→ Morten Hovd

2311-17
S. SUGGESTED

Exercise controllability for simple unstable plant

Problem

Derive bounds on S (M_S) for the following four plants. Which one is most difficult to control? Discuss your results.

$$G_a = \frac{s-2}{(s+2)(s-4)}$$

$$G_b = \frac{s-2}{(s+2)(s-1)}$$

$$G_c = \frac{e^{-s}}{(s-4)} \qquad (\approx G_a \text{ if we use Padé})$$

$$G_d = \frac{e^{-s}}{(s-1)} \qquad (\approx G_b \text{ if we use Padé})$$

Solution

Bound on $|T|$: $M_T \geq \frac{z+p}{z-p} e^{p\theta}$ (5.18) ← gives indirect bound on M_S; $M_S \geq M_T - 1$

Bound on $|S|$: $M_S \geq \frac{z+p}{z-p}$ (5.15) ←

z: RHP-zero

p: RHP-pole

	$M_T \geq$	$M_S \geq$	
a	$\frac{2+4}{4-2} = 3$	3	} seem to be equally difficult to control, but if we want tight control at low frequency (integral action) then G_a is more difficult than G_b.
b	$\frac{4+2}{\lvert 2-1\rvert} = 3$	3	
c	$e^4 = 54.6$	53.6	
d	$e^1 = 2.71$	1.71	

TIGHT!

Comment:

G_c is clearly the most difficult to control according to these bounds. However, note that according to the Padé-approximation $e^{-s} \approx \frac{(s-2)}{(s+2)}$ so G_a should be similar. The Padé-approximation fails to show that we can only have tight control at low frequencies ($\omega < 1/\theta$).