**Solution Problem 1 (Dialysis)**

**a) Max 5 p.**

Figure 13.2-1, a is the correct answer. It is crucial that the dotted line through the membrane is lying below the $c_{1i}$ and $c_{2i}$ points because K'=0.75 is given. Likewise, it is important to indicate that $c_1$ and $c_2$ are the bulk concentrations, and not the concentrations at the surface.

**b) Max 8 p.**
**c) Max 8 p.**

(b) Max 8p.

$$N_A = \frac{C_1 - C_2}{\frac{1}{kc_1} + \frac{1}{P_M} + \frac{1}{kc_2}} \qquad P_M = \frac{P_{AB} K'}{L}$$

$$\frac{1}{P_M} = \frac{L}{D_{AB} \cdot K'}$$

$\Rightarrow$ side 1; $\frac{1}{k_{c_1}} = \frac{1}{3.5 \cdot 10^{-5}} = 2.9 \cdot 10^4 \frac{s}{m}$

$\frac{1}{k_{c_2}} = \frac{1}{2.1 \cdot 10^{-5}} = 4.8 \cdot 10^4 \frac{s}{m}$

$\frac{1}{P_M} = = 0.61 \cdot 10^{11} \cdot 10^{-5} = 6.1 \cdot 10^5 \frac{s}{m}$

$\Rightarrow$ Total resistance $= (0.29 + 0.48 + 6.1) \cdot 10^5$

$= 6.87 \cdot 10^5 \, s/m$

Membrane resistance : $\frac{6.1}{6.87} \cdot 100 = 88.7\%$

© Flux at steady state:

Max 8p

$N_A = \frac{(2 - 0.3) \cdot 10^{-2}}{6.87 \cdot 10^5} = 0.247 \cdot 10^{-7} \frac{kmol \, A}{s \cdot m^2}$

$= 2.47 \cdot 10^{-8} \frac{kmol \, A}{s \cdot m^2}$

area for transfer 0.01 kmol A/h

$\Rightarrow 1 / \frac{2.47 \cdot 10^{-8} \cdot 3600}{0.01} = \frac{0.01}{2.47 \cdot 10^{-8} \cdot 3.6}$
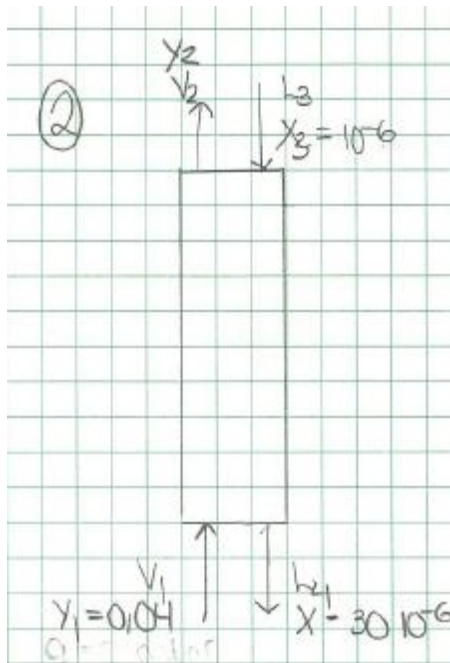
$= 1.1246 \cdot 10^{-3} \cdot 10^5 = 1.12 \cdot 10^2 \, m^2$

eller $112 \, m^2$

**d) Max 4 p.**

The flow rate is doubled on both sides – however, the main resistance is through the membrane as documented in b), hence the flux will increase slightly, but not significantly

**Solution Problem 2**



(i)  We assume ideal gas. Then volume flow at the inlet is

V = nRT/p = 10^4*8.31*283/e5 = 235 m3/s

V = v*A, where it is given that v=1m/s, so

A = V/v = 233.5 m3/s / 1 m/s = 235 m2.

and D = √(4A/π) = 17.2 m

Comment: Some students commented that this is not realistic, but actually it is. This is the size of the absorption column required for a large power plant. However, the liquid flow is unrealistic as we will see next.

(ii) We set up balances for the absorption column.

Overall mass balance [mol/s]:

(1)  V1 + L3 = V2 + L4

 Balance CO2 [mol CO2/s],

(2)  V1*y1 + L3*x3 = V2*y2 + L4*x4

Balance water (assume water is "inert" with respect to phase transfer, i.e., we neglect evaporation of water)

(3)  L3*(1-x3) = L4*(1-x4)          = L' (flow of "inert" water)

Comment: We can neglect vaporization of water because of the low temperature of 10C.

 Balance inerts in gas ("inerts" with respect to phase transfer, i.e., neglect condensation of these)

(4)  V1*(1-y1) = V2*(1-y2)          = V'  (flow of "inert" gases)

Comment: (2), (3) and (4) can be combined to get
            V'*y1/(1-y1) + L'*x3/(1-x3) = V'*y2/(1-y2) + L'*x4/(1-x4)
but this equation is not used here.

Given data:

V1 =10000 mol/s,

y1 = 0.04,

x3=1.e-6

x4=30.e-6

$V_2*y_2 = 0.1*V_1*y_1 = 0.1*10000*0.04 = 40$ mol/s ("10% of incoming")

From this we can find all flows and compositions.

The liquid flow L' can be found from the CO2-balance (2):

$400$ mol/s $+ L'*x_3/(1-x_3) = 40$ mol/s $+ L'*x_4/(1-x_4)$

And with the given data for x3 and x4 we find

L' = 12.41 e6 mol/s (water flow)

Comment: You do not need to set up all the equations. A simple solution is the following: The amount of CO2 transferred to the liquid is 0.9*V1*y1 =360 mol/s. Because of the small values for x3 and x4, we can safely assume L'=L3=L4. The mass balance for the liquid is then L(x3-x4)=360 mol/s, and we derive L = 360/29e-6 = 12.41 e6 mol/s.

This gives the water flow: L = 12.41 e6 mol/s *18e-3 kg/mol = 224.28 e3 kg/s or L = 224.28 m3/s. (These are VERY large flows).

Heat duty in cross heat exchanger: Q = m4 cp (T5-T4) = 224.28 e3 kg/s * 4.18 kJ/kg,K * 80 K = 75000000 kJ/s = 75000 MW

Comment: This is unrealistic in practice. The power plant only produces 170 MW of electricity, and this heat exchanger has a duty which is 441 times larger !!!

(iii) Number of equilibrium stages in absorber.

**Equilibrium line:** From Henry's law we have for CO2: $P_{co2} = y*p = Hx$ where p is the total pressure (1 bar). We get in the absorber where the temperature is 10C:

y = m x,  where m =H/p = 1000 bar/1bar = 1000

**Operating line:** The operating line goes through the points (x4,y1)=(30e-6,0.04) (btm) and (x3,y2)= (1-e-6,0.00417) (top; exact).
The "exact" value y2=0.00417 follows from the material balance on the gas side:

$V_2*y_2 = 40$ mol/s, where V2 = V'/(1-y2)  and V'=V1(1-y1)=10e3*0.96 = 9600 mol/s.

*Comment: If we are a bit less accurate, and assume V=10  kmol/s is constant through the column, then we find y2=0.004 (approx.); this is also accepted as a correct answer and will actually be used in the following)*

See the McCabe-Thiele diagram for the solution.
We assume that L/V is constant through the column because this gives straight operating line, and we assume that the operating line goes through (x3,y2)= (1-e-6,0.004) (top; approx).
We find that we need about 5.2 stages.
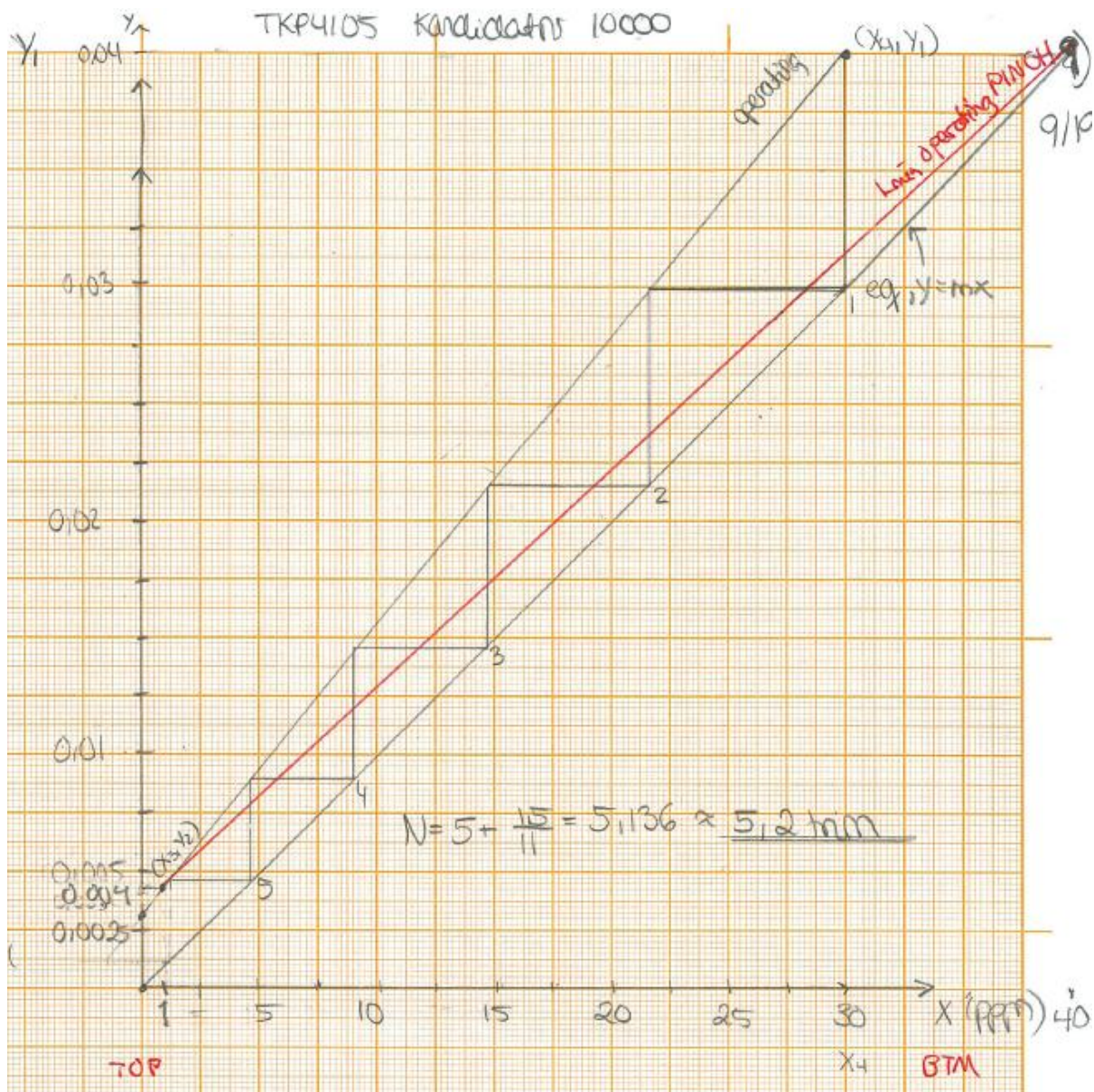
Comment 1: As mentioned, the "exact" McCabe-Thiele diagram should start from (x3,y2)=(1.e-6,0.00417) instead of (1.e-6,0.004). In addition, the operating line will curve a little upwards. Both these effects give a slight reduction in the "exact" number of stages; see also Example 10.6-2 in the book for how to make curved operating lines.

Comment 2: Let us check the answer with the Kremser equations, which should give the same result.
Define a =(y0-xN)/(x0-yN+1/m) = (1.e-6 – 30e-6)/(1-e-6-0.04/1000)=0.743
A = (L/V)/m = 1241 / 1000 = 1.241 (note: I am here assuming that V=1.e4 mol/s in the whole column)
N = ln(1-a)/(1-aA) / lnA = ln(1-0.743)/(1-0.922) / ln 1.241 = 5.52  (which is OK!)

(iv) Packing height.

End 1 (top): $y_1 = 0.004$, $y_1^*=mx_1 = 0.001$. End 2 (btm): $y_2=0.04$, $y_2^* = mx_2 = 0.03$.

So we get: $(y-y^*)_{LM} = 0.00581$.

Amount of Co2 transferred from vapor to liquid: $N_A = 0.9*V_1*y_1 = 360$ mol/s.,

Other data: S = 235 m2, Kya = 66 mol/s,m3

Get: $z = N_A / K_y \, a \, S \, (y-y^*)_{LM}$ = 360/(66*235*0.00581) = 3.99 m

Comment 1: The height is small compared to the diameter, so it may be difficult to get a good liquid and vapor distribution in this column.

Comment 2: Note that HoG = V / Kya S = 1.e4/66*233.5 = 0.65 m, NOG = (y1-y2)/(y-y*)LM = 6.2, and we see that NOG is similar to the number of equilibrium stages found above.

(v) Minimum liquid

Min. liquid is with infinite number of stages. With infinite number of stages we have equilibrium at bottom of the column $x_N = x_N^*$,

Here $x_N^*$ = yN+1*/m = 0.04/1000 = 40e-6

So with infinite number of stages we get a higher concentration of CO2 (40 ppm rather than 30 ppm) and need less water to remove the required amount of C02 (which is 360 mol/s). We assume that the incoming water still contains 1 ppm of CO2.

From a mass balance for CO2 on the liquid side (here N is stream 4 and 0 is stream 3):

Lmin (xN – x0) = Lmin (40e-6 – 1.e-6) = 360 mol/s ->  Lmin = 360/39e-6 = 9.23 e6 mol/s

(whereas the actual amount of water is 12.41 e6 mol/s). Corresponds to Lmin = 166 m3/s.

(b**) Flash at top of stripper**.
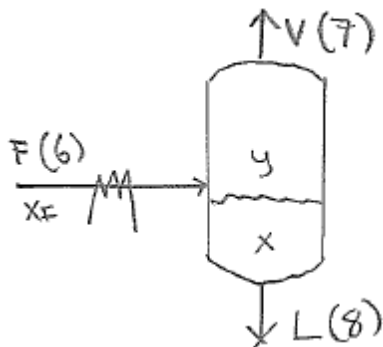


(let F=stream 6, L=stream 8, V=stream 7

Total balance:  F= V + L  (eq.1)

CO2 balance:  xF*F = y*V + x*L  (eq.2)

Water balance: xFw = yw*V + xw*L

(Water balance is actually the same as total balance since y+yw=1, etc.)

Given:  xFw = pw/ptot=1 bar/1.1 bar = 0.90909 (partial pressure of water is 1 bar)

So $xF = 1-xFw = 0.0909$ (eq.3)

$y*V = 360$ mol/s (amount of CO2 in stream 7) (eq.4)

We assume equilibrium at 10C between streams 7 (V)and 8 (L).

Water equilibrium (Raoult's law): $yw*p = xw*psatw$ or

$(1-y)p = (1-x)*psatw$ (eq. 5) (where p=1.1 bar = total pressure and psatw=0.012bar)

CO2 equilibrium (Henry's law): $y*p = H*x$ (eq.6) (where p=1.1 bar and H = 1000 bar)

We have 6 unknowns (F,L,V,z,x,y) and 6 equations so this is solvable!

The last two equilibrium equations contain only y and x:

$y = x*1000/1.1$

$(1-y) = (1-x)*0.012/1.1$

And solution gives: $x = 0.00104$, $y=0.98908$

From the given amount of CO2 in the product, eq. 4, we then get

$V = 360/y = 364.0$ mol/s

The CO2 balance, eq. 2, then gives, when inserting the total balance:

$F*xF = Vy + Lx$

$F*0.09091 = 360 + (F-360)*0.00104$ -> F = 4002 mol/s


Summary

| Stream | Total [mol/s] | C02 (%) | Water(%) |
|---|---|---|---|
| F (6) | 4002 | 9.09% | 90.9% |
| V (7) | 364 | 98.9% | 1.09% |
| L (8) | 3638 | 0.104% | 99.9% |


**Cooling duty condenser:**

Condense to get L (condense at 100C and cool liquid to 10C, assume pure water for simplicity):
$L*(dhvap + cp*dT) = 3638*0.909(40700 + 4.18*18*(100-10)) = 3638*47500 = 171.5$ MW

Cool to get gas V (assume pure CO2): $V*cp*dT = 364*37.3*90 = 1.22$ MW

Sum duty: 172.7 MW

**Solution Problem 3.**

    (a) CVs: h1, h2, p1, p2, pG (given in text)

MVs: 3 liquid flows (valves, zF,z1,z2) + 2 compressors (compressor speed or power, W1 and W2)

DV: Main disturbances are probably pF + feed composition

(b) To find the control structure (pairing) one may write the "process matrix" but in this case the best pairings are fairly obvious:
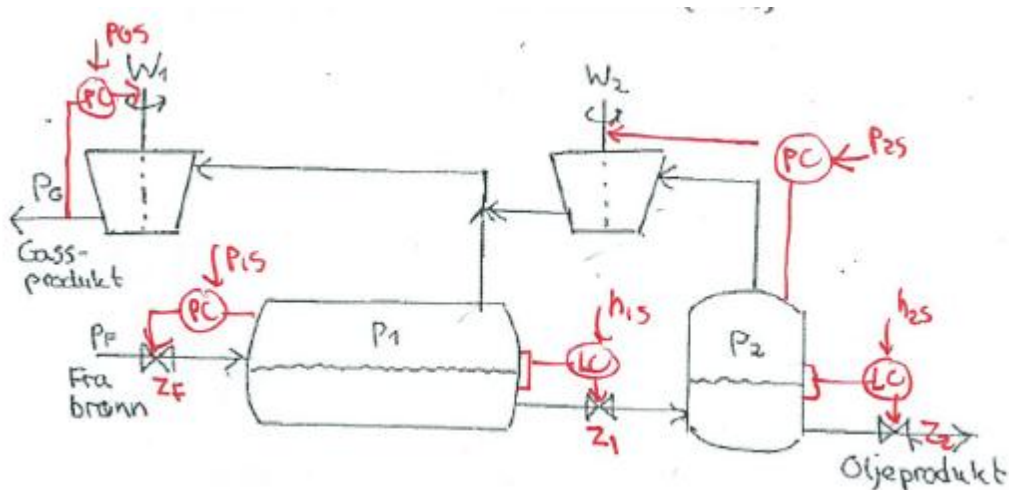


Comment: Where is the throughput set in this process? It is not so obvious, but the throughput for this process can be changed by changing the setpoint for pG. A higher pG makes more gas leave the process, and to keep the pressure p1 up, we need to open the "production valve" (zF). It may be that the operators do not like this indirect way of setting the throughput. If we the operators manually set the production valve (zF), which is common in practice, then we must use compressor 1 (W1) to control p1, so we must give up keeping a fixed pressure pG.

    c) The main purpose of the integral action is to remove steady-state offset. Tau can be found experimentally by making a step change in the input (u). tau is then the time (after the initial delay) for the output (y) to reach 63% of its steady-state change

**Solution Problem 4**

    **a) Max 5 p**

The figure 9.5-1 b) is the expected curve to be drawn. The candidate should also explain the dominating drying process taking place from start (A) to B, from B – C, from C – D and from D – E

    **b) Max 5 p.**

See the attached Figure. Humidity of incoming gas is H = 0,026 kg water / kg dry air, and temperature of air at outlet with 90% humidity will be 37°C

    **c) Max 7 p.**      (see attached)
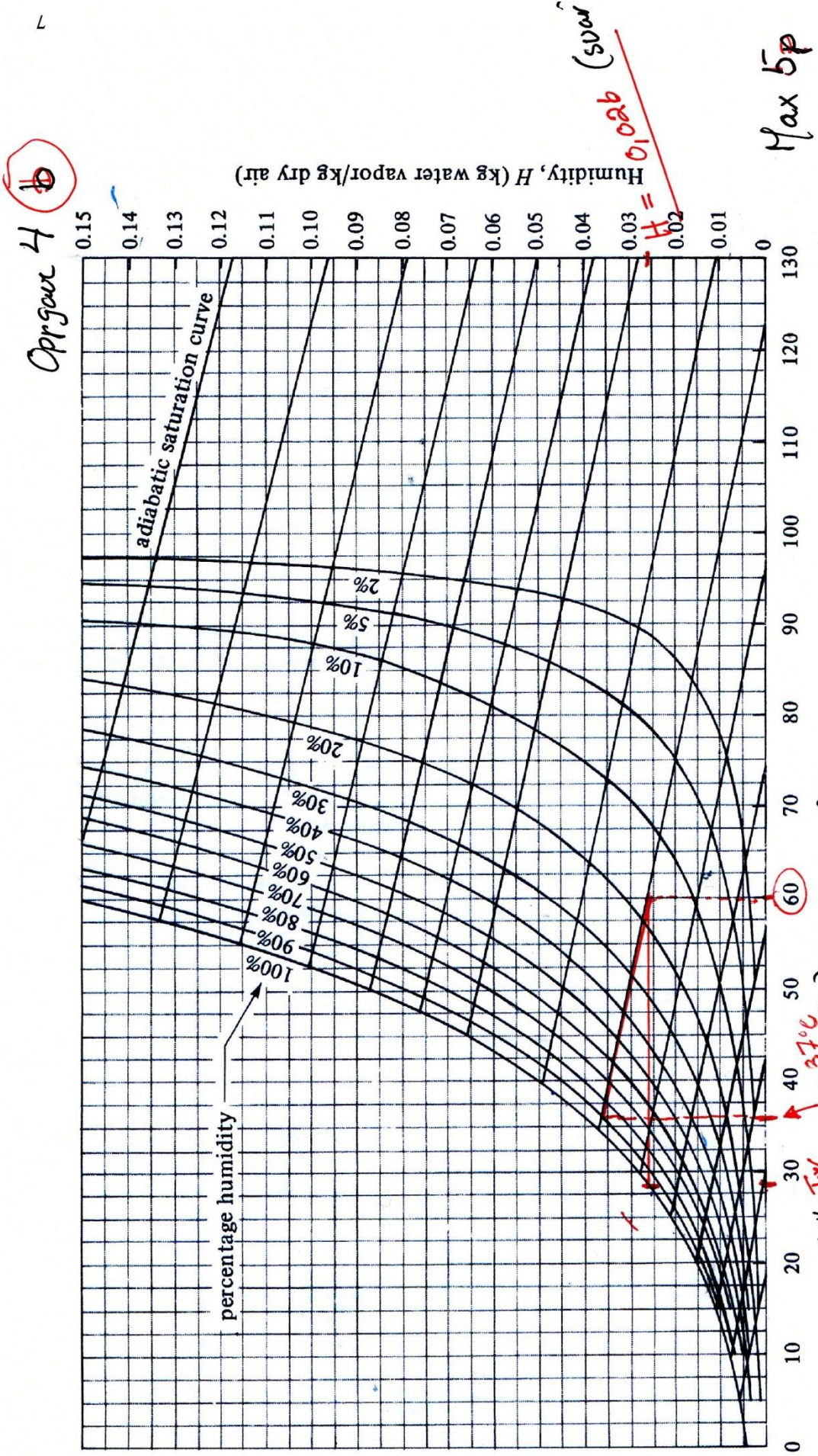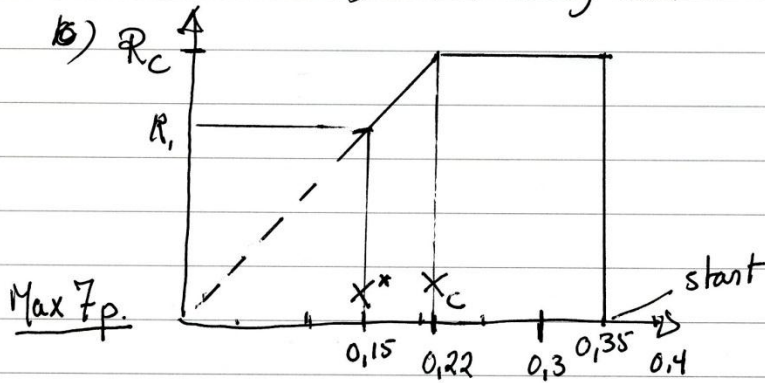    **d) Max 8 p.**      (see attached)

FIGURE 9.3-2. *Humidity chart for mixtures of air and water vapor at a total pressure of 101.325 kPa (760 mm Hg). (From R. E. Treybal, Mass-Transfer Operations, 3rd ed. New York: McGraw-Hill Book Company, 1980. With permission.)*

a) Book on page 577 **5p**

b) on diagram **5p**

b) $R_c$



Max 7p.

0,15   0,22   0,3  0,35  0,4

$\underbrace{\phantom{xx}}_{2} \quad \underbrace{\phantom{xxxxxx}}_{0,5\ time}$

Skisse **3p**

$R = -\frac{L}{A} \cdot \frac{dX}{dt}$

$t = \frac{L_s}{A \cdot R_c}(X_1 - X_c)$

$0,5 = \frac{12}{1 \cdot R_c}(0,35 - 0,22$

$R_c = \frac{12}{0,5}(0,35 - 0,22)$

**4p** $= 24(0,13)$

$= 3.12 \; \frac{kg\ H_2O}{h \cdot m^2}$

d)

Max 8p

See book p.587

$t = \frac{L_s}{A} \int_{X_2}^{X_1} \frac{dX}{R}$ where $R = aX + b$

$\Rightarrow t = \frac{L_s}{aA} \int_{R_c}^{R_1} \frac{dR}{R} = \frac{L_s}{aA} \cdot \ln \frac{R_1}{R_c}$

for drying through origo: $t = \frac{L_s X_c}{A \cdot R_c} \ln \frac{X_c}{X_2}$

**4p**

$\underbrace{\phantom{xxxxx}}_{0,3825}$

$t = 0,324$

$\Rightarrow$ Total tørketid $= \boxed{0,824\ h} \approx 50\ minutter$