**Exam 2013. Solution**

**Solution Problem 1**

(5%) Rule 1. Economic objective is generally

Min\_u J(u,d) subject to constraints

At the optimum, some constraints are usually active while the remaining degrees of freedom (u) are unconstrained. In terms of operation, and looking at small deviations from the optimal point, the loss is linear in the active constraints, but only quadratic in the unconstrained degrees of freedom.

Loss = lambda\*|c-cactive| + (c-copt)’ Jcc (c-copt) + …

Here lambda is the Lagrange multiplier. This means that the loss is generally largest for deviations from the active constraints, so these should be selected as CVs.

(5%) Rule 2. There are two reasons: 1) Avoid product give-way (“selg gråstein som gull”). 2) Overpurification generally increases the cost (typically requires more energy).

(5%) Rule 3. For example, never try to control directly the cost J. Assume we want to minimize J (e.g., J = energy) - and we make the stupid choice of selecting CV = J - Then setting J < Jmin: Gives infeasible operation (cannot meet constraints) - and setting J > Jmin: Forces us to be nonoptimal (which may require strange operation; see Exercise 3 on evaporators)

An example is the heat exchanger split in Problem 3. Trying to control T (final temperature), which should be maximized, will not work! If you set it too high, then there is no feasible split. If you set it too low, then you do not which side it is on, so you do not know the sign of then gain and how to change the split.

**Solution Problem 2**

1. 5% I. Top Down

* Step S1: Define operational objective (cost) and constraints
* Step S2: Identify degrees of freedom and optimize operation for disturbances
* Step S3: Implementation of optimal operation (select economic CVs)
  + Control active constraints
  + Remaining unconstrained: control self-optimizing variables
* Step S4: Where set the production rate (TPM)? (Inventory control)
  + Locate at bottleneck (economics)
  + Avoid using variable that may optimally saturate

1. 8% S1. Objective is minimize the pumping cost (J=W), which is achieved with the valve being as open as possible, that is, can instead use J=-z.

There are three constraints mentioned in the problem formulation:

Constraint 1: F=Fs (equality, so always active),

Constraint 2: Pressure before booster pump, p > pmin.

Constraint 3: Valve, z<zmax=1 (valve fully open)

+ other constraints: valve closed (zmin=0), max. pump speed.

S2. Degrees of freedom are main pump speed (u1) and valve opening (u2).

BUT there is only one remaining DOF with the equality constraint F=Fs.

The main disturbance is usually the throughput, which in this case is F=Fs.

Optimal operation: Active constraint will depend on flowrate (F).

Low F: Booster pump will give a large pressure increase and p may drop below pmin. To avoid this we need to partially close the valve.

High F: Booster pump will give a smaller pressure increase so p>pmin. It is then optimal to keep the valve fully open.

Conclusion optimal operation

Low F:

P = pmin (active constaaint)

z <zmax (partially closed valve to increase pressure after booster pump)

High F:

P > pmin

Z = zmax (active constraint, fully open valve)

S3. Choice of CVs. Should make sure that we control the active constraints

S4. TPM = MV used for control of throughput (F). Because the valve may optimally saturate (fully open), the valve should not be used as TPM. In addition, the TPM should be located at the bottleneck which is the max. speed for the main pump. Conclusion: Use main pump (TPM) to control flow.

1. 7% . The proposed control structure is shown below. The main pump is the TPM and controls the flow (at all flowrates). The setpoint for the pressure controller is set at pmin. This will “automatically” (without any need for logic) give the most open valve and thereby optimal economic operation:

Low flowrate: p=pmin, valve partially open

High flowrate: Valve is fully open (we “lose control” of p, but this is not a problem since we will get p>pmin).

Note: Make sure to avoid integral windup for the PC



Comment 1: It is maybe not so obvious that the solution works, at least if we consider going from low flowrates (where we control p=pmin) to high flowrates (where we get a fully open valve; why is there not a problem of “losing control” of p?)..… It is much more obvious if we go from high to low flowrates: At high flowrate p>pmin and we can keep the valve fully open (which is the optimal). If the flowrate drops, then p drops, and if p goes below pmin, then we close the valve, that is, we control p=pmin using the valve….. so this is obvious… and going back is just the reverse.

(One way of thinking about the reverse, is that we close to valve to increase the pressure, so when it reaches fully open it means that it is no longer needed to increase pressure - so we are “losing control” in the direction we are allowed to lose control).

Comment 2: The fact that it must work is related to the rule “pair an MV that may (optimally) saturate with a CV that may be given up”, because when a MV is optimally saturated, then we lose a degree of freedom, and there must be a CV which should no longer be (optimally) controlled. In our case, when the MV saturates (valve fully open) we should no longer control p.

Comment 3: The alternative solution is the reverse pairing. It will work at low flowrates where p=pmin is the active constraint. However, at higher flowrates, the valve will become fully open and we lose control of the flowrate (and can no longer increase it), because we keep controlling p=pmin, which is no longer an active constraint. In conclusion, the reverse pairing is not acceptable, because it is not able to give large production rates.

**Problem 3. Solution**

1. (4%) From the energy balance we derive an expression for T, which we differentiate to obtain Ju = dT/dalpha. Further simplification gave the final form given above.
2. (4%) Because we always have Ju=0 at the optimal operating point, independent of disturbances. It is therefore the ideal self-optimizing variable, if we can obtain it. We also do not need to find the optimal setpoint as we normally have to do for a self-optimizing variable (like optimal temperature setpoint). (However, note that measurement error/noise is not considered.)
3. (4%) Compute c=Ju = JT1 – JT2 based on the 5 temperature measurements and adjust alpha using a simple DT (Jäschke temperature difference) controller.
4. (4% ) Because it corresponds to controlling c=J, which may lead to infeasibility or the best non-optimal operation; see problem 1, rule 3.
5. (5%) The idea behind the nullspace method is that copt should be independent of d, that is, dcopt/dd=0. We let c=Hy (combination of measurements), and then, dcopt=H dyopt = HF d where F = dyopt/dd is the optimal sensitivity. The nullspace method is to select HF=0 so that dcopt=0 for any d.
6. (5%) In our case, there are 6 disturbances and 5 measurements so F is a 6x5 matrix. We could obtain F from the model, by reoptimizing wrt. alpha to find yopt for each of the 6 disturbances; F = dyopt/dd.

To be able to use the nullspace method (to find H such that HF=0), we need at least nu+nd=1+6=7 measurements, but we have only 5 it seems the nullspace method cannot be used. (This is accepted as a correct solution. It assumes that the 7 input + disturbances are independent, but from the analytical expression for Ju there must be some dependency, so in practice it would work after all, that is, we can make HF=0 even with only 5 measurements).

1. (4%) The “exact local method” minimizes the loss J-Jopt for the set of expected disturbances, and it can be applied with any number of measurements (as long as ny >= nu), so it would work for the example. It also has the advantage of including measurement noise, which none of the above methods do.

Problem 4. Control of flow and pressure.

TPM is at feed valve.

1. and (c) don’t work because pressure control is opposite direction of flow (do not follow radiation rule)
2. and (d) are OK (follow radiation rule)

Comment: get 2 points for correct answer and -2 for wrong (unless there is some good reason for the wrong answer); but cannot end up with negative points…

**Problem 5. Control of offshore process. Solution:**

1. (4%) No, it is not “consistent”, so it is not workable. For example, if we add some gas in tank 1 so that p1 increases, then pG increases and compressor 1 will reduce its power, which is the opposite of what it should do.

Also note that the pressure loop for pG is against the direction of flow (“radiating rule”).

It is NOT possible to get a workable control system with the given objectives, so your boss has given you an impossible job! The reason is that when we set the delivery pressure pG then we are indirectly setting the gasflow out (the gas product goes to a pipeline where the pressure p0 at the end of the pipeline is given, and with a fixed pG and p0 the gasflow will be given), so the setpoint for pG is really a TPM. This cannot be combined with a given production flow (two TPMs on the same flow is not possible).

A simple “fix” to make it workable is to let W1 control p1, but we are then not controlling pG as we were told to do.

1. (4%) Here we are no longer told to control the total flow. We can then replace the FC for the inlet valve by a PC that controls p1. This will be consistent.
2. (4%) TPM = throughput manipulator = degree of freedom that sets throughput in the process.

As explained, in the first case, we actually have two TPMs (total feedrate and setpoint for pG), which is not possible.

In the last case the TPM is the setpoint for pG (by increasing pGs we will increase the throughput)

**Problem 5. Solution**

1. (7%) “By squeezing the variance you may shift the setpoint closer to the active constraint”.

The basis is when you have active hard constraints which require backoff to get feasibility. The magnitude backoff is given by the sum of the control error and the measurement error. The control error can be reduced by improving control (MPC), but to get the economic benefit one also need to shift the setpoint (using RTO or simply a good operator).

1. (8%) For this case, PID is recommended because we have tau2=6.5 which is larger than theta=1.

PID-controller (series form) with tauc=theta:

Kc = 1/k’(tauc+theta) = 1/3\*(1+1) = 1/6 = 0.17

taui = 4(tauc+theta)=8

taud = 6.5