Effective Implementation of optimal operation using Self-optimizing control

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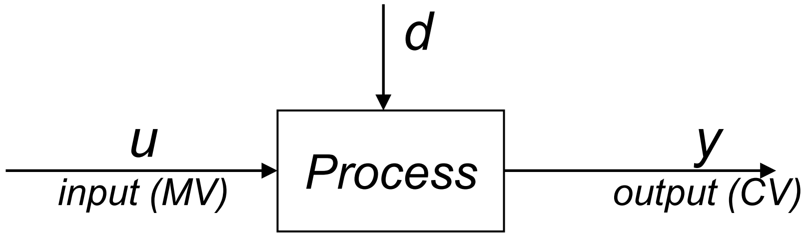
Classification of variables



Independent variables ("the cause"):

(a) Inputs (MV, u): Variables we can adjust (valves)

(b) Disturbances (DV, d): Variables outside our control

Dependent (output) variables ("the effect or result"):

(c) Primary outputs (CV1, y_1): Variables we want to keep at a given setpoint (economics)

(d) Secondary outputs (CV2,y_2): Extra outputs that we control to improve control (e.g., stabilizing loops)

Plantwide control = Control structure design

- *Not* the tuning and behavior of each control loop,
- But rather the *control philosophy* of the overall plant with emphasis on the ***structural decisions***:
  - *Selection of controlled variables (“outputs”)*
  - *Selection of manipulated variables (“inputs”)*
  - *Selection of (extra) measurements*
  - *Selection of control* ***configuration*** (structure of overall controller that interconnects the controlled, manipulated and measured variables)
  - *Selection of controller type* (LQG, H-infinity, PID, decoupler, MPC etc.).
- That is: **Control structure design** includes all the decisions we need make to get from ``PID control’’ to “PhD” control

Main objectives control system

1. Economics: Implementation of acceptable (near-optimal) operation
2. Regulation: Stable operation

ARE THESE OBJECTIVES CONFLICTING?

- *Usually NOT*
  - *Different time scales*
    - *Stabilization fast time scale*
  - *Stabilization doesn't "use up" any degrees of freedom*
    - *Reference value (setpoint) available for layer above*
    - *But it "uses up" part of the time window (frequency range)*

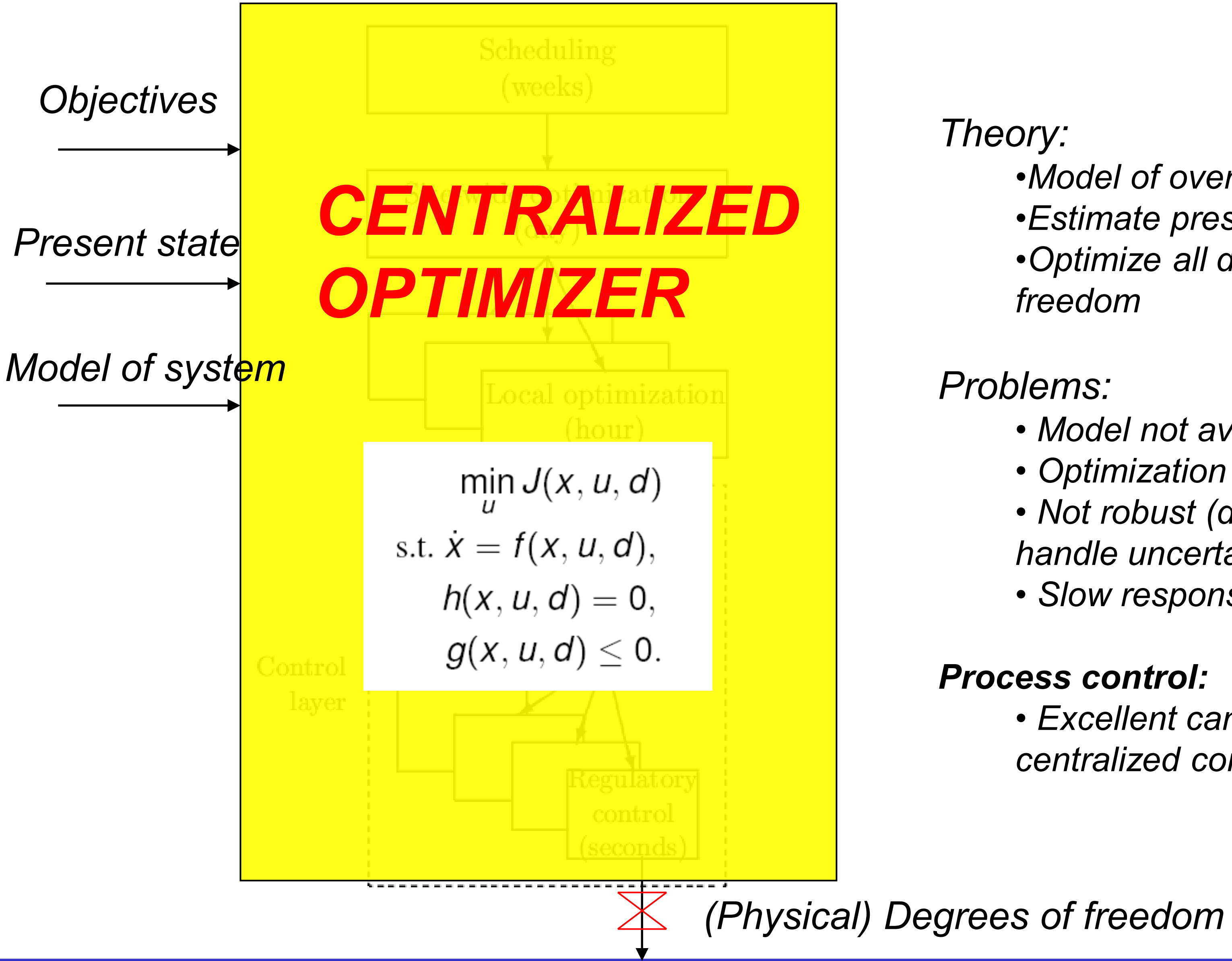
Optimal operation (economics)

Example of systems we want to operate optimally

- Process plant
  - minimize J=economic cost
- Runner
  - minimize J=time
- «Green» process plant
  - Minimize J=environmental impact (with given economic cost)

- General multiobjective:
  - Min J (scalar cost, often $)
  - Subject to satisfying constraints (environment, resources)

Theory: Optimal operation



Theory:

- *Model of overall system*
- *Estimate present state*
- *Optimize all degrees of freedom*

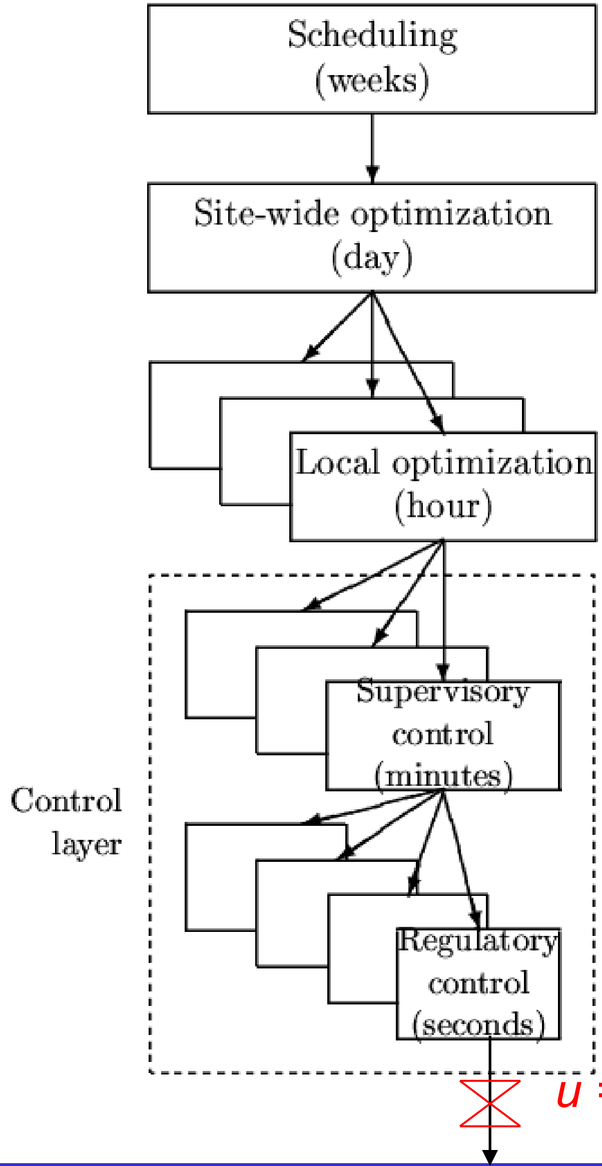
Problems:

- *Model not available*
- *Optimization complex*
- *Not robust (difficult to handle uncertainty)*
- *Slow response time*

Process control:

- *Excellent candidate for centralized control*

Practical operation: Hierarchical structure



u = valves

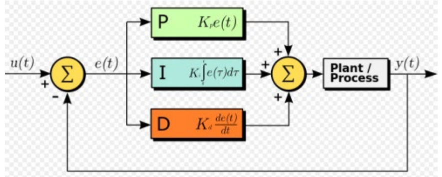
Manager

Process engineer

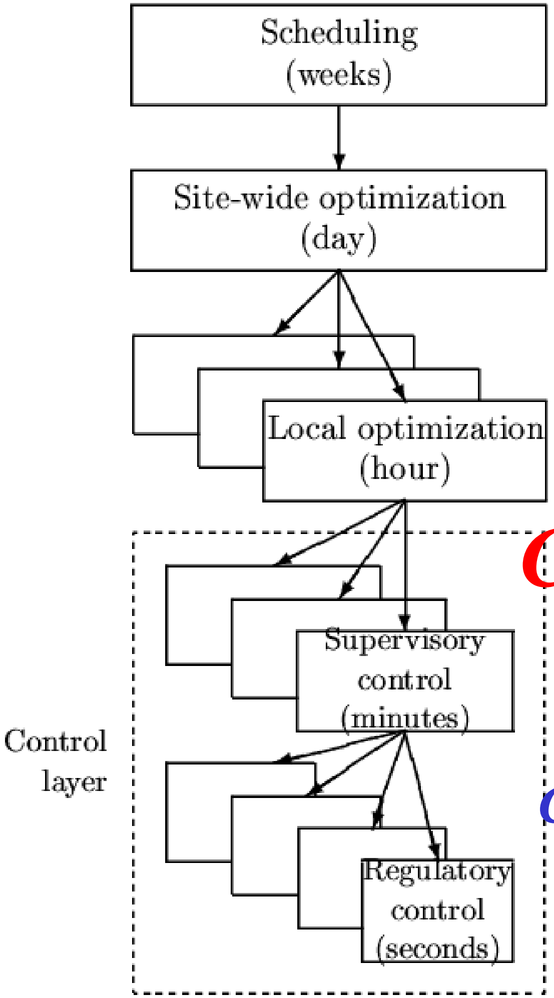
Operator/RTO

Operator/"Advanced control"/MPC

PID-control



Translate optimal operation into simple control objectives: What should we control?



CV_1 = c ? (economics)

CV_2 = ? (stabilization)

Outline

- Skogestad procedure for control structure design

  I Top Down

  - Step S1: Define operational objective (cost) and constraints
  - Step S2: Identify degrees of freedom and optimize operation for disturbances
  - Step S3: Implementation of optimal operation
    - What to control ? (primary CV's) (self-optimizing control)
  - Step S4: Where set the production rate? (Inventory control)

  II Bottom Up

  - Step S5: Regulatory control: What more to control (secondary CV's) ?
  - Step S6: Supervisory control
  - Step S7: Real-time optimization

<u>Step S1</u>. Define optimal operation (economics)

- What are we going to use our degrees of freedom (u=MVs) for?
- Typical cost function in process industry*:

$$J = cost\ feed + cost\ energy - value\ products$$

- *No need to include fixed costs (capital costs, operators, maintainance) at ”our” time scale (hours)
- Note: J=-P where P= Operational profit

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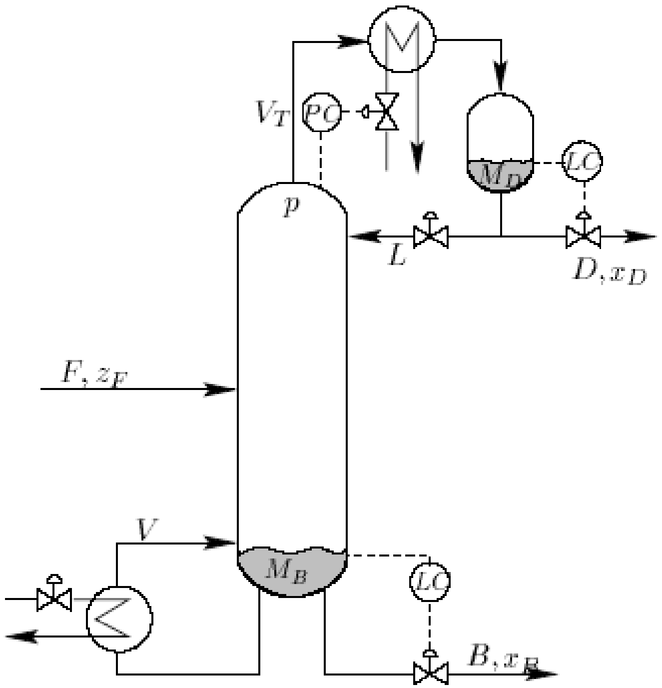
Example: distillation column

Cost J [$s] to be minimized (economics):

cost energy (heating + cooling)

$$J = -P \quad \text{where} \quad P = \underbrace{p_D D + p_B B}_{\text{value products}} - p_F F - p_V V$$

value products *cost feed*



Subject to Constraints:

Purity D: For example,	$x_{D,\ impurity} \leq \max$
Purity B: For example,	$x_{B,\ impurity} \leq \max$
Flow constraints:	$\min \leq D, B, L$ etc. $\leq \max$
Column capacity (flooding):	$V \leq V_{max}$, etc.

- Optimal operation: Minimize J with respect to steady-state degrees of freedoms (inputs u)
  - u = reflux L + heat input V

Step S2. Optimize

(a) Identify degrees of freedom
(b) Optimize for expected disturbances

- Need good model, usually steady-state
- Optimization is time consuming! But it is offline
- Main goal: Identify ACTIVE CONSTRAINTS
- A good engineer can often guess the active constraints

Step S3: Implementation of optimal operation

- Have found the optimal way of operation. How should it be implemented?
- **What to control ?** (primary CV's).

  CV(c) = H y

  1. **Active constraints**
  2. **Self-optimizing variables** (for **unconstrained** degrees of freedom)

Optimal operation of runner

- Cost to be minimized, J=T
- One degree of freedom (u=power)
- What should we control?

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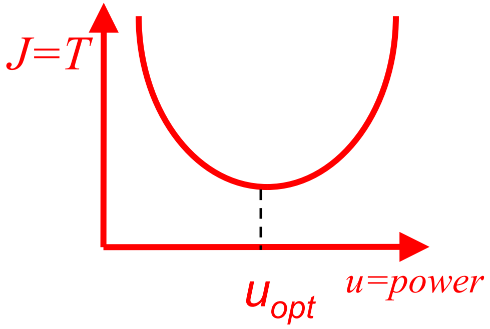
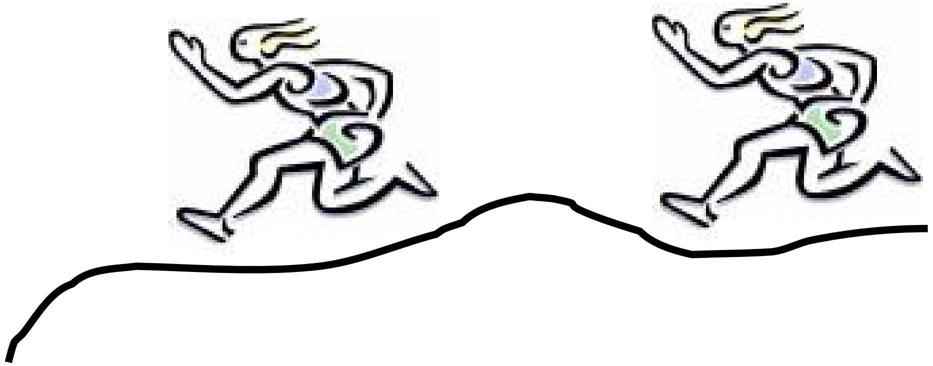
1. Optimal operation of Sprinter

- 100m. J=T
- Active constraint control:
  - Maximum speed (”no thinking required”)
  - CV = power (at max)

2. Optimal operation of Marathon runner

- 40 km. J=T
- What should we control? CV=?
- Unconstrained optimum



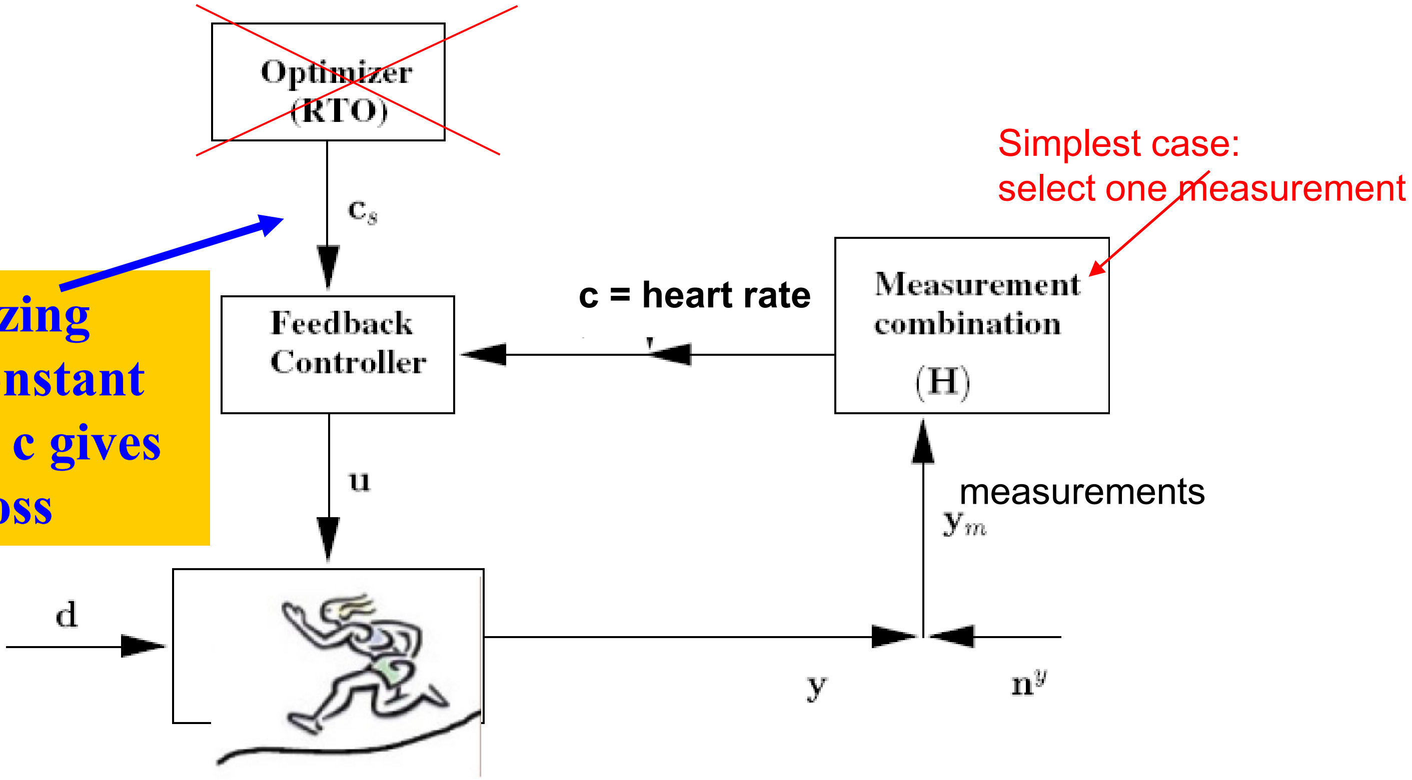


Self-optimizing control: Marathon (40 km)

- Any self-optimizing variable (to control at constant setpoint)?
  - c_1 = distance to leader of race
  - c_2 = speed
  - c_3 = heart rate
  - c_4 = level of lactate in muscles

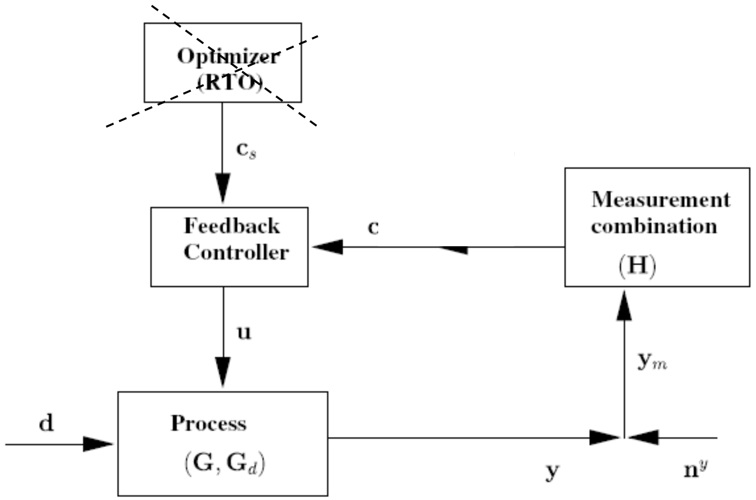
Implementation:
Feedback control of Marathon runner, J=T

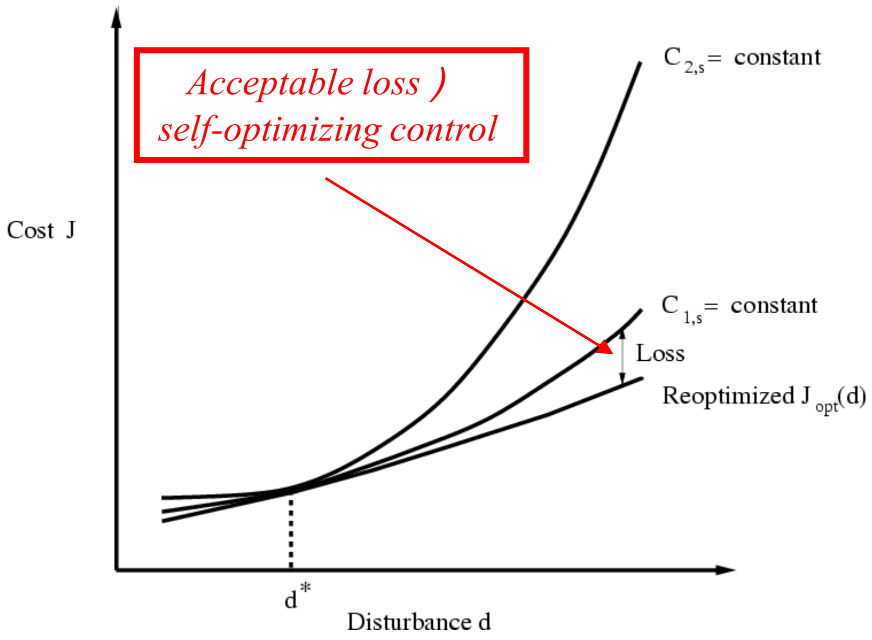
"Self-optimizing control:" Constant setpoints for c gives acceptable loss



- Simple and robust implementation
- Disturbances are indirectly handled by keeping a constant heart rate
- May have infrequent adjustment of setpoint (heart rate)

Definition of self-optimizing control





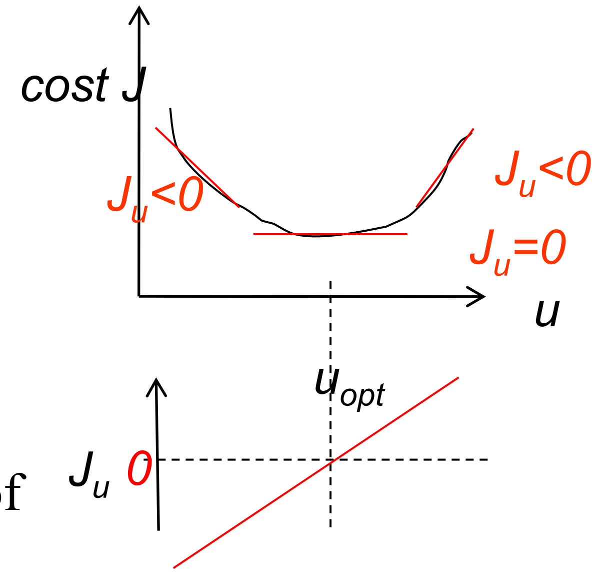
"Self-optimizing control is when we achieve acceptable loss (in comparison with truly optimal operation) with constant setpoint values for the controlled variables (without the need to reoptimize when disturbances occur)."

Reference: S. Skogestad, "Plantwide control: The search for the self-optimizing control structure", Journal of Process Control, 10, 487-507 (2000).

Ideal "Self-optimizing" variables

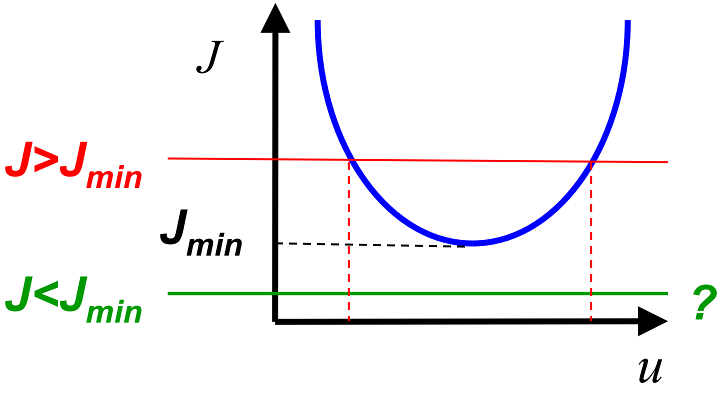
- **Operational objective: Minimize cost function J(u,d)**
- The ideal "self-optimizing" variable is the gradient
  (first-order optimality condition) (Halvorsen and Skogestad, 1997; Bonvin et al., 2001; Cao, 2003):

$$c = \alpha J_u; \quad J_u = \frac{\partial J}{\partial u}$$



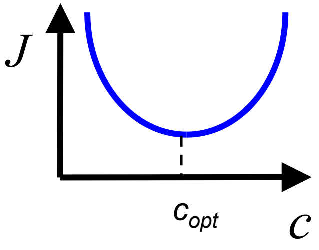
- BUT: Gradient can not be measured in practice
- Possible approach: Estimate gradient J_u based on measurements y
- **Approach here**: Look directly for c as a function of measurements y (c=Hy) without going via gradient

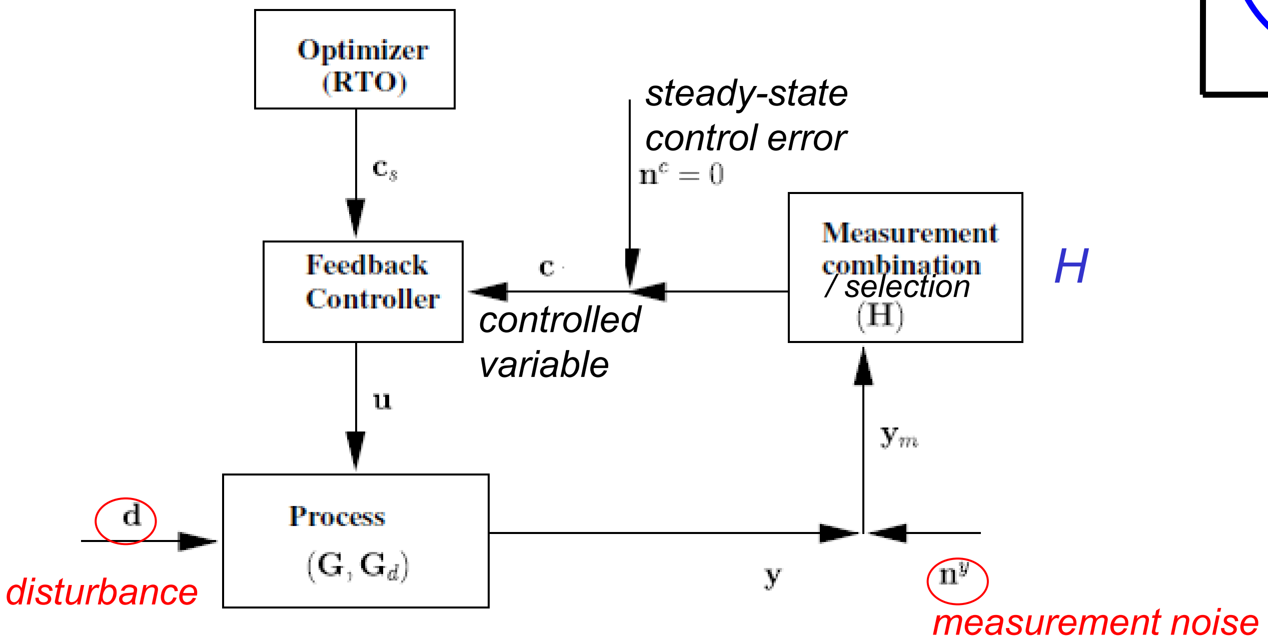
I.J. Halvorsen and S. Skogestad, ``Optimizing control of Petlyuk distillation: Understanding the steady-state behavior'', *Comp. Chem. Engng.*, **21**, S249-S254 (1997)
Ivar J. Halvorsen and Sigurd Skogestad, ``Indirect on-line optimization through setpoint control'', AIChE Annual Meeting, 16-21 Nov. 1997, Los Angeles, Paper 194h. .



Unconstrained optimum: NEVER try to control the cost J (or a variable that reaches max or min at the optimum)

- In particular, never try to control directly the cost J
- Assume we want to minimize J (e.g., J = V = energy) - and we make the stupid choice os selecting CV = V = J
  - **Then setting J < Jmin: Gives infeasible operation (cannot meet constraints)**
  - **and setting J > Jmin: Forces us to be nonoptimal (two steady states: may require strange operation)**





Ideal: $c = J_u$

In practise: $c = H y$

- Single measurements:

$$\mathbf{c} = \mathbf{H}\mathbf{y} \qquad \mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

- Combinations of measurements:

$$\mathbf{c} = \mathbf{H}\mathbf{y} \qquad \mathbf{H} = \begin{bmatrix} h_{11} & h_{12} & h_{13} & h_{14} \\ h_{21} & h_{22} & h_{23} & h_{24} \end{bmatrix}$$

Candidate controlled variables c for self-optimizing control

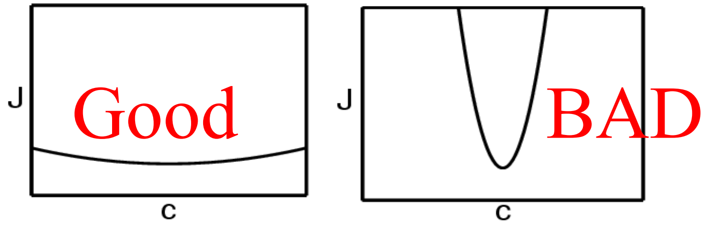
Intuitive

1. The *optimal value* of c should be *insensitive* to disturbances

2. Optimum should be flat (to avoid sensitivity noise).

   Equivalently: *Value* of c should be *sensitive* to degrees of freedom u.

   - "Want large gain", |G|
   - Or more generally: Maximize minimum singular value, $\underline{\sigma}(G)$



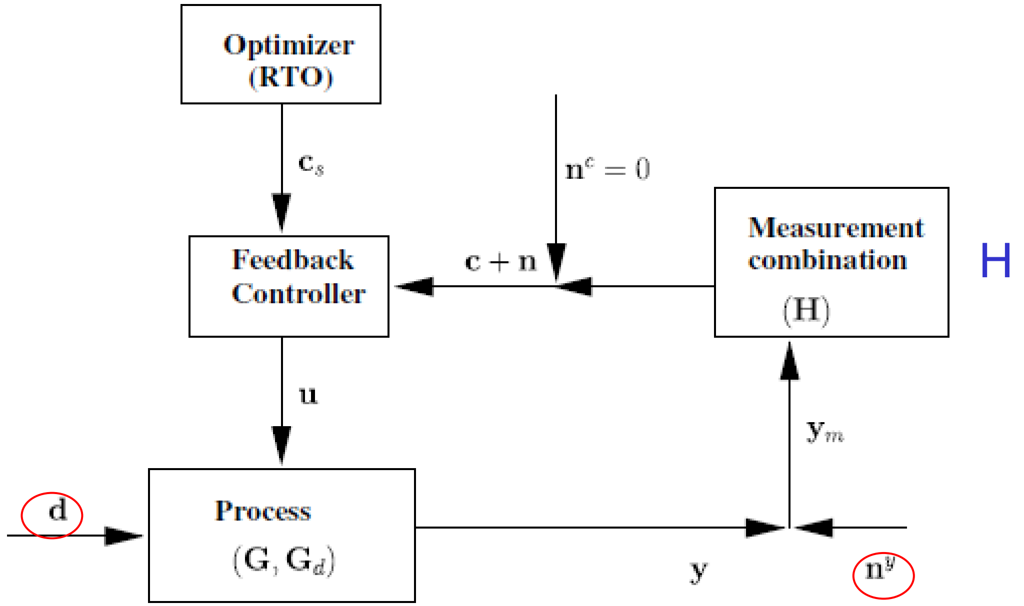
(b) Flat optimum: Implementation easy

(c) Sharp optimum: Sensitive to implementation erros

Optimal measurement combination

$$\Delta c = h_1 \Delta y_1 + h_2 \Delta y_2 + \cdots = H \Delta y$$

•Candidate measurements (y): Include also inputs u



Linear measurement combinations, c = Hy

c=Hy is approximate gradient J_u

Two approaches

1. **Nullspace method** (HF=0): Simple but has limitations
   - Need many measurements if many disturbances (ny = nu + nd)
   - Does not handle measurement noise

2. Generalization: **Exact local method**

$$H = G^{yT}(YY^T)^{-1}$$

   + Works for any measurement set y
   + Handles measurement error / noise

+

No measurement noise ($n^y=0$)

Nullspace method

Theorem

Given a sufficient number of measurements ($n_y \geq n_u + n_d$) and no measurement noise, select $\mathbf{H}$ such that

$$\mathbf{HF} = 0$$

where

$$\mathbf{F} = \frac{\partial \mathbf{y}^{opt}}{\partial \mathbf{d}}$$

Controlling $\mathbf{c} = \mathbf{Hy}$ to zero yields locally zero loss from optimal operation.

No measurement noise (n^y=0)

Proof nullspace method

Basis: Want optimal value of c to be independent of disturbances

$$\Rightarrow \quad \Delta c_{\text{opt}} = 0 \cdot \Delta d$$

- Find optimal solution as a function of d: $u_{opt}(d)$, $y_{opt}(d)$
- Linearize this relationship: $\Delta y_{opt} = F \, \Delta d$
- Want: $\Delta c_{opt} = H \Delta y_{opt} = H F \Delta d = 0$
- To achieve this for all values of Δ d:

$$HF = 0 \;\; \Rightarrow \;\; H \in \mathcal{N}(F^T)$$

Amazingly simple!

Sigurd is told how easy it is to find H

- To find a F that satisfies HF=0 we must require

$$n_y \geq n_u + n_d$$

- *Optimal* when we disregard implementation error (n)

V. Alstad and S. Skogestad, ``Null Space Method for Selecting Optimal Measurement Combinations as Controlled Variables'',
Ind.Eng.Chem.Res, 46 (3), 846-853 (2007).

CV=Measurement combination

Alternative proof

Nullspace method (HF=0) gives J_u=0

Proof:

$$J_u(u,d) = \underbrace{J_u(u_{opt}(d),d)}_{=0} + J_{uu}\cdot(u-u_{opt})$$

$u - u_{opt} = (HG^y)^{-1}(c - c_{opt})$

Here: $c - c_{opt} = \Delta c - \Delta c_{opt}$

where we have introduced deviation variables around a nominal optimal point (c^*, d^*) (where $c^* = c_{opt}(d^*)$)

Assume perfect control of c (no noise): $\Delta c = 0$

Optimal change: $\Delta c_{opt} = H\Delta y_{opt} = HF\Delta d$

Gives: $J_u = -J_{uu}(HG^y)^{-1}HF\Delta d$

$\Rightarrow HF = 0$ gives $J_u = 0$ for any disturbance Δd

- Proof. Appendix B in: Jäschke and Skogestad, "NCO tracking and self-optimizing control in the context of real-time optimization", *Journal of Process Control*, 1407-1416 (2011)

Example. Nullspace Method for Marathon runner

u = power, d = slope [degrees]

y_1 = hr [beat/min], y_2 = v [m/s]

$F = dy_{opt}/dd = [0.25 \ \ -0.2]'$

$H = [h_1 \ \ h_2]]$

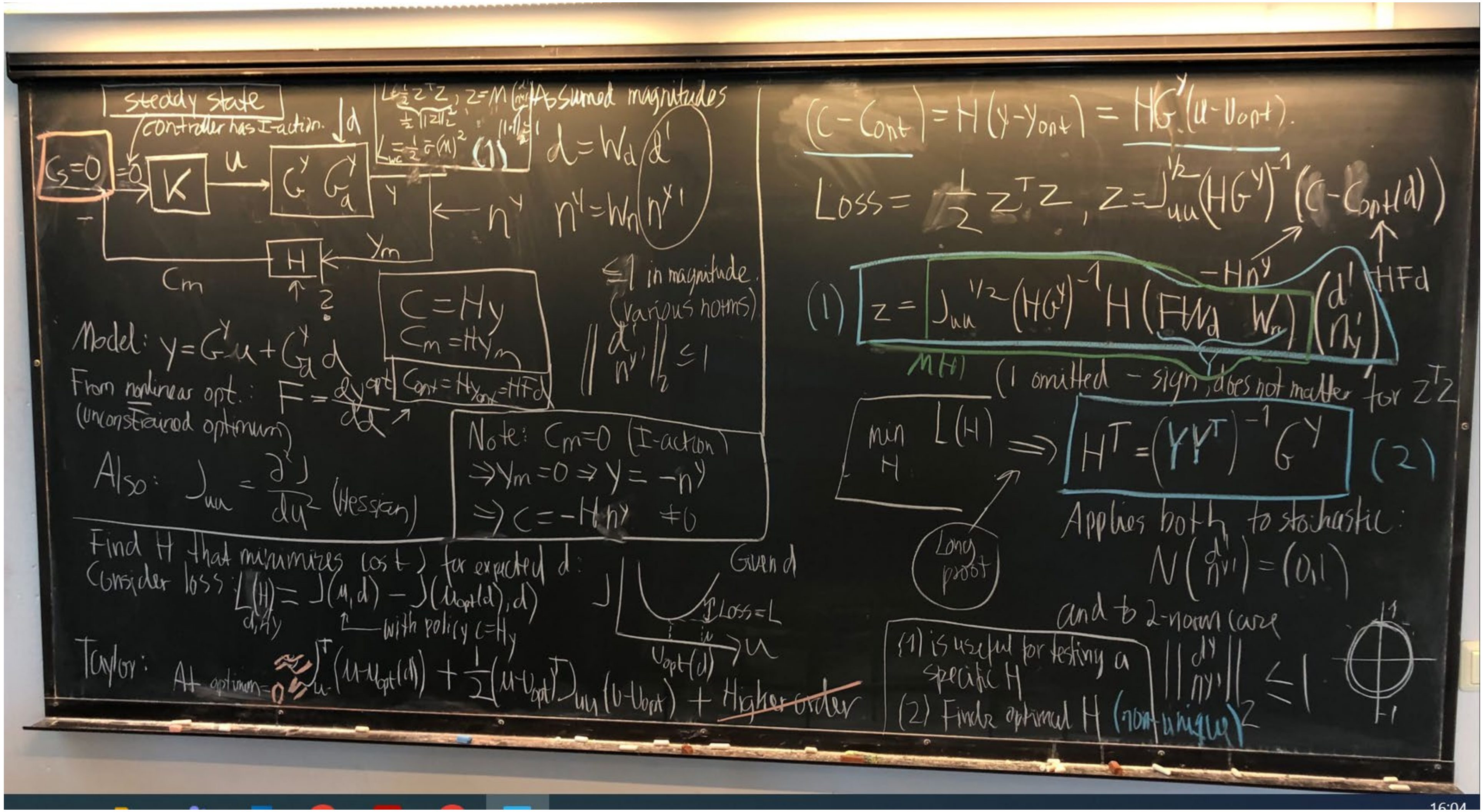
HF = 0 -> $h_1 f_1 + h_2 f_2 = 0.25 h_1 - 0.2 h_2 = 0$

Choose $h_1 = 1$ -> $h_2 = 0.25/0.2 = 1.25$

Conclusion: **c = hr + 1.25 v**

Control **c = constant** -> hr increases when v decreases (OK uphill!)

«Exact local method»



Optimal H ("exact local method"): Problem definition

Given that for any disturbance d, we select u such that

$$H \underbrace{(y + n^y)}_{y_m} = c_s \quad \text{(constant, } = 0 \text{ nominally)}$$

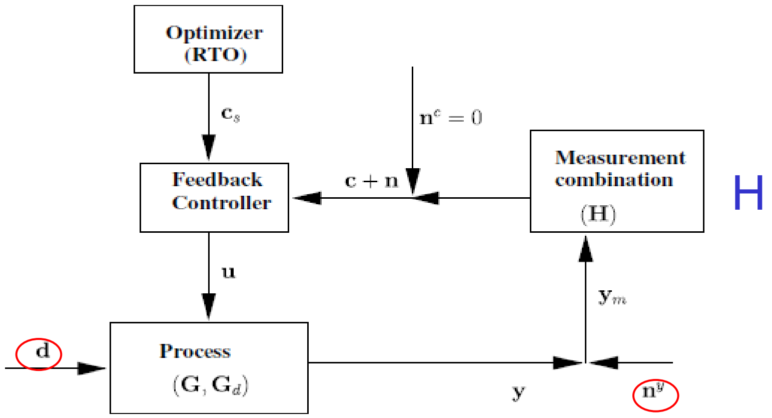
find the optimal H such that "magnitude" of the loss

$$L = J(u,d) - J_{\text{opt}}(d)$$

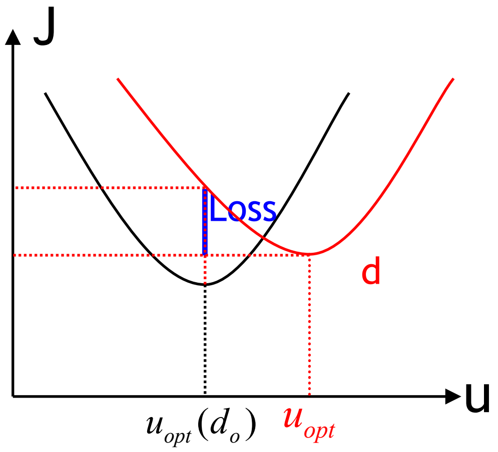
is minimized for the "expected" d and n^y:

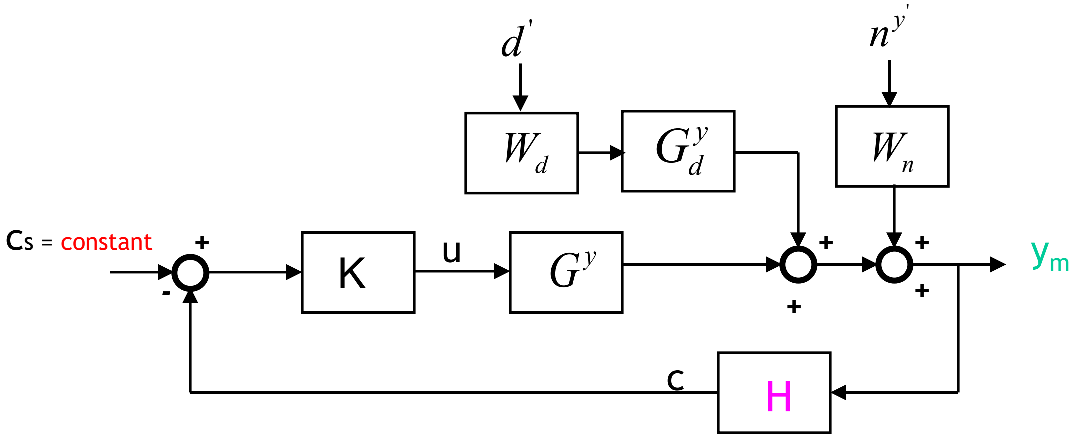
$$d = W_d d', \ n^y = W_{n^y} n^{y'}, \ \left\| \begin{matrix} d' \\ n^{y'} \end{matrix} \right\|_2 \leq 1$$

- Worst-case loss, $L_{wc} = \max_{\left\| \begin{matrix} d' \\ n^{y'} \end{matrix} \right\|_2 \leq 1} L$



Loss evaluation (with measurement noise)





Controlled variables, $c = Hy$

$$L = J(u,d) - J_{opt}(u_{opt}, d)$$

$$J(u,d) = J(u_{opt}, d) + J_u(u - u_{opt}) + \frac{1}{2}(u - u_{opt})^T J_{uu}(u - u_{opt}) + \zeta^3$$

$$L_{wc} = \tfrac{1}{2}\bar{\sigma}(J_{uu}^{1/2}(HG^y)^{-1}HY)^2 = \tfrac{1}{2}\bar{\sigma}(M)^2$$

Loss with $c=Hy_m=0$ due to
(i) Disturbances d
(ii) Measurement noise n^y
Book: Eq. (10.12)

$$Y = [FW_d \quad W_n],$$
$$F = G^y J_{uu}^{-1} J_{ud} - G_d^y$$

Proof: See handwritten notes

Optimal H (with measurement noise)

$$\min_H \left\| J_{uu}^{1/2} (HG^y)^{-1} HY \right\|_F$$

1. Kariwala: Can minimize 2-norm (Frobenius norm) instead of singular value
2. BUT seemingly Non-convex optimization problem (Halvorsen et al., 2003), see book p. 397

Have extra degrees of freedom

$$H_1 = DH$$ D : any non-singular matrix

$$(H_1 G_y)^{-1} H_1 = (DHG_y)^{-1} DH = (HG_y)^{-1} D^{-1} DH = (HG_y)^{-1} H$$

Improvement (Alstad et al. 2009)

$$\min_H \|HY\|_F$$

st $HG^y = J_{uu}^{1/2}$

Convex optimization problem

Global solution

Analytical solution

$$H^T = (YY^T)^{-1} G^y \, \cancel{(G^{yT} (YY^T)^{-1} G^y)^{-1} J_{uu}^{1/2}}$$

$$Y = [F W_d \quad W_{n^y}]$$

- Do not need J_{uu}
- Analytical solution applies when YY^T has full rank (w/ meas. noise):

NEW

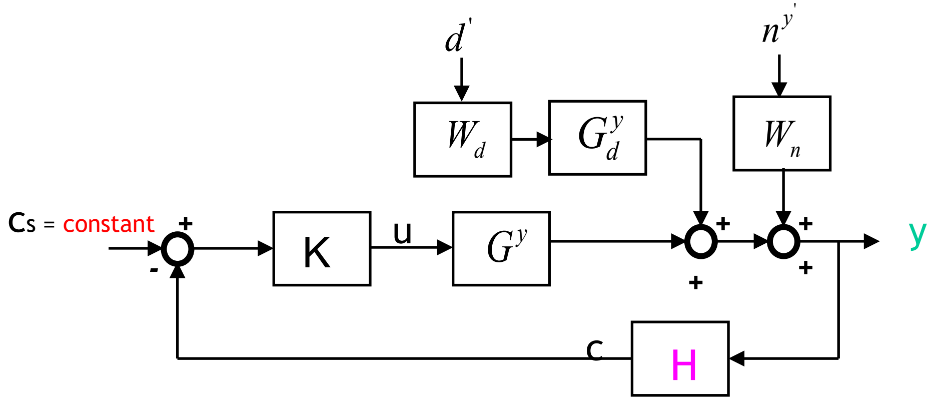
Marathon runner: Exact local method

- Wd = 1, Wny = [1 0; 0 1]
- F = [0.25; -0.2]
- Y = [Wd*F Wny]
- Gy = [2 1]'
- H = (inv(Y *Y')*Gy)'

- Get H = [1.932   1.054]
- Or normalized H1 = D*H = [1   0.55]

- Note: Gives same as nullspace when Wny is small

Special cases



$$\min_H \|J_{uu}^{1/2}(HG^y)^{-1}H[FW_d \quad W_{n^y}]\|_F$$

- No noise ($n^y=0$, $W_{ny}=0$):

  Optimal is HF=0 (Nullspace method)

  - But: If many measurement then solution is not unique

- No disturbances ($d=0$; $W_d=0$) + same noise for all measurements ($W_{ny}=I$):

  Optimal is $H^T=G^y$ ("control sensitive measurements")

  - Proof: Use analytic expression

$$H^T = (YY^T)^{-1}G^y$$
$$Y = [FW_d \quad W_{n^y}]$$

New 2024:

Nullspace method to estimate gradient

Optimal measurement-based estimate of the cost gradient for real-time optimization

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The optimal local gradient estimate for use in steady-state real-time optimization is simply $\hat{J}_u = H^J y_m - c_s$ with H^J as in (12) and $c_s = H^J y^* - J_u^*$ (Theorem 1). This gradient estimate is optimal also in the constraint case when used with the KKT optimality conditions (2) (Theorem 2). The gradient estimate

$$H^J = J_{uu}\left[G^{yT}\left(YY^T\right)^{-1}G^y\right]^{-1}G^{yT}\left(YY^T\right)^{-1} \qquad (12)$$

$$Y = [FW_d \quad W_n],$$
$$F = G^y J_{uu}^{-1} J_{ud} - G_d^y$$

Some references

S. Skogestad, ``**Control structure design for complete chemical plants'',** *Computers and Chemical Engineering*, **28** (1-2), 219-234 (2004).

V. Alstad and S. Skogestad, ``Null Space Method for Selecting Optimal Measurement Combinations as Controlled Variables'', *Ind.Eng.Chem.Res*, **46** (3), 846-853 (2007).

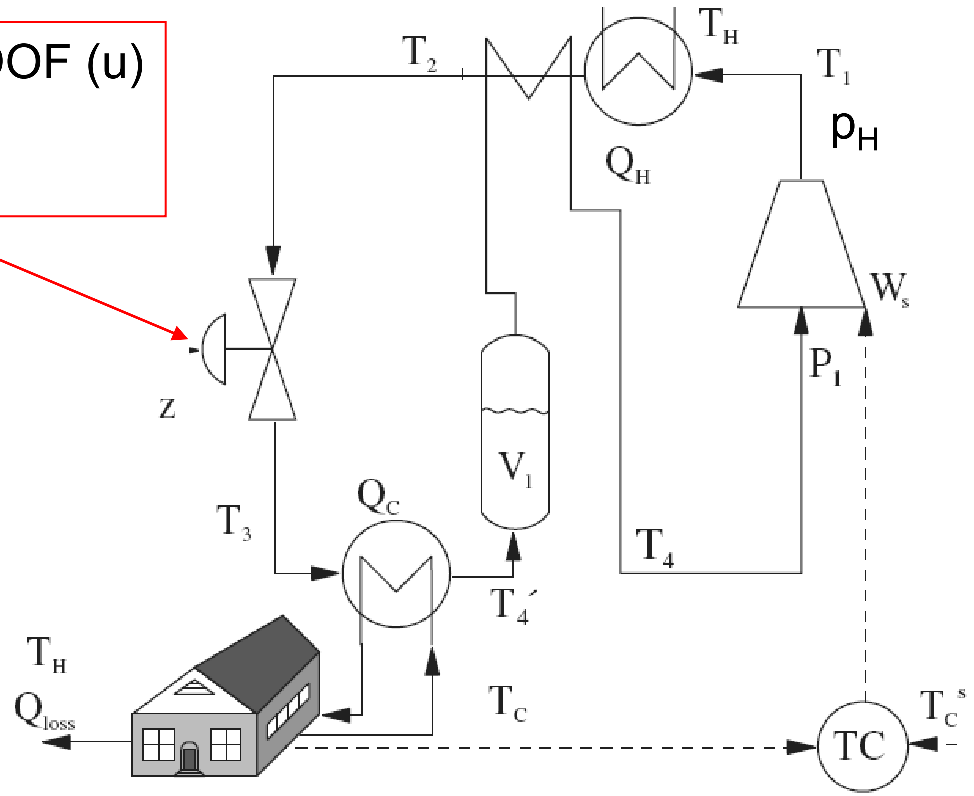
V. Alstad, S. Skogestad and E.S. Hori, ``Optimal measurement combinations as controlled variables'', *Journal of Process Control*, **19**, 138-148 (2009)

Ramprasad Yelchuru, Sigurd Skogestad, Henrik Manum , MIQP formulation for Controlled Variable Selection in Self Optimizing Control *IFAC symposium DYCOPS-9*, Leuven, Belgium, 5-7 July 2010

Download papers: Google "Skogestad"

Example: CO_2 refrigeration cycle

Unconstrained DOF (u)
Control what?
c=?



CO2 refrigeration cycle

Step 1. One (remaining) degree of freedom (u=z)

Step 2. Objective function. $J = W_s$ (compressor work)

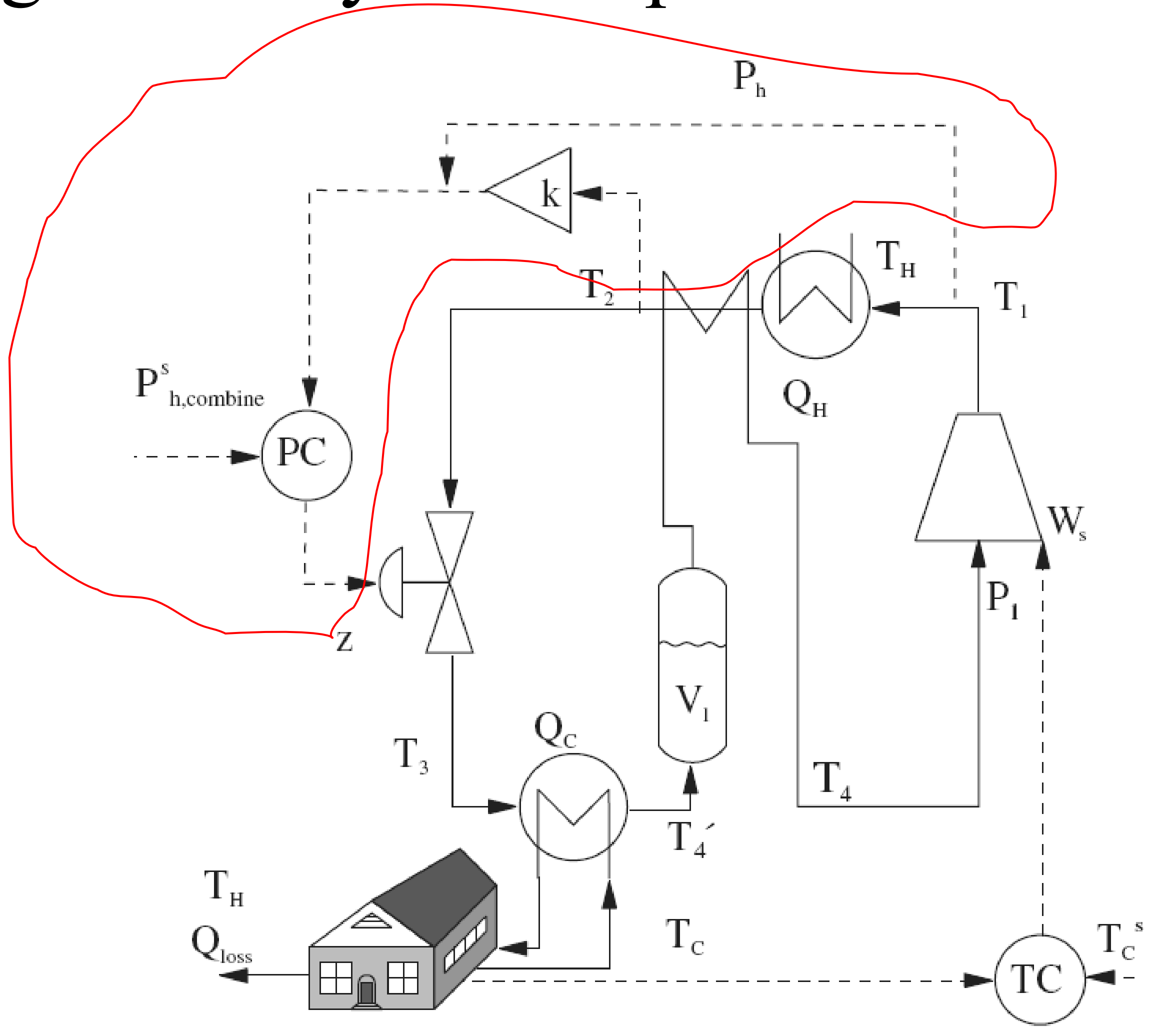
Step 3. Optimize operation for disturbances ($d_1=T_C$, $d_2=T_H$, $d_3=UA$)

- Optimum always unconstrained

Step 4. Implementation of optimal operation

- No good single measurements (all give large losses):
  - p_h, T_h, z, …
- Nullspace method: Need to combine $n_u+n_d=1+3=4$ measurements to have zero disturbance loss
- Simpler: Try combining two measurements. Exact local method:
  - $c = h_1 p_h + h_2 T_h = p_h + k T_h$; $k = -8.53$ bar/K
- Nonlinear evaluation of loss: OK!

Refrigeration cycle: Proposed control structure



Control c= “temperature-corrected high pressure”

Conclusion optimal operation

ALWAYS:

1. Control active constraints and control them tightly!!
   - Good times: Maximize throughput -> tight control of bottleneck

2. Identify "self-optimizing" CVs for remaining unconstrained degrees of freedom

- Use offline analysis to find expected operating regions and prepare control system for this!
  - One control policy when prices are low (nominal, unconstrained optimum)
  - Another when prices are high (constrained optimum = bottleneck)

ONLY if necessary: consider RTO on top of this