4.2

~~2.18~~. Denote by $F(x)$ the distribution function of X. Suppose that $mX < c$. Then

$$E|X-c| = \int_{-\infty}^{\infty} |x-c|dF(x) = \int_{-\infty}^{mX} |x-c|dF(x) + \int_{mX}^{c} |x-c|dF(x) + \int_{c}^{\infty} |x-c|dF(x)$$

$$= c\int_{-\infty}^{mX} dF(x) - \int_{-\infty}^{mX} xdF(x) + c\int_{mX}^{c} dF(x) - \int_{mX}^{c} xdF(x) + \int_{c}^{\infty} xdF(x) - c\int_{c}^{\infty} dF(x)$$

and

$$E|X-mX| = mX\int_{-\infty}^{mX} dF(x) - \int_{-\infty}^{mX} xdF(x) - mX\int_{mX}^{c} dF(x) + \int_{mX}^{c} xdF(x)$$
$$+\int_{c}^{\infty} xdF(x) - mX\int_{c}^{\infty} dF(x),$$

hence

$$E|X-c| - E|X-mX|$$
$$= (c-mX)\int_{-\infty}^{mX} dF(x) - (c-mX)\int_{c}^{\infty} dF(x) - (c-mX)\int_{mX}^{c} dF(x) + 2\int_{mX}^{c} (c-x)dF(x)$$
$$= (c-mX)\big(P(X \le mX) - P(X > mX)\big) + 2\int_{mX}^{c} (c-x)dF(x) \ge 0.$$

In case $mX > c$, the proof is analogous.