PID+ TRACKING IN A LEADER-FOLLOWER SPACECRAFT FORMATION

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ABSTRACT

In this paper we present two PID+ solutions to the problem of relative translational and rotational tracking in a leader-follower spacecraft formation, based on the concept of integrator backstepping. By using an integrator augmentation technique, we include integral action in the controllers to account for unknown orbital perturbations. Two methods for including integral action in the control loop are presented, and the equilibrium points in the two closed-loop systems are proved to be uniformly exponentially stable and uniformly asymptotically stable, respectively. Finally, simulation results are presented to show the controller performance.

Key words: Spacecraft formation; integrator backstepping; PID+ control.

1. INTRODUCTION

Synchronization, coordination and cooperative control are new and promising trends within mechanical systems technology. Replacing complex single units with several simpler and less expensive agents makes it possible to achieve larger operational areas with greater flexibility and performance. In the space industry, the concept makes way for new and better applications, such as improved monitoring of the Earth and its surrounding atmosphere, geodesy, deep-space imaging and exploration and even in-orbit spacecraft servicing and maintenance.

Synchronized control of relative attitude in spacecraft formations has received increased attention over the last years. State feedback tracking control laws for relative position and attitude were developed in [1, 2], and these solutions were proved to result in exponentially stable equilibrium points in the closed loop system. These latter results were however based on an assumption of known orbital perturbations, which is seldom the case. In [3], a nonlinear tracking controller for both relative position and attitude was presented, including an adaptation law to account for unknown mass and inertia parameters of the spacecraft. The controller ensures global asymptotic convergence of position and velocity errors, and the stability result was proved using a Lyapunov framework and standard signal-chasing arguments. Semiglobal asymptotic convergence of relative position and attitude errors was proved in [4] for an adaptive output feedback controller using relative position only, tracing the steps of [3].

In this paper, we present two PID+ tracking controllers for relative translation and rotation in a leader-follower spacecraft formation, derived using integrator backstepping. The controllers are extensions of earlier works in [5] on relative attitude control, where the equilibrium points in the closed-loop system were proved to be uniformly asymptotically stable (UAS) based on an assumption of known orbital perturbations. In this paper, we extend the solution to both relative translation and rotation, and relax the latter assumption to unknown, but constant, perturbations. By using an integrator augmentation technique (*cf.* [6]), we include integral action in the controllers to account for the unknown orbital perturbations. The difference between the controllers are the inclusion point of the integral action. The first controller renders the equilibrium points in the closed-loop system UES, but requires measurement of relative acceleration, while the second controller results in UAS equilibrium points, without this requirement.

The rest of the paper is organized as follows: Section 2 defines the different reference frames used and presents the mathematical models of relative attitude dynamics and kinematics in a leader-follower spacecraft formation. The controller design is performed in Section 3, and simulation results for the derived controllers are presented in Section 4. Concluding remarks are given in Section 5.

2. MODELLING

In the following, we denote by $\dot{\mathbf{x}}$ the time derivative of a vector $\mathbf{x}$, i.e. $\dot{\mathbf{x}} = d\mathbf{x}/dt$. Moreover, $\ddot{\mathbf{x}} = d^2\mathbf{x}/dt^2$. We denote by $\|\cdot\|$ the Euclidian norm of a vector and the induced $\mathcal{L}_2$ norm of a matrix. Coordinate reference frames are denoted by $\mathcal{F}_{\cdot}$, and in particular, the standard Earth-centered inertial (ECI) frame is denoted $\mathcal{F}_i$. We denote by $\omega_{b,a}^c$ the angular velocity of $\mathcal{F}_a$ relative to $\mathcal{F}_b$, referenced in $\mathcal{F}_c$. Matrices representing rotation or coordinate transformation between $\mathcal{F}_a$ and $\mathcal{F}_b$ are denoted $\mathbf{R}_a^b$. When the context is sufficiently explicit, we may omit to write arguments of a function, vector or matrix.

2.1. Cartesian coordinate frames

To form the basis of our relative attitude model, we use the standard definition of the Earth-Centered Inertial (ECI) frame $\mathcal{F}_i$, with z axis towards celestial north. In addition, we employ a standard LVLH-definition of the leader orbit reference frame $\mathcal{F}_l$, with unit vectors defined as

$$\mathbf{e}_r = \frac{\mathbf{r}_l}{r_l}, \qquad \mathbf{e}_\theta = \mathbf{e}_h \times \mathbf{e}_r \qquad \text{and} \qquad \mathbf{e}_h = \frac{\mathbf{h}}{h}, \quad (1)$$

where $\mathbf{r}_l$ is the vector from the center of the Earth to the leader spacecraft, $\mathbf{h} = \mathbf{r}_l \times \dot{\mathbf{r}}_l$ is the angular momentum vector of the leader orbit, and $h = |\mathbf{h}|$. Moreover, we define a follower orbit reference frame $\mathcal{F}_f$ with origin specified by the relative orbit position vector

$$\mathbf{p} = \mathbf{r}_f - \mathbf{r}_l = x\mathbf{e}_r + y\mathbf{e}_\theta + z\mathbf{e}_h \quad (2)$$

and with unit vectors aligned with the unit vectors in $\mathcal{F}_l$ at all times. We also define leader and follower body frames $\mathcal{F}_{lb}$ and $\mathcal{F}_{fb}$ respectively, with origin in the corresponding centers of mass and axes fixed to the spacecraft body.

The rotation matrix describing rotations from an orbit frame $\mathcal{F}_s$ to a body frame $\mathcal{F}_{sb}$ can be described by

$$\mathbf{R}_s^{sb} = [\mathbf{c}_1\ \mathbf{c}_2\ \mathbf{c}_3] = \mathbf{I} + 2\eta\mathbf{S}(\varepsilon) + 2\mathbf{S}^2(\varepsilon) \quad (3)$$

where the superscript/subscript s is used in general to denote the spacecraft in question, so $s = l, f$ for the leader and follower spacecraft, respectively. The elements $\mathbf{c}_i$ are directional cosine vectors,

$$\mathbf{q} = \begin{bmatrix} \eta & \varepsilon^\top \end{bmatrix}^\top, \quad \eta^2 + \varepsilon^\top\varepsilon = 1 \quad (4)$$

are the Euler parameters, and the matrix $\mathbf{S}(\varepsilon) = \varepsilon\times$ is the cross product operator. The inverse rotation is given by the complex conjugate of $\mathbf{q}$ as $\bar{\mathbf{q}} = \begin{bmatrix}\eta, & -\varepsilon^\top\end{bmatrix}^\top$, and the quaternion product is defined as (cf. [7])

$$\mathbf{q}_1 \otimes \mathbf{q}_2 := \begin{bmatrix} \eta_1\eta_2 - \varepsilon_1^\top\varepsilon_2 \\ \eta_1\varepsilon_2 + \eta_2\varepsilon_1 + \mathbf{S}(\varepsilon_1)\varepsilon_2 \end{bmatrix}. \quad (5)$$

2.2. Relative translation

From the fundamental differential equation of the two-body problem (*cf.* [8]), the nonlinear position dynamics can be represented in $\mathcal{F}_l$ on the form (*cf.* [9, 10])

$$m_f\dot{\mathbf{v}} + \mathbf{C}_t(\dot{\nu})\mathbf{v} + \mathbf{D}_t(\dot{\nu},\ddot{\nu},r_f)\mathbf{p} + \mathbf{n}_t(r_l,r_f) = \mathbf{F}_a + \mathbf{F}_d \quad (6)$$

where $\mathbf{v} = \dot{\mathbf{p}}$, ν is the true anomaly of the leader spacecraft, μ is a constant of gravity,

$$\mathbf{C}_t(\dot{\nu}) = 2m_f\dot{\nu}\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \in SS(3) \quad (7)$$

is a skew-symmetric Coriolis-like matrix,

$$\mathbf{D}_t(\dot{\nu},\ddot{\nu},r_f)\mathbf{p} = m_f\begin{bmatrix} \frac{\mu}{r_f^3} - \dot{\nu}^2 & -\ddot{\nu} & 0 \\ \ddot{\nu} & \frac{\mu}{r_f^3} - \dot{\nu}^2 & 0 \\ 0 & 0 & \frac{\mu}{r_f^3} \end{bmatrix}\mathbf{p} \quad (8)$$

may be viewed as a time-varying potential force, and

$$\mathbf{n}_t(r_l,r_f) = m_f\mu\begin{bmatrix} \frac{r_l}{r_f^3} - \frac{1}{r_l^2}, & 0, & 0 \end{bmatrix}^\top. \quad (9)$$

The composite perturbation force $\mathbf{F}_d$ and the relative control force $\mathbf{F}_a$ are given by

$$\mathbf{F}_d = \mathbf{f}_{df} - \frac{m_f}{m_l}\mathbf{f}_{dl} \qquad \mathbf{F}_a = \mathbf{f}_{af} - \frac{m_f}{m_l}\mathbf{f}_{al} \quad (10)$$

where $\mathbf{f}_{dl}, \mathbf{f}_{df} \in \mathbb{R}^3$ are the perturbation terms due to external effects and $\mathbf{f}_{al}, \mathbf{f}_{af} \in \mathbb{R}^3$ are the actuator inputs.

2.3. Relative rotation

From the standard kinematics and dynamical relations of a spacecraft in orbit (*cf.* [8]), we obtain by using the quaternion product that the relative attitude kinematics can be expressed as (*cf.* [10])

$$\dot{\mathbf{q}} = \mathbf{T}(\mathbf{q})\,\omega, \quad \omega = \omega_{i,fb}^{fb} - \mathbf{R}_{lb}^{fb}\omega_{i,lb}^{lb} \quad (11)$$

where $\mathbf{q} = \mathbf{q}_f \otimes \bar{\mathbf{q}}_l$ and ω is the relative angular velocity between the leader and the follower spacecraft. Moreover, the relative attitude dynamics can be written

$$\mathbf{J}_f\dot{\omega} + \mathbf{C}_r(\omega)\,\omega + \mathbf{n}_r(\omega) = \Upsilon_d + \Upsilon_a \quad (12)$$

where $\mathbf{J}_l, \mathbf{J}_f$ are the leader and follower moments of inertia

$$\begin{aligned}\mathbf{C}_r(\omega) = \mathbf{J}_f\mathbf{S}\left(\mathbf{R}_{lb}^{fb}\omega_{i,lb}^{lb}\right) + \mathbf{S}\left(\mathbf{R}_{lb}^{fb}\omega_{i,lb}^{lb}\right)\mathbf{J}_f \\ - \mathbf{S}\left(\mathbf{J}_f\left[\omega + \mathbf{R}_{lb}^{fb}\omega_{i,lb}^{lb}\right]\right)\end{aligned} \quad (13)$$

is a skew-symmetric matrix, $\mathbf{C}_r(\omega) \in SS(3)$,

$$\begin{aligned}\mathbf{n}_r(\omega) = \mathbf{S}\left(\mathbf{R}_{lb}^{fb}\omega_{i,lb}^{lb}\right)\mathbf{J}_f\mathbf{R}_{lb}^{fb}\omega_{i,lb}^{lb} \\ - \mathbf{J}_f\mathbf{R}_{lb}^{fb}\mathbf{J}_l^{-1}\mathbf{S}\left(\omega_{i,lb}^{lb}\right)\mathbf{J}_l\omega_{i,lb}^{lb}\end{aligned} \quad (14)$$

is a nonlinear term, and

$$\Upsilon_d = \tau_{df}^{fb} - \mathbf{J}_f\mathbf{R}_{lb}^{fb}\mathbf{J}_l^{-1}\tau_{dl}^{lb}, \quad \Upsilon_a = \tau_{af}^{fb} - \mathbf{J}_f\mathbf{R}_{lb}^{fb}\mathbf{J}_l^{-1}\tau_{al}^{lb} \quad (15)$$

are the relative perturbation torques and relative actuator torques, respectively.

2.4. Total model

To write the total 6DOF model of relative translation and rotation in the spacecraft formation, we define the state vectors

$$\mathbf{x}_1 := \left[\mathbf{p}^\top \; \mathbf{q}^\top\right]^\top \quad \text{and} \quad \mathbf{x}_2 := \left[\mathbf{v}^\top \; \omega^\top\right]^\top . \tag{16}$$

Based on (6) and (12), the total model of the relative translational and rotational motion between the leader and the follower spacecraft can now be expressed

$$\dot{\mathbf{x}}_1 = \Lambda(\mathbf{x}_1)\mathbf{x}_2 \tag{17}$$

$$\mathbf{M}_f\dot{\mathbf{x}}_2 + \mathbf{C}(\dot{\nu},\omega)\mathbf{x}_2 + \mathbf{D}(\dot{\nu},\ddot{\nu},r_f)\mathbf{x}_1 + \mathbf{n}(\omega,r_l,r_f) = \mathbf{U}+\mathbf{W} \tag{18}$$

where $\mathbf{M}_f = \operatorname{diag}\left\{m_f\mathbf{I},\ \mathbf{J}_f\right\}$, $\Lambda(\mathbf{x}_1) = \operatorname{diag}\{\mathbf{I},\ \mathbf{T}(\mathbf{q})\}$ is the coupling term between the first and second order dynamics,

$$\mathbf{C}(\dot{\nu},\omega) = \operatorname{diag}\{\mathbf{C}_t(\dot{\nu}),\ \mathbf{C}_r(\omega)\} \tag{19}$$

$$\mathbf{D}(\dot{\nu},\ddot{\nu},r_f)\mathbf{x}_1 = \operatorname{diag}\left\{\mathbf{D}_t(\dot{\nu},\ddot{\nu},r_f),\ \mathbf{0}\right\} \tag{20}$$

$$\mathbf{n}(\omega,r_l,r_f) = \left[\mathbf{n}_t^\top(r_l,r_f),\ \mathbf{n}_r^\top(\omega)\right]^\top \tag{21}$$

are the collections of (7)-(9) and (13)-(14), and finally

$$\mathbf{U} = \left[\mathbf{F}_a^\top,\ \Upsilon_a^\top\right]^\top \quad \text{and} \quad \mathbf{W} = \left[\mathbf{F}_d^\top,\ \Upsilon_d^\top\right]^\top \tag{22}$$

contains the relative input forces and orbital perturbations, respectively.

3. CONTROLLER DESIGN

Having established the 6DOF mathematical model of relative motion in a leader-follower formation, we now present a solution to the control problem.

3.1. Problem statement

The control problem is to design a controller that makes the state $\mathbf{x}_1$ converge to a time-varying smooth trajectory $\mathbf{x}_d(t)$. The desired trajectory can be specified as

$$\mathbf{x}_{d1}(t) = \begin{bmatrix} \mathbf{p}_d(t) \\ \mathbf{q}_d(t) \end{bmatrix} \qquad \mathbf{x}_{d2}(t) = \begin{bmatrix} \mathbf{v}_d(t) \\ \omega_d(t) \end{bmatrix} \tag{23}$$

so that

$$\dot{\mathbf{x}}_{d1} = \Lambda(\mathbf{x}_{d1})\mathbf{x}_{d2}\,. \tag{24}$$

The relative translation error is defined as $\tilde{\mathbf{p}} = \mathbf{p} - \mathbf{p}_d$, the relative rotation error $\tilde{\mathbf{q}} = \left[\tilde{\eta}\ \tilde{\varepsilon}^\top\right]^\top$ is given from the quaternion product $\tilde{\mathbf{q}} = \mathbf{q}\otimes\bar{\mathbf{q}}_d$, and analogues to (11) we may represent the rotation error kinematics as

$$\dot{\tilde{\mathbf{q}}} = \mathbf{T}(\tilde{\mathbf{q}})\,\tilde{\omega} \tag{25}$$

Note that, due to the redundancy in the quaternion representation, we have that $\tilde{\mathbf{q}}$ and $-\tilde{\mathbf{q}}$ represents the same physical orientation, however one is rotated 2π relative to the other about an arbitrary axis. Accordingly, we have two equilibrium points in the closed-loop system, namely $\tilde{\mathbf{q}}_+ = \left[1,\mathbf{0}^\top\right]^\top$ and $\tilde{\mathbf{q}}_- = \left[-1,\mathbf{0}^\top\right]^\top$. The choice of equilibrium point for a given initial condition should aim at minimizing the path length for the desired rotation, and this can be ensured by choosing the equilibrium point corresponding to the sign of $\tilde{\eta}$ in the initial condition. Hence, we choose $\tilde{\mathbf{q}}_+$ if $\tilde{\eta} \geq 0$, and $\tilde{\mathbf{q}}_-$ otherwise. We define the position/attitude error as

$$\mathbf{e}_{1\pm} := [\tilde{\mathbf{p}},\ 1\mp\tilde{\eta},\ \tilde{\varepsilon}] \tag{26}$$

for the positive equilibrium point $(\mathbf{0},\ \tilde{\mathbf{q}}_+)$ and negative equilibrium point $(\mathbf{0},\ \tilde{\mathbf{q}}_-)$, respectively. We also define the relative angular velocity error as

$$\mathbf{e}_2 := \mathbf{x}_2 - \mathbf{x}_{d2} = \left[\tilde{\mathbf{v}}^\top,\ \tilde{\omega}^\top\right]^\top \tag{27}$$

with $\tilde{\mathbf{v}} = \mathbf{v} - \mathbf{v}_d$ and $\tilde{\omega} = \omega - \omega_d$, and in accordance with (17) and (24) we have

$$\dot{\mathbf{e}}_{1\pm} = \Lambda_e(\mathbf{e}_{1\pm})\mathbf{e}_2 \tag{28}$$

with

$$\Lambda_e(\mathbf{e}_{1\pm}) = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \frac{1}{2}\begin{bmatrix} \pm\tilde{\varepsilon}^\top \\ \tilde{\eta}\mathbf{I} + \mathbf{S}(\tilde{\varepsilon}) \end{bmatrix} \end{bmatrix}. \tag{29}$$

for $\mathbf{e}_{1+}$ and $\mathbf{e}_{1-}$, respectively.

Note that, for both cases, $\Lambda_e^\top\Lambda_e = \overline{\Lambda}_e = \operatorname{diag}\{\mathbf{I},\ \mathbf{I}/4\}$, which can be shown by direct calculation and using $\mathbf{S}(\tilde{\varepsilon})^\top = -\mathbf{S}(\tilde{\varepsilon})$, $\mathbf{S}(\tilde{\varepsilon})^\top\mathbf{S}(\tilde{\varepsilon}) = \tilde{\varepsilon}^\top\tilde{\varepsilon}\mathbf{I} - \tilde{\varepsilon}\tilde{\varepsilon}^\top$ and (4). Note also that $\Lambda_e^\top(\mathbf{e}_{1\pm})\mathbf{e}_{1\pm} = \left[\tilde{\mathbf{p}}^\top\ \pm\tilde{\varepsilon}^\top\right]^\top$.

When nothing else is explicitly mentioned, we assume that the leader spacecraft is perfectly controlled in its orbit, so that $\tau_{al}^{lb} = -\tau_{dl}^{lb}$ and $\mathbf{f}_{al} = -\mathbf{f}_{dl}$. Accordingly, we have that $\mathbf{U} = \mathbf{u}_f = [\mathbf{f}_{af}^\top,\ (\tau_{af}^{fb})^\top]^\top$ and $\mathbf{W} = \mathbf{w}_f = [\mathbf{f}_{df}^\top,\ (\tau_{df}^{fb})^\top]^\top$.

3.2. PID+ controller I

Theorem 1 *Assuming that the orbital perturbations* $\mathbf{w}_f = [\mathbf{f}_{df}^\top,\ (\tau_{df}^{fb})^\top]^\top$ *working on the follower spacecraft are constant, and that the desired relative position/attitude* $\mathbf{x}_{d1}(t)$, *desired relative velocity/angular velocity* $\mathbf{x}_{d2}(t)$ *and desired relative acceleration/angular acceleration* $\dot{\mathbf{x}}_{d2}(t)$ *are all bounded functions, the dual equilibrium points* $(\mathbf{e}_{1\pm},\mathbf{e}_2) = (\mathbf{0},\mathbf{0})$ *of the system (17)-(18), in closed loop with the control law*

$$\mathbf{u}_f = \mathbf{C}\mathbf{x}_2 + \mathbf{D}\mathbf{x}_1 + \mathbf{n} + \mathbf{M}_f\dot{\mathbf{x}}_{d2} + \mathbf{M}_f\dot{\alpha}_1 - \mathbf{K}_2\mathbf{z}_2 - \mathbf{z}_1 \tag{30}$$

$$\alpha_1 = -\mathbf{K}_1\mathbf{z}_1 - \mathbf{z}_0 - \dot{\Lambda}_e^\top\mathbf{e}_1 + \dot{\alpha}_0 \tag{31}$$

$$\alpha_0 = -\mathbf{K}_0\mathbf{z}_0 \tag{32}$$

where $\dot{\mathbf{z}}_0 = \Lambda_e^\top \mathbf{e}_1$, $\mathbf{z}_1 = \Lambda_e^\top \mathbf{e}_1 - \alpha_0$ and $\mathbf{z}_2 = \mathbf{e}_2 - \alpha_1$ are auxiliary state variables, and $\mathbf{K}_0 = \mathbf{K}_0^\top > \mathbf{0}$, $\mathbf{K}_1 = \mathbf{K}_1^\top > \mathbf{0}$ and $\mathbf{K}_2 = \mathbf{K}_2^\top > \mathbf{0}$ are feedback gain matrices, are uniformly exponentially stable (UES).

Sketch of proof:

The controller structure in (30)-(32) is designed using integrator backstepping, and closing the loop of (17)-(18) leaves the auxiliary system dynamics

$$\dot{\mathbf{z}}_0 = -\mathbf{K}_0\mathbf{z}_0 + \mathbf{z}_1 \tag{33}$$

$$\dot{\mathbf{z}}_1 = -\mathbf{K}_1\mathbf{z}_1 - \mathbf{z}_0 + \mathbf{z}_2 \tag{34}$$

$$\mathbf{M}_f\dot{\mathbf{z}}_2 = -\mathbf{K}_2\mathbf{z}_2 - \mathbf{z}_1 \, . \tag{35}$$

The integral action is included in the controller by augmentation of the auxiliary state variables with $\dot{\mathbf{z}}_0 = \Lambda_e^\top \mathbf{e}_1$, and the proof of the stability properties of the dual equilibrium points are performed in two steps. For the first step, assume that no orbital perturbations are working on the follower spacecraft, such that $\mathbf{w}_f = \mathbf{0}$. For this case, using the radially unbounded, positive definite Lyapunov functions

$$V_0 = \frac{1}{2}\mathbf{z}_0^\top \mathbf{z}_0 \qquad V_1 = V_0 + \frac{1}{2}\mathbf{z}_1^\top \mathbf{z}_1 \tag{36}$$

$$V_2 = V_1 + \frac{1}{2}\mathbf{z}_2^\top \mathbf{M}_f \mathbf{z}_2 \tag{37}$$

with the corresponding Lyapunov function derivative

$$\dot{V}_2 = -\mathbf{z}_0^\top \mathbf{K}_0\mathbf{z}_0 - \mathbf{z}_1^\top \mathbf{K}_1\mathbf{z}_1 - \mathbf{z}_2^\top \mathbf{K}_2\mathbf{z}_2 \tag{38}$$

we find from standard Lyapunov theorems that the dual equilibrium points $(\mathbf{z}_0, \mathbf{z}_1, \mathbf{z}_2) = (\mathbf{0}, \mathbf{0}, \mathbf{0})$ of the closed-loop auxiliary system (33)-(35) are uniformly exponentially stable (UES). This analysis holds for both equilibrium points, and using the relations between the original system and the auxiliary system, it then follows that the dual equilibrium points $(\mathbf{e}_{1\pm}, \mathbf{e}_2) = (\mathbf{0}, \mathbf{0})$ are also UES. For the second part of the proof, we relax our assumptions on the follower orbital perturbations to be constant. The controller structure in (30)-(32) will then result in the closed-loop system (error dynamics) given by (33), (34) and

$$\mathbf{M}_f\dot{\mathbf{z}}_2 = -\mathbf{K}_2\mathbf{z}_2 - \mathbf{z}_1 + \mathbf{w}_f \tag{39}$$

and by analyzing the steady state behavior of the auxiliary system dynamics, we find that the dual equilibrium points $(\mathbf{e}_{1\pm}, \mathbf{e}_2) = (\mathbf{0}, \mathbf{0})$ are still UES, but the auxiliary closed-loop z-system will converge to the non-zero equilibrium points of the error dynamics.

3.3. PID+ controller II

Theorem 2 *Assuming that the orbital perturbations $\mathbf{w}_f = [\mathbf{f}_{df}^\top,\ (\tau_{df}^{fb})^\top]^\top$ working on the follower spacecraft are constant, and that the desired relative position/attitude $\mathbf{x}_{d1}(t)$, desired relative velocity/angular velocity $\mathbf{x}_{d2}(t)$ and desired relative acceleration/angular acceleration $\dot{\mathbf{x}}_{d2}(t)$ are all bounded functions, the dual equilibrium points $(\mathbf{e}_{1\pm}, \mathbf{e}_2) = (\mathbf{0}, \mathbf{0})$ of the system (17)-(18), in closed loop with the control law*

$$\mathbf{u}_f = \mathbf{C}\mathbf{x}_2 + \mathbf{D}\mathbf{x}_1 + \mathbf{n} + \mathbf{M}_f\dot{\mathbf{x}}_{d2} + \mathbf{M}_f\dot{\alpha}_1 - \mathbf{K}_2\mathbf{z}_2 - \Lambda_e^\top \mathbf{z}_1 \tag{40}$$

$$\alpha_1 = -\mathbf{K}_1\Lambda_e^\top \mathbf{z}_1 - \mathbf{K}_0\mathbf{z}_0 \tag{41}$$

where $\dot{\mathbf{z}}_0 = \Lambda_e^\top \mathbf{e}_1$, $\mathbf{z}_1 = \mathbf{e}_1$ and $\mathbf{z}_2 = \mathbf{e}_2 - \alpha_1$ are auxiliary state variables, and $\mathbf{K}_0 = \mathbf{K}_0^\top > \mathbf{0}$, $\mathbf{K}_1 = \mathbf{K}_1^\top > \mathbf{0}$ and $\mathbf{K}_2 = \mathbf{K}_2^\top > \mathbf{0}$ are feedback gain matrices, are uniformly asymptotically stable (UAS).

Sketch of proof:

The controller structure in (40)-(41) is designed using integrator backstepping, and closing the loop of (17)-(18) leaves the auxiliary system dynamics

$$\dot{\mathbf{z}}_0 = \Lambda_e^\top \mathbf{z}_1 \tag{42}$$

$$\dot{\mathbf{z}}_1 = -\Lambda_e\mathbf{K}_1\Lambda_e^\top \mathbf{z}_1 - \Lambda_e\mathbf{K}_0\mathbf{z}_0 + \Lambda_e\mathbf{z}_2 \tag{43}$$

$$\mathbf{M}_f\dot{\mathbf{z}}_2 = -\mathbf{K}_2\mathbf{z}_2 - \Lambda_e^\top \mathbf{z}_1 \, . \tag{44}$$

As in Theorem 1, the integral action is included in the controller by augmentation of the auxiliary state variables with $\dot{\mathbf{z}}_0 = \Lambda_e^\top \mathbf{e}_1$, and the proof of the stability properties of the dual equilibrium points are performed in two steps. For the first step, assume that no orbital perturbations are working on the follower spacecraft, such that $\mathbf{w}_f = \mathbf{0}$. For this case, using the radially unbounded, positive definite Lyapunov functions

$$V_1 = \frac{1}{2}\mathbf{z}_0^\top \mathbf{K}_0\mathbf{z}_0 + \frac{1}{2}\mathbf{z}_1^\top \mathbf{z}_1 \tag{45}$$

$$V_2 = V_1 + \frac{1}{2}\mathbf{z}_2^\top \mathbf{M}_f \mathbf{z}_2 \tag{46}$$

we end up with the corresponding Lyapunov function derivative

$$\dot{V}_2 = -\mathbf{z}_1^\top \Lambda_e\mathbf{K}_1\Lambda_e^\top \mathbf{z}_1 - \mathbf{z}_2^\top \mathbf{K}_2\mathbf{z}_2 \tag{47}$$

By further defining the auxiliary function

$$\mathbf{W}(\mathbf{z}) = \mathbf{z}_0^\top \Lambda_e^\top \mathbf{z}_1 \tag{48}$$

we obtain that

$$\dot{\mathbf{W}} = -\frac{1}{2}\mathbf{z}_0^\top \Lambda_e^\top \Lambda_e\mathbf{K}_0\mathbf{z}_0 \tag{49}$$

in the set $E : \{\dot{V}_2 = 0\} = \left\{\Lambda_e^\top \mathbf{z}_1 = \mathbf{0}, \mathbf{z}_2 = \mathbf{0}\right\}$. Moreover, since $4\Lambda_e^\top \Lambda_e = \mathrm{diag}\{4\mathbf{I}, \mathbf{I}\}$, we have that

$$\|\dot{\mathbf{W}}\| \geq \frac{1}{4}k_0\|\mathbf{z}_0\|^2 \tag{50}$$

Thus, all the conditions in Matrosovs theorem and the accompanying Lemma 2 in [11] are satisfied, so it follows that the dual equilibrium points $(\mathbf{z}_0, \mathbf{z}_1, \mathbf{z}_2) = (\mathbf{0}, \mathbf{0}, \mathbf{0})$ of

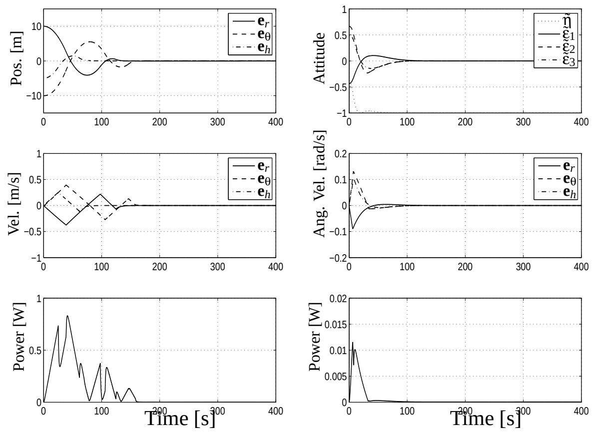

Figure 1. PID 1: Relative translation, rotation and power consumption in the settling phase.

the closed-loop auxiliary system (42)-(44) are uniformly asymptotically stable (UAS). This analysis holds for both equilibrium points, and using the relations between the original system and the auxiliary system, it then follows that the dual equilibrium points $(\mathbf{e}_{1\pm}, \mathbf{e}_2) = (\mathbf{0}, \mathbf{0})$ are also UAS. For the second part of the proof, we follow the same approach as in the proof of Theorem 1 to show that constant orbital perturbations does not affect the stability properties.

Remark 3 *Note that the even though Theorems 1 and 2 are stated for constant perturbations, the results also hold for slowly-varying perturbations as long as the integral action is faster than the variation of perturbation. We stress this point since the external perturbations in space can be expected to be slowly varying.*

4. SIMULATIONS

In this section, simulation results are presented to illustrate the performance of the presented control law. In the simulations, the orbital perturbations are set to $\tau_{df}^{fb} = [-1.5,\ -2.5,\ 0.011] \cdot 10^{-2}$ and $\mathbf{f}_{df} = [-13.7,\ 1,\ 1] \cdot 10^{-3}$. Both the leader and the follower spacecraft have masses 100 kg, and moments of inertia given as $\mathbf{I} = \mathrm{diag}\{\ 4.350 \quad 4.3370 \quad 3.6640\ \}$ kgm^2. The leader spacecraft is assumed to follow an equatorial orbit with a perigee altitude of 250 km and eccentricity $e = 0.3$. The follower spacecraft is assumed to have available continuous actuation in/about all body axes, with a maximum force and torque of 1 N and 0.1 Nm, respectively, and the controller gains $\mathbf{K}_0 = \mathbf{K}_2 = 0.1\mathbf{I}$ and $\mathbf{K}_1 = 20\mathbf{I}$ have been used.

The initial relative positions and attitudes are standstill at $[0,\ -10,\ 0]$ m and $[-75^\circ,\ -175^\circ,\ 70^\circ]$, respectively. The latter corresponds to the quaternion values

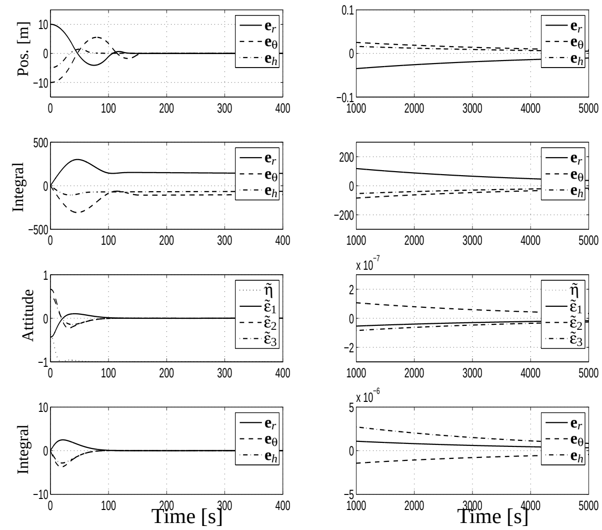

Figure 2. PID 1: Relative position, attitude and corresponding error integrals over one orbit.

$[-0.3772,\ -0.4329,\ 0.6645,\ 0.4783]$. The follower spacecraft is commanded to follow smooth sinusoidal trajectories around the origin with velocity and angular velocity profiles

$$\mathbf{v}_*(t) = [10c_o\sin(c_o t), 20c_o\cos(2c_o t), -15c_o\sin(3c_o t)]^\top$$

$$\omega_*(t) = \left[-c_o\sin(2c_o t), \frac{8}{5}c_o\sin(4c_o t), \frac{4}{5}c_o\sin(2c_o t)\right]^\top$$

where $c_o = \frac{\pi}{T_o}$ is a leader orbital period constant.

4.1. Results

The simulation results for a leader-follower spacecraft incorporating the controller in Theorem 1 is shown in Figures 1-2, where the former figure shows the convergence of relative translation and rotation to the desired trajectories, while the latter shows convergence of the same over one orbit together with error integrals, both with constant orbital perturbations working on the follower. The results in Figure 1 indicate that both relative translation and relative rotation state errors converge exponentially towards zero within 200 seconds, however with a significant initial overshoot. In addition, Figure 2 shows the integral action in the control loop, and the error integrals are also seen to converge. The initial overshoot is probably caused by the increased magnitude of control effort as a result of the added integral action, leading to stiffness in the controller. This is also indicated by the velocity profiles, revealing that the actuator force limit was reached. The power consumption for the given maneuver is 559.785 W and 5.189 W for relative translation and rotation; it should however be noted that the perturbations

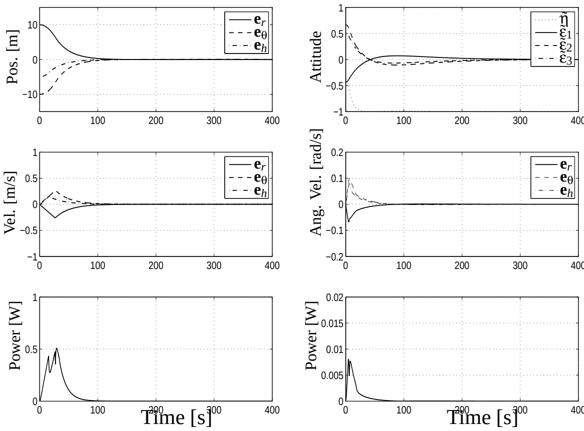

Figure 3. PID 2: Relative translation, rotation and power consumption in the settling phase.

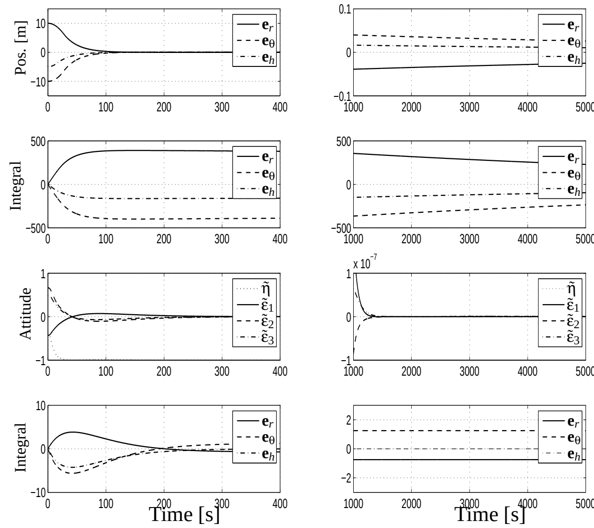

Figure 4. PID 2: Relative position, attitude and corresponding error integrals over one orbit.

in the simulations were approximately ten times larger than expected values ($|\mathbf{w}_f| \leq 1 \cdot 10^{-3}$ N).

Similar results for a spacecraft formation incorporating the controller in Theorem 2 are shown in Figures 3-4. The results indicate that this controller gives a significantly better initial convergence with less overshoot, although with an increased settling time for the relative rotation. This is also reflected in the power consumption, with a total of 407.238 W and 4.267 W for relative translation and rotation, respectively. On the other hand, the results show a slower convergence of the error integrals.

5. CONCLUSION

We have presented two PID+ tracking controllers for relative translation and rotation in a leader-follower spacecraft formation. The results are derived using integrator backstepping, and rest on an assumption of constant, but unknown, orbital perturbations. By using an integrator augmentation technique, we included integral action in the controllers to withstand unknown orbital perturbations. The equilibrium points in the closed-loop system were proved UES and UAS for the respective controllers.

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