EXAM 2013

13.4 COMPLETE-MIXING MODEL FOR GAS SEPARATION BY MEMBRANES

13.4A Basic Equations Used

In Fig. 13.4-1 a detailed process flow diagram is shown for complete mixing. When a separator element is operated at a low recovery (i.e., where the permeate flow rate is a small fraction of the entering feed rate), there is a minimal change in composition. Then the results derived using the complete-mixing model provide reasonable estimates of permeate purity. This case was derived by Weller and Steiner (W4).

The overall material balance (Fig. 13.4-1) is as follows:

$$q_f = q_o + q_p \tag{13.4-1}$$

where q_f is total feed flow rate in cm^3(STP)/s; q_o is outlet reject flow rate, cm^3(STP)/s; and q_p is outlet permeate flow rate, cm^3(STP)/s. The cut or fraction of feed permeated, θ, is given as

$$\theta = \frac{q_p}{q_f} \tag{13.4-2}$$

The rate of diffusion or permeation of species A (in a binary of A and B) is given below by an equation similar to Eq. (6.5-8) but which uses cm^3(STP)/s as rate of permeation rather than flux in kg mol/s · cm^2.

$$\frac{q_A}{A_m} = \frac{q_p y_p}{A_m} = \left(\frac{P'_A}{t}\right)(p_h x_o - p_l y_p) \tag{13.4-3}$$

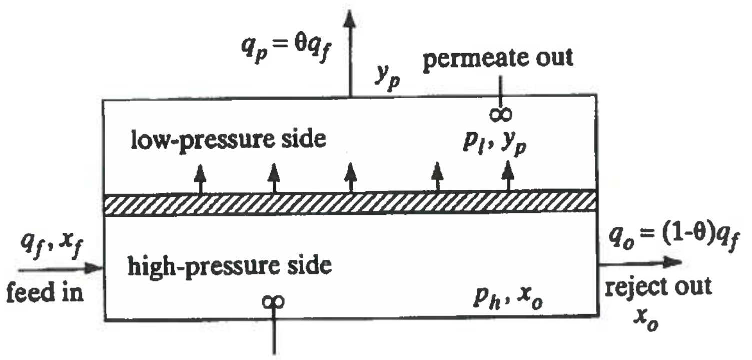

FIGURE 13.4-1. *Process flow for complete mixing case.*

where P'_A is permeability of A in the membrane, $cm^3(STP) \cdot cm/(s \cdot cm^2 \cdot cm\ Hg)$; q_A is flow rate of A in permeate, $cm^3(STP)/s$; A_m is membrane area, cm^2; t is membrane thickness, cm; p_h is total pressure in the high-pressure (feed) side, cm Hg; p_l is total pressure in the low-pressure or permeate side, cm Hg; x_o is mole fraction of A in reject side; x_f is mole fraction of A in feed; and y_p is mole fraction of A in permeate. Note that $p_h x_o$ is the partial pressure of A in the reject gas phase.

A similar equation can be written for component B.

$$\frac{q_B}{A_m} = \frac{q_p(1-y_p)}{A_m} = \left(\frac{P'_B}{t}\right)[p_h(1-x_o) - p_l(1-y_p)] \tag{13.4-4}$$

where P'_B is permeability of B, $cm^3(STP) \cdot cm/(s \cdot cm^2 \cdot cm\ Hg)$. Dividing Eq. (13.4-3) by (13.4-4)

$$\frac{y_p}{1-y_p} = \frac{\alpha^*[x_o - (p_l/p_h)y_p]}{(1-x_o) - (p_l/p_h)(1-y_p)} \tag{13.4-5}$$

This equation relates y_p, the permeate composition, to x_o, the reject composition, and the ideal separation factor α^* is defined as

$$\alpha^* = \frac{P'_A}{P'_B} \tag{13.4-6}$$

Making an overall material balance on component A

$$q_f x_f = q_o x_o + q_p y_p \tag{13.4-7}$$

Dividing by q_f and solving for the outlet reject composition,

$$x_o = \frac{x_f - \theta y_p}{(1-\theta)} \quad \text{or} \quad y_p = \frac{x_f - x_o(1-\theta)}{\theta} \tag{13.4-8}$$

Substituting $q_p = \theta q_f$ from Eq. (13.4-2) into Eq. (13.4-3) and solving for the membrane area, A_m,

$$A_m = \frac{\theta q_f y_p}{(P'_A/t)(p_h x_o - p_l y_p)} \tag{13.4-9}$$

13.4C Minimum Concentration of Reject Stream

If all of the feed is permeated, then $\theta = 1$ and the feed composition $x_f = y_p$. For all values of $\theta < 1$, the permeate composition $y_p > x_f$ (H1). Substituting the value $x_f = y_p$ into Eq. (13.4-5) and solving, the minimum reject composition x_{oM} for a given x_f value is obtained as

$$x_{oM} = \frac{x_f\left[1 + (\alpha^* - 1)\dfrac{p_l}{p_h}(1-x_f)\right]}{\alpha^*(1-x_f) + x_f} \tag{13.4-12}$$

Hence, a feed of x_f concentration cannot be stripped lower than a value of x_{oM} even with an infinitely large membrane area for a completely mixed system. To strip beyond this limiting value a cascade-type system could be used. However, a single unit could be used which is not completely mixed but is designed for plug flow.