT denotes T on 1st, H on 2nd, T on 3rd and T on 4th. There are $2^4 = 16$ such sample points.

b. The number of damaged leaves is a nonnegative integer. So we might use $S = \{0, 1, 2, \ldots\}$.

c. We might observe fractions of an hour. So we might use $S = \{t : t \geq 0\}$, that is, the half infinite interval $[0, \infty)$.

d. Suppose we weigh the rats in ounces. The weight must be greater than zero so we might use $S = (0, \infty)$. If we know no 10-day-old rat weighs more than 100 oz., we could use $S = (0, 100]$.

e. If n is the number of items in the shipment, then $S = \{0/n, 1/n, \ldots, 1\}$.

1.2 For each of these equalities, you must show containment in both directions.

a. $x \in A \backslash B \Leftrightarrow x \in A$ and $x \notin B \Leftrightarrow x \in A$ and $x \notin A \cap B \Leftrightarrow x \in A \backslash (A \cap B)$. Also, $x \in A$ and $x \notin B \Leftrightarrow x \in A$ and $x \in B^c \Leftrightarrow x \in A \cap B^c$.

b. Suppose $x \in B$. Then either $x \in A$ or $x \in A^c$. If $x \in A$, then $x \in B \cap A$, and, hence $x \in (B \cap A) \cup (B \cap A^c)$. Thus $B \subset (B \cap A) \cup (B \cap A^c)$. Now suppose $x \in (B \cap A) \cup (B \cap A^c)$. Then either $x \in (B \cap A)$ or $x \in (B \cap A^c)$. If $x \in (B \cap A)$, then $x \in B$. If $x \in (B \cap A^c)$, then $x \in B$. Thus $(B \cap A) \cup (B \cap A^c) \subset B$. Since the containment goes both ways, we have $B = (B \cap A) \cup (B \cap A^c)$. (Note, a more straightforward argument for this part simply uses the Distributive Law to state that $(B \cap A) \cup (B \cap A^c) = B \cap (A \cup A^c) = B \cap S = B$.)

c. Similar to part a).

d. From part b).
$A \cup B = A \cup [(B \cap A) \cup (B \cap A^c)] = A \cup (B \cap A) \cup A \cup (B \cap A^c) = A \cup [A \cup (B \cap A^c)] = A \cup (B \cap A^c)$.

1.4 a. "A or B or both" is $A \cup B$. From Theorem 1.2.9b we have $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

b. "A or B but not both" is $(A \cap B^c) \cup (B \cap A^c)$. Thus we have

$$\begin{aligned} P((A \cap B^c) \cup (B \cap A^c)) &= P(A \cap B^c) + P(B \cap A^c) && \text{(disjoint union)} \\ &= [P(A) - P(A \cap B)] + [P(B) - P(A \cap B)] && \text{(Theorem1.2.9a)} \\ &= P(A) + P(B) - 2P(A \cap B). \end{aligned}$$

c. "At least one of A or B" is $A \cup B$. So we get the same answer as in a).

d. "At most one of A or B" is $(A \cap B)^c$, and $P((A \cap B)^c) = 1 - P(A \cap B)$.

1.9 a. Suppose $x \in (\cup_\alpha A_\alpha)^c$, by the definition of complement $x \notin \cup_\alpha A_\alpha$, that is $x \notin A_\alpha$ for all $\alpha \in \Gamma$. Therefore $x \in A_\alpha^c$ for all $\alpha \in \Gamma$. Thus $x \in \cap_\alpha A_\alpha^c$ and, by the definition of intersection $x \in A_\alpha^c$ for all $\alpha \in \Gamma$. By the definition of complement $x \notin A_\alpha$ for all $\alpha \in \Gamma$. Therefore $x \notin \cup_\alpha A_\alpha$. Thus $x \in (\cup_\alpha A_\alpha)^c$.

b. Suppose $x \in (\cap_\alpha A_\alpha)^c$, by the definition of complement $x \notin (\cap_\alpha A_\alpha)$. Therefore $x \notin A_\alpha$ for some $\alpha \in \Gamma$. Therefore $x \in A_\alpha^c$ for some $\alpha \in \Gamma$. Thus $x \in \cup_\alpha A_\alpha^c$ and, by the definition of union, $x \in A_\alpha^c$ for some $\alpha \in \Gamma$. Therefore $x \notin A_\alpha$ for some $\alpha \in \Gamma$. Therefore $x \notin \cap_\alpha A_\alpha$. Thus $x \in (\cap_\alpha A_\alpha)^c$.

1.11 We must verify each of the three properties in Definition 1.2.1.

a. (1) The empty set $\emptyset \in \{\emptyset, S\}$. Thus $\emptyset \in \mathcal{B}$. (2) $\emptyset^c = S \in \mathcal{B}$ and $S^c = \emptyset \in \mathcal{B}$. (3) $\emptyset \cup S = S \in \mathcal{B}$.

b. (1) The empty set $\emptyset$ is a subset of any set, in particular, $\emptyset \subset S$. Thus $\emptyset \in \mathcal{B}$. (2) If $A \in \mathcal{B}$, then $A \subset S$. By the definition of complementation, A^c is also a subset of S, and, hence, $A^c \in \mathcal{B}$. (3) If $A_1, A_2, \ldots \in \mathcal{B}$, then, for each i, $A_i \subset S$. By the definition of union, $\cup A_i \subset S$. Hence, $\cup A_i \in \mathcal{B}$.

c. Let $\mathcal{B}_1$ and $\mathcal{B}_2$ be the two sigma algebras. (1) $\emptyset \in \mathcal{B}_1$ and $\emptyset \in \mathcal{B}_2$ since $\mathcal{B}_1$ and $\mathcal{B}_2$ are sigma algebras. Thus $\emptyset \in \mathcal{B}_1 \cap \mathcal{B}_2$. (2) If $A \in \mathcal{B}_1 \cap \mathcal{B}_2$, then $A \in \mathcal{B}_1$ and $A \in \mathcal{B}_2$. Since $\mathcal{B}_1$ and $\mathcal{B}_2$ are both sigma algebra $A^c \in \mathcal{B}_1$ and $A^c \in \mathcal{B}_2$. Therefore $A^c \in \mathcal{B}_1 \cap \mathcal{B}_2$. (3) If $A_1, A_2, \ldots \in \mathcal{B}_1 \cap \mathcal{B}_2$, then $A_1, A_2, \ldots \in \mathcal{B}_1$ and $A_1, A_2, \ldots \in \mathcal{B}_2$. Therefore, since $\mathcal{B}_1$ and $\mathcal{B}_2$ are both sigma algebra, $\cup_{i=1}^{\infty} A_i \in \mathcal{B}_1$ and $\cup_{i=1}^{\infty} A_i \in \mathcal{B}_2$. Thus $\cup_{i=1}^{\infty} A_i \in \mathcal{B}_1 \cap \mathcal{B}_2$.