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Integration of vehicle yaw stabilisation and rollover prevention through nonlinear hierarchical control allocation

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This work presents an approach to rollover prevention that takes advantage of the modular structure and optimisation properties of the control allocation paradigm. It eliminates the need for a stabilising roll controller by introducing rollover prevention as a constraint on the control allocation problem. The major advantage of this approach is the control authority margin that remains with a high-level controller even during interventions for rollover prevention. In this work, the high-level control is assigned to a yaw stabilising controller. It could be replaced by any other controller. The constraint for rollover prevention could be replaced by or extended to different control objectives. This work uses differential braking for actuation. The use of additional or different actuators is possible. The developed control algorithm is computationally efficient and suitable for low-cost automotive electronic control units. The predictive design of the rollover prevention constraint does not require any sensor equipment in addition to the yaw controller. The method is validated using an industrial multi-body vehicle simulation environment.

Keywords: active safety; control allocation; integrated vehicle dynamics control; rollover

1. Introduction

Control systems for active safety are among the major advances in automotive technology over the past decades. In the wake of this success, the number and complexity of automatic control systems designed to improve the safety and comfort of the vehicle steadily increases.

This remarkable growth makes the integration of the single sub-systems an increasingly relevant and challenging task that is being discussed in the vehicle dynamics community for decades.[1–3] Particularly at the limits of stability, interferences between the sub-systems are a safety-critical issue. The control design needs to rule out such interferences destabilise the vehicle. Moreover, the design objective should be set to use any available actuation to optimise the overall vehicle performance in any possible driving situation.

Electronic stability control (ESC) systems to stabilise the yaw motion and to prevent over- and understeering accidents are standard in modern road vehicles. Another class of active safety systems is designed to prevent vehicle rollover. Vehicles with a high centre of gravity, such as commercial or sport utility vehicles, show a notable vulnerability towards such

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accidents. According to the US National Highway Traffic Safety Administration (NHTSA), rollover is the second most dangerous type of accident.[4]

Active rollover prevention is subject to intense research. Control systems have been successfully designed to mitigate rollover by using brakes,[5–7] steering,[8,9] anti-roll bars,[10,11] or a combination of different actuators.[12–14] The majority of approaches quantifies the rollover threat by an appropriate index and switches the control authority to a roll stabilising controller when indicated. To mitigate rollover without affecting the yaw rate, Rajamani and Piyabongkarn [15] propose to counter-steer the vehicle into the negative roll angle equilibrium during cornering.

The relationship between yaw stabilisation and rollover prevention is complex. Both are related to the lateral acceleration; yaw instability is mainly a low friction phenomenon, whereas untripped rollover mainly occurs at high friction. Thus, both control objectives are generally not in a direct conflict. However, depending on the driving situation, a control design can be more favourable for one of the problems than for the other.

This problem can be illustrated by discussing an oversteering scenario, considering differential braking for actuation. In such case, the decreased radius of curvature of the vehicle path increases the lateral acceleration at the centre of gravity of the vehicle. If the tyre–road interaction allows sufficiently large lateral tyre forces to oppose this acceleration, the resulting roll torque might cause rollover.

A yaw stabilising control action, such as braking the outside turn front wheel as typically performed by ESC systems,[16] generates a yaw torque that counteracts the oversteering, increasing the radius of curvature, decreasing the lateral acceleration, and thus mitigating potential rollover threat as well.[5] In addition, the lateral acceleration is reduced via the decreasing longitudinal velocity. The braking further limits the lateral tyre force of that wheel, which is favourable in terms of rollover prevention.

However, the braking action commanded by a yaw stabilising control system might not be sufficient to prevent rollover. On the other hand, an increased braking action that successfully prevents rollover might be too aggressive in terms of yaw and destabilise the yaw motion. As shown by Rajamani and Piyabongkarn,[15] it is not possible to reduce the rollover threat by reallocation of tyre forces without changing the yaw rate or vehicle speed.

Moreover, in addition to a braking of the outside turn front wheel, rollover can be effectively mitigated by braking all wheels. Taking advantage from the friction circle, lateral tyre forces can be minimised by maximised longitudinal forces, that is, full brake application.[5] This allows the vehicle to skid sideways and thereby redirects the influence of the lateral acceleration from a rotation about the roll axis to a lateral movement. In addition, the braking forces reduce the lateral acceleration via a decreasing longitudinal velocity. The vertical tyre forces are larger on the outside turn wheels than on the inside, especially when the latter are close to lift-off. Therefore, the yaw torque resulting from a braking of all wheels counteracts oversteering. However, due to the intended skidding, the lateral stability suffers and the vehicle might leave the track. Therefore, this strategy is less favourable for yaw stabilisation than a braking of the outside turn front wheel only. Reducing the lateral forces even further by provoking a wheel lock-up is diametrically opposed to the objectives of the anti-lock braking system (ABS).

This brief discussion gives an impression of the subtle challenges of a control design for both yaw stabilisation and rollover prevention. Without doubt, a single heuristically designed ESC-like controller can be an effective and robust measure to mitigate both yaw instability and rollover. However, due to the large number of different driving situations that need to be considered and tested, such approach is expensive in terms of both time and resources. Moreover, if at a later stage an additional control objective or actuation is added, it might be necessary to start the design and testing process from scratch again. Finally, due to possible compromises that have to be made, the resulting system might not prevent any accident that could have been prevented.

The design of two separate stabilising controllers or control modes for yaw and roll is an established alternative.[7,14] As discussed previously, the control authority can be switched from the yaw to the roll controller when the rollover threat is high. Advantages of this approach are the modular structure of the control architecture and the optimised design of both controllers for the respective application.

This work presents an approach to rollover prevention that takes advantage of the modular structure and optimisation properties of the control allocation paradigm. It eliminates the need for a stabilising roll controller by introducing rollover prevention as a constraint on the control allocation problem. The major advantage of this approach is the control authority margin that remains with a high-level controller even during interventions to prevent rollover. In line with the model predictive control of Beal and Gerdes,[17] the use of constraints can be seen as an envelope controller that defines stability boundaries for the operational region accessible for a high-level control.

In this work, the high-level control is assigned to an existing yaw stabilising controller.[18] It could be replaced by any other controller or, as done by Beal and Gerdes,[17] by the requests of the human driver. The constraint for rollover prevention could be replaced by or extended to different control objectives. This work uses differential braking for actuation. The use of additional or different actuators is possible. The developed control allocation algorithm is computationally efficient and suitable for low-cost automotive electronic control units. The predictive design of the rollover prevention constraint does not require any sensor equipment in addition to the yaw controller. The method is validated using an industrial multi-body vehicle simulation environment. This paper extends the results from [19].

Section 2 describes the vehicle model used for control design, divided in a high- and a low-level part as required by the control allocation approach. Section 3 discusses the control architecture, in which the new algorithm is embedded. High- and low-level control and the estimation of non-measured signals are briefly introduced. Section 4 presents the developed control allocation algorithm, which is the main result of this work. Finally, Section 5 shows the behaviour of the control system in a multi-body simulation environment.

2. Modelling

The control allocation paradigm, which is introduced in Section 4, divides vehicle modelling into two parts: a high-level model with a virtual input of forces and torques, and a low-level model providing the relationship between this virtual input and the actual control input. The high-level dynamics describe the chassis motion, while the low-level model is mainly related to tyre–road interaction and the braking system. An overview of all model variables and parameters is given in Table 1 and illustrated in Figure 1.

In order to describe the chassis dynamics, this work relies on a nonlinear two-track model [20] with roll dynamics:

$$\begin{pmatrix} \dot{v} \\ \dot{\beta} \\ \dot{r} \end{pmatrix} = -\begin{pmatrix} 0 \\ r \\ 0 \end{pmatrix} + \begin{pmatrix} \frac{1}{m}\cos\beta & \frac{1}{m}\sin\beta & 0 \\ -\frac{1}{mv}\sin\beta & \frac{1}{mv}\cos\beta & 0 \\ 0 & 0 & \frac{1}{I_z} \end{pmatrix} \begin{pmatrix} F_x \\ F_y \\ M_z \end{pmatrix}, \tag{1}$$

$$\begin{pmatrix} \dot{\phi} \\ \ddot{\phi} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ \dfrac{mgh - c_\phi}{I_x} & -\dfrac{d_\phi}{I_x} \end{pmatrix} \begin{pmatrix} \phi \\ \dot{\phi} \end{pmatrix} - \begin{pmatrix} 0 \\ \dfrac{h}{I_x} \end{pmatrix} F_y. \tag{2}$$

The yaw and roll dynamics are assumed to be mutually independent. The only relationship taken into account is the lateral tyre force F_y being an input to both systems.

This work uses differential braking for actuation, thus, the actual control input consists of the braking pressures. But due to the utilisation of a brake control system able to track commanded longitudinal tyre forces, the low-level model is related to the longitudinal tyre

Table 1. Model variables and parameters.

α_i	Tyre side-slip angle, $\tan\alpha_i := -\frac{v_{y,i}}{v_{x,i}}$
β	Vehicle side-slip angle
δ_i	Steering angle
μ	Tyre–road friction coefficient
ϕ	Roll angle
ψ	Yaw angle
ω_i	Wheel speed
B_i, D_i	Pacejka stiffness and peak factor
$C_{\alpha,i}$	Cornering stiffness
F_x, F_y	Total longitudinal/lateral tyre force
$F_{x,i}, F_{y,i}, F_{z,i}$	Longitudinal/lateral/vertical tyre force
$F_{y,i,\max}$	Maximum lateral tyre force
M_z	Total torque about the yaw axis
a_x, a_y	Vehicle longitudinal/lateral acceleration
i	Subscript for each wheel (Figure 1(a))
r	Yaw rate
v	Absolute vehicle velocity at the centre of gravity
$v_{x,i}, v_{y,i}$	Wheel longitudinal/lateral velocity
$p_{b,i}$	Braking pressure
C, E	Pacejka shape factors
I_x, I_z	Moment of inertia about the roll/yaw axis
$b_\mathrm{F}, b_\mathrm{R}$	Track width front/rear
c_ϕ, d_ϕ	Total roll stiffness/damping
c_1	Maximum cornering stiffness
c_2	Load at the maximum cornering stiffness
g	Gravitational acceleration
h	Height of the centre of gravity above the roll axis
$l_\mathrm{F}, l_\mathrm{R}$	Distance of the front/rear axle to the centre of gravity
m	Total vehicle mass

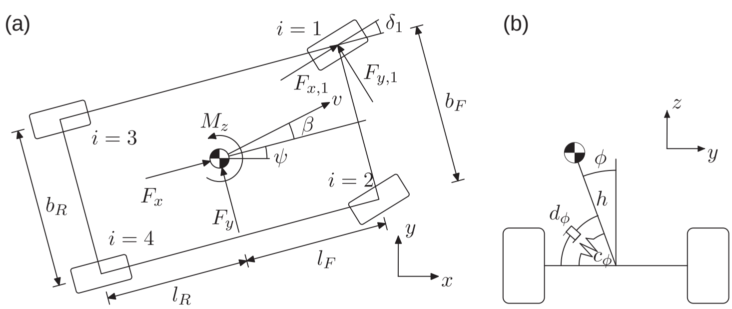

Figure 1. Vehicle schematic in the horizontal and vertical plane.

forces. By

$$\begin{pmatrix} F_x \\ F_y \end{pmatrix} = \sum_{i=1}^{4} \begin{pmatrix} \cos\delta_i & -\sin\delta_i \\ \sin\delta_i & \cos\delta_i \end{pmatrix} \begin{pmatrix} F_{x,i} \\ F_{y,i} \end{pmatrix} \tag{3}$$

and

$$\begin{aligned} M_z &= \sum_{i=1}^{4} \begin{pmatrix} b_i \\ l_i \end{pmatrix}^{\mathrm{T}} \begin{pmatrix} \cos\delta_i & -\sin\delta_i \\ \sin\delta_i & \cos\delta_i \end{pmatrix} \begin{pmatrix} F_{x,i} \\ F_{y,i} \end{pmatrix}, \\ b_{1,2} &:= \mp\frac{b_{\mathrm{F}}}{2}, \quad b_{3,4} := \mp\frac{b_{\mathrm{R}}}{2}, \quad l_{1,2} := l_{\mathrm{F}}, \quad l_{3,4} := l_{\mathrm{R}}, \end{aligned} \tag{4}$$

the virtual control vector can be obtained from the individual tyre forces.[20] The available lateral tyre forces are related to the longitudinal forces via the friction ellipse [21]:

$$F_{y,i} = F_{y,i,\max}\sqrt{1 - \left(\frac{F_{x,i}}{\mu F_{z,i}}\right)^2}, \tag{5}$$

and the maximum lateral tyre forces can, for instance, be determined by Pacejka's model [22]:

$$F_{y,i,\max} = D_i \sin[C \arctan\{B_i\alpha_i - E(B_i\alpha_i - \arctan(B_i\alpha_i))\}], \tag{6}$$

where

$$\begin{aligned} B_i &= \frac{C_{\alpha,i}}{CD}, \\ D_i &= \mu F_{z,i}, \\ C_{\alpha,i} &= c_1 \sin\left(2\arctan\frac{F_{z,i}}{c_2}\right), \end{aligned}$$

or other common tyre models. The friction coefficient μ is assumed to be uniform for all wheels.

Additional or alternative actuators can be included in the presented set-up by the introduction of a respective low-level model.

3. Control architecture

The architecture of the presented control system is illustrated in Figure 2. It consists of following elements.

Estimation. In order to generate estimates of vehicle velocity and side-slip angle based on yaw rate, wheel speed, acceleration, and steering angle measurements, a nonlinear observer based on [23] is utilised, also providing an estimate for the tyre–road friction coefficient.

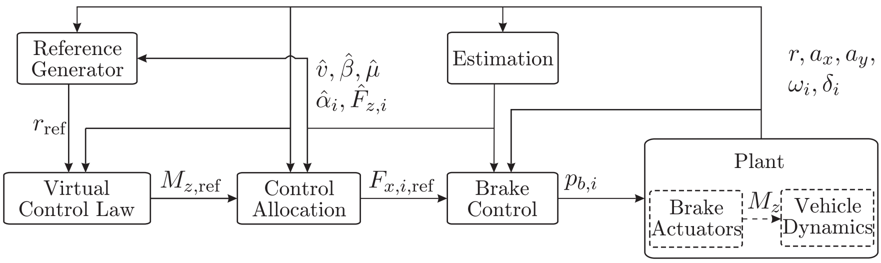

Figure 2. Control architecture.

Using the estimates of vehicle velocity and side-slip angle and measurements of yaw rate and steering angle, the tyre side-slip angles can be approximated [20]:

$$\begin{aligned} \hat{\alpha}_i &= \delta_i - \arctan\left(\frac{v\sin\beta + l_i r}{v\cos\beta + b_i r}\right), \\ b_{1,2} &:= \mp\frac{b_{\mathrm{F}}}{2}, \quad b_{3,4} := \mp\frac{b_{\mathrm{R}}}{2}, \quad l_{1,2} := l_{\mathrm{F}}, \quad l_{3,4} := -l_{\mathrm{R}}. \end{aligned} \tag{7}$$

The vertical tyre forces can be determined as a function of the longitudinal and lateral accelerations,[20]

$$\hat{F}_{z,1,2} = m\left(\frac{l_{\mathrm{R}}g}{l_{\mathrm{F}}+l_{\mathrm{R}}} - \frac{h}{l_{\mathrm{F}}+l_{\mathrm{R}}}a_x\right)\left(\frac{1}{2} \mp \frac{h}{b_{\mathrm{F}}g}a_y\right), \tag{8a}$$

$$\hat{F}_{z,3,4} = m\left(\frac{l_{\mathrm{F}}g}{l_{\mathrm{F}}+l_{\mathrm{R}}} + \frac{h}{l_{\mathrm{F}}+l_{\mathrm{R}}}a_x\right)\left(\frac{1}{2} \mp \frac{h}{b_{\mathrm{R}}g}a_y\right). \tag{8b}$$

Reference generator and virtual control law. As presented in [18], a yaw stabilising reference for the yaw rate can be derived from the vehicle side-slip dynamics. Based on the model (1),

$$\dot{\beta} = -r - \frac{F_x}{mv}\sin\beta + \frac{F_y}{mv}\cos\beta, \tag{9}$$

a desired steady-state zero side-slip angle $\dot{\beta} = \beta = 0$ is associated to a yaw rate

$$r_{\text{ref}} := \frac{1}{mv}F_{y,\text{ref}}, \tag{10}$$

where the total lateral force reference $F_{y,\text{ref}}$ can be determined from the tyre model (e.g. Equations (3)–(6)), by letting $F_{x,i} = 0$, $\forall i$, and using the tyre side-slip angles obtained from Equation (7) for $\beta = 0$. The friction parameter can be used to influence the yaw motion generated by the yaw rate reference.[18]

To track the yaw rate reference, based on the yaw dynamics (1), a virtual control law

$$M_{z,\text{ref}} := -K_P\tilde{r} - K_I\tilde{\psi} + I_z\dot{r}_{\text{ref}} \tag{11}$$

in the form of a PI controller is designed such that under the highly simplifying assumptions of the model, the origin of the error dynamics described by $\tilde{r} = r - r_{\text{ref}}$ and $\dot{\tilde{\psi}} = \tilde{r}$ is uniformly globally asymptotically stable for $M_z = M_{z,\text{ref}}$ and $K_P, K_I > 0$.[18]

Brake control. The control system utilises a non-trivial proprietary brake control system to determine the braking pressures based on demanded longitudinal tyre forces $F_{x,i,\text{ref}}$. The system further takes the role of an ABS. Beyond the scope of this work, the ABS function could be integrated in the form of a third objective along yaw stabilisation and rollover prevention.

Control allocation. This work develops a nonlinear control allocation algorithm for the hierarchical allocation problem of mapping a yaw stabilising virtual input under a constraint representing rollover prevention. It defines dynamic update laws for the desired longitudinal tyre forces $F_{x,i,\mathrm{ref}}$, tracked by the brake control system, such that the yaw stabilising virtual control $M_{z,\mathrm{ref}}$ is tracked under consideration of a constraint designed to prevent rollover. Further, a minimum braking objective and the actuator constraints are incorporated in the form of an instantaneous cost function.

4. Control allocation

Due to its modular structure and optimisation properties, control allocation-based approaches appear well suited for the integration of different control objectives.

As illustrated in Figure 3, such approach splits the design process into two parts: first, a high-level controller is designed on the basis of a model considering a virtual control input $\tau \in \mathbb{R}^p$ of forces and torques. Then, in a second step named control allocation, this virtual control is mapped to the actual control input $u \in \mathbb{R}^q$. Particularly interesting is control allocation in case of over-actuated systems, that is, $q > p$, where a configuration of actuators that generates a certain control action is not necessarily unique. For such systems, it is the task of the allocation to find the solution that fits previously defined criteria best, under consideration of actuator constraints.

An additional advantage of control allocation approaches is the simple adaptability to an alternative or extended set of actuators. The low-level actuator model can be changed without affecting the high-level control. Actuator failures can be handled by reallocating the control input to the remaining actuators.

The feasibility of the allocation problem can be limited by the actuator constraints. If, for example, an emergency lane change was to be executed by brake forces only, then the allocation is not instantaneously feasible.

An overview of control allocation methods and applications is given in [24]. Being well established in flight control and marine vessels control, various computational algorithms addressing the control allocation problem have been developed. Typically, the allocation task is formulated as an optimisation problem and solved by suitable algorithms. Due to the fast dynamics and an implementation on low-cost hardware, for vehicle applications it is of great importance to consider computationally efficient methods.

Existing studies in vehicle dynamics control adopted the control allocation approach to ground vehicles, with various choices of actuators. Traditionally, ESC systems are already designed in a similar architecture as shown in Figure 3.[16] Yaw stabilising control is thus a natural application for the control allocation theory and has been successfully realised.[18,25,26] Similar approaches that use control allocation to coordinate the actuation for integrated vehicle dynamics control systems have been studied by Andreasson and Bünte,[27] Acarman,[28] and Tagesson et al.[29] A control allocation approach for rollover

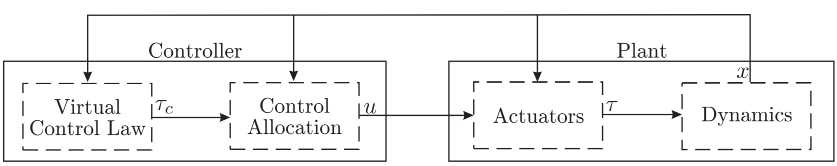

Figure 3. Control architecture with control allocation.

prevention without yaw stabilisation is presented by Johansson and Gräfvert.[30] Possible benefits for electric vehicles are discussed by Chen and Wang.[31]

An allocation-based control system that integrates yaw stabilisation and rollover prevention has been developed by Schofield.[7] Rollover prevention is achieved by a threshold-switched controller with a threshold on the lateral acceleration. Mapping this virtual control to the actual control input is prioritised over a yaw stabilising virtual control by a respectively chosen weighting matrix in the control allocation. The allocation algorithm is based on active set programming methods.

4.1. *Problem formulation*

As introduced in Section 1, this work formulates rollover prevention by a constraint on the vehicle motion.

The control allocation of the yaw stabilising high-level control is an optimisation problem. By reformulating the yaw control allocation as a minimisation problem subject to the constraint for rollover prevention, the interferences between both objectives can be minimised. This work develops a suitable control allocation algorithm for this problem set-up.

To allow a nonlinear formulation of the yet to be defined constraint representing rollover prevention, nonlinear control allocation methods have to be considered. The presented control allocation in the form of a dynamic update law is based on the nonlinear optimising allocation algorithm developed in [32,33]. It considers an over-actuated nonlinear system of the form

$$\dot{x} = f(t, x, \tau), \tag{12a}$$

$$\tau = h(t, x, u), \tag{12b}$$

where $t \geq 0$ denotes time, $x \in \mathbb{R}^n$ the state vector, $u \in \mathbb{R}^q$ the control input vector, and $\tau \in \mathbb{R}^p$ the vector of virtual controls. The high-level dynamics $f(t, x, \tau)$ consider the virtual control vector τ as an available input, which is obtained from the actual control input u by the low-level actuator model $h(t, x, u)$.

The common control allocation problem of mapping a requested virtual input under minimum cost to the control input is expressed by the nonlinear static minimisation problem

$$\min_u J(t, x, u) \quad \text{s.t. } \tau_c - h(t, x, u) = 0, \tag{13}$$

where $\tau_c \in \mathbb{R}^p$ represents the virtual control generated by a high-level controller and $J(t, x, u)$ an instantaneous cost function.

For the given application, the control input $u_i = F_{x,i,\text{ref}}$ refers to the desired longitudinal tyre forces, $\tau_c = M_{z,\text{ref}}$ to the yaw stabilising virtual control (11), and $h(t, x, u) = M_z$ to the low-level model (4). For a simple tyre model and an inaccurately known friction coefficient, tracking of the yaw rate reference can be improved by considering only the yaw torque created by the longitudinal tyre forces. Omitting the torque created by the lateral forces provides a more direct relationship between control and virtual inputs and avoids noise due to the tyre model. The price paid is a substantial simplification of the yaw dynamics. The states of the high-level dynamics and the additional arguments of the low-level model are treated as known time-varying signals, being constructed by a suitable observer or state estimator.

Hence, mapping of the virtual control addresses yaw stabilisation, while the cost function is defined to minimise the braking and to respect the actuator constraints $u_{\min} \leq u_i \leq u_{\max}$,

$$J(u) := u^{\mathrm{T}} H u - w_u \sum_{i=1}^{4} \ln(u_i - u_{\min}) - w_u \sum_{i=1}^{4} \ln(-u_i + u_{\max}), \tag{14}$$

where $H = H^{\mathrm{T}} > 0$ and $w_u > 0$ are weighting factors.

Introducing rollover prevention in the form of a constraint $C(t, x, u) \in \mathbb{R}^s$ that is prioritised over the mapping of the virtual control changes the allocation problem (13) to

$$u = \arg \min_{u \in \Omega(t,x)} J(t, x, u), \tag{15a}$$

$$\Omega(t, x) = \arg \left\{ \min_u \|\tau_c - h(t, x, u)\| \text{ s.t. } C(t, x, u) = 0 \right\}, \tag{15b}$$

where $\Omega(t, x)$ is the possibly uncountable set of all solutions u to Equation (15b).

4.2. *Rollover constraint design*

In literature, there are various approaches to determine indices that quantify how close the vehicle is to a rollover event.[6,34,35]

In order to be compatible to the proposed allocation algorithm, the constraint for rollover prevention $C(t, x, u)$ has to satisfy the controllability-like condition that for all $C(t, x, u) \neq 0$ there exists a constant $\kappa > 0$ such that

$$\frac{\partial C}{\partial u}(t, x, u) \frac{\partial C}{\partial u}^{\mathrm{T}}(t, x, u) \geq \kappa \mathbb{I}_s, \tag{16}$$

where $\mathbb{I}_s$ denotes the $s \times s$ identity matrix.[32,33] Hence, dependency of the constraint on the control input is necessary.

Based on a truncated Taylor series expansion, this work uses the predicted roll angle $\bar{\phi}(t, x, u)$ after a time step T_p for the constraint definition

$$\bar{\phi}(t, x, u) := \hat{\phi}(t) + T_p \cdot \hat{\dot{\phi}}(t) + \tfrac{1}{2} T_p^2 \cdot \ddot{\phi}(t, x, u), \tag{17}$$

where $\ddot{\phi}(t, x, u)$ is determined by the linear roll model (2) connected to the low-level model. The model-based roll acceleration $\ddot{\phi}(t, x, u)$ is the only term with non-zero partial derivatives with respect to the control input, determining the control action of the allocation algorithm in case of a possible constraint violation. In contrast, the estimates of the roll angle $\hat{\phi}(t)$ and the roll rate $\hat{\dot{\phi}}(t)$ are only required to quantify the rollover threat. Therefore, sampled updates of the roll model are sufficient to obtain these estimates and additional sensors can be avoided.

The constraint for rollover prevention is defined to limit the absolute value of the predicted roll angle to a critical value $\phi_{\text{crit}} \geq 0$:

$$C(t, x, u) := w_C \cdot \Theta(|\bar{\phi}(t, x, u)| - \phi_{\text{crit}}) \cdot (|\bar{\phi}(t, x, u)| - \phi_{\text{crit}})^2, \tag{18}$$

where $\Theta(\cdot)$ denotes the Heaviside step function and $w_C > 0$. For $|\bar{\phi}(t, x, u)| \leq \phi_{\text{crit}}$ the constraint is satisfied and has no influence on the allocation. Equation (18) is twice continuously differentiable since it contains a non-smooth factor.

For this work, the constraint for rollover prevention was designed under the objective of a modest complexity. Its performance can be limited, for instance, if bounce and pitch motions are occurring simultaneously. Advanced definitions of the constraint are possible. Due to actuator limitations and the dynamic nature of the algorithm it is not always feasible to prevent a temporary violation of the constraint. Such event need not necessarily be an actual rollover. Therefore, $C(t, x, u)$ should be defined and finite in a region around the origin.

4.3. *Control allocation algorithm*

Solving the hierarchical allocation problem (15) by iterative numerical optimisation as its formulation implies is very demanding in terms of computational resources. To allow a cost-efficient implementation for real-time operation, the problem set-up (15) should be approximated in a single optimisation step.

There are various approaches towards such approximation. For instance, the yaw rate reference tracking could be included in the cost function instead of its formulation as a constraint. Alternatively, $C(t,x,u)$ could be introduced in Equation (13) as a second constraint along $\tau_c - h(t,x,u) = 0$ and prioritised by a weighting parameter. Taking advantage of the structure of the allocation algorithm, this section shows another approach that emphasises on a systematic set-up of the algorithm.

The common control allocation problem (13) can be reformulated by introducing Lagrange multipliers $\pi_1 \in \mathbb{R}^p$ and the Lagrangian

$$\ell_1(t,x,u,\pi_1) := J(t,x,u) + (\tau_c - h(t,x,u))^{\mathrm{T}}\pi_1. \tag{19}$$

In the following, arguments are left implicit to simplify the notation. Local minima of Equation (13) satisfy the first-order optimality conditions for ℓ_1. As shown in [32,33], this can be utilised to design a control Lyapunov function

$$V_1(t,x,u,\pi_1) := \sigma V_0(t,x) + \frac{1}{2}\left(\frac{\partial \ell_1}{\partial u}^{\mathrm{T}}\frac{\partial \ell_1}{\partial u} + \frac{\partial \ell_1}{\partial \pi_1}^{\mathrm{T}}\frac{\partial \ell_1}{\partial \pi_1}\right), \tag{20}$$

where $\sigma > 0$, attracting the total state to the desired optimal solution of Equation (13). $V_0 : [0,\infty) \times \mathbb{R}^n \to \mathbb{R}$ represents a suitable differentiable Lyapunov function proving that the virtual controller $\tau_c = k(t,x)$ stabilises the origin of the error dynamics.

In analogy to the Lyapunov design of Equation (20), Equation (15) can be approximated by

$$\min_{u,\pi_1} \frac{1}{2}\left(\frac{\partial \ell_1}{\partial u}^{\mathrm{T}}\frac{\partial \ell_1}{\partial u} + \frac{\partial \ell_1}{\partial \pi_1}^{\mathrm{T}}\frac{\partial \ell_1}{\partial \pi_1}\right) \quad \text{s.t.} \quad C(t,x,u) = 0, \tag{21}$$

minimising the Lyapunov function (20) subject to a constraint $C(t,x,u)$. Thus, if no constraint would be present, any feasible solution to this problem would control the Lyapunov function to the origin, which is equivalent to solving Equation (13). Subject to a constraint $C(t,x,u)$, the solution is attracting (x,u) to the optimum of Equation (13) within the limits set by the constraint. Equation (15) can thus be approximated in a single optimisation step.

Since Equations (13) and (21) both are instances of a general class of problems

$$\min_{\tilde{u}} \mathcal{J}(t,x,\tilde{u}) \quad \text{s.t.} \quad \mathcal{C}(t,x,u) = 0, \tag{22}$$

where in Equation (13) $\tilde{u} = u$ and in Equation (21) $\tilde{u} = (u,\pi_1)$, it is possible to extend the results of [32,33] to problems of the form shown in Equation (21).

Introducing Lagrange multipliers $\pi_2 \in \mathbb{R}^s$, Equation (21) is reformulated by the Lagrangian

$$\ell_2(t,x,u,\pi_1,\pi_2) := \frac{1}{2}\left(\frac{\partial \ell_1}{\partial u}^{\mathrm{T}}\frac{\partial \ell_1}{\partial u} + \frac{\partial \ell_1}{\partial \pi_1}^{\mathrm{T}}\frac{\partial \ell_1}{\partial \pi_1}\right) + C^{\mathrm{T}}\pi_2. \tag{23}$$

As shown in [32,33], Equation (20) can be used to derive an optimising control allocation algorithm in the form of a dynamic update law, solving Equation (13) for a general class of nonlinear systems. For a new Lyapunov function candidate in analogy to Equation (20)

$$V_2(t,x,u,\pi_1,\pi_2) := \sigma V_0(t,x) + \frac{1}{2}\left(\begin{pmatrix}\dfrac{\partial \ell_2}{\partial u}\\ \dfrac{\partial \ell_2}{\partial \pi_1}\end{pmatrix}^{\mathrm{T}}\begin{pmatrix}\dfrac{\partial \ell_2}{\partial u}\\ \dfrac{\partial \ell_2}{\partial \pi_1}\end{pmatrix} + \frac{\partial \ell_2}{\partial \pi_2}^{\mathrm{T}}\frac{\partial \ell_2}{\partial \pi_2}\right), \tag{24}$$

the time derivative $\dot{V}_2(t,x,u,\pi_1,\pi_2)$ can be maintained negative definite by defining for u, π_1, and π_2 the dynamic update laws

$$\begin{pmatrix}\dot{u}\\ \dot{\pi}_1\end{pmatrix} = -\Gamma\begin{pmatrix}\dfrac{\partial V_2}{\partial u}\\ \dfrac{\partial V_2}{\partial \pi_1}\end{pmatrix} + \zeta, \tag{25}$$

$$\dot{\pi}_2 = -W\frac{\partial V_2}{\partial \pi_2} + \xi, \tag{26}$$

where $\Gamma = \Gamma^{\mathrm{T}} > 0$ and $W = W^{\mathrm{T}} > 0$. The vector signals $\zeta(t) \in \mathbb{R}^{q+p}$ and $\xi(t) \in \mathbb{R}^{s}$ are chosen to cancel the indefinite terms in $\dot{V}_2(t,x,u,\pi_1,\pi_2)$ and provide a feedforward-like compensation to maintain a time-varying optimum. Applying these update laws, it can be shown under additional technical assumptions that the optimal equilibrium set defined by the first-order optimality conditions is uniformly asymptotically stable (for a detailed derivation and discussion see [32,33]).

Following the steps of this Lyapunov design, the dynamic control allocation solving Equation (21) is defined in the form of the Newton-like update law

$$\begin{pmatrix}\dot{u}\\ \dot{\pi}_1\\ \dot{\pi}_2\end{pmatrix} = -\gamma(\mathbb{H}_2^{\mathrm{T}}\mathbb{H}_2 + \varepsilon\mathbb{I}_m)^{-1}\mathbb{H}_2\left(\begin{pmatrix}\mathbb{H}_1 & 0\\ 0 & \mathbb{I}_s\end{pmatrix}\begin{pmatrix}\dfrac{\partial J}{\partial u} - \dfrac{\partial h}{\partial u}^{\mathrm{T}}\pi_1\\ \tau_c - h\\ C\end{pmatrix} + \begin{pmatrix}\dfrac{\partial C}{\partial u}^{\mathrm{T}}\pi_2\\ 0\\ 0\end{pmatrix}\right), \tag{27}$$

where $\gamma = \gamma^{\mathrm{T}} > 0$, $\varepsilon \geq 0$, and $m = q + p + s$. The matrices $\mathbb{H}_{1,2}$ are defined by

$$\mathbb{H}_1(t,x,u) := \begin{pmatrix}\dfrac{\partial^2 J}{\partial u^2} & -\dfrac{\partial h}{\partial u}^{\mathrm{T}}\\ -\dfrac{\partial h}{\partial u} & 0\end{pmatrix}, \tag{28}$$

$$\mathbb{H}_2(t,x,u) := \begin{pmatrix}\mathbb{H}_1^{\mathrm{T}}\mathbb{H}_1 & \begin{matrix}\dfrac{\partial C}{\partial u}^{\mathrm{T}}\\ 0\end{matrix}\\ \begin{matrix}\dfrac{\partial C}{\partial u} & 0\end{matrix} & 0\end{pmatrix}. \tag{29}$$

For the given application, the feedforward-like terms $\zeta(t)$ and $\xi(t)$ for the update law of [32,33] are small and can be neglected to improve the computational efficiency. Further, in the definitions (Equations (28) and (29)) of $\mathbb{H}_{1,2}$, the term $\partial^2\ell_1/\partial u^2$ is approximated by $\partial^2 J/\partial u^2$ and the Hessian of $\ell_2(t,x,u,\pi_1,\pi_2)$ with respect to (u,π_1) by $\mathbb{H}_1^{\mathrm{T}}\mathbb{H}_1$, enforcing non-singularity and boundedness of these matrices in addition to the computational advantage.

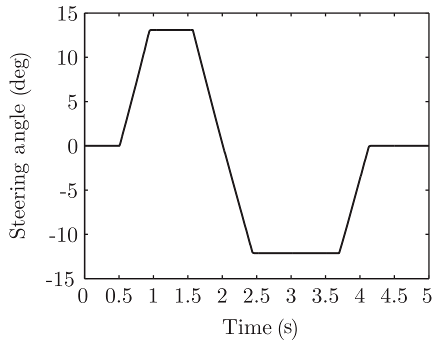

Figure 4. Steering angle input of the rollover critical simulation scenario.

5. Simulation results

The presented control system has been tested in a simulation environment based on Daimler's proprietary multi-body simulation software CASCaDE (Computer Aided Simulation of Car, Driver and Environment) for Matlab/Simulink, which uses a six degrees of freedom rigid body vehicle model with wheel and suspension dynamics. The simulated vehicle is a passenger car. Such a vehicle is typically not very prone to rollover, to increase the rollover threat it is therefore burdened with a load of four passengers, 100 kg roof load, and 100 kg trunk load.

The control algorithm is operated with a sampling time of 5 ms. The brake pressure commands are delayed by 20 ms in order to simulate the effect of communication and computer delays. Both measurement and actuation signals are subject to noise, and the brake pressure build up time is 0.4 s. The rollover critical threshold for the roll angle is defined $\phi_{\text{crit}} = 9^\circ$, and the prediction horizon is $T_p = 50$ ms.

The behaviour of the presented control system has been compared to a yaw stabilising controller without rollover prevention. Such has been generated by replacing the presented hierarchical control allocation by a conventional control allocation [32,33] solving Equation (13) without a rollover prevention constraint. All parameters are chosen identically. Without rollover threat, the presented control system exactly resembles the behaviour of the conventional approach and successfully maintains yaw stability in critical low friction scenarios.

A rollover critical scenario has been generated by a fishhook manoeuvre with a maximum steering angle at 13° and high friction $\mu = 1.2$. The steering angle input during the manoeuvre is shown in Equation (4). The initial vehicle speed is 120 km/h.

The yaw stabilising control system is only designed to address over- and understeering in low friction scenarios. In the rollover critical manoeuvre, it is not sufficient in terms of rollover prevention. As shown in Figure 5(a), the inside turn wheels lift off and the roll angle increases up to 35°. Without changing the high-level controller, adding the rollover prevention constraint in the control allocation successfully maintains the roll stability. As shown in Figure 5(b), at high rollover threat the allocation increases the yaw torque towards the reference commanded by the yaw stabilising high-level controller. This increase is generated by a higher brake force at the outside turn front wheel (Figure 6). By an additional braking of the inside turn rear wheel, the resulting yaw torque is obtained at reduced lateral forces. Furthermore, the vehicle speed is reduced, though this effect is only minor. The system thus combines both rollover prevention strategies discussed in Section 1.

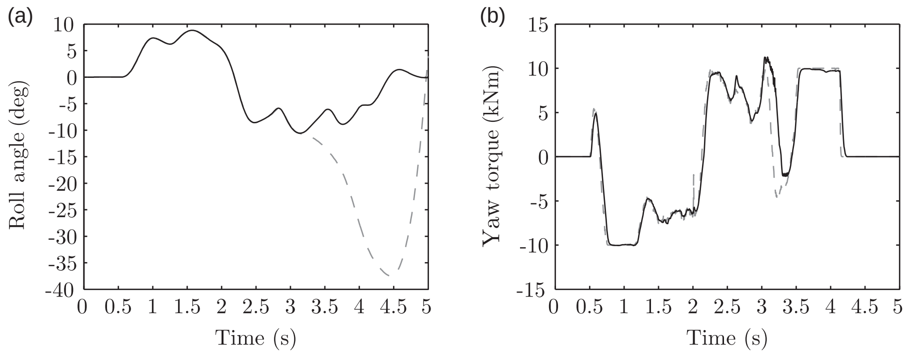

Figure 5. Simulation results. The yaw stabilising control system without rollover prevention is not sufficient to prevent wheel lift-off. The control system with rollover prevention maintains the roll stability by increasing the yaw torque, deviating from the yaw stabilising reference. (a) Roll angle. Dark solid: result for yaw stabilisation with rollover prevention; light dashed: result for yaw stabilisation without rollover prevention. (b) Yaw torque using the control system with rollover prevention. Dark solid: actual value; light dashed: yaw stabilising reference.

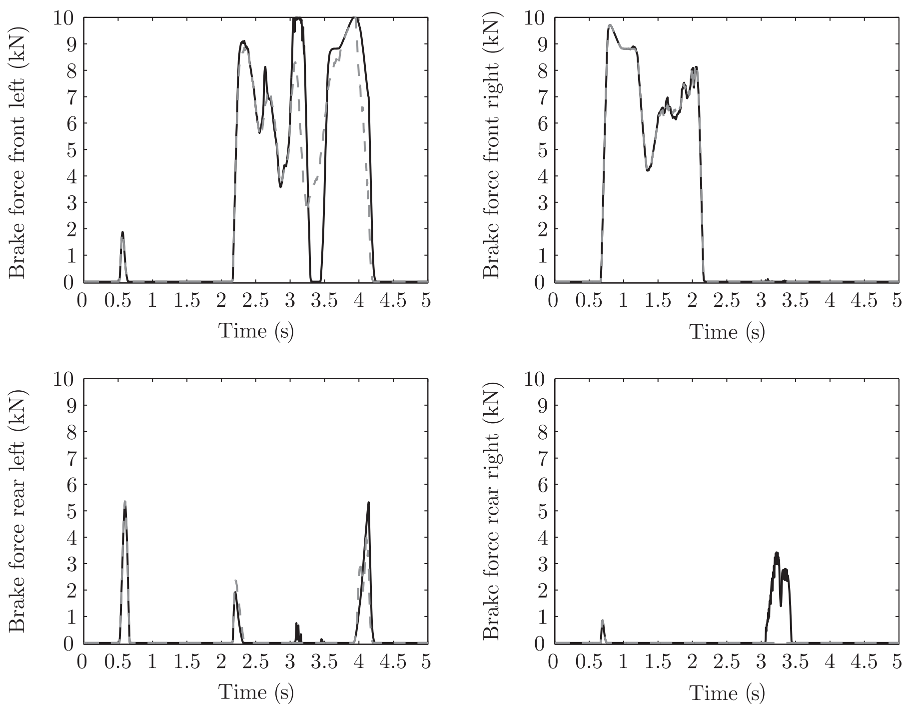

Figure 6. Commanded brake inputs. Dark solid: result for yaw stabilisation with rollover prevention; light dashed: result for yaw stabilisation without rollover prevention. At imminent rollover threat between $t = 3\,\text{s}$ and $t = 3.5\,\text{s}$, the control system with rollover prevention increases the yaw torque by additional braking of the outside turn front wheel. By additionally applying the brakes of the inside turn rear wheel, the resulting yaw torque is generated at reduced lateral forces and vehicle speed.

The presented control system has been successfully tested in various further scenarios in which the yaw stabilising system fails to prevent rollover. In the presented scenario, the steering angle can be increased by almost 50% without rollover taking place.

6. Concluding remarks

This work presented an approach to rollover prevention that takes advantage of the modular structure and optimisation properties of the control allocation paradigm.

Rollover prevention was introduced in the form of a nonlinear constraint on the control allocation problem. The design of a stabilising controller for the roll motion was thus not necessary. The major advantage of this approach is the control authority margin that remains with a high-level controller even during interventions for rollover prevention.

In this work, the high-level control was assigned to a yaw stabilising controller. It could be replaced by any other controller or by the requests of the human driver. The constraint for rollover prevention could be replaced by or extended to different control objectives. Differential braking was used for actuation. The use of additional or different actuators is possible.

This work developed a nonlinear control allocation algorithm for the hierarchical allocation problem of mapping a virtual input under a constraint. It defined dynamic update laws for the actuation. A minimum braking objective and the actuator constraints were incorporated in the form of an instantaneous cost function. The developed control algorithm is computationally efficient and suitable for low-cost automotive electronic control units. The proposed design of the rollover prevention constraint does not require any sensor equipment in addition to the yaw controller.

The developed control system was tested using an industrial multi-body vehicle model. It successfully maintained the roll stability in situations where the original yaw stabilising controller fails. The algorithm requires measurements of yaw rate, lateral and longitudinal acceleration, wheel speeds, and steering angle.

Further research could improve the constraint design. More sophisticated models of the roll dynamics and estimation techniques for the roll angle and the roll rate could be investigated. The slip estimation and the tyre model can be improved. To prevent rollover by minimised lateral forces, short-term high slip operation that overrides the ABS can be analysed in more detail. The derived approach should be applied to different high-level controllers, additional control objectives, and additional actuators. In particular, it could be used to design an envelope control for the requests of the human driver, similar to the model predictive control of Beal and Gerdes.[17] A control allocation approach could provide the benefit of being computationally less expensive than model predictive control.

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