Passivity based compressor surge control using a close-coupled valve

Jan Tommy Gravdahl and Olav Egeland
Department of Engineering Cybernetics
Norwegian University of Science and Technology
O.S. Bragstads plass 8, N-7034 Trondheim, Norway
Email: {Tommy.Gravdahl,Olav.Egeland}@itk.unit.no

Abstract

Passivity-based control of a second order compressor surge model is investigated using a close-coupled valve. The analysis shows that the system has certain passivity properties which leads to a simple controller for the close-coupled valve.

1 Introduction

If the flow through a compressor is throttled to the surge line, the flow becomes unstable. Dependent on the system geometry the instability can take the form of either rotating stall, surge or both. Surge, which will be considered here, is a axisymmetric oscillation of the flow. These oscillations severely reduces the compressor efficiency and can possibly damage the compressor. A number of approaches to control of surge (and rotating stall) have been proposed. A review of the different approaches can be found in [1].

The use of a close-coupled valve for compressor surge control was studied in [3, 4] where this strategy was compared to a number of other possible methods of actuation and sensing. It was concluded that the use of feedback from the mass flow measurement to a close-coupled valve, or alternatively to an injector, has certain advantages compared to other methods. In this paper we will study the use of a close-coupled valve.

Simon and Valavani [3] introduced a Lyapunov function candidate termed the incremental energy to investigate the stability of a compressor with closed-coupled valve control. In this paper the incremental energy function is used as a starting point for controller design based on a passivity analysis of the system. The resulting controller presented in this paper is simpler than the one presented by [3], and the passivity analysis offers some additional insight into the problem which will be briefly discussed in the following.

The notation concerning passivity and L_2 is found in e.g. [5]. A system $u \mapsto y$ with input $u \in L_{2e}$ and output $y \in L_{2e}$ is said to be passive if there exists a β so that $\int_0^T u(t)y(t)dt \geq \beta$ for all $u \in L_{2e}$ and all $T \geq 0$. The inner product on L_{2e} is $\langle u, y\rangle_T = \int_0^T u(t)y(t)dt$, and the truncated norm is $\|u\|_T^2 = \langle u, u\rangle_T$.

2 Modelling

A compressor in series with a close-coupled valve, a plenum and a throttle is studied. The term close-coupled implies that there is no significant mass storage between compressor and the close-coupled valve. The mathematical model is taken from Greitzer [2]. The compressor characteristic is written $\Psi_c(\phi)$ and the close-coupled valve characteristic is written $\Psi_v(\phi)$ where ϕ is the flow coefficient. The resulting or equivalent characteristic of the compressor and valve is accordingly given by

$$\Psi_e(\phi) = \Psi_c(\phi) - \Psi_v(\phi), \tag{1}$$

The compressor characteristic is assumed to be given by

$$\Psi_c(\phi) = -k_3\phi^3 - k_2\phi^2 - k_1\phi, \tag{2}$$

where $k_1 = \frac{3H\phi_0}{2W^2}\left(\frac{\phi_0}{W} - 2\right)$, $k_2 = \frac{3H}{2W^2}\left(\frac{\phi_0}{W} - 1\right)$ and $k_3 = \frac{H}{2W^3}$. Obviously $k_3 > 0$, while $k_1 \leq 0$ if the equilibrium is in the unstable region of the compressor map and $k_1 > 0$ otherwise. The sign of k_2 may vary. The model is [2]

$$\begin{aligned} \dot{\phi} &= B(\Psi_e(\phi) - \psi) \\ \dot{\psi} &= \frac{1}{B}(\phi - \Phi(\psi)), \end{aligned} \tag{3}$$

where ψ is the pressure coefficient, $\Phi(\psi)$ is the throttle characteristic and B is termed the "B-parameter" as defined in [2]. All quantities have been nondimensionalized. It is assumed that the system has been transformed so that $\phi = \psi = 0$ is an equilibrium. Without the valve, the surge line will pass through the local maximum of the compressor characteristics. As in [3], the control variable is assumed to be pressure coefficient Ψ_v of the close-coupled valve.

The throttle is assumed to be a passive component, and the characteristic satisfies the sector condition

$$\Phi(\psi)\psi \geq \gamma\|\psi\|_T^2, \tag{4}$$

which is indeed the case for a typical throttle characteristic.

Our aim will be to design a control law $\Psi = \Psi_v(\phi)$ for the valve such that the compressor also can be operated on the left side of this original surge line without going into surge. That is, we are going to use feedback to move the systems surge line towards lower values of ϕ. This allows for rapid stabilization of the compressor even if the combination of compressor speed and throttle opening results in an equilibrium point to the left of the compressor surge line. An interesting advantage with this solution compared to actuation using the throttle is that there may be significant acoustic dynamics between the compressor and the throttle which in that case limits the performance of the feedback controller.

3 Flow dynamics

Consider the nonegative function

$$V_1(\phi) = \frac{1}{2B}\phi^2 \tag{5}$$

The time derivative along solution trajectories is

$$\dot{V}_1 = -\psi\phi + \Psi_e(\phi)\phi. \tag{6}$$

Then, it is evident that

$$\langle -\psi, \phi \rangle_T = \langle \Psi_e(\phi), \phi \rangle_T + V_1(T) - V_1(0) \geq \langle \Psi_e(\phi), \phi \rangle_T - V_1(0) \tag{7}$$

Hence, the flow dynamics $-\psi \mapsto \phi$ can be given certain passivity properties if the equivalent compressor characteristic $\Phi_e(\phi)$ can be shaped appropriately by selecting the valve control law $\Phi_v(\phi)$.

4 Passivity of pressure dynamics

Proposition 1 *The pressure dynamics $\phi \mapsto \psi$ are passive.*

Proof 1 *Consider the nonnegative function*

$$V_2(\psi) = \frac{B}{2}\psi^2. \tag{8}$$

Differentiating V_2 along the solution trajectories of (3) gives

$$\dot{V}_2 = \psi\phi - \Phi(\psi)\psi. \tag{9}$$

In view of the sector condition (4) and V_2 being nonnegative, it follows that $G_2 : \phi \mapsto \psi$ is passive, and, moreover,

$$\langle \psi, \phi \rangle_T = \langle -\Phi(\psi), \psi \rangle_T + V_2(T) - V_2(0) \geq \gamma \|\psi\|_T^2 - V_2(0) \tag{10}$$

Remark: In [3], the incremental energy

$$V = \frac{B}{2}\psi^2 + \frac{1}{2B}\phi^2 \tag{11}$$

was used as a Lyapunov function candidate. The selection of the functions V_1 and V_2 used here is obviously inspired by this function.

5 Control law

The following simple control law is proposed.

$$\Psi_v = c\phi \tag{12}$$

where $c > \frac{k_2^2}{4k_3} - k_1 + \delta$ and δ is a design parameter.

Proposition 2 *Let the control law be given by (12). Then the equivalent compressor characteristic $-\Psi_e(\phi)$ will satisfy the sector condition*

$$\langle -\Psi_e(\phi), \phi \rangle_T \geq \int_0^T \delta\phi^2(t)dt = \delta\|\phi\|_T^2 \tag{13}$$

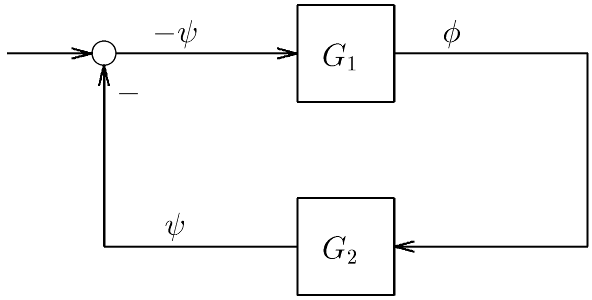

Figure 1: Feedback interconnection

Proof 2 *The compressor characteristics is given according to equation (2). The equivalent compressor characteristic* $\Psi_e(\phi)$ *is then given by*

$$\Psi_e(\phi) = -k_3\phi^3(t) - k_2\phi^2(t) - (k_1 + c)\phi(t) \tag{14}$$

Consider

$$\begin{aligned} \langle -\Psi_e(\phi), \phi \rangle_T &= \int_0^T \phi(t) \left(k_3\phi^3(t) + k_2\phi^2(t) + (k_1 + c)\phi(t) \right) dt \\ &= \int_0^T \phi^2(t) \left(k_3\phi^2(t) + k_2\phi(t) + (k_1 + c) \right) dt \end{aligned} \tag{15}$$

then with

$$c \geq \frac{k_2^2}{4k_3} - k_1 + \delta \tag{16}$$

it it follows that

$$k_3\phi^2(t) + k_2\phi(t) + (k_1 + c) \geq \delta \tag{17}$$

By inserting (17) in (15) we get

$$\langle -\Psi_e(\phi), \phi \rangle_T \geq \int_0^T \delta\phi^2(t) dt = \delta \|\phi\|_T^2, \tag{18}$$

6 Stability analysis

The system is a feedback interconnection of the two systems $G_1 : \phi \mapsto \psi$ and $G_2 : -\psi \mapsto \phi$. In view of (7,10,13) the two operators satisfy

$$\langle \phi, G_1\phi \rangle \geq \delta \|G_1\phi\|_T^2 - V_1(0) \tag{19}$$

$$\langle \psi, G_2\psi \rangle \geq \gamma \|G_2\psi\|_T^2 - V_2(0) \tag{20}$$

$$\tag{21}$$

for all $T \geq 0$ and all $\phi, \psi \in L_{2e}$. Then according to Theorem 2.2.6 in [5], the system is stable.

7 Discussion

The control of compressor pressure rise using mass flow measurements makes sense for a compressor as the reason for compressor instability is related to incidence losses at low mass flows. Thus, the destabilizing effect of incidence losses can be compensated for by the close-coupled valve when feedback from mass flow is used. Also, it is of interest in this connection that mass flow times pressure rise equals power, and this implies that there are inherent passivity properties between pressure rise and mass flow.

It is straightforward to show that the stability result is still valid if the control law (12) is changed as long as the sector condition (13) hold.

Similar results when bleed valves are used are not available. In the case of bleed valves, the control enters differently in the equations, and it seems that the resulting dynamics becomes somewhat more involved. It would be of great interest to investigate further the nonlinear problem when bleed valves are used. Also, the inclusion of more detailed dynamics will be the topic of future research.

References

[1] B. de Jager. Rotating stall and surge control : A survey. In *Proceedings of the 35th Conference on Decision and Control*, pages 1857–1862, New Orleans, LA, 1995.

[2] E.M. Greitzer. Surge and rotating stall in axial flow compressors, Part I: Theoretical compression system model. *Journal of Engineering for Power*, 98:190–198, 1976.

[3] J.S. Simon and L. Valavani. A Lyapunov based nonlinear control scheme for stabilizing a basic compression system using a close-coupled control valve. In *Proceedings of the 1991 American Control Conference*, pages 2398–2406, 1991.

[4] J.S. Simon, L. Valavani, A.H. Epstein, and E.M. Greitzer. Evaluation of approaches to active compressor surge stabilization. *Journal of Turbomachinery*, 115:57–67, 1993.

[5] A. van der Schaft. *L2-gain and passivity techniques in nonlinear control.* Springer, 1996.