

**Exam Sep.tek. Dec. 2013. Solution**

**Problem 2. Distillation (25%)**

(a) (17%) **Upper operating line:** Mass balances over the top of the column give:

$$\text{Total: } V = D + L \quad (1)$$

$$\text{Component: } Vy = D yD + L x \quad (2)$$

Inserting  $D=V-L$ , (2) gives the top operation line

$$y = (L/V) x + (1-L/V) yD$$

Inserting  $L/V=0.86$  and  $yD=0.90$  this gives

$$y = 0.86 x + 0.126.$$

This line goes through  $(yD, yD)=(0.9, 0.9)$  and the point  $(0, 0.126)$  and is plotted on the xy-diagram.

**Feed line.** The feed is liquid, and the feed line is therefore a vertical line through the point  $(xF, xF)=(0.5, 0.5)$ ,

**Lower operating line.** The lower operating line starts from the intersection between the feed line and the upper operating line and ends in  $(xB, xB)$ , but we do not yet know  $xB$ .

(i) To find  $xB$ , we need to do a "trial-and-error" approach, where we guess  $xB$  and keep changing it until we can fit in 7 stages in the McCabe-Thiele staircase; see the plot.

With 3 stages (partial condenser + 2 plates) above the feed, we find that the feed is about optimally located, and by trial and error we find  $xB=0.07$  that fits in 4 stages in the bottom.

We can now find  $D$  and  $B$  from the overall material balances

$$F = D + B$$

$$FxF = DyD + BxB$$

We get:

$$10 \cdot 0.5 = D \cdot 0.9 + (10 - D) \cdot 0.07$$

and  $D = 5.18 \text{ kmol/h}$  and  $B = 4.82 \text{ kmol/h}$

(ii) With the feed moved down there 4 stages above the feed and we find that the feed is no longer optimally located (compare 4 with 4'), so we get a slightly higher value for  $xB$ , maybe around  $xB=0.08$  (see stages 4', 5' 6' and 7' in figure).



(b) **Control (8%).**

Inputs for control (MVs): L, VB (QB), D, B, Lcw (Qc)

Controlled variables (CVs): L/V (said to fixed), yD, MD (level), MB (level), pressure

Main disturbances: F, xF

The pairing is not obvious, but one suggestion is given in the Figure:

First one rather obvious pairing:

$B \leftrightarrow MB$

The other are less obvious, but one possibility is:

$L \leftrightarrow L/V$  (see figure)

$D \leftrightarrow p$  (since D is a vapor product it has a direct effect on pressure, but only an indirect effect on level, so this is why we do not use D for controlling MB)

$Lcw (QC) \leftrightarrow MD$  (this is reasonable since the liquid that condenses increases the level)

$VB (QB) \leftrightarrow yD$  (this is a bit by elimination; but although VB seems a bit far away from yD its effect is very fast)

There are also other possibilities, for example, L to control composition (xD) and VB(QB) to control L/V. This makes composition control more "direct" (in the right end)

**Solution Problem 3. Absorption (25%)**

Flows: We have a dilute mixtures so we assume  $V = V1 = V2$  and  $L = L1 = L2$  (throughout column).

Equilibrium: From  $p_{SO_2} = Hx$  (which applies to dilute mixtures) we derive

$$y = (H/p) x = mx$$

where y is the vapor mole fraction of SO<sub>2</sub>. At p=1 bar and 25C, we get  $m=46\text{bar}/1\text{bar} = 46$ .

- (a) (10%) The "problem" is that the liquid flow rate L is too low so that cannot absorb enough SO<sub>2</sub> even with equilibrium at the bottom of the column. Note also that the absorption factor is less than 1:  $A = (L/V)/m = (90000/3000)/46 = 0.652$ .

*Calculations:*

Assume that we are able to remove 95% of the SO<sub>2</sub>. The resulting x<sub>1</sub> would then be:

$$x_1 = 0.95 * Vy_1/L = 5.7/90000 = 0.000063$$

However, with equilibrium at the bottom (which requires an infinite number of stages) we have that

$$x_1^* = y_1/m = 0.002/46 = 0.0000435.$$

Since  $x_1 > x_1^*$  we must "exceed" equilibrium to remove 95% so the necessary x<sub>1</sub> is impossible.

So how much SO<sub>2</sub> are we able to remove (find y<sub>2</sub>)?

Assume equilibrium in the bottom, so  $x_1=x_1^*=0.0000435$  (this is the best we can get).

From the mass balance we can then compute y<sub>2</sub>:

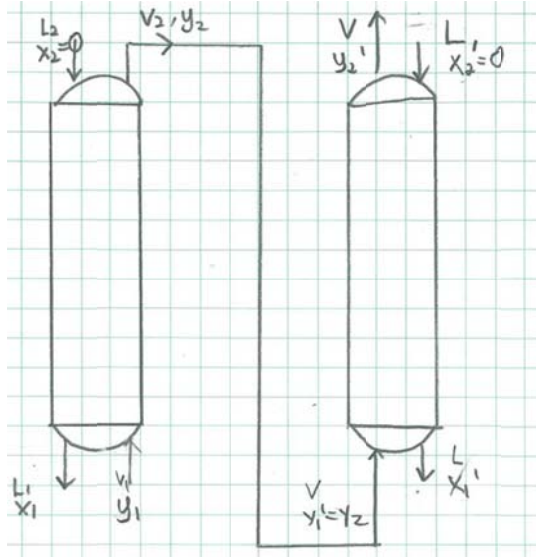
$$\text{In SO}_2 = \text{Out SO}_2$$

$$V y_1 + L x_2 = V y_2 + L x_1$$

Here  $x_2=0$  and  $x_1=x_1^*=y_1/m$  so we derive

$$y_2 = y_1 - (L y_1 / (V m)) = y_1 (1 - A) = 0.002 * 0.348 = 0.000696$$

which is about 7 times too large.



- (b) (5%) With another identical column we send  $y_2$  as the feed to the second column (so  $y_1' = y_2 = 0.000696$ ). This helps a little, but still we have  $A=0.652 < 1$  so it is not enough.

We get (similar to above):

$$y_2' = y_1' (1-A) = 0.000696 * 0.348 = 0.000242, \text{ which is still a factor 2.5 too large.}$$

- (c) (6%) Instead of building a second column, the following modifications for the original design have been suggested:

- Increase the sea water flow by a factor 2
- Decrease the temperature from 25C to 5C.
- Increase the pressure from 1 bar to 2 bar

All of these increase  $A=(L/V)/m$  by a factor of about 2 and will remove  $SO_2$  effectively (see detailed calculation below).

- $L$  is increased by a factor 2, so  $A=1.304$
- $m$  is decreased by a factor 2.09, so  $A = 1.363$
- $m = H/p$  is decreased by a factor 2, so  $A=1.304$

All of these are OK as we find that  $x_1 (= 0.95 y_1 V/L)$  corresponding to 95% removal is smaller than  $x_1^*=y_1/m$  (which is the value achieved with infinite stages):

- $x_1(95\%) = 3.17 e-5, x_1^* = 4.35e-5$
- $x_1(95\%) = 6.33 e-5, x_1^* = 8.7 e-5$
- $x_1(95\%) = 6.33e-5, x_1^* = 8.7 e-5$

**Extra. More detailed calculation of  $y_2$  for case a ( $N=10$ ,  $L=90000 \cdot 2$ ,  $A=1.304$ ).**

From the Kremser equation we have

$$(y_1 - y_1^*) / (y_2 - y_2^*) = A^N = 1.304^{10} = 14.2$$

In addition we have the mass balance

$$V y_1 + L x_2 = V y_2 + L x_1$$

And equilibrium relationships,  $y_1^* = m x_1$ ,  $y_2^* = m x_2$ .

We have given  $y_1 = 0.002$  and  $x_2 = 0$  so we have enough information to find  $y_2$  and  $x_1$ .

From the mass balance we get since  $x_2 = 0$

$$x_1 = (V/L) (y_1 - y_2) \text{ or } y_1^* = (y_1 - y_2) / A$$

Putting this into the Kremser equation using  $y_2^* = 0$  gives

$$y_1 / y_2 - (y_1 / y_2 - 1) / A = A^N$$

$$y_1 / y_2 (1 - 1/A) = A^N - A$$

and we derive

$$y_2 = y_1 (1 - 1/A) / (A^N - A) = 0.002 * (1 - 1/1.304) / (14.2 - 1.304) = 0.000036$$

which is a factor 3 lower than the requirement of  $y_2 < 0.000100$ , so we are OK

Which of the three modifications is the best?

All three give about the same  $A$  and the same  $\text{SO}_2$ -removal so practical issues is probably the main reason for choosing one. Increasing the pressure is expensive as it requires compressing the feed gas, so I would suggest one of the other alternatives. If there is no cold sea water, then the best alternative is to increase the seawater flow,

- (d) The Kremser equation (1) is on the form given in the lectures, which it is equivalent to the versions given in the book.
- (a) The assumptions are (i) dilute mixtures with  $m$  constant (straight equilibrium line). (ii)  $L/V$  constant (straight operating line, i.e. constant molar flows) and (iii) equilibrium on all stages.
  - (b)  $y_1^* = m x_1$  and  $y_2^* = m x_2$  are the "imaginary" vapor compositions in equilibrium with  $x_1$  and  $x_2$ .
  - (c) Make a McCabe-Thiele figure:
    - Straight equilibrium line  $y = m x$  (goes through origin)
    - Straight operating line with slope  $L/V$  (goes through  $(x_1, y_1)$  and  $(x_2, y_2)$ ).

From the figure you see easily that  $(y_2 - y_1) / (x_2 - x_1) = L/V$  (operating line) and  $(y_2^* - y_1^*) / (x_2 - x_1) = m$  (equilibrium line); and it follows that  $(y_2 - y_1) / (y_2^* - y_1^*) = (L/V) / m = A$ ; See also lecture notes (slide).