PROCESS DESIGN AND CONTROL

Consistent Inventory Control

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Inventory or material balance control is an important part of process control. In the literature, many rules have been proposed to help in designing such systems, but their justification is often unclear. The main contribution of this paper is to propose the more general local-consistency rule for evaluating inventory control systems. Consistency means that the steady-state mass balances (total, component, and phase) for the individual units and the overall plant are satisfied. In addition, *local consistency* is a desired property, meaning that the local mass balances are satisfied with local inventory loops only.

Introduction

One of the more elusive aspects of process control education is inventory or material balance control. An engineer with some experience can usually immediately say if a proposed inventory control system is workable. However, for a student or newcomer to the field, it is not obvious, and even for an experienced engineer there may be cases where experience and intuition are not sufficient. The objective of this paper is to present concise results on inventory control, relate to previous work, tie up loose ends, and provide some good illustrative examples. The main result (consistency rule) can be regarded as obvious; nevertheless we have not seen it presented in this way before.

The main result is a simple rule to check whether an inventory control system is *consistent*. Here, "consistency" means that the mass balances for the entire plant and units are satisfied.[1] In addition, we usually want the inventory control system to be *local consistent*. ["Local consistency" is closely related to the term "self-consistency" used by Price and Georgakis.[1,2]] Local consistency requires, in addition to consistency, that all inventories are regulated locally, without the need to rely on control loops outside the unit. Consistency is a required property, because the mass balances must be satisfied in a plant, whereas local consistency is a desirable property of an inventory control system. In practice, an inconsistent control structure will lead to a situation with a fully open or closed control valve and the associated control loop cannot fulfill or attain the control set point.[3]

In most plants, we want the inventory control system to use simple proportional−integral controllers and be part of the basic (regulatory) control layer. This is because it is generally desirable to separate the tasks of regulatory (stabilizing) control and supervisory (economic) control. From this it follows that the structure of the inventory control system is usually difficult to change later.

The importance of consistency of inventory control structures is often overlooked. Our work is partly inspired by the many examples of Kida, who has given industrial courses in Japan on control structures for many years. In a personal communication[3] he states that "most process engineers, and even academic people, do not understand the serious problem of inconsistency of plantwide control configurations. When writing a paper, you have to clearly explain this point and make them convinced at the very outset. Otherwise they will not listen to or read through your detailed statements, but skip them all."

A very good early reference on inventory control in a plantwide setting is by Buckley.[4] He states that material balance control must be in the direction of flow downstream a given flow and opposite the direction of flow upstream a given flow. Price et. al[1,2] extended this and state that the inventory control must "radiate" outward from the point of a given flow (throughput manipulator). As shown in this paper, these statements are a consequence of requiring the inventory control system to be locally consistent.

Downs[5] provides a very good discussion of material balance control in a plantwide control environment, with many clarifying examples. However, it is somewhat difficult for the reader to find a general rule or method that can be applied to new cases.

Luyben et al.[6] propose a mainly heuristic design procedure for plantwide control. Luyben's procedure consists of, among others, "Step 6. Control inventories (pressures and levels) and fix a flow in every recycle loop." Possible limitations of this guideline are discussed in the present paper. Another guideline of Luyben et al.[6] is to "ensure that the overall component balances for each chemical species can be satisfied either through reaction or exit streams by accounting for the component's composition or inventory at some point in the process". This guideline is a bit limited because entrance (feed) streams are not considered.

Specific guidelines for designing inventory control structures are presented by Price and Georgakis.[1,2] They propose a set of heuristic guidelines for inventory control design in a plantwide environment and also discuss consistency. The authors also state the importance of a consistent inventory control structure: "Self-consistency appears to be the single most important characteristic governing the impact of the inventory control structure on system performance."

As already mentioned, Fujio Kida from JGC Corporation in Japan has developed a lot of teaching material[3] and written several papers on inventory control.[7] Unfortunately, the work is published in Japanese only; nevertheless it is clear that there are many detailed rules and some require detailed calculations.

In summary, the literature provides a number of specific rules of designing inventory control systems, but the justification and limitations of these rules are often unclear. The main result of this paper is to present the simple local-consistency rule for evaluating inventory control systems, which applies to all cases and only requires structural information.

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The outline of the paper is as follows. First the terms "consistency" and "local consistency" are defined, and then we present and derive the local-consistency rule. We then apply the rule to some flow networks, such as units in series and recycle systems. This is followed by a derivation of some more specific rules before we end up with some more complex examples, including distillation and recycle reactors. Note that the present paper focuses on analysis of a given control structure. The design of the inventory control system, which in particular is related to the placement of the throughput manipulator, is discussed in more detail by Aske.[8,9]

Remark on Notation: In this paper, when a flow (valve) is left unused or with a flow controller (FC), this indicates that this is a *given* flow. By the term "given flow" we mean that the flow is *not* used for inventory control but rather is given by conditions outside the inventory control system. Specifically, in this paper a "given flow" can be

1. a throughput manipulator (TPM)
2. a flow that comes from another part of the plant (disturbance for our part)
3. a fixed flow
4. a flow that is used for other control tasks (e.g., control of composition or temperature).

Definition of Consistent Inventory Control

The dynamic mass balance for total or component mass in any unit or process section can be written:[5]

$$\frac{dI}{dt} = \text{rate of change in inventory} = \text{inflow} + \text{generation} - \text{outflow} - \text{consumption} \quad (1)$$

During operation we must have "inventory regulation", meaning that the inventories of total, component, and phase mass are kept within acceptable bounds. To achieve this we need a "consistent" and preferably "local-consistent" inventory control system.

Definition 1: *Consistency.* *An inventory control system is consistent if it can achieve acceptable inventory regulation for any part of the process, including the individual units and the overall plant.*

More precisely, by "it can achieve acceptable inventory regulation" we mean that there exist controller settings such that one can keep the inventories of total, component, and phase mass within acceptable bounds.

Remark 1: The use of mass balances for a phase may seem odd, and is discussed in more detail in the next section.

Remark 2: Consistency requires that the steady-state mass balances (total, components, and phases) are "satisfied", meaning at steady state there is a balance between in-terms (inflow + generation) and out-terms (outflow + consumption) such that $dI/dt = 0$. In addition, we must require that the inventories can be kept within acceptable bounds ($I_{min} \le I \le I_{max}$). For example, if we have a process where a component has no specified exit, then it will eventually have to exit somewhere (at steady state), but the value of its steady-state inventory I (and composition) may not be acceptable, so we may still not have consistency.

Remark 3: Since the mass balance must be satisfied for the overall plant, it follows that a consistent inventory control system must be *"able to propagate a production rate change throughout the process and in particular if such a change produces changes in the flow rates of major feed and product streams"*.[1]

In most cases, we want the inventory control system also to be "local consistent", meaning that the consistent inventory control system involves only local loops.

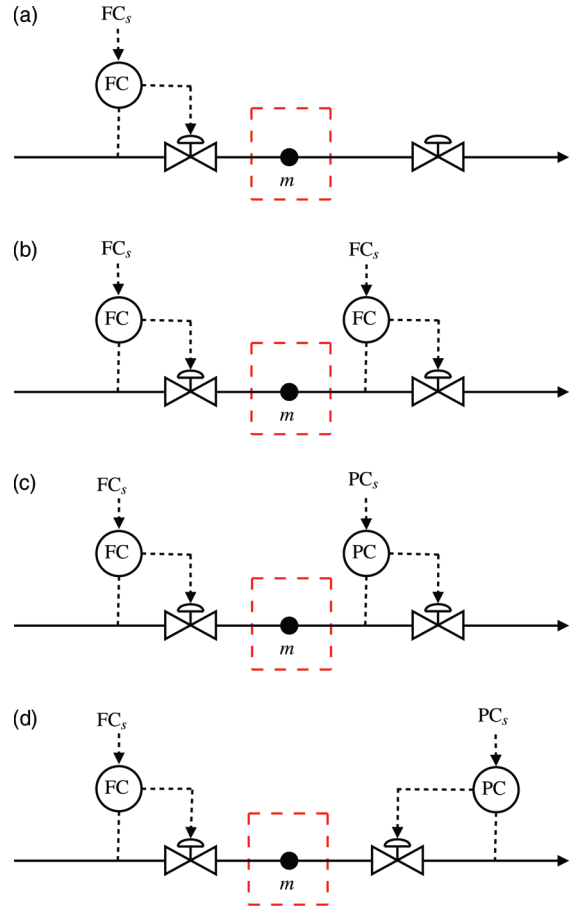

Figure 1. Four different control structures with two valves and set inflow. (a) OK (consistent self-regulating structure since outflow of dashed box depends on inventory m). (b) Not consistent control structure since outflow is set. (c) OK (consistent control structure since outflow depends on inventory m). (d) Not consistent control structure since outflow does not depend correctly on inventory m. See dynamic simulations in Figure 2. Note: For the flow controllers (FC) it does not matter whether the valve is downstream (as shown above) or upstream of the flow measurement, but it does matter for the pressure controllers (PC).

Definition 2: *Local Consistency.* *A consistent inventory control system (see Definition 1) is local consistent if the inventory of each unit is regulated (controlled) by only its in- or outflows (with no manipulated variables outside the unit).*

For a process consisting of just one unit (as in Figure 1 where the unit is the black dot), local consistency and consistency are the same, but not in general. To understand the difference between consistency and local consistency, consider the serial process in Figure 3, which is discussed in more detail later in Example 2. Here, control of inventory in the last unit involves the main feed flow which is outside the unit, so we have consistency but not local consistency. The main problem with not having local consistency in this case is that the last unit in Figure 3 cannot be operated by itself (independently of the other units). Another problem is that the "long" inventory control loop (with a large effective time delay from the feed valve to the last unit) may result in poor control of the inventory in the last unit.

Consistency Rule

From the above definitions it follows that local consistency is equivalent to requiring local regulation of all inventories, and we can derive the following rule.

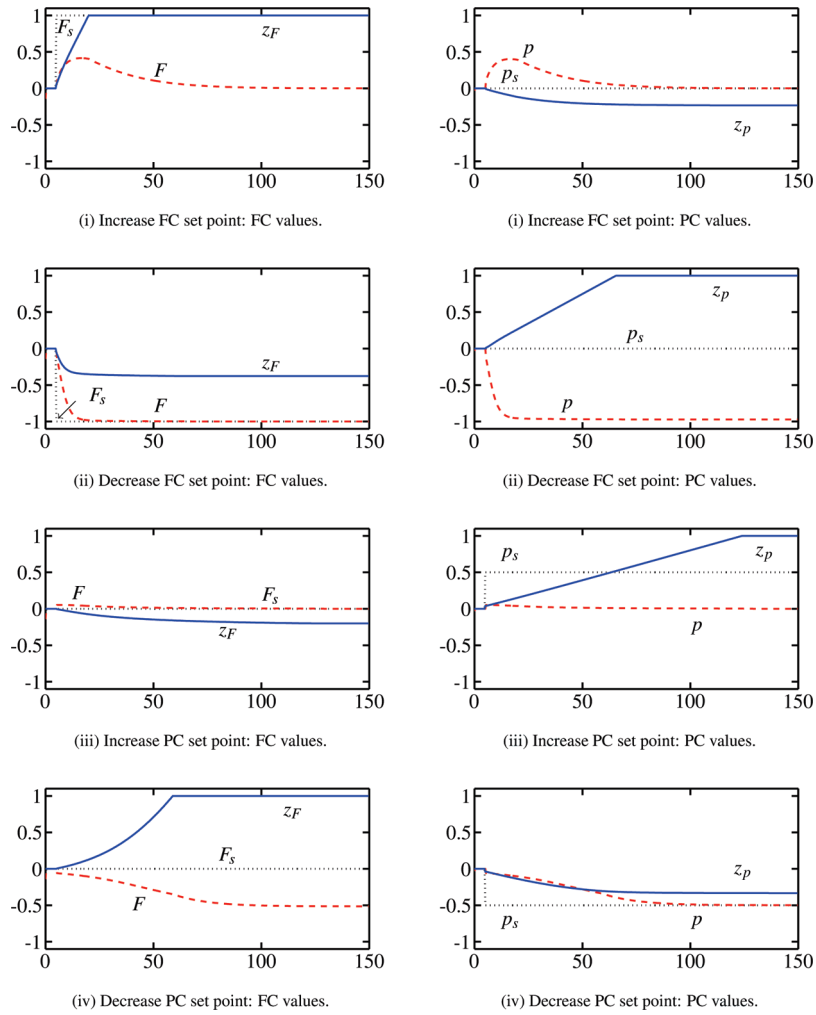

Figure 2. Dynamic simulations of inconsistent configuration in Figure 1d. Left column: flow controller (FC). Right column: pressure controller (PC). In all cases, one of the two valves moves to fully open. z_F, inlet valve opening (from −1 to 1); z_P, outlet valve opening.

Rule 1: ***"Local-Consistency Rule".*** *Local consistency (acceptable inventory regulation using only local control loops) requires that*

1. *The* ***total inventory (mass)*** *of any part of the process (unit) must be regulated by its in- or outflows, which implies that at least one flow in or out of any part of the process (unit) must depend on the inventory inside that part of the process (unit).*

2. *For systems with several components, the inventory of each* ***component*** *of any part of the process must be regulated by its in- or outflows or by chemical reaction.*

3. *For systems with several phases, the inventory of each* ***phase*** *of any part of the process must be regulated by its in- or outflows or by phase transition.*

Remark 1: The above requirement must be satisfied for "any part of the process". In practice, one should at least consider the individual units plus the overall process.

Remark 2: By the term "regulated" we mean "kept within acceptable bounds". This usually requires a control system ("active control"), but some inventories can be "self-regulated" ("passive control"). "Active" inventory control usually involves a level controller (LC) (liquid) or pressure controller (PC) (gas and in some cases liquid), but it may also be a temperature controller (TC) or composition controller (CC). On the other hand, a flow controller (FC) cannot be used for inventory control because flow is not a measure of inventory.

Remark 3: It is possible to extend the local-consistency rule to energy inventory, but this is not done here. We also doubt if such an extension is very useful, because in most cases the energy balance will maintain itself by self-regulation (without active control), for example because a warmer inflow in a tank leads to a warmer outflow.

The above rule may seem obvious. Nevertheless, a more formal proof is useful and may clarify some issues.

Proof of Local-Consistency Rule. 1. A boundary (control volume) may be defined for any part of the process. Let m [kg] denote the inventory inside the control volume and let $\dot{m}_{in}$ and $\dot{m}_{out}$ [kg/s] denote in- and outflows. Then the (total) mass balance is

$$\frac{dm}{dt} = \sum \dot{m}_{in} - \sum \dot{m}_{out} \quad [kg/s] \tag{2}$$

If all terms are independent of the inventory m, then this is an integrating process where m will drift out of bounds ($dm/dt \neq 0$ at steady state) when there is a disturbance in one of the terms ($\dot{m}_{in}$ or $\dot{m}_{out}$). To regulate the inventory, $\dot{m}_{in}$ or $\dot{m}_{out}$ must depend on the inventory (m), such that m is kept within acceptable bounds. More precisely, we need negative feedback, so $\dot{m}_{in}$ must decrease when m increases or $\dot{m}_{out}$ must increase when m increases.

2. Similarly, let n_A [mol of A] denote the inventory of component A inside the control volume and let $\dot{n}_{A,in}$ and $\dot{n}_{A,out}$ [mol of A/s] denote the in- and outflows. The mass balance for component A is

$$\frac{dn_A}{dt} = \sum \dot{n}_{A,in} - \sum \dot{n}_{A,out} + G_A \quad \text{[mol of A/s]} \qquad (3)$$

where G_A is the net amount generated by chemical reaction. To self-regulate the component inventory, $\dot{n}_{A,in}$, $\dot{n}_{A,out}$ or G_A must depend on n_A such that n_A is kept within acceptable bounds in spite of disturbances.

An example where the inventory n_A may be self-regulated (by the reaction term G_A) is the irreversible reaction A + B → P, where B is in excess and A is the limiting reactant. In this case, an increase in inflow of A ($\dot{n}_{A,in}$) will be counteracted by an increased consumption of A in the chemical reaction.

3. The rule for the individual phase follows by simply defining the control volume as the parts of the process that contain a given phase P and applying the mass balance to this control volume. Let m^P [kg] denote the inventory of the given phase inside the control volume and let $\dot{m}^P_{in}$ and $\dot{m}^P_{out}$ [kg/s] denote the in- and outflows. The mass balance for a given phase is then

$$\frac{dm^P}{dt} = \sum \dot{m}^P_{in} - \sum \dot{m}^P_{out} + G^P \quad \text{[kg/s]} \qquad (4)$$

where G^P is the net phase transition over the phase boundary. To regulate the phase inventory, $\dot{m}^P_{in}$, $\dot{m}^P_{out}$ or G^P must depend on the inventory (m^P) such that m^P is kept within acceptable bounds in spite of disturbances.

An example where we need to consider individual phases is a flash tank where a two-phase feed is separated into gas and liquid; see Figure 4b.

Example 1: Stream with Two Valves. To demonstrate the local-consistency rule on a very simple example, consider a single stream with two valves; see Figure 1a. There is only a single (small) inventory (hold-up) m in this simple process (illustrated by the big dot), so consistency and local consistency are the same. In this case, the pressure p is a direct measure of inventory m (for a liquid the dependency is very strong; for an ideal gas it is $p = mRT/V$). Thus, regulation of pressure is the same as regulation of inventory. To apply the local-consistency rule, we define a control volume (dashed box) as shown in Figure 1 and note that the inflow is given (on flow control) in all four cases; that is, the inflow is independent of the inventory m. Thus, according to Rule 1, to have consistency (acceptable regulation), the outflow must depend on the inventory m (pressure p) and more specifically the outflow must increase when m (p) increases.

Four different control structures are displayed in Figure 1. According to Rule 1, the structure in Figure 1a is consistent since the outflow through the valve increases when the inventory m (pressure p) increases. Thus, we have self-regulation with no need for active control.

The control structure in Figure 1b is not consistent because the second flow controller makes the outflow independent of the inventory m. Even if the set points for the two flow controllers were set equal, any error in the actual flow would lead to an imbalance, which would lead to accumulation or depletion of mass and the inventory would not be regulated within acceptable bounds.

The structure in Figure 1c is consistent because the pressure controller increases the outflow when the inventory m (pressure p) increases.

Finally, the control structure in Figure 1d is not consistent because the pressure controller is configured such that the outflow depends on the inventory m (and pressure) in the wrong (opposite) manner. To understand this, consider a decrease in inflow, which will lead to a decreased pressure inside the control volume. A lower differential pressure over the pressure-controlled valve leads to a smaller flow through the valve and the pressure at the downstream measuring point will decrease, leading the pressure controller to open the valve. The result is a further pressure decrease in the control volume, so the pressure controller is actually working in the wrong direction. The opening of the pressure-controlled valve will also affect the flow-controlled valve and, depending on the set point of the controllers, either the flow-controlled valve or the pressure-controlled valve will move to fully open. The other pressure-controlled valve or flow-controlled valve will continue to control pressure or flow. This is confirmed by dynamic simulations in Figure 2i−iv:

(i) 10% increase in FC set point: The FC valve saturates at fully open and the PC maintains its set point.

(ii) 10% decrease in FC set point: The FC maintains its set point and the PC valve saturates at fully open.

(iii) 5% increase in PC set point: The FC maintains its set point and the PC valve saturates at fully open.

(iv) 5% decrease in PC set point: The FC valve saturates at fully open and the PC maintains its set point.

The simulations were performed with the flow sheet simulator Aspen HYSYS, and in all cases the system was initially at steady state. It should also be noted that the pressure control loop in Figure 1d is in the direction opposite to flow, which is not correct when the inflow is given (see further discussion in the next section).

Remark about the Sign of Controllers: For closed-loop stability the controller and the plant should overall give a negative feedback loop:

1. Flow control. Opening a valve always increases the flow (positive gain), so a flow controller is always "reverse acting" (with a negative feedback sign).

2. Level and pressure control. The controller sign depends on the location of the valve relative to the inventory (level or pressure). If control is in the direction of flow (with the inventory measurement for level or pressure upstream the valve), then the controller must be "direct acting" (positive feedback sign); if control is in the opposite direction of flow, then it must be "reverse acting".

These remarks were used when choosing the sign for controller gains for the dynamic simulations in Figure 2.

Example 2: Units in Series. To understand the difference between the terms "consistency" (Definition 1) and "local consistency" (Definition 2), consider inventory control of the series process in Figure 3. The control structure is *consistent* and is able to propagate a production rate change to a change in the feed rate. However, the in- and outflows for the last unit (dashed box) do not depend directly on the inventory inside the unit and the control volume is therefore *not local consistent* according to the "consistency rule" (Rule 1). Also, the inventory controllers are not in the opposite direction to flow as they should be for a local-consistent process with a given product rate (see also next section). To make the structure consistent,

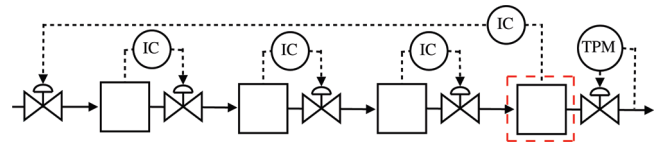

Figure 3. Consistent, but not local-consistent, inventory control structure.

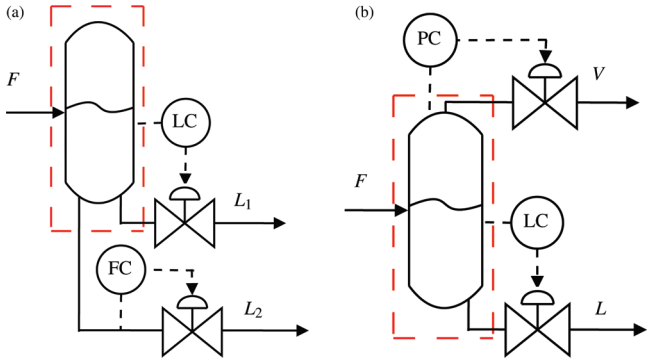

Figure 4. Local-consistent inventory control of split with one and two phases. (a) Single-phase tank: adjustable split. (b) Two-phase tank: inventory control determines split.

we have in Figure 3 introduced a "long loop" where the inflow to the first unit is used to control the inventory in the last unit.

Example 3: Phase Inventories. In some cases, phase inventories need to be considered for local consistency. Consider Figure 4 where the inflow F is given. Thus, according to Rule 1, to have consistency the outflow must depend on the inventory in the tank.

In Figure 4a, the inflow is a single phase (liquid) and the outflow from the single-phase tank is split in two liquid streams (L_1 and L_2). There is one inventory, so for local consistency, one of the outflows must be on inventory control whereas the other outflow can be given, for example by a flow controller. That is, we have an adjustable split.

In Figure 4b the inflow is two-phase (liquid and vapor) and there are two inventories (liquid and vapor) that need to be regulated. To have a consistent inventory control structure, both outflows (vapor and liquid) must be used for inventory control. In Figure 4b this is illustrated by the LC (liquid inventory) and PC (vapor inventory). In this case, the split is not adjustable in practice because the split is indirectly determined by the feed quality (fraction of vapor).

Local Consistency of Flow Networks

Throughput Manipulator. In a flow network there is at least one degree of freedom, called the throughput manipulator (TPM), which sets the network flow. More generally, *a TPM is a degree of freedom that affects the network flow and which is not directly or indirectly determined by the control of the individual units, including their inventory control.*[8] Typically, a given flow (e.g., flow controller with an adjustable set point) is a TPM. As discussed in more detail below, the location of the TPM is very important. In particular, if the flow network has no splits or junctions, then, for a given placement of the TPM, there is only one *local-consistent* inventory control structure.

Flow Split. In most cases a flow split is adjustable and this introduces an extra degree of freedom for control of flow and inventory at the network level.[3] However, a flow split does not always introduce a degree of freedom for network flow as illustrated by the flash tank in Figure 4b, where the two outflows are indirectly determined by the feed enthalpy (phase distribution). Another example where a split does not give an extra degree of freedom for control of network flow is a distillation column where the outflows (distillate flow D and bottoms flow B) are indirectly determined by the feed composition.

For an adjustable split or junction (e.g., multiple feeds) that introduces an extra degree of freedom for control of flow and

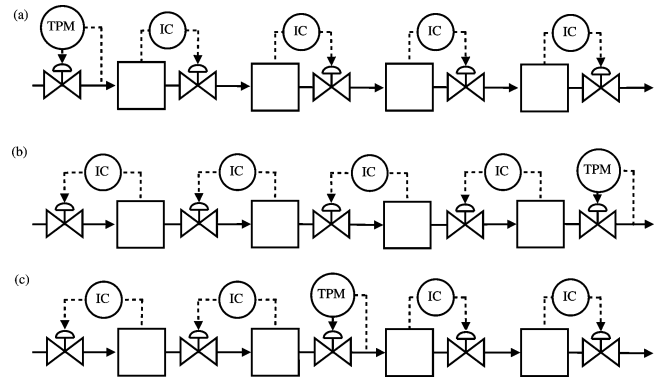

Figure 5. Local consistency requires a radiating inventory control around a given flow (TPM). (a) TPM at inlet (feed): inventory control in direction of flow. (b) TPM at outlet (on demand): inventory control in direction opposite to flow. (c) General case with TPM inside the plant: radiating inventory control.

inventory at the network level, a common choice is to use the largest flow for inventory control.[6] For example, with a given feed, the largest product stream may be used for inventory control with the flow rates of the smaller product streams used for quality control. Similarly, with a given production rate, the largest feed rate is often used for inventory control and the smaller feed flows are set in ratio relative to this, with the ratio set point possibly adjusted for quality control.

The objective is now to apply the local-consistency rule to analyze inventory control structures for real processes (flow networks). We consider three network classes:

1. units in series
2. recycle systems
3. closed systems.

A series network may have splits, provided the flow is still in the same direction. Note that each single-phase split introduces one extra degree of freedom (the split ratio; see Figure 4). A recycle system contains one or more splits that are (partly) fed back to the system. A closed system has total recycle with no feeds or products.

Units in Series ("Radiating Rule"). As mentioned above, if there are no splits or junctions, the location of the throughput manipulator determines the local-consistent inventory control structure. Specifically, a direct consequence of the local-consistency rule is

(i) *Inventory control must be in the direction of flow downstream the location of a given flow (TPM).*

(ii) *Inventory control must be in the direction opposite to flow upstream the location of a given flow (TPM).*

More generally, we have

Rule 2: *Radiation Rule*[1]. *A local-consistent inventory control structure must be radiating around the location of a given flow (TPM).*

These rules are further illustrated in Figure 5.

Recycle Systems. A recycle system has a flow split that usually introduces an extra degree of freedom for control of network flow and inventory. A simple recycle flow network with a given feed flow is considered in Figure 6 (there is a pump or compressor in the recycle loop which is not shown). The flow split introduces a degree of freedom, which means that we may fix one of the three remaining flows, and four alternative inventory control structures are shown. Figures 6a and 6b have consistent inventory control structures, because the outflows from units 1 and 2 depend on the inventory inside each unit. In both cases one flow in the recycle loop is set (flow controlled with an adjustable set point that may be used for other purposes

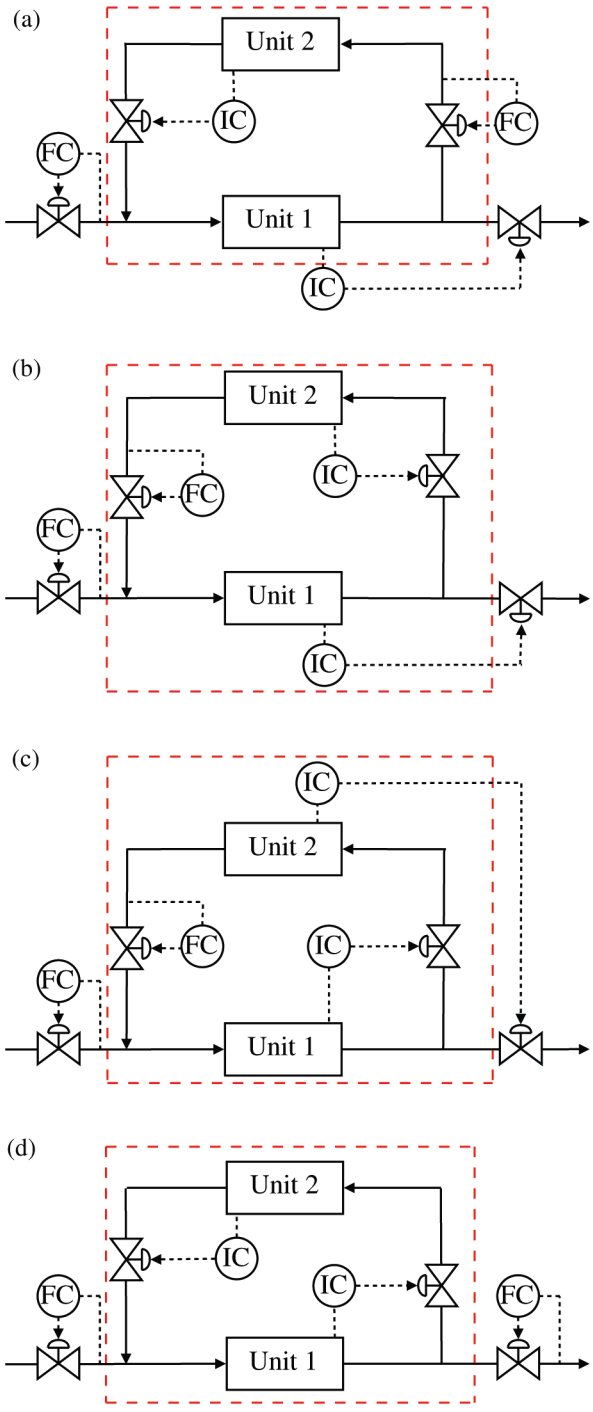

Figure 6. Inventory control of simple recycle process with given feed. (a) Local-consistent inventory control. (b) Local-consistent inventory control. (c) Consistent, but not local-consistent inventory control. (d) Not consistent inventory control.

than inventory control). Note that *the inventory control in the recycle loop can be either in the direction of flow* (Figure 6a) *or in the direction opposite to flow* (Figure 6b), because the flow rate can be set at any location in the recycle loop.

In Figure 6c the inventory loops for units 1 and 2 are paired opposite compared to Figure 6b. This structure is consistent (as the material balances are satisfied), but not local consistent because inventory control of unit 2 involves a flow that is not one of "its in- or outflows" and thus violates Rule 1.

Finally, in Figure 6d the inventory control of each unit is acceptable and involves only local flows. Nevertheless, this structure is clearly not consistent (and also not local consistent, which is a stricter requirement) since both the feed and product rates to the overall process are given. In particular, the inflow and outflow to the dashed box (which represents the overall process) do not depend on the inventory inside this part of the process, which violates Rule 1. Thus, for consistency it is not sufficient to check only the individual units; one should also check the overall process.

Closed Systems. Closed systems require particular attention. First, the total inventory is constant. Second, since there are no in or out streams our previously derived rule (Rule 1) does not really apply to the overall system. As an example, consider in Figure 7 a closed system with two inventories. In Figure 7a we follow Rule 1 and attempt to control both inventories, but the two loops will "fight each other" and will drift to a solution with either a fully open or fully closed valve. For example, a (feasible) solution is to have zero flow in the cycle. The problem is that the flow is not set anywhere in the loop. To get a consistent inventory control structure, *one must let one of the inventories be uncontrolled*, as shown in Figure 7b,c. *The corresponding unused degree of freedom (flow) sets the flow rate ("load", throughput) of the closed system.*

To be able to use our local-consistency rule (Rule 1) for closed systems, there are two alternative "fixes":

1. Let the total inventory be uncontrolled, which is how such systems are usually operated in practice. Typically the largest single inventory is uncontrolled. However, the remaining inventories must be regulated, as usual, to have local consistency of the inventory control system.

2. Introduce a "dummy" (small) stream that keeps the total inventory constant. This corresponds to allowing for filling (charging) or emptying the system. In practice, this stream may be a make-up stream line that refills or empties the largest inventory, e.g., on a daily or monthly basis.

Both approaches allow for disturbances, such as leaks or supply. The inventory control system can then be analyzed using the normal local-consistency rule (Rule 1). Figure 7a is clearly not allowed by fix 1 as the total inventory is not left uncontrolled. Figure 7a is also not consistent by fix 2, since for local consistency the dummy inlet stream must be used for inventory control instead of one of the two flows in the recycle loop.

Example 4: Absorber−Regenerator Example. In this example, the local-consistency rule (Rule 1) is used for an individual phase (liquid), which forms a closed system. Consider the absorber and regenerator example in Figure 8,[3] where a component (e.g., CO_2) is removed from a gas by absorption. The inlet gas flow (feed) is indirectly given because there is a pressure control in the direction of flow at the inlet. The gas outlet flows are on pressure control in the direction of flow and thus depend on the gas holdup in the plant. Therefore the gas-phase inventory control is consistent. However, the liquid flows between the absorber and regenerator constitute a "closed system" (except for minor losses). There is a flow controller for the recycled liquid, but its set point is set by the inventory in the regenerator; hence all inventories in the closed system are on inventory control, which violates the rule just derived. To get a consistent inventory control structure, we must break the level-flow cascade loop and let the inventory in the bottom of the regenerator remain uncontrolled. Alternatively, let the absorber liquid inventory be uncontrolled and break the level-

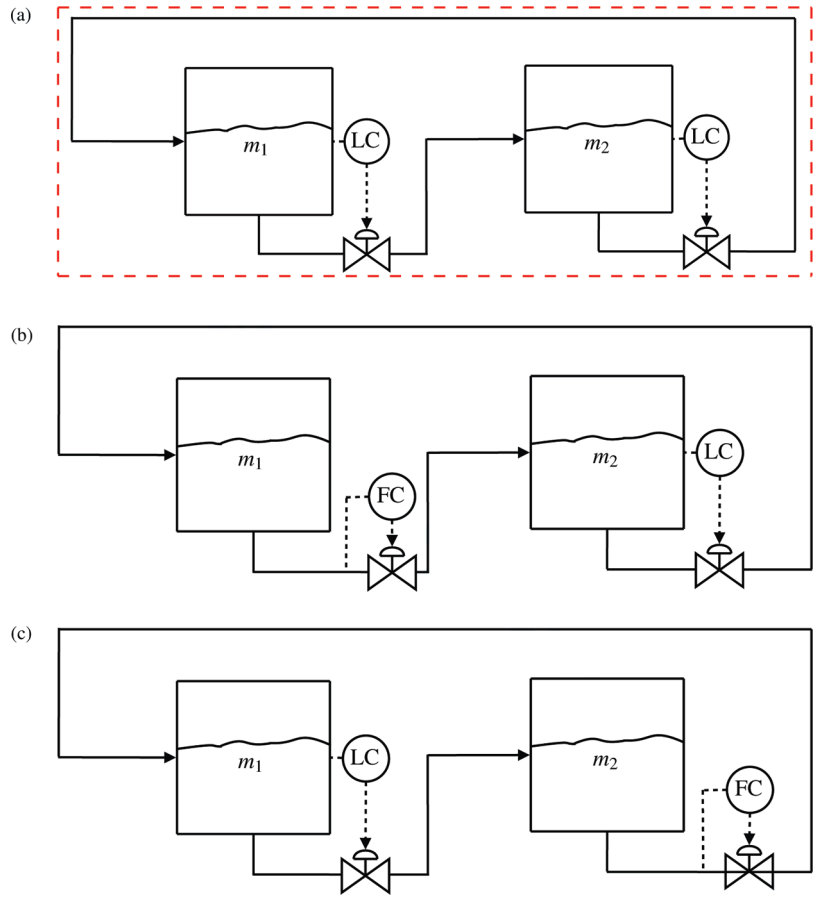

Figure 7. Inventory control for closed system. (a) Not consistent (because there is no uncontrolled inventory). (b) Local consistent (inventory m_1 is uncontrolled). (c) Local consistent (inventory m_2 is uncontrolled).

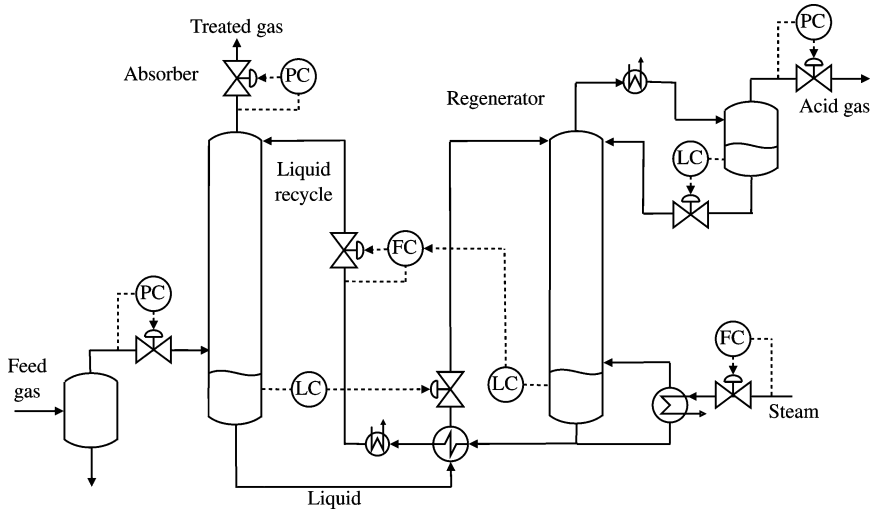

Figure 8. Absorber and regenerator example: not consistent liquid inventory control.

flow cascade loop and let the feed into the regenerator control the regenerator liquid inventory.

Summary and Discussion of Specific Rules

In the literature there are many rules that deal with the inventory control structure. In addition to the radiating rule (Rule 2), some useful rules that can be developed from the local-consistency rule (Rule 1) are as follows:

1. *All systems must have at least one given flow.*

Proof. Assume there is no given flow such that all flows are on inventory control. This will not result in a unique solution, for example, zero flow will be an allowed solution.

2. *Component balance rule (Downs,*[5] *p 414): Each component, whether important or insignificant, must have its inventory controlled within each unit operation and within the whole process.* This is also referred to as "Downs' drill" in, Luyben et al.,[10] p 56.

Proof. This comes from the requirement of component local consistency (Rule 1).

3. *A stream cannot be flow controlled more than once; that is, a structure with two flow controllers on the same stream is not consistent.*

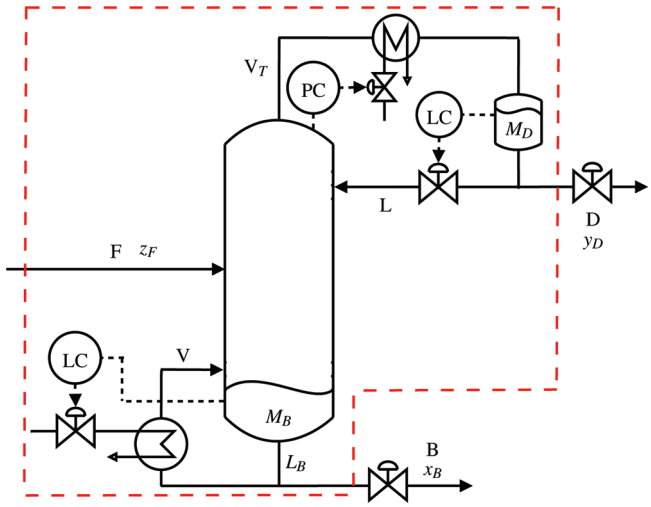

Figure 9. Example of inconsistent inventory control: distillation column with *DB* configuration.

Table 1. Proposed Guidelines for Design of Local-Consistent Inventory Control System[a]

1 Choose the location of the throughput manipulator.
2 Identify inventories that need to be controlled including
  (a) total mass
  (b) components
  (c) individual phases
3 Identify manipulators suitable for adjusting each inventory.
4 Design a local-consistent radiation inventory control system that controls all the identified inventories. This means
  (a) inventory control in the direction of flow downstream the throughput manipulator
  (b) inventory control in the direction opposite to flow upstream the throughput manipulator
5 At junctions or splits a decision has to be made on which flow to use for inventory control. Typically, the largest flow is used, or both streams are changed such that their ratio is held constant (the ratio itself is often set by a slower outer composition loop).
6 Recycles require special consideration. Make a block (control volume) around the entire section and make sure that there is local consistency for total mass, (individual) components, and phases (if relevant).
7 Assign control loops for any process external flow that remains uncontrolled. Typically, "extra" feed rates are put on ratio control with the ratio set point being set by an outer composition loop.

[a] In case of doubt, consult the general local-consistency rule (Rule 1).

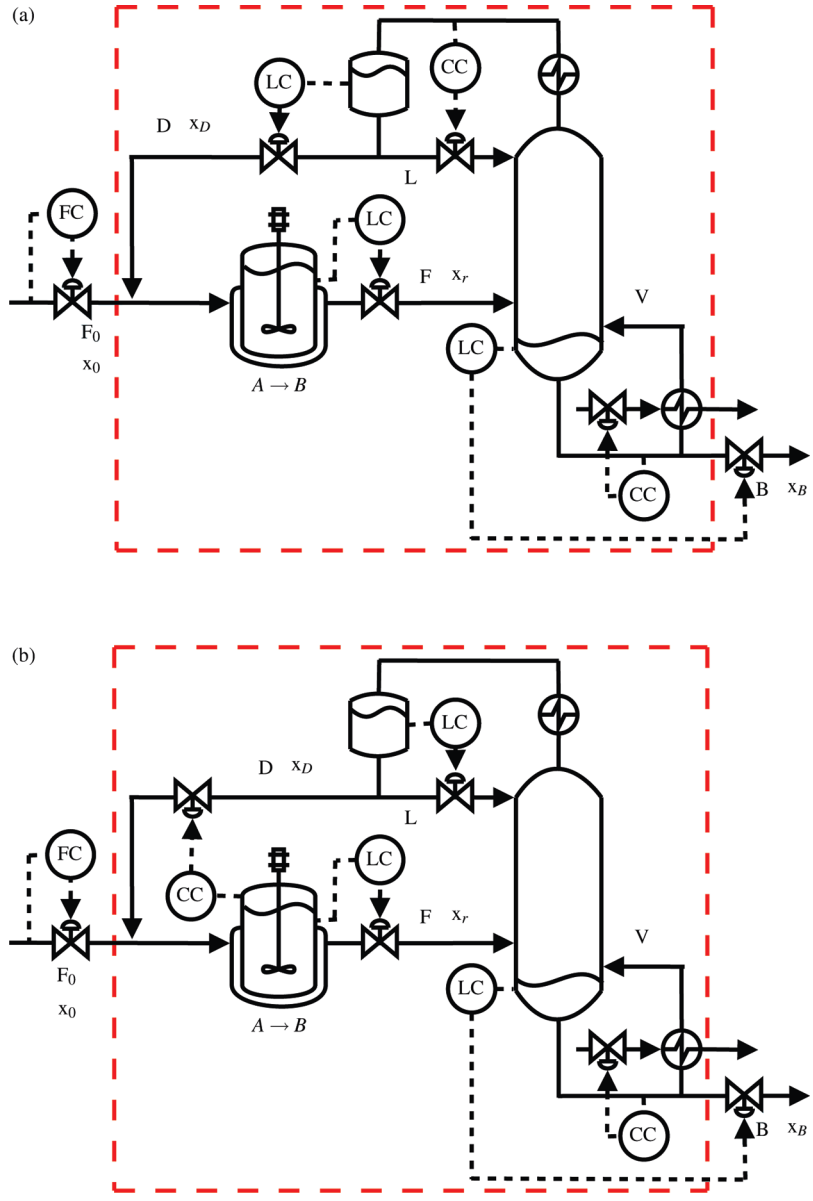

Figure 10. Reactor−separator−recycle process with one reactant (A). (a) Conventional: local-consistent inventory control structure. (b) Composition control of reactor composition: not consistent for component A.

Proof. Make a control volume with the two flow-controlled streams as in- and outflows. Then neither the inflow nor the outflow depends on the control volume and the inventory is not regulated. This is demonstrated in Figure 1b.

4. *If a change in the throughput manipulator does not result in a change in the main feed flow, then the control structure is inconsistent* (Price and Georgakis,[1] p. 2699).

Justification. This follows from the total steady-state mass balances.

5. *A local-consistent inventory control structure must use the feed or the product (or both) for inventory control* (generalized from Price and Georgakis,[1] p. 2699).

Justification. This follows from the total steady-state mass balance. This is also discussed in the section Units in Series ("Radiating Rule"), and a clear illustration of this statement is found in Figure 5.

6. *For closed systems: One inventory must be left uncontrolled and one flow in the closed system must be used to set the load.*

Justification. This follows from that all systems must have at least one given flow to be unique. To be able to set the load for a closed system, one inventory must be uncontrolled.

The rules are summarized by the proposed procedure for inventory control system design in Table 1, which is inspired by the inventory control guidelines of Price et al.[2]

Remark. Luyben[11] provides the rule to "fix a flow in each recycle". If we interpret the term "fix a flow" to mean "do not use a flow for inventory control", then this rule follows from the requirement of local consistency provided the recycle loop contains a split that introduces an extra degree of freedom. However, if no degree of freedom is introduced by the recycle, as is in the case if we have a separator or flash where the split is (indirectly) fixed by the feed properties (e.g., see Figure 4b), then this rule is *not* a requirement, (e.g., see the self-consistent control structure in Figure 10a, where all the flows in the recycle loop are on inventory control).

Examples

In this section we apply the local-consistency rule to some more difficult examples from the academic literature where local consistency of component inventory is an issue.

Example 5: Distillation Column with *DB* Configuration. An example of a recycle system is a distillation column. As seen from Figure 9, a distillation column has one split in the condenser (V_T splits into L and D) and one split in the reboiler (L_B splits into B and V). In both cases one of the streams is recycled to the column (L and V, respectively). The two splits introduce two degrees of freedom and this gives rise to many possible inventory control structures ("configurations"), as has been discussed widely in the literature (see Skogestad[12] for a summary of this discussion).

Figure 9 displays the unusual *DB* configuration, which uses reflux L and boilup V for inventory control (condenser and reboiler level control), such that the flows of D and B remain as degrees of freedom for other purposes. The *DB* configuration has earlier been labeled "impossible", "unacceptable", or "infeasible" by distillation experts.[13,14] This inventory control system also violates Luyben's rule of "fixing a flow in the recycle loop", and it is indeed true that this inventory control system is not local consistent. To see this, consider the dashed box in Figure 9, where we note that none of the flows in or out of the column (F, D, and B) depend on the inventory inside the column. However, an inconsistent inventory control system can usually be made consistent by adding control loops outside the local units (which here are the tanks with M_D and M_B) and the *DB* configuration is workable (and consistent) provided one closes at least one extra loop, for example by using D to control a temperature inside the column.[15,16] Thus, labeling the *DB* configuration as "impossible" is not appropriate. In summary, the *DB* configuration is not local consistent, but it can be made consistent by adding a temperature (or composition) control loop.

Remark 1: An example of a local-consistent inventory control structure for distillation is the common *LV* configuration, where the two level loops have been interchanged such that D and B are used for level control and L and V remain as degrees of freedom (e.g., on flow control). In the *LV* configuration, inventory is controlled in the direction of flow, as expected since the feed is given.

Remark 2: An additional inventory issue for distillation columns is related to the split between light and heavy components (component inventory). One may regard the column as a "tank" with light component in the upper part and heavy component in the lower part. Thus, one is not really free to set the split between D and B, and to avoid a "drifting" composition profile (with possible "breakthrough" of light component in the bottom or of heavy component in the top), one must in practice close a quality (e.g., temperature) loop to achieve component local consistency.[12] For example, for the *LV* configuration one may use the boilup V to control a temperature inside the column. This consideration about controlling the column profile also applies to the *DB* configuration. Thus, in practice, the *DB* configuration requires closing *two* quality loops to maintain mass and component balances, otherwise the split D/F (or B/D) will be fixed and there is no adjustment to changes in feed composition. This means that both D and B must be used for quality control for the *DB* configuration, rather than only one (L or V) for the *LV* configuration.

Example 6: Reactor−Separator−Recycle Example with One Reactant. A common recycle example from the academic literature is the reactor−separator−recycle system in Figure 10. The system has a continuously stirred-tank reactor (CSTR) with an irreversible, isothermal, first-order reaction A → B, followed by separation (distillation) and recycle of the unreacted feed component back to the reactor (e.g., refs 1 and 17−19). Note that in this case the recycle does not introduce an extra degree of freedom for control of flow at the network level because the split in the distillation column is indirectly determined by the column feed composition.

The feed (F_0) is pure reactant A and the steady-state component mass balances give

$$\text{Component A:}\quad F_0 = \underbrace{k(T)\cdot x_{r,A}\cdot V}_{-G_A=G_B} + B\cdot x_{B,A}$$

$$\text{Component B:}\quad \underbrace{k(T)\cdot x_{r,A}\cdot V}_{G_B} = B\cdot x_{B,B}$$

where x is the mole fraction, V is the reactor volume, and $k(T)$ is the reaction rate constant. Note that $B = F_0$ [mol/s] at steady state. Component A enters the process in the feed stream and its conversion (consumption) in the reactor increases with the amount of A. The inventory of component A is therefore expected to be self-regulated by the reaction. Component B is produced in the reactor (G_B) and exits the process in stream B. Component B is not self-regulated by the reaction (because the reaction rate is independent of the amount of B) and thus requires a controller to adjust its inventory.

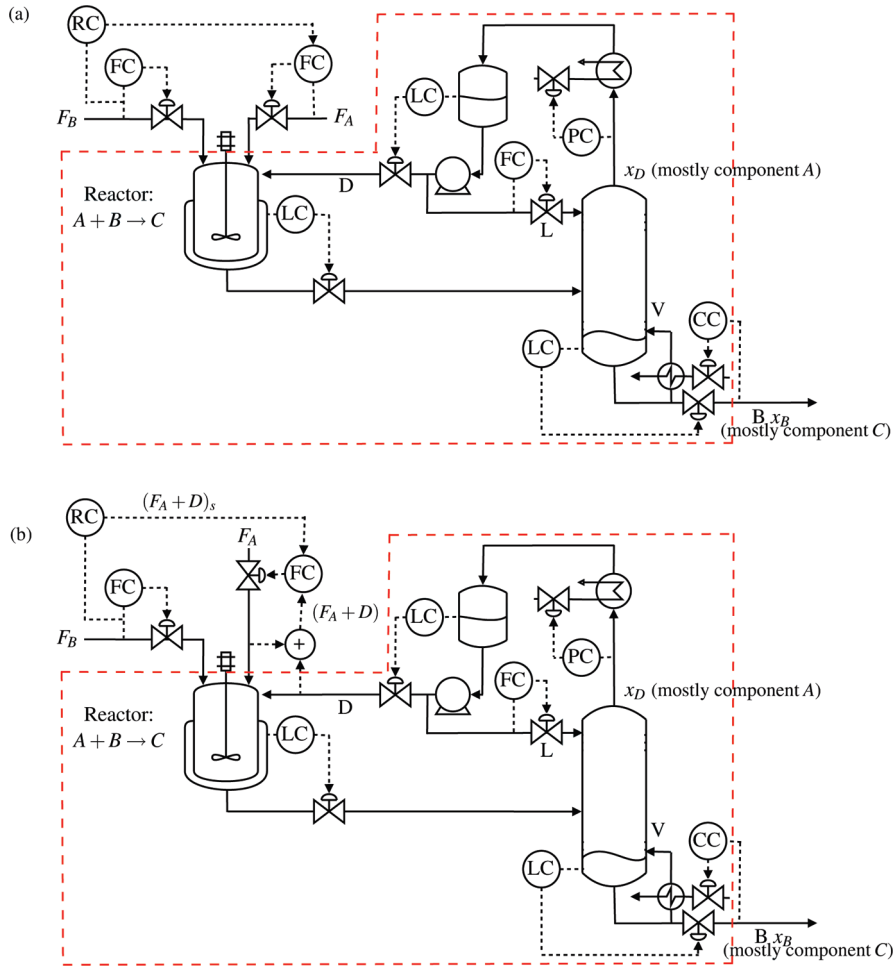

Figure 11. Reactor−recycle system with two reactants (A + B). (a) Inconsistent structure with both reactant flows given. (b) Local-consistent structure where feed of reactant A depends on inventory of A (as reflected by the recycle D).

Two different control structures for the reactor−separator−recycle process are displayed in Figure 10. Both have given feed (F_0) and inventory control is in the direction of flow. Thus, both of them are local consistent in total mass, because the outflow B from the process depends on the inventory inside the process (indicated by the dashed control volume) (Rule 1). Since the outflow B mainly consists of component B, this implies that both structures are also consistent with respect to the inventory of component B. The difference between the two structures is related to the control of component A. The "conventional" structure in Figure 10a uses the LV configuration for the distillation column where the reflux (L) controls the composition in the recycle (distillate) D. The structure in Figure 10b uses the DV configuration for the column where the reactor composition $x_{r,A}$ is controlled instead of the recycle (distillate) composition.

For the "conventional" structure in Figure 10a, the inventory of component A is expected to be self-regulated by the reaction A → B. More precisely, the amount that reacts is $-G_A = kx_{r,A}V$ and the composition $x_{r,A}$ will "self-regulate" such that at steady state (assuming $x_{B,A} \approx 0$) $F_0 \approx -G_A$; that is, $x_{r,A} \approx F_0/(kV)$. However, the structure with control of reactor composition in Figure 10b is not consistent because control of reactor composition eliminates the self-regulation: The amount of A that reacts is given by $-G_A = G_B = k(T)x_{r,A}V$ and with given T, V and $x_{r,A}$ there is no self-regulation. The inconsistency of this control structure has also been noticed previously.[5,20]

Remark 1: The control structures in Figure 10 would both be local consistent without composition control (CC), that is, with (a) L given or (b) D given. The reason for closing these composition loops is therefore not to achieve consistent inventory control but rather for other (economic) reasons.[19] The interesting point to note is that closing an extra loop can in some cases make the system inconsistent (Figure 10b).

Remark 2: Luyben[20] has proposed to make the system in Figure 10b consistent by letting the reactor volume float (adjustable reactor volume), but this is not generally a good solution, because we want to use maximum reactor volume for economic reasons (including energy saving).[19]

Remark 3: We already noted that setting $x_{r,A}$ (Figure 10b) breaks the self-regulation and makes the system inconsistent. A related problem, which applies to Figure 10a, is when the reactor volume V is too small relative to the feed F_0, such that the required $x_{r,A}$ exceeds 1, which is impossible. In practice, if we increase the feed rate F_0 and approach this situation, we will experience "snowballing",[11] where the recycle D becomes very large, and also the boilup V becomes very large. Eventually, V may reach its maximum value, and we lose composition control and we will get "breakthrough" of A in the bottom product. Snowballing is therefore the result of too small a reactor.

Remark 4: Consider the same process (Figure 10), but assume that the fresh feed (F_0) contains an inert component I in addition to the reactant A. If I is more volatile than component B, then component I will be recycled back to the reactor and will accumulate in the process. None of the inventory control systems in Figure 10 are consistent for the inert I. To make the

system local consistent for the inert, a purge stream must be introduced where part of stream D is taken out as a byproduct.

Example 7: Reactor−Separator−Recycle Process with Two Reactants. Another well-studied recycle example is a reactor−separator−recycle process where two reactants A and B react according to the reaction A + B → C (e.g., ref 21). Component B is the limiting reactant as the recycle D contains mostly component A. Two different control structures are displayed in Figure 11. In both cases the distillate flow D (recycle of A) is used to control the condenser level (main inventory of A).

In Figure 11a, both fresh reactant feeds (F_A and F_B) are flow controlled into the reactor, where reactant A is set in ratio to reactant B such that $F_A/F_B = 1$. This control strategy is not consistent because none of the two feeds depend on the inventory inside. This follows because it is not possible to feed exactly the stoichiometric ratio of the two reactants[10] and any imbalance will over time lead to a situation where the recycle of A either goes toward zero or goes toward infinity.

To get a consistent inventory control structure, the first requirement is that one of the feed rates (F_A or F_B) must depend on what happens inside the process, such that at steady state we can achieve $F_A = F_B$. One solution is to set F_B (the limiting reactant) and adjust F_A such that the desired excess of A is achieved, resulting in the local-consistent control structure in Figure 11b. Here F_A depends on the inventory of A as reflected by the recycle flow D by keeping the reactor feed ratio $(F_A + D)/F_B$ constant at a given value (larger than 1 to make B the limiting reactant). The structure is consistent for all components: C has an outlet in the bottom of the column, B is self-regulated by reaction because it is the limiting reactant, and the feed of A depends on the inventory of A.

There exist also other consistent inventory control structures.[10] For example, one strategy is to keep the recycle D constant and use F_A to control the condenser level (main inventory of A), but this structure is not local consistent and the dynamics for this "long" level loop are not favorable.

Conclusion

Consistency is a required property since all inventories must be regulated (kept within bounds). An additional desired property is to have local consistency where all inventories are regulated using only local loops. Local consistency of a given control system can be checked by using the proposed local-consistency rule (Rule 1). The local-consistency rule follows from mass balances for total mass, component, and individual phases, and its use for control design is summarized in Table 1.

The local-consistency rule may be regarded as "obvious", but has nevertheless proven to be very useful in many applications, and it agrees with previously proposed rules. For example, a direct consequence of the local-consistency rule is the "radiation rule",[1] which states that the inventory control structure must be radiating around the location of a given flow.

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