Attitude Determination for the Student Satellite nCubeII: Kalman Filter

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Abstract

In this paper we present the design of a modified Extended Kalman Filter (EKF) for the Norwegian student satellite nCubeII. The EKF determines the attitude of the satellite from available measurements which are the position of the Sun and the magnetic field of the Earth. By using the Newton-Euler optimalization algorithm, the two measurements are combined to produce a single measurement that exactly determines the estimation problem. Simulations show that the modified filter not only maintain previous performance, but improves it. The increased performance is the result of the determination system now exploiting static observability instead of dynamic.

Introduction

The purpose of this paper is to further develop the Extended Kalman Filter designed for the Norwegian cubesat satellite, nCubeII. The filter is based on observations of the Sun's position and the Earth's magnetic field. There exist numerous papers and articles on the problem of attitude determination of spacecrafts, where the EKF is one among the many presented. Because of its small size, the nCube satellite shown in Figure 1 has limited computational power and as a result, previous versions of attitude determination schemes have been proven too computational demanding. In order to overcome this limitation, we propose in this paper to solve the problem using an optimization algorithm to combine the two vector measurements into one quaternion based attitude measurement.

Figure 1: The nCube satellite Photo:NTNU Info/Nina E. Tveter.

Satellite Model

When modeling the satellite, the following frames of reference will be used:

- ECI - Earth-centered inertial frame, $\mathscr{F}_i$
- ECEF - Earth-centered Earth fixed frame, $\mathscr{F}_e$
- Orbit-fixed reference frame, $\mathscr{F}_o$
- Body-fixed reference frame, $\mathscr{F}_b$

Kinematics

To ensure global solutions, we will describe the attitude kinematics in the form of Euler parameters. The parameters are defined from the angle-axis parameters $\boldsymbol{\theta}$ and $\mathbf{k}$ and defines the rotation matrix

$$\mathbf{R}_e(\eta,\boldsymbol{\varepsilon}) = 1 + 2\eta\boldsymbol{\varepsilon}^\times + 2\boldsymbol{\varepsilon}^\times\boldsymbol{\varepsilon}^\times, \tag{1}$$

The satellites kinematic equations may now be expressed as

$$\dot{\eta} = -\frac{1}{2}\boldsymbol{\varepsilon}^{\mathrm{T}}\boldsymbol{\omega}_{ob}^b \tag{2}$$

$$\dot{\boldsymbol{\varepsilon}} = \frac{1}{2}[\eta\mathbf{I} + \boldsymbol{\varepsilon}^\times]\boldsymbol{\omega}_{ob}^b \tag{3}$$

where $\mathbf{q} = (\eta, \boldsymbol{\varepsilon})^{\mathrm{T}}$, can be treated as a unit quaternion.

Dynamics

Derived from elementary ridgid-body mechanics, the satellite's dynamic equations of motion is given by

$$\dot{\boldsymbol{\omega}}_{ib}^b = (\mathbf{I})^{-1}[-(\boldsymbol{\omega}_{ib}^b)^\times\mathbf{I}\boldsymbol{\omega}_{ib}^b + \boldsymbol{\tau}_e^b] \tag{4}$$

where $^\times$ denotes the vector cross product operator, $\mathbf{I}$ is the satellite's inertia given in the body frame $\mathscr{F}_b$, $\boldsymbol{\omega}_{ib}^b$ is the angular velocity of the body frame relative to the ECI frame $\mathscr{F}_i$, $\boldsymbol{\tau}_e^b$ is the external torques acting on the satellite. We assume that the only noticeably external torques are the actuator torque, $\boldsymbol{\tau}_a^b$, and the gravety gradient torque ,$\boldsymbol{\tau}_g^b$, defined as

$$\boldsymbol{\tau}_a^b = (\mathbf{m}^b)^\times\mathbf{B}^b(t) \tag{5}$$

$$\boldsymbol{\tau}_g^b = 3\omega_o^2\mathbf{c}_2^\times\mathbf{I}\mathbf{c}_3 \tag{6}$$

where $\mathbf{m}^b$ is the magnetic moment exerted by the magnetic torques, $\mathbf{B}^b(t)$ is the geomagnetic field and $\mathbf{c}_i$ is the direct cosine of the rotation matrix , where $\mathbf{R}_o^b = \begin{bmatrix}\mathbf{c}_1 & \mathbf{c}_2 & \mathbf{c}_3\end{bmatrix}$.

Discrete satellite model

In order to use the above presented nonlinear model in the EKF design, it has to be linearized. By defining the state vector, $\mathbf{x} = \begin{bmatrix}\mathbf{q} & \boldsymbol{\omega}_{ob}^b\end{bmatrix}^{\mathrm{T}}$, the satellite model can be written in the state space form $\dot{\mathbf{x}} = \mathbf{f}(t,\mathbf{x},\boldsymbol{\tau}_e^b)$. Because there are no control torque while attitude determination is performed, we only linearize the model with respect to the state vector to obtain

$$\Delta\dot{\mathbf{x}} = \mathbf{F}\Delta\mathbf{x} \tag{7}$$

where

$$\mathbf{F} = \frac{\partial\mathbf{f}(t,\mathbf{x},\boldsymbol{\tau}_e^b)}{\partial\mathbf{x}} \tag{8}$$

The linearized satellite model may now be expressed in a general discrete form as

$$\mathbf{x}_{k+1} = \boldsymbol{\Phi}_k\mathbf{x}_k + \mathbf{E}_k\mathbf{w}_k \tag{9}$$

where

$$\mathbf{E}_k = \mathbf{E}\Delta t \tag{10}$$

$$\boldsymbol{\Phi}_k = e^{\mathbf{F}_k\Delta t} = \sum_{k=0}^{\infty}\frac{\mathbf{F}^k\Delta t^k}{k!} \simeq \mathbf{I} + \mathbf{F}_k\Delta t + \frac{1}{2}\mathbf{F}_k^2\Delta t^2 \tag{11}$$

and Δt is the step size and k is the step number.

Sensor Models

Light

The Sun sensor consists of six Light Dependent Resistors (LDR) placed on each of the sides of the satellite, and measures the Suns position relative to the satellites body frame. In order to use the measurement, knowledge of the Suns position relative to the satellites orbital position is needed for comparison. The Sun's position relative to the satellite may be given by

$$\mathbf{s}^o = \mathbf{R}_i^o\mathbf{s}^i \tag{12}$$

where $\mathbf{s}_0^i$ is the Sun's position when the Earth passes vernal equinox, $\mathbf{R}_i^o$ is the rotation matrix defined by the time-varying Sun orbit parameter, λ_s, and the elevation of the Sun, $\boldsymbol{\varepsilon}_s$.

IGRF

The magnetic field over the Earth's surface is highly varying and a simplified version is needed to be of practical use. The IGRF model is a series of mathematical models of the Earth's main field and its annual rate of change. The vector $\mathbf{B}^e$ given by the IGRF model is transformed into $\mathbf{B}^o$ using the following relations

$$\mathbf{B}^{oc} = \mathbf{R}_z(\omega)\mathbf{R}_x(i)\mathbf{R}_z(\Omega - \theta)\mathbf{B}^e \tag{13}$$

$$\mathbf{B}^o = \mathbf{R}_x(\frac{\pi}{2})\mathbf{R}_z(\nu + \frac{\pi}{2})\mathbf{B}^{oc} \tag{14}$$

where ω, i, θ, and ν are the satellites Keplerian elements. Discrete measurements are produced by rotating the references $\mathbf{s}^o$ and $\mathbf{B}^o$ with the satellites actual attitude and adding disturbances as

$$\mathbf{y}_{m,k}^b = \begin{bmatrix}\mathbf{s}_k^b \\ \mathbf{B}_k^b\end{bmatrix} = \begin{bmatrix}\mathbf{R}_o^b\mathbf{s}_k^o \\ \mathbf{R}_o^b\mathbf{B}_k^o\end{bmatrix} + \mathbf{v}_k \tag{15}$$

where $\mathbf{v}_k$ is measurement noise.

EKF Design

Attitude EKF

When employing unit quaternion in representing the attitude it is crucial to maintain the constraint on the quaternion norm. For this, we will use quaternion normalization

$$\bar{\mathbf{q}}_{k+1} = \frac{\bar{\mathbf{q}}_{k+1}}{||\bar{\mathbf{q}}_{k+1}||} \tag{16}$$

Due to numerical round-offs, the introduction of (16) leads to difficulties in maintaining a singular covariance matrix, $\mathbf{P}_k$. The solution is to reduce the dimension of $\mathbf{P}_k$ by one, and is done by removing $\boldsymbol{\eta}$, denoted by the subscript r, from the state vector. By using the unit quaternion property,

$$\mathbf{p}^{\mathrm{T}}\mathbf{p} = \eta^2 + \boldsymbol{\varepsilon}^{\mathrm{T}}\boldsymbol{\varepsilon} = 1, \tag{17}$$

an update for the complete quaternion may be achieved by the following EKF algorithm

$$\mathbf{K}_{r,k} = \bar{\mathbf{P}}_{r,k}\mathbf{H}_{r,k}^{\mathrm{T}}[\mathbf{H}_{r,k}\bar{\mathbf{P}}_{r,k}\mathbf{H}_{r,k}^{\mathrm{T}} + \mathbf{R}_r]^{-1} \tag{18a}$$

$$\boldsymbol{\nu}_k = \mathbf{y}_{m,k}^b \times \bar{\mathbf{y}}_k^b \tag{18b}$$

$$\hat{\mathbf{q}}_k = \bar{\mathbf{q}}_k \otimes \begin{bmatrix}\sqrt{1 - ||\mathbf{K}_{\boldsymbol{\varepsilon},k}\boldsymbol{\nu}_k||^2} \\ \boldsymbol{\nu}_k\end{bmatrix} \tag{18c}$$

$$\hat{\boldsymbol{\omega}}_{ob,k}^b = \bar{\boldsymbol{\omega}}_{ob,k}^b + \mathbf{K}_{\boldsymbol{\omega},k}\boldsymbol{\nu}_k \tag{18d}$$

$$\bar{\mathbf{x}}_{k+1} = \boldsymbol{\Phi}_k\hat{\mathbf{x}}_k \tag{18e}$$

$$\bar{\mathbf{q}}_{k+1} = \frac{\bar{\mathbf{q}}_{k+1}}{||\bar{\mathbf{q}}_{k+1}||} \tag{18f}$$

$$\bar{\mathbf{P}}_{k+1} = \boldsymbol{\Phi}_{r,k}\hat{\mathbf{P}}_{r,k}\boldsymbol{\Phi}_{r,k}^{\mathrm{T}} + \mathbf{Q}_r \tag{18g}$$

where $\mathbf{K}_{\boldsymbol{\omega},k}$ and $\mathbf{K}_{\boldsymbol{\varepsilon},k}$ are submatrices of the Kalman gain matrix, $\mathbf{K}_{r,k}$, $\mathbf{R}_k$ and $\mathbf{Q}_k$ are design matrices describing the expected covariance of the measurement noise, $\mathbf{v}_k$, and the process noise, $\mathbf{w}_k$. The measurement matrix, $\mathbf{H}_k$, is defined and approximated as

$$\mathbf{H}_k = \frac{\partial}{\partial\mathbf{x}}(\bar{\mathbf{y}}_k^b)\bigg|_{\mathbf{x}=\bar{\mathbf{x}}} \simeq \begin{bmatrix}2(\bar{\mathbf{S}}_k^b)^\times & 0_{3\times3} \\ 2(\bar{\mathbf{B}}_k^b)^\times & 0_{3\times3}\end{bmatrix} \tag{19}$$

where $\bar{\mathbf{y}}_k^b$ is (15) with respect to predicted states.

Problems with the EKF

Several researchers have reported poor EKF performance for system that exploits dynamic observability. It is a significant factor in estimator design when the necessary variations of the system is slow compared to the rate at which new measurements are acquired, and it implies that long intervals of subsequent measurements do not generate observability of the complete state vector.

While the above presented attitude determination problem is overdetermined, the two measurements are individually under determined. By using the approximated measurement matrix, (19), the above presented EKF treats the two measurements separately, making the overall system dynamically observable. Dynamic observability occurs when the system relies on variations in the system states or time-varying measurement matrices to achieve observability. It reduces the performance of the EKF in two ways. First, the uncertainty of the state estimates due to linearization increases the corruption of the covariance. Secondly, the accuracy of the covariance matrix's ability to capture important information from past measurements reduces. This information is crucial because combined with later measurements it generates observability.

Modified Attitude EKF

The Gauss-Newton algorithm is a numerical optimization algorithm that uses line search in minimizing the squared-error function

$$\mathbf{Q}^o = \boldsymbol{\zeta}^{\mathrm{T}}\boldsymbol{\zeta} = (\mathbf{y}_r^o - \mathbf{M}\mathbf{y}_m^b)^{\mathrm{T}}(\mathbf{y}_r^o - \mathbf{M}\mathbf{y}_m^b) \tag{20}$$

where

$$\mathbf{M} = \begin{bmatrix}\mathbf{R}_b^o & 0 \\ 0 & \mathbf{R}_b^o\end{bmatrix} \tag{21}$$

By applying the Gauss-Newton algorithm on the two vector measurements we obtain a quaternion, $\hat{\mathbf{q}}_{g,k}$, which is now used as measurement. The EKF's measurement matrix are now reduced to a constant 3×6 matrix and the resulting EKF's calculation load is reduced by almost two thirds. The modified EKF is given by

$$J = -\left[\left(\frac{\partial\mathbf{M}}{\partial\eta_{g,k}}\mathbf{y}_m^b\right)\left(\frac{\partial\mathbf{M}}{\partial\varepsilon_{g,1,k}}\mathbf{y}_m^b\right)\left(\frac{\partial\mathbf{M}}{\partial\varepsilon_{g,2,k}}\mathbf{y}_m^b\right)\left(\frac{\partial\mathbf{M}}{\partial\varepsilon_{g,3,k}}\mathbf{y}_m^b\right)\right] \tag{22a}$$

$$\hat{\mathbf{q}}_{g,k+1} = \hat{\mathbf{q}}_{g,k} - \left[J^{\mathrm{T}}(\hat{\mathbf{q}}_{g,k})J(\hat{\mathbf{q}}_{g,k})\right]^{-1}J^{\mathrm{T}}(\hat{\mathbf{q}}_{g,k})\boldsymbol{\zeta}^o(\hat{\mathbf{q}}_{g,k}) \tag{22b}$$

$$\mathbf{K}_{r,k} = \bar{\mathbf{P}}_{r,k}\mathbf{H}_{r,k}^{\mathrm{T}}[\mathbf{H}_{r,k}\mathbf{P}_{r,k}\mathbf{H}_{r,k}^{\mathrm{T}} + \mathbf{R}_r]^{-1} \tag{22c}$$

$$\Delta\mathbf{q}_k = \hat{\mathbf{q}}_{g,k} \otimes \bar{\mathbf{q}}_k^{-1} \tag{22d}$$

$$\hat{\mathbf{q}}_k = \bar{\mathbf{q}}_k \otimes \begin{bmatrix}\sqrt{1 - ||\mathbf{K}_{\boldsymbol{\varepsilon},k}\Delta\boldsymbol{\varepsilon}_k||^2} \\ \Delta\boldsymbol{\varepsilon}_k\end{bmatrix} \tag{22e}$$

$$\bar{\Delta}\boldsymbol{\varepsilon}_k = \hat{\boldsymbol{\varepsilon}}_{g,k} - \bar{\boldsymbol{\varepsilon}}_k \tag{22f}$$

$$\hat{\boldsymbol{\omega}}_{ob,k}^b = \bar{\boldsymbol{\omega}}_{ob,k}^b + \mathbf{K}_{\boldsymbol{\omega},k}\bar{\Delta}\boldsymbol{\varepsilon}_k \tag{22g}$$

$$\bar{\mathbf{x}}_{k+1} = \boldsymbol{\Phi}_k\hat{\mathbf{x}}_k \tag{22h}$$

$$\bar{\mathbf{q}}_{k+1} = \frac{\bar{\mathbf{q}}_{k+1}}{||\bar{\mathbf{q}}_{k+1}||} \tag{22i}$$

$$\bar{\mathbf{P}}_{k+1} = \boldsymbol{\Phi}_{r,k}\hat{\mathbf{P}}_{r,k}\boldsymbol{\Phi}_{r,k}^{\mathrm{T}} + \mathbf{Q}_r \tag{22j}$$

where

$$\mathbf{H}_{r,k} = \begin{bmatrix}\mathbf{I}_{3\times3} & 0_{3\times3}\end{bmatrix} \tag{23}$$

EKF Performance

The attitude of the nCube is investigated by simulating the satellite as a gravity-gradient stabilized satellite actuated by means of electromagnetic torques. The satellite is simulated in continuous-time, while the attitude determination is, as it would in real life, run in discrete-time. The distinction is performed in order to make the simulations as close to real life as possible, and thus obtain a more actual performance analysis. The measurement disturbance is assumed uncorrelated white Gaussian noise with magnitude of $4.3312\cdot10^{-6}$ for the magnetometer and $1.6374\cdot10^{-6}$ for the sun sensor. The process disturbance contains several different components, whit the resulting disturbance modeled as $8.5\cdot10^{-9}$ for ε_1, ε_2, and ε_3 and $8.5\cdot10^{-12}$ for $\boldsymbol{\omega}_{ob,x}^b$, $\boldsymbol{\omega}_{ob,y}^b$, and $\boldsymbol{\omega}_{ob,z}^b$.

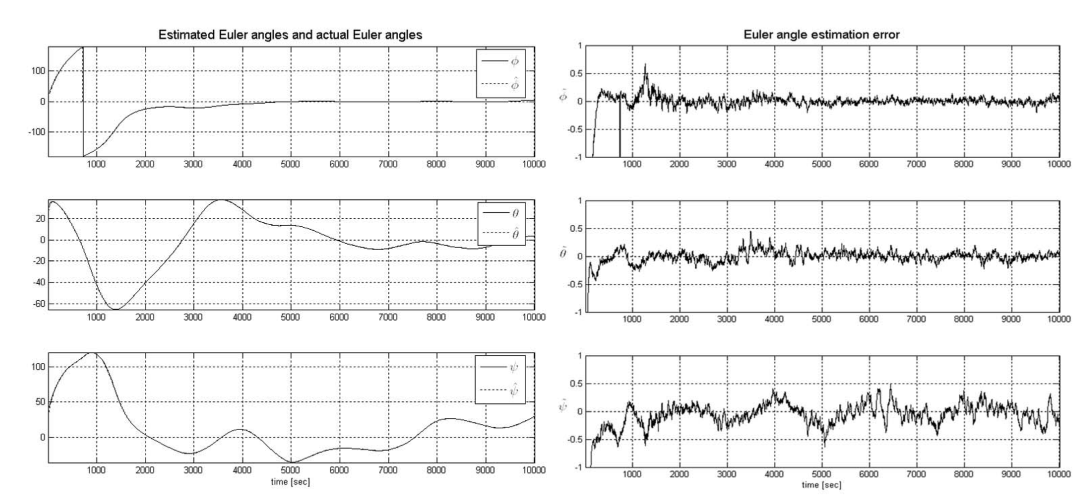

Figure 2: Euler angles and estimation errors

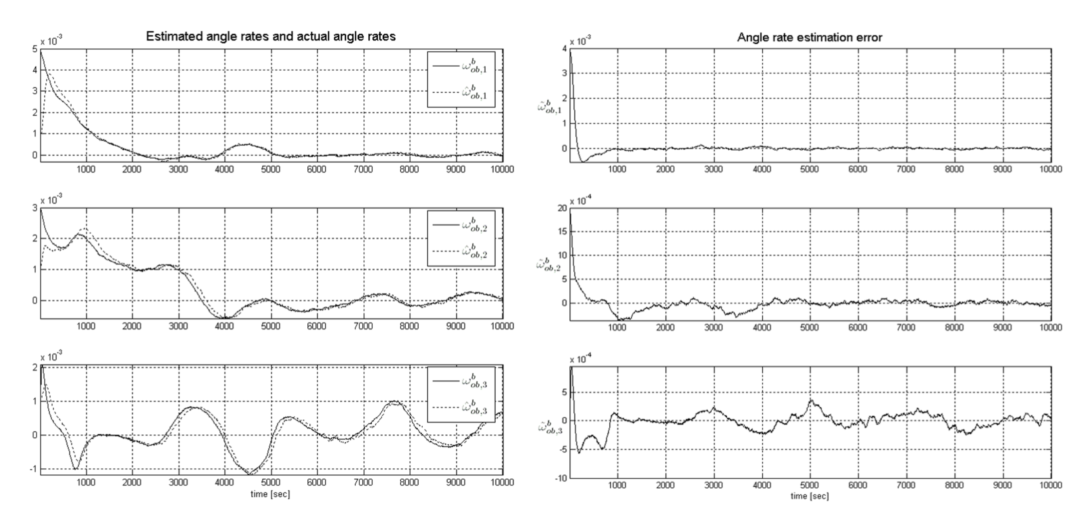

Figure 3: Angular velocities and estimation errors