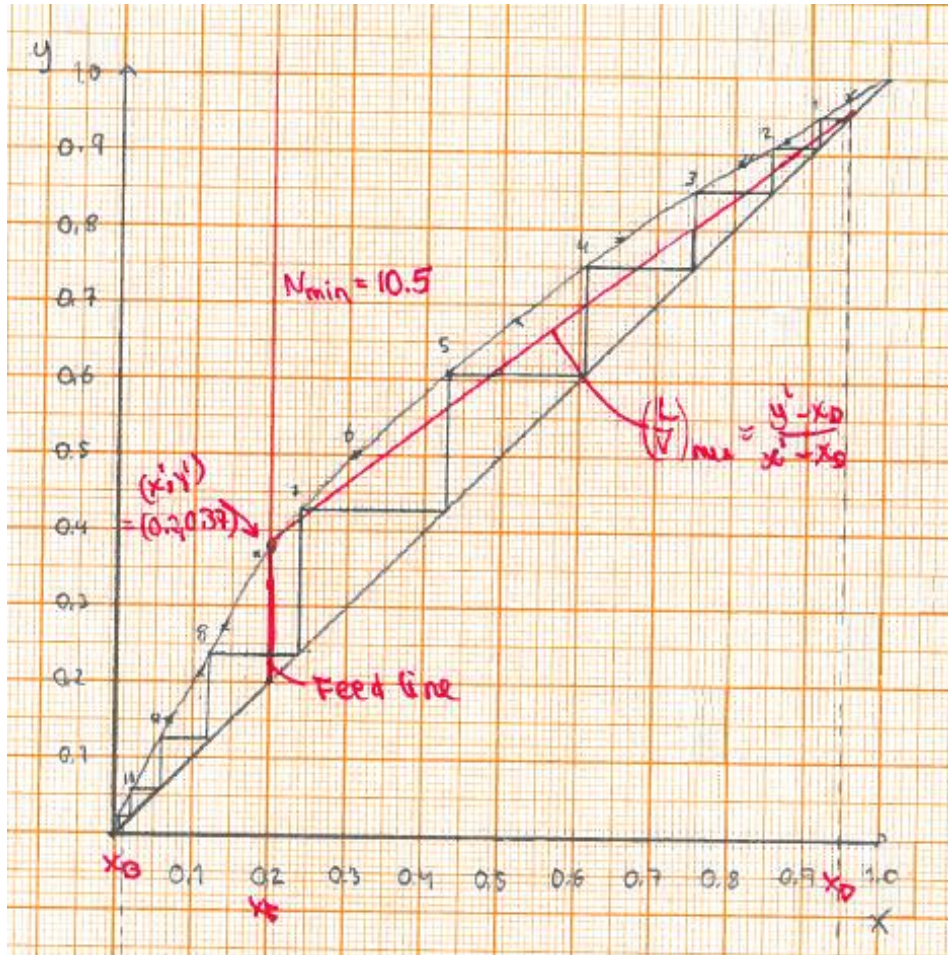


Distillation Solution (problem 1).

- (a) Material balances:  $Fz_F = D x_D + B x_B$  for light component and  $F=D+B$  for total flow. Combining gives with  $F=1$ ,  $z_F=0.2$ ,  $x_D=0.95$ ,  $x_B=0.01$ :  $D = 0.19/0.94=0.202$ ,  $B=1-D=0.798$



- (b) Minimum no. of stages is with infinite reflux, so  $(L/V)=1$  and the operating line is along the diagonal. From the Figure we find:  $N_{min} = 10.5$
- (c) See Figure (McCabe-Thiele): The feed line is vertical for a saturated liquid feed. We identify the point  $(x', y') = (0.2, 0.37)$  where it crosses the equilibrium line. The slope of the operating line in top with infinite number of stages is then:  
 $(L/V)_{min} = (x_D - y') / (x_D - x') = (0.95 - 0.37) / (0.95 - 0.2) = 0.773$   
 Here  $V=L+D$ , so  $L/V=R/(R+1)$  where  $R=L/D$ , and we get  $R_{min}=L_{min}/D = 0.773 / (1 - 0.773) = 3.41$  and we get  $L_{min} = 3.41 * 0.202 = 0.69$
- (d) The feed is saturated liquid so the vapor flow in the bottom (generated by the reboiler) is  $V_{min} = L_{min} + D = 0.69 + 0.202 = 0.89$  mol/s. The minimum reboiler duty is then:  $Q_{min} = V_{min} * dH_{vap} = 0.89$  mol/s \* 45 kJ/mol = 40.0 kW .  
 Comments: 1. We assume that the heat of vaporization for the mixture with 99% isopropanol is the same as for pure isopropanol. 2. With the actual number of stages close

to  $N=2.5 N_{min}$  the real reboiler duty  $Q$  should be less than 10% above  $Q_{min}$ ; see next question.

(e) I would typically recommend  $N = 2.5 N_{min} = 26.2$  stages. With this value  $L$  should be less than 10% higher than  $L_{min}$ .

(f)  $\alpha$  is the relative volatility and  $S$  is the separation factor,  $S = (x_1/x_2)_D / (x_1/x_2)_B$ . Both formulas assume constant  $\alpha$  (which generally holds for an ideal mixture with components with similar heat of vaporization). For the Fenske equation there is no additional assumption. For the King's formula, we have in addition assumed high-purity products and a saturated liquid feed.

Derivation: Both formulas can be derived by considering the graphical solution and using the constant relative volatility expression to represent the equilibrium. For Fenske we make use of the fact that  $y=x$  between the stages at infinite reflux and we then find that the "separation"  $x_1/x_2$  increases by a factor  $\alpha$  for each stage, so overall  $S = \alpha^N$ . For King we use  $x'=x_F$  (feed liquid) and use the constant relative volatility formula to express  $y'=y_F$  and then do some algebra and make the assumption that  $x_D=1$  and  $x_B=0$  ... then  $x_F$  and  $y_F$  disappear and we get the desired expression! See slides for details.

(g) Relative volatility  $\alpha = (y_1/x_1) / (y_2/x_2) = y^*(1-x) / x^*(1-y)$

$x=0.068, y=0.146, \alpha=2.34$

$x=0.524, y=0.684, \alpha=1.97$

$x=0.947, y=0.967, \alpha=1.64$

We see that constant relative volatility is not a very good assumption in this case. If we for simplicity assume that  $\alpha = 2$  then we get:

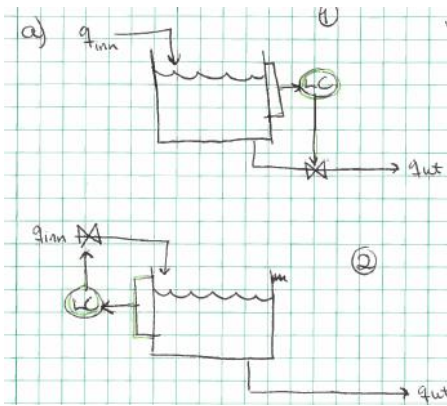
Fenske:  $S = (x_1/x_2)_D / (x_1/x_2)_B = (0.95/0.05)/(0.01/0.99) = 1881$ . Get  $N_{min} = \ln S / \ln \alpha = 10.9$  (which is close to the correct value of 10.5 found graphically)

King:  $L_{min} = F/(\alpha-1) = 1/1 = 1$  mol/s (This is quite far from the correct solution, mainly because  $\alpha$  is not constant).

Comment: We should use  $\alpha$  at the feed stage ( $y', x'$ ) = (0.37, 0.2) which gives  $\alpha = (0.37/0.2) / (0.63/0.8) = 2.35$  and  $L_{min} = F/(\alpha-1) = 1/1.35 = 0.74$  mol/s. This is still slightly higher than the value 0.69 mol/s found graphically because the actual top purity is 0.95 and not 1).

(h) We use  $\alpha=1.64$  (top) to find the increase. Note that we can divide up the column in sections since  $\ln S = \ln S_1 * S_2 = \ln S_1 + \ln S_2 = N_{1min} + N_{2min}$ . Here  $S_1$  is the original separation factor and  $S_2$  the change due to increased purity. That is,  $S_2 = (0.9999/0.0001) / (0.95/0.05) = 526$  and with  $\alpha=1.64$  we find the increase  $N_{2min} = \ln S_2 / \ln \alpha = \ln 526 / \ln 1.64 = 12.7$  stages.

### Various Solution (Problem 3):



(a) See the figure for the two alternative structures. 1) If the inflow is given (a disturbance) then we need to use the outflow for level control. 2) If the outflow is given (a disturbance) then we need to use the inflow for level control. Both structures are expected to work well.

(b) 1. Raoult's law is  $p_A = x_A P_{\text{Asat}}(T)$ . It applies to *ideal (liquid) mixtures* where the partial pressure is proportional to the mole fraction in the liquid multiplied by the pure component vapor pressure.

Comment: Introducing  $p_A = y_A P$  this gives  $y_A/x_A = p_{\text{Asat}}(T)/p$

2.  $y = Kx$  with  $K = \text{constant}$  is a variant of Henry's law which applies to a dilute mixture at given pressure.

Comment 1: Often we use the symbol  $m$  instead of  $K$ .

Comment 2: The "original" Henry's law says that  $p_A = H(T) x_A$  where  $H(T)$  depends on temperature only. Introducing  $p_A = y_A p$  this gives  $y_A/x_A = K = H(T)/p$ .

Comment 3: At given  $p$  we have that  $K = H(T)/p$  is constant because  $T$  is approximately constant for a dilute mixture at constant  $p$ .

Since  $K = H(T)/p$  we expect that  $K$  drops by a factor of about 2 when  $p$  increases from 1 atm to 2 atm.

(c) 1. The general molar balance equation without reaction is:  $dn/dt = In - Out$  (mol/s) where  $n$  is the inventory (total or component). In our case there is no inflow, so we get:

$$\text{Total mass: } dL/dt = 0 - V$$

$$\text{Component mass: } d(xL)/dt = 0 - yV$$

$$2. \text{ The component balance gives: } x dL/dt + L dx/dt = -yV$$

$$\text{Multiply by } dt \text{ on both sides: } x dL + L dx = -yVdt$$

$$\text{Introduce } -Vdt = dL \text{ from the total material balance: } xdL + Ldx = y dL$$

$$\text{Rearrange: } dL/L = dx/(y-x)$$

Integration of this gives the Rayleigh equation

3. We assume vapor-liquid equilibrium (that is, we have perfect mixing in the liquid phase and no diffusion limitation). With  $y = Kx$  (equilibrium) we then get

$$\int dx/(y-x) = \int dx/(x(K-1)) = \ln x / (K-1).$$

So we get  $\ln(L1/L0) = \ln(x1/x0) / (K-1) = \ln(1.E-4/1.E-2) / 99 = \ln(0.01)/99 = -0.04562$ . With  $L0=1000$  mol this gives  $L1 = 1000*\exp(-0.04652)=954.6$  mol. Thus the reduction in L is 45.5 mol and since  $V=0.1$  mol/s this takes 455 s.