Proceedings of the American Control Conference
Albuquerque, New Mexico June 1997
0-7803-3832-4/97/$10.00 © 1997 AACC

Compressor surge control using a close-coupled valve and backstepping

Jan Tommy Gravdahl and Olav Egeland
Department of Engineering Cybernetics
Norwegian University of Science and Technology
O.S. Bragstads plass 8, N-7034 Trondheim, Norway
{Tommy.Gravdahl,Olav.Egeland}@itk.ntnu.no

Abstract

In this paper we propose anti surge controllers for a close coupled valve in a compression system. The valve modifies the characteristic of the compressor, and allows for stable operation beyond the original surge line. The design tool used is backstepping and global uniform asymptotic stability is proven. Damping terms are included in the controllers, and in the presence of both mass flow and pressure disturbances, global uniform boundedness and convergence to a set is ensured. Under the assumption of decaying disturbances the controller ensures convergence to the origin.

1. Introduction

Compressor surge occurs if the flow is throttled beyond the surge line. Dependent on the system geometry the instability can take the form of either rotating stall, surge or both. In this paper we focus on surge which is an axisymmetric oscillation of the flow. These oscillations severely reduces the compressor efficiency and can possibly damage the compressor. A number of approaches to control of surge (and rotating stall) have been proposed. A review of the different approaches can be found in [1].

The use of a close-coupled valve (CCV) for control of compressor surge was studied in [8], and experimental results of compressor surge control using a CCV was reported in [2]. In [9], this strategy was compared, using linear theory, to a number of other possible methods of actuation and sensing. The conclusion was that the most promising methods of surge control is to actuate the system with feedback from the mass flow measurement to a CCV or an injector. Here we will study the use of a CCV.

In [8] the stability of a compressor with CCV control was studied using a Lyapunov function termed the incremental energy. The control law developed in [8] requires knowledge of the compressor characteristic, and additional adjustments to the controller dictated by the Lyapunov analysis is performed in order to avoid a discontinuous controller.

Here we will use backstepping [4] to derive a control law for a CCV which gives a GUAS equilibrium beyond the original surge line. As in [8], disturbances in the pressure rise will be considered and in addition we will also consider disturbances in the plenum outflow. In the case of only pressure disturbances, we will derive a controller that only requires knowledge of an upper bound on the slope of the compressor characteristic in order to guarantee stability. Discontinuousity is not a problem with this controller. Under mild assumptions on the disturbances, global uniform boundedness and convergence will be proven in the presence of both pressure and mass flow disturbances.

Backstepping was used in [5] and [6] to design anti surge and anti stall controllers when the throttle is the control variable. Here, we use the pressure drop across the CCV as the control variable. In [5] and [6] the controller uses feedback from mass flow and pressure. As will be shown, the application of the backstepping procedure to CCV control, in the case of no mass flow disturbances, results in a control law which uses feedback from mass flow only.

2. Compressor and throttle model

The differential equations describing pressure and mass flow oscillations in a compressor-plenum-throttle system is found in [3]. The model is

$$\begin{aligned} \dot{\phi} &= B(\Psi_c(\phi) - \psi) \\ \dot{\psi} &= \frac{1}{B}(\phi - \Phi(\psi)), \end{aligned} \tag{1}$$

where ϕ is the mass flow coefficient (annulus averaged, axial velocity divided by wheel speed, [7]), ψ is the non dimensional plenum pressure or pressure coefficient (pressure divided by density and the square of wheel speed), $\Phi(\psi)$ is the throttle characteristic and B[1] is the "B-parameter" defined in [3]. The time variable t used throughout this text is also nondimensional, and is referred to time for the wheel to rotate one radian.

The compressor characteristic can be modelled as [7],

$$\Psi_c(\phi) = \psi_{c0} + H\left[1 + \frac{3}{2}\left(\frac{\phi}{W} - 1\right) - \frac{1}{2}\left(\frac{\phi}{W} - 1\right)^3\right], \tag{2}$$

where the parameters $\psi_{c0} > 0$, $H > 0$ and $W > 0$ are defined in [7]. The throttle mass flow $\Phi(\psi)$ is given by the throttle characteristic

$$\Phi(\psi) = \gamma\sqrt{\psi} \tag{3}$$

where γ is the throttle gain. The compressor is in equilibrium when $\dot{\psi} = \dot{\phi} = 0$. The steady state values of mass flow ϕ_0 and plenum pressure ψ_0 are found from the intersection of the throttle characteristic with the compressor characteristic as shown in figure 1. As the compressor is throttled, that is, as γ is decreased, the equilibrium point moves along the compressor characteristic towards lower values of ϕ. This is shown in figure 1 for $\gamma = 0.5$ and $\gamma = 0.65$.

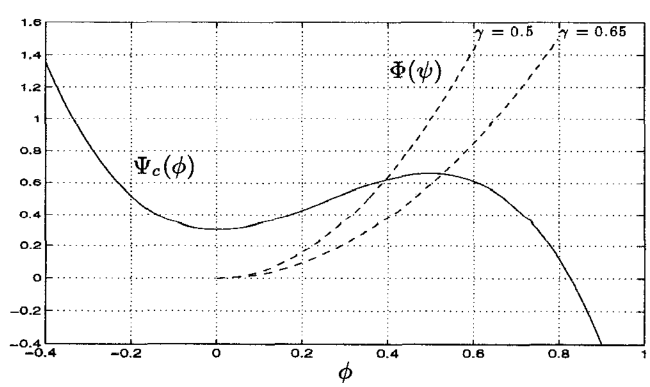

Figure 1: Compressor and throttle characteristic.

[1] $B = \frac{U}{2a_s}\sqrt{\frac{V_p}{A_c L_c}}$, where U is compressor speed, a_s is the speed of sound, V_p is the plenum volume, A_c is the flow area and L_c is the length of ducts and compressor

The nondimensionalization employed, transforms the usual family of curves in the compressor map, one for each compressor speed, to one single characteristic given by (2). The surge line, which passes through the local maxima of the family of curves is transformed to the local maximum of (2). Equilibria to the right of this local maximum are stable, and equilibria to the left are unstable. That is, if the throttle line crosses the compressor characteristic in an area of positive slope, the compressor will go into surge. The objective of this paper is to design control laws that stabilizes these unstable equilibria.

3. Actuation

A compressor in series with a CCV will be studied in the following. The system is shown in figure 3. With close-coupled is understood that the distance between the compressor outlet and the valve is so small that no significant mass storage can take place [8]. The equivalent characteristic of the compressor

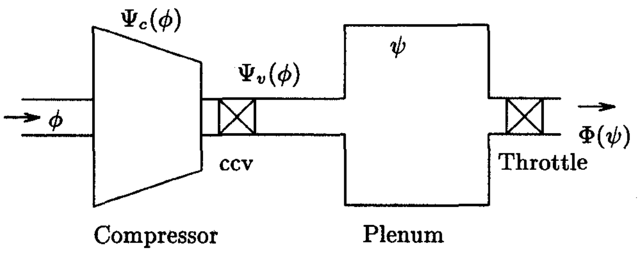

Figure 2: Compression system

and valve is given as

$$\Psi_e(\phi) = \Psi_c(\phi) - \Psi_v(\phi), \tag{4}$$

where $\Psi_c(\phi)$ and $\Psi_v(\phi)$ are the compressor pressure rise and valve pressure drop respectively. The model of [3] can now be written as [8]

$$\begin{aligned} \dot{\phi} &= B(\Psi_e(\phi) - \psi) \\ \dot{\psi} &= \frac{1}{B}(\phi - \Phi(\psi)), \end{aligned} \tag{5}$$

as the compressor in series with the valve is treated as an equivalent compressor.

4. Change of coordinates

In order to simplify the analysis, the approach of [8] is followed where the system is transformed so that the origin becomes the equilibrium under study. This is done by a change of variables to

$$\left.\begin{aligned} \hat{\phi} &= \phi - \phi_0 \\ \hat{\psi} &= \psi - \psi_0 \\ \hat{\Psi}_e(\hat{\phi}) &= \Psi_e(\hat{\phi} + \phi_0) - \Psi_e(\phi_0) \\ \hat{\Psi}_c(\hat{\phi}) &= \Psi_c(\hat{\phi} + \phi_0) - \Psi_c(\phi_0) \\ u = \hat{\Psi}_v(\hat{\phi}) &= \Psi_v(\hat{\phi} + \phi_0) - \Psi_v(\phi_0) \\ \hat{\Phi}(\hat{\psi}) &= \Phi(\hat{\psi} + \psi_0) - \Phi(\psi_0), \end{aligned}\right\} \tag{6}$$

where ϕ_0, ψ_0 now is the equilibrium values of ϕ, ψ in (5). Applying the transformations (6) to the model (5), and using (2) and (3) results in the transformed equations

$$\dot{\hat{\psi}} = \frac{1}{B}\left(\hat{\phi} - \hat{\Phi}(\hat{\psi})\right) \tag{7}$$

$$\dot{\hat{\phi}} = B\left(\hat{\Psi}_c(\hat{\phi}) - \hat{\psi} - u\right), \tag{8}$$

where

$$\hat{\Phi}(\hat{\psi}) = \gamma\sqrt{\hat{\psi} + \psi_0} - \gamma\sqrt{\psi_0}, \tag{9}$$

$$\hat{\Psi}_c(\hat{\phi}) = -k_3\hat{\phi}^3 - k_2\hat{\phi}^2 - k_1\hat{\phi}, \tag{10}$$

$k_1 = \frac{3H\phi_0}{2W^2}\left(\frac{\phi_0}{W} - 2\right)$, $k_2 = \frac{3H}{2W^2}\left(\frac{\phi_0}{W} - 1\right)$ and $k_3 = \frac{H}{2W^3}$. Obviously $k_3 > 0$, while $k_1 \leq 0$ if the equilibrium is in the unstable region of the compressor map and $k_1 > 0$ otherwise. The sign of k_2 may vary.

As in [8], the control variable $u = \hat{\Psi}_v$ is the pressure drop across the valve. Our aim will be to design a control law u for the valve such that the compressor can be operated also on the left side of the original surge line without going into surge. That is, we are going to use feedback to move the systems surge line towards lower values of ϕ.

As the pressure difference across the valve always will be a pressure drop, the valve must be partially closed during stable operation in order for the control u to attain both positive and negative values. A further discussion of the steady state pressure loss can be found in [8].

5. Backstepping

The backstepping methodology of [4] will now be employed in designing a control law for the CCV.

Step 1. Two error variables are defined as $z_1 = \hat{\psi}$ and $z_2 = \hat{\phi} - \alpha$. The control Lyapunov function (clf) for this step is chosen as

$$V_1 = \frac{B}{2}z_1^2 \tag{11}$$

with time derivative

$$\dot{V}_1 = z_1\left(-\hat{\Phi}(z_1) + z_2 + \alpha\right). \tag{12}$$

The load is assumed passive, that is $\hat{\psi}\hat{\Phi}(\hat{\psi}) \geq 0 \; \forall \hat{\phi}$. We have

$$\hat{\psi}\hat{\Phi}(\hat{\psi}) \geq 0 \; \Rightarrow \; -z_1\hat{\Phi}(z_1) \leq 0 \tag{13}$$

As it is desirable to avoid cancellation of useful nonlinearities in (12), the stabilizing function α is not needed and accordingly $\alpha = 0$, which gives

$$\dot{V}_1 = -\hat{\Phi}(z_1)z_1 + z_1 z_2. \tag{14}$$

Although the virtual control α is not needed here, in the interest of consistency with the following sections this notation is kept.

Step 2. The derivative of z_2 is

$$\dot{z}_2 = B\hat{\Psi}_c(z_2) - Bz_1 - Bu. \tag{15}$$

The clf for this step is

$$V_2 = V_1 + \frac{1}{2B}z_2^2 \tag{16}$$

with derivative

$$\dot{V}_2 = -z_1\hat{\Phi}(z_1) + z_2\left(\hat{\Psi}_c(z_2) - u\right). \tag{17}$$

Notice that V_2 as defined by (16) is identical to the incremental energy of [8].

Control law. The control variable u will be chosen so that (17) is made negative definite. To this end we define the *linear* control law

$$u = c_2 z_2, \tag{18}$$

where the controller gain $c_2 > 0$ is chosen so that

$$z_2\Psi_c(z_2) - c_2 z_2^2 < 0. \tag{19}$$

Using (10) this implies that c_2 must satisfy

$$-k_3 z_2^2\left(z_2^2 + \frac{k_2}{k_3}z_2 + \frac{k_1 + c_2}{k_3}\right) < 0. \tag{20}$$

Finding the roots of the above bracketed expression, it is seen that (20) is satisfied if c_2 is chosen according to

$$c_2 > \frac{k_2^2}{4k_3} - k_1. \tag{21}$$

Although (21) implies that the compressor characteristic must be known in order to determine c_2, it can be shown that the knowledge of a bound on the positive slope of the characteristic is sufficient. Differentiating (10) twice wrt $\hat{\phi}$ reveals that the maximum positive slope occurs for $\hat{\phi} = \hat{\phi}_m = -\frac{k_2}{3k_3}$ and is given by

$$a = \left.\frac{d\hat{\Psi}_c(\hat{\phi})}{d\hat{\phi}}\right|_{\hat{\phi}=\hat{\phi}_m} = \frac{k_2^2}{3k_3} - k_1. \tag{22}$$

Assuming that only an upper bound a_m on the positive slope of $\hat{\Psi}_c(\hat{\phi})$ is known, a conservative condition for c_2 is

$$c_2 > a_m \geq a > \frac{k_2^2}{4k_3} - k_1. \tag{23}$$

Thus the price paid for not knowing the exact coefficients of the compressor characteristic is a somewhat conservative condition for the controller gain c_2. Notice also that no knowledge of Greitzer's B-parameter or its upper bound is required in formulating the controller. The final expression for $\dot{V}_2$ is then

$$\dot{V}_2 = -z_1\hat{\Phi}(z_1) + \hat{\Psi}_c(z_2)z_2 - c_2z_2^2 = -W(z_1,z_2) \leq 0. \tag{24}$$

The closed loop system can be written as

$$\dot{z}_1 = \frac{1}{B}(-\hat{\Phi}(z_1) + z_2) \tag{25}$$

$$\dot{z}_2 = B(-z_1 + \hat{\Psi}(z_2) - c_2z_2). \tag{26}$$

It then follows from the LaSalle-Yoshizawa theorem that the equilibrium point $z_1 = z_2 = 0$ is globally uniformly asymptotically stable (GUAS).

5.1. Sensing requirements

Up to this point the pressure drop $\hat{\Psi}_v$ across then CCV has been considered the control variable. It is now assumed that the CCV has a characteristic of the form

$$\Psi_v(\phi) = \frac{1}{\gamma_{cc}^2}\phi^2, \tag{27}$$

where $\gamma_{cc} > 0$ is proportional to the valve opening. Notice that the assumption of no mass storage between the CCV and the compressor (hence close-coupled) implies that the same mass flow ϕ is seen by both the compressor and the CCV. According to (6) and (27) u is given by

$$u = \hat{\Psi}_v(\hat{\phi}) = \Psi_v(\hat{\phi} + \phi_0) - \Psi_v(\phi_0)$$

$$u = \frac{1}{\gamma_{cc}^2}\phi^2 - \Psi_v(\phi_0). \tag{28}$$

Inserting

$$u = c_2\hat{\phi} = c_2(\phi - \phi_0) \tag{29}$$

in (28) and solving for γ_{cc} gives a control law for γ_{cc}:

$$\gamma_{cc} = \frac{\phi}{\sqrt{c_2(\phi - \phi_0) + \Psi_v(\phi_0)}}. \tag{30}$$

This control law requires only sensing of the mass flow ϕ.

6. Disturbances

As in [8] the effect of a pressure disturbance $\hat{\Psi}_d(t)$, and a flow disturbance $\hat{\Phi}_d(t)$ is considered. The pressure disturbance will accelerate the flow, and the flow disturbance is modelling unsteady plenum outflow. In the analysis of [8] $\hat{\Phi}_d(t)$ is set to zero. Here, both disturbances will be considered. The disturbances are time varying, and the only assumption made at this point is boundedness, that is $\|\hat{\Phi}_d\|_\infty$ and $\|\hat{\Psi}_d\|_\infty$ exists. With these disturbances the model is:

$$\dot{\hat{\psi}} = \frac{1}{B}\left(\hat{\phi} - \hat{\Phi}(\hat{\psi}) - \hat{\Phi}_d(t)\right)$$

$$\dot{\hat{\phi}} = B\left(\hat{\Psi}_c(\hat{\phi}) - \hat{\psi} + \hat{\Psi}_d(t) - u\right). \tag{31}$$

To ensure boundedness of the system states, damping is included in the controller design.

6.1. Pressure disturbances

First, pressure disturbances will be considered. That is, $\hat{\Phi}_d(t)$ is set to zero as in [8]. The backstepping procedure is as follows:

Step 1. Identical to Step 1 in section 5.

Step 2. The derivative of z_2 is

$$\dot{z}_2 = B\hat{\Psi}_c(\hat{\phi}) - Bz_1 + B\hat{\Psi}_d(t) - Bu. \tag{32}$$

V_2 is chosen as

$$V_2 = V_1 + \frac{1}{2B}z_2^2 \tag{33}$$

where $\dot{V}_2$ can be bounded according to

$$\dot{V}_2 = -\hat{\Phi}(z_1)z_1 + z_2\left(\hat{\Psi}_c(\hat{\phi}) + \hat{\Psi}_d(t) - u\right). \tag{34}$$

Control law. To counteract the effect of the disturbance, a damping factor $d_2 > 0$ is included and u is chosen as

$$u = c_2z_2 + d_2z_2. \tag{35}$$

c_2 is chosen so that (23) is satisfied. Inserting (35) in (34) gives

$$\dot{V}_2 = -z_1\hat{\Phi}(z_1) + \hat{\Psi}_c(z_2)z_2 - c_2z_2^2 + \hat{\Psi}_d(t)z_2 - d_2z_2^2. \tag{36}$$

Use of Young's inequality gives

$$z_2\hat{\Psi}_d(t) \leq d_2z_2^2 + \frac{\hat{\Psi}_d^2(t)}{4d_2} \leq d_2z_2^2 + \frac{\|\hat{\Psi}_d\|_\infty^2}{4d_2}, \tag{37}$$

and $\dot{V}_2$ can be bounded according to

$$\dot{V}_2 \leq -W(z_1,z_2) + \frac{\hat{\Psi}_d^2(t)}{4d_2} \leq -W(z_1,z_2) + \frac{1}{4d_2}\|\hat{\Psi}_d\|_\infty^2 \tag{38}$$

where

$$W(z_1,z_2) = z_1\hat{\Phi}(z_1) - (\hat{\Psi}_c(z_2)z_2 - c_2z_2^2) \tag{39}$$

is radially unbounded and positive definite. This implies that $\dot{V}_2 < 0$ outside a set $\mathcal{R}_1$ in the $z_1 - z_2$ plane.

According to [4], the fact that $V_2(z_1,z_2)$ and $W(z_1,z_2)$ is positive definite and radially unbounded, and $V_2(z_1,z_2)$ is smooth, implies that there exists class-$\mathcal{K}_\infty$ functions β_1, β_2 and β_3 such that

$$\left.\begin{array}{l}\beta_1(|z|) \leq V_2(z) \leq \beta_2(|z|)\\ \beta_3(|z|) \leq W(z)\end{array}\right\} \tag{40}$$

where $z = (z_1 \;\; z_2)^{\mathrm{T}}$. It now follows from lemma 2.26 in [4], that $z(t)$ is globally uniformly bounded and that $z(t)$ converges to the set

$$\mathcal{R}_1 = \left\{z \;:\; |z| \leq \beta_1^{-1} \circ \beta_2 \circ \beta_3^{-1}\left(\frac{\|\hat{\Psi}_d\|_\infty^2}{4d_2}\right)\right\}. \tag{41}$$

Since $\alpha = 0 \Rightarrow z_1 = \hat{\psi}$ and $z_2 = \hat{\phi}$ the global uniform boundedness and convergence for $\hat{\psi}(t)$ and $\hat{\phi}(t)$ and even $\psi(t)$ and $\phi(t)$ follows.

Notice that the controller (35) is essentially the same as (18), with the only difference being that (35) requires a larger gain in order to suppress the disturbance. Consequently, the same sensing requirements as described in section 5.1. apply.

6.1.1. Convergence to the origin

In this section we show that an additional assumption on the disturbance ensures that the controller (35) not only makes the states globally uniformly bounded, but also guarantees convergence to the origin. It is now assumed that the disturbance term $\hat{\Psi}_d(t)$ is upper bounded by a monotonically decreasing non-negative function $\overline{\Psi}_d(t)$ such that

$$|\hat{\Psi}_d(t)| \leq \overline{\Psi}_d(t) \; \forall t \geq 0 \tag{42}$$

and

$$\lim_{t\to\infty} \overline{\Psi}_d(t) = 0. \tag{43}$$

Inspired by the calculations starting on p. 75 in [4] for a simple scalar system, we introduce the signal $V_2(z)e^{ct}$ for use in the convergence proof. Notice that the positive constant c introduced at this point is used for analysis only, and is *not* included in the implementation of the control law. It now follows that

$$\frac{d}{dt}\left\{V_2(z)e^{ct}\right\} = \left(\dot V_2(z)+cV_2(z)\right)e^{ct}$$
$$\leq \left(-W(z)+\frac{\hat\Psi_d^2(t)}{4d_2}+cV_2(z)\right)e^{ct}$$
$$\leq \left(-\beta_3(|z|)+c\beta_2(|z|)\right)e^{ct}+\frac{\hat\Psi_d^2(t)}{4d_2}e^{ct}. \quad (44)$$

By choosing c according to

$$c \leq \beta_2^{-1}\circ\beta_3(|z|) \leq \beta_2^{-1}\circ\beta_3(\|z\|_\infty), \quad (45)$$

where the existence of $\|z\|_\infty$ follows from (41), (44) gives

$$\frac{d}{dt}\left\{V_2(z)e^{ct}\right\} \leq \frac{\hat\Psi_d^2(t)}{4d_2}e^{ct}. \quad (46)$$

By integrating (46) and using an argument similar to the one in the proof of lemma 2.24 in [4], it can be shown that

$$V_2(z(t)) \leq V_2(z(0))e^{-ct}+\frac{1}{4cd_2}\left(\overline\Psi_d^{\,2}(0)e^{-\frac{ct}{2}}+\overline\Psi_d^{\,2}(t/2)\right). \quad (47)$$

Since $\lim_{t\to\infty}\overline\Psi_d^{\,2}(t/2)=0$ it follows that

$$\lim_{t\to\infty} V_2(z(t)) = 0. \quad (48)$$

As V_2 is positive definite it follows that

$$\lim_{t\to\infty} z(t) = 0. \quad (49)$$

Thus we have shown that under the additional assumptions (42) and (43) on the disturbance term, $z(t)$ converges to the origin. This also implies that $\hat\phi$ and $\hat\psi$ converges to the origin and that $\phi(t)$ and $\psi(t)$ converges to the point of intersection of the compressor and throttle characteristic.

6.2. Pressure and flow disturbances

At this point we include the flow disturbance $\hat\Phi_d(t)$ in the analysis. The backstepping procedure is as follows:

Step 1. As before two error variables z_1 and z_2 are defined as $z_1=\hat\psi$ and $z_2=\hat\phi-\alpha$. Again, V_1 is chosen as

$$V_1 = \frac{B}{2}z_1^2, \quad (50)$$

with derivative

$$\dot V_1 = z_1\left(-\hat\Phi(z_1)+z_2-\hat\Phi_d(t)+\alpha\right), \quad (51)$$

where (31) is used. The virtual control α is chosen as

$$\alpha = -d_1 z_1, \quad (52)$$

where $-d_1z_1$ is a damping term to be used to counteract the disturbance $\hat\Phi_d(t)$. $\dot V_1$ can now be written as

$$\dot V_1 = -d_1z_1^2+z_1z_2-\hat\Phi_d(t)z_1-\hat\Phi(z_1)z_1, \quad (53)$$

and upper bounded according to

$$\dot V_1 \leq -\hat\Phi(z_1)z_1+z_1z_2+\frac{\|\hat\Phi_d\|_\infty^2}{4d_1}. \quad (54)$$

To obtain the bound in (54), Young's inequality has been used to obtain

$$-\hat\Phi_d(t)z_1 \leq d_1z_1^2+\frac{\hat\Phi_d^2(t)}{4d_1} \leq d_1z_1^2+\frac{\|\hat\Phi_d\|_\infty^2}{4d_1}. \quad (55)$$

Step 2. The derivative of z_2 is

$$\dot z_2 = B\hat\Psi_c(\hat\phi)-Bz_1+B\hat\Psi_d(t)-\frac{\partial\alpha}{\partial z_1}\frac{1}{B}\left(-\hat\Phi(z_1)+\hat\phi\right)$$
$$+\frac{1}{B}\frac{\partial\alpha}{\partial z_1}\hat\Phi_d(t)-Bu. \quad (56)$$

From (52) it is seen that

$$\frac{\partial\alpha}{\partial z_1} = -d_1. \quad (57)$$

V_2 is chosen as

$$V_2 = V_1+\frac{1}{2B}z_2^2. \quad (58)$$

Using (54) and (56), an upper bound on $\dot V_2$ is

$$\dot V_2 \leq -\hat\Phi(z_1)z_1+\frac{\|\hat\Phi_d\|_\infty^2}{4d_1}+z_2\Big(\hat\Psi_c(\hat\phi)+\hat\Psi_d(t)$$
$$+\frac{d_1}{B^2}\left(-\hat\Phi(z_1)+\hat\phi\right)-\frac{d_1}{B^2}\hat\Phi_d(t)-u\Big). \quad (59)$$

Control law. To counteract the effect of the disturbances, a damping factor d_2 must be included and u is chosen as

$$u = c_2z_2-k_3\left(\alpha^3+3\alpha z_2^2\right)-k_2\hat\phi^2-k_1\alpha$$
$$+\frac{d_1}{B^2}\left(-\hat\Phi(z_1)+\hat\phi\right)+d_2z_2\left(1+\frac{d_1^2}{B^2}\right). \quad (60)$$

The parameter c_2 is now chosen according to

$$c_2 > |k_1|. \quad (61)$$

Notice that this control law requires knowledge of the coefficients in the compressor characteristic, the throttle characteristic and the B-parameter. Inserting (60) in (59) gives

$$\dot V_2 \leq -(c_2+k_1)z_2^2-k_3(z_2^4+3\alpha^2z_2^2)-\hat\Phi(z_1)z_1+\frac{\|\hat\Phi_d\|_\infty^2}{4d_1}$$
$$-d_2z_2^2+|z_2|\|\hat\Psi_d\|_\infty+\frac{d_1}{B^2}|z_2|\|\hat\Phi_d\|_\infty-\frac{d_1^2}{B^2}d_2z_2^2. \quad (62)$$

Using Young's inequality twice gives

$$|z_2|\|\hat\Psi_d\|_\infty \leq d_2z_2^2+\frac{\|\hat\Psi_d\|_\infty^2}{4d_2} \quad (63)$$

$$\frac{d_1}{B^2}|z_2|\|\hat\Phi_d\|_\infty \leq \frac{1}{B^2}\left(d_1^2d_2z_2^2+\frac{\|\hat\Phi_d\|_\infty^2}{4d_2}\right). \quad (64)$$

The final upper bound for V_2 can now be written as

$$\dot V_2 \leq -W(z_1,z_2)+\frac{1}{\kappa_1}\|\hat\Phi_d\|_\infty^2+\frac{1}{\kappa_2}\|\hat\Psi_d\|_\infty^2 \quad (65)$$

where

$$\frac{1}{\kappa_1}=\left(\frac{1}{4d_1}+\frac{1}{4B^2d_2}\right),\ \frac{1}{\kappa_2}=\frac{1}{4d_2} \quad (66)$$

and

$$W(z_1,z_2) = (c_2+k_1)z_2^2+k_3(z_2^4+3\alpha^2z_2^2)+\hat\Phi(z_1)z_1 \quad (67)$$

is radially unbounded and positive definite. This implies that $\dot V_2<0$ outside a set $\mathcal{R}_2$ in the z_1-z_2 plane. As in section 6.1, the functions $V_2(z)$ and $W(z)$ exhibits the properties in (40). Again, it can be shown that this implies that $z(t)$ is globally uniformly bounded and that $z(t)$ converges to the set

$$\mathcal{R}_2=\left\{z:|z|\leq\beta_1^{-1}\circ\beta_2\circ\beta_3^{-1}\left(\frac{\|\hat\Psi_d\|_\infty^2}{\kappa_1}+\frac{\|\hat\Phi_d\|_\infty^2}{\kappa_2}\right)\right\}. \quad (68)$$

Comment: Once the bounds on the disturbances $\|\hat\Phi_d\|_\infty$ and $\|\hat\Psi_d\|_\infty$ are known, the size of the set $\mathcal{R}_2$ in the z_1-z_2 plane can be made arbitrary small by choosing the damping factors d_1 and d_2 sufficiently large. The same comment applies to the set $\mathcal{R}_1$ defined in (41).

6.2.1. Convergence to the origin

In order to prove convergence of $z(t)$ to the origin, we make the following assumption on $\hat\Phi_d(t)$:

$$|\hat\Phi_d(t)| \leq \overline\Phi_d(t)\ \forall t\geq 0 \quad (69)$$

and

$$\lim_{t\to\infty}\overline\Phi_d(t)=0. \quad (70)$$

$\overline\Phi_d(t)$ is a monotonically decreasing non-negative function. The assumptions on $\hat\Psi_d(t)$ in equations (69) and (70) are also used. By using the same arguments as in section 6.1.1., but with two disturbance terms, it can be shown that

$$V_2(z(t)) \leq V_2(z(0))e^{-ct}+\frac{1}{c\kappa_1}\left(\overline\Phi_d^{\,2}(0)e^{-\frac{ct}{2}}+\overline\Phi_d^{\,2}(t/2)\right)$$
$$+\frac{1}{c\kappa_2}\left(\overline\Psi_d^{\,2}(0)e^{-\frac{ct}{2}}+\overline\Psi_d^{\,2}(t/2)\right). \quad (71)$$

Now $\lim_{t\to\infty} \overline{\Psi}_d^{\,2}(t/2) = 0$ and $\lim_{t\to\infty} \overline{\Phi}_d^{\,2}(t/2) = 0$ implies that $\lim_{t\to\infty} V_2(z(t)) = 0$ and by the positive definiteness of V_2 it follows that

$$\lim_{t\to\infty} z(t) = 0. \tag{72}$$

Thus we have shown that under the assumptions (42),(43), (69) and (70) on the disturbance terms, $z(t)$ converges to the origin. This also implies that $\hat{\phi}(t)$ and $\hat{\psi}(t)$ converges to the origin and that $\phi(t)$ and $\psi(t)$ converges to the point of intersection of the compressor and throttle characteristic.

7. Simulations

The characteristic of [7] and $B = 1$ is used in all simulations. In figure 3, an equilibrium located in the unstable area of the compressor map is stabilized. The throttle gain is set to $\gamma = 0.5$ so that the intersection of the throttle line and the compressor characteristic is located on the part of the characteristic that has positive slope, and thus the equilibrium is unstable, see figure 3. In the two plots to the left in figure 3, it is shown how the controller (18) with $c_2 = 6$ stabilizes the system, while in the two plots to the right noise has been added, and the system is stabilized by the controller (60) with $d_1 = 1$ and $c_2 = d_2 = 6$.

In figure 4 a step in the throttle gain occurs at t=30. The gain changes from $\gamma = 0.65$ to $\gamma = 0.5$ and, consequently, the compressor goes into surge as shown in the two leftmost plots in figure 4. In this simulation a pressure disturbance is included. In the two rightmost plots, the controller (35) with $c_2 = d_2 = 3$ is active. As can bee seen the compressor remains stable after the throttle change. The damping of the disturbance can also be observed.

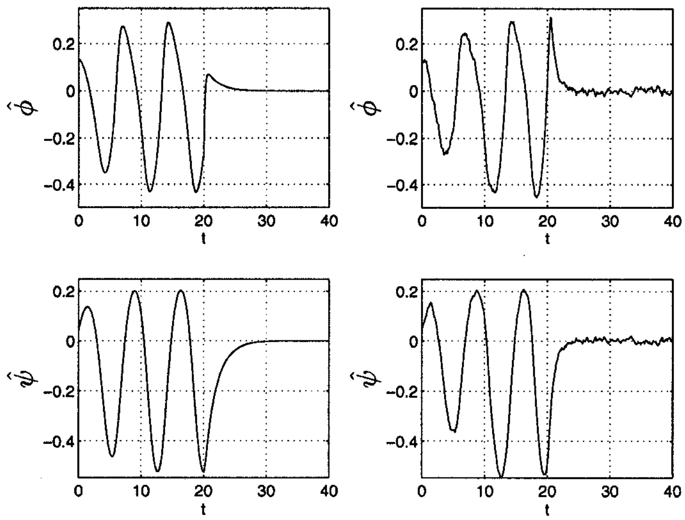

Figure 3: The throttle gain is set to $\gamma = 0.5$, and the compressor is surging. The controllers are switched on at $t = 20$. The pressure disturbance is white noise of amplitude 0.15, and the mass flow disturbance is white noise of amplitude 0.1.

8. Conclusion

Anti surge controllers for a close-coupled valve in series with a compressor have been developed. By the application of the backstepping methodology, a control law which uses feedback from mass flow only has been derived. Only a upper bound on the slope of the compressor characteristic is required to implement this controller. The controller is used both in the case of no disturbances and in the presence of pressure disturbances.

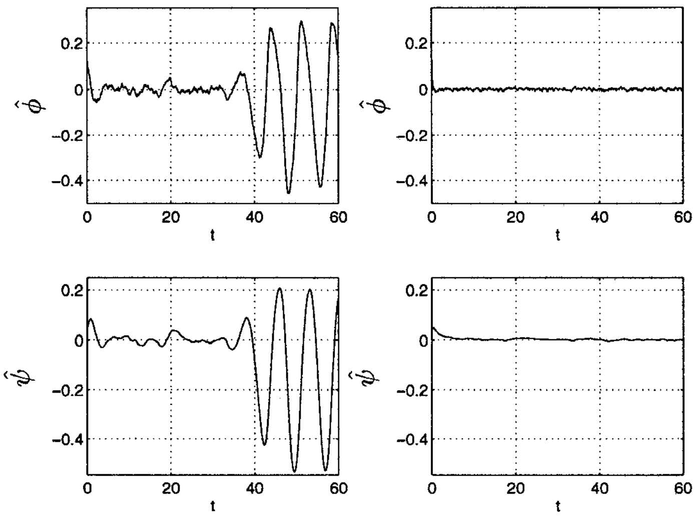

Figure 4: The compressor is throttled from $\gamma = 0.65$ to $\gamma = 0.5$ at t=30. The pressure disturbance is white noise of amplitude 0.10.

A more complicated control law is derived for the case of both pressure- and mass flow disturbances. In order to implement this controller, the compressor characteristic and the B-parameter must be known.

The control laws stabilizes the undisturbed Moore-Greizer model in the previously unstable area of the compressor map. In the presence of disturbances globally uniformly boundedness of both mass flow and pressure is ensured. Under the assumption of decaying disturbances, convergence to the origin is proved.

References

[1] B. de Jager. Rotating stall and surge control : A survey. In *Proceedings of the 35th Conference on Decision and Control*, pages 1857–1862, New Orleans, LA, 1995.

[2] J.L. Dussourd, G.W. Pfannebecker, and S.K. Singhania. An experimental investigation of the control of surge in radial compressors using close coupled resistances. *Journal of Fluids Engineering*, Vol 99:64–76, 1977.

[3] E.M. Greitzer. Surge and rotating stall in axial flow compressors, Part I: Theoretical compression system model. *Journal of Engineering for Power*, Vol 98:190–198, 1976.

[4] M. Krstić, I. Kanellakopoulos, and P.V. Kokotović. *Nonlinear and Adaptive Control Design.* John Wiley & Sons Inc., 1995.

[5] M. Krstić and P.V. Kokotović. Lean backstepping designs for jet engine compressor model. In *Proceedings of the 4th IEEE Conference on Control Applications*, pages 1047–1052, 1995.

[6] M. Krstić, J.M. Protz, J.D. Paduano, and P.V. Kokotović. Backstepping designs for jet engine stall and surge control. In *Proceedings of the 35th Conference on Decision and Control*, pages 3049–3055, New Orleans, LA, 1995.

[7] F.K. Moore and E.M. Greitzer. A theory of post-stall transients in a axial compressor systems: Part I–Development of equations. *Journal of Engineering for Gas Turbines and Power*, Vol 108:68–76, 1986.

[8] J.S. Simon and L. Valavani. A Lyapunov based nonlinear control scheme for stabilizing a basic compression system using a close-coupled control valve. In *Proceedings of the 1991 American Control Conference*, pages 2398–2406, 1991.

[9] J.S. Simon, L. Valavani, A.H. Epstein, and E.M. Greitzer. Evaluation of approaches to active compressor surge stabilization. *Journal of Turbomachinery*, Vol 115:57–67, 1993.