Problem 2 – Distillation of air (20%)

A feed with two components (79 mole% N_2 and 21% O_2) is to be separated by continuous distillation.
(a) Compute equilibrium data (y,x) at 1 atm for N_2-O_2 at x=0, 0.2, 0.4, 0.6, 0.8 and 1, given the relative volatility α.= 3.5.
(b) Find the minimum number of theoretical stages to get products with 99.99% N_2 (distillate, top) and 99.5% O_2 (bottom)? (You may find this graphically, but it is simpler to use the Fenske formula N=lnS/lnα)
(c) What is the minimum reflux (L_{min}/F) and corresponding minimum boilup (V_{min}/F) when the feed F is saturated liquid and saturated vapor, respectively? (You may find this graphically)
Given: Relative volatility, $\alpha = (y_L/x_L) / (y_H/x_H) = y(1-x) / x(1-y)$, where y and x are the mole fractions of light component. The normal boiling point is 77.4 K for N_2 and 90.2 K for O_2.

Solution

(a) Note from the boiling points that Nitrogen is the light component (x,y).

$$\alpha = (y/x) / ((1-y)/(1-x)) = 3.5$$

Solve with respect to y:

$$y = \alpha x / (1 + (\alpha-1)x)$$

Get with α=3.5:

x	0	0.2	0.4	0.6	0.8	1
y	0	0.47	0.7	0.84	0.93	1

(b) Minimum stages (Nmin) is obtained with infinite reflux, so L/V=1 and the operating line is on diagonal in the xy-diagram. Note that the feed composition and feed condition (liquid, vapor) does not matter when obtaining Nmin.

Separation factor,

$$S = (x_L/x_H)_D / (x_L/x_H)_B = (0.9999/0.0001) / (0.005/0.995) = 1989801$$

$$\ln S = 14.51$$

$$\ln \alpha = 1.25$$

Fenske: $N_{min} = \ln S/\ln \alpha = 11.59$

(You can also do this graphically on the xy-diagram by making stair case between the diagonal and equilibrium curve, but it is very difficult to get accurate values in the corners).

(c) Want to find minimum reflux (Lmin), corresponding to infinite stages. This will correspond to a pinch at the feed.

Overall mass balance for the column

$$F = D + B$$

$$z_F F = x_D D + x_B B$$

Assume F=1, so all flows are relative to F. Introduce B=1-D. Get

$$z_F = x_D D + x_B (1-D)$$

or

$$D = (z_F - x_B) / (x_D - x_B) = (0.79-0.005)/(0.9999-0.005) = 0.789$$

Minimum flows in top section (see figure)

$$(LT/VT)_{min} = (xD-y') / (xD-x')$$

where xD = 0.9999 and (x',y') is at crossing between feed line and equilibrium curve (see Figure).

Note here that: R = L/D = L/(V-L) = L/V / (1 – L/V)

And then with known Rmin and D we can find Lmin and Vmin (see Figure of column below).

Feed liquid (see figure): x' = zF = 0.79 and y' = α x' / (1 + (α-1)x') = 0.9294

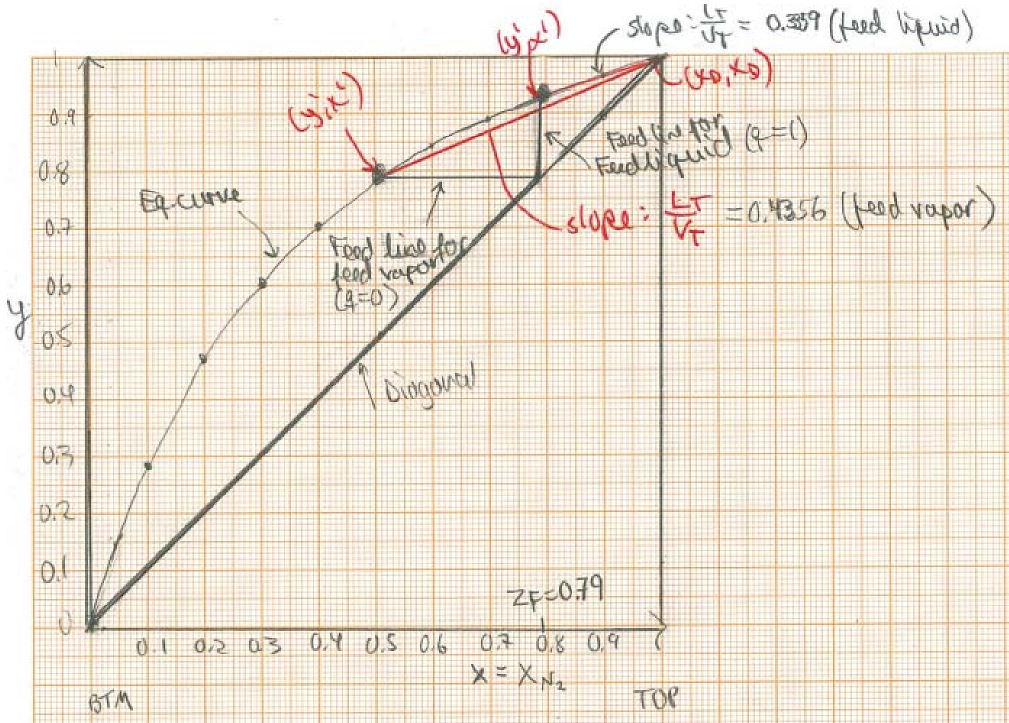
$$(LT/VT)_{min} = (xD-y') / (xD-x') = (0.9999 – 0.9294)/(0.9999-0.79) = 0.3359$$

Rmin = (L/D)min = 0.3359 / (1-0.3359) = 0.5058
Lmin = (L/D)min * D = 0.5058 * 0.789 = **0.399** (in top of column)
Vmin = D+L = 0.789 + 0.399 = **1.188** (in top and btm of column)

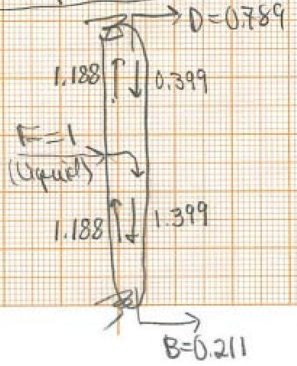
Feed vapor: y' = zF = 0.79 and x'=0.518 (in equilibrium with y')
$(LT/VT)_{min}$ = (xD-y') / (xD-x') = (0.9999 – 0.79)/(0.9999-0.0.518) = 0.4356
Rmin = (L/D)min = 0.4356 / (1-0.4356) = 0.7718
Lmin = (L/D)min * D/F = 0.7718 * 0.789 = **0.609** (in top of column)
VTmin = D + L = 0.789 + 0.609 = 1.398 (in top of column, see figure)
VBmin = VTmin - F = **0.398** (in btm of column, see figure)

The resulting minimum flows are summarized in the figure.

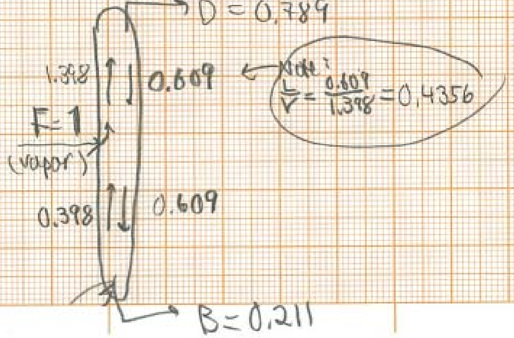
Comment: For feed liquid, Kings's formula from the lectures gives (assuming pure products)
Feed liquid: $L_{min}/F = 1/(\alpha-1) = 1/ 2.5 = 0.4$ (which agrees with the above)
Feed vapor: $L_{min}/F = 1/(\alpha-1) + B/F = 0.611$ (which agrees) or equivalently
$V_{min}/F = 1/(\alpha-1) = 1/ 2.5 = 0.4$ (which also agrees)



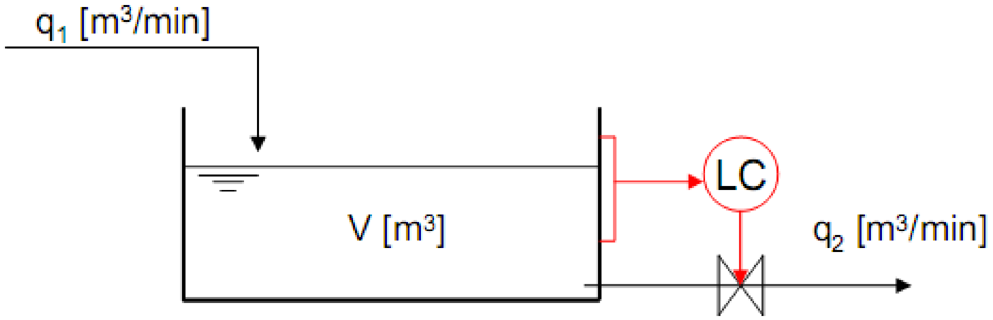
Feed liquid. Min flows



Feed vapor. Min flows



Problem 3 –Prosess control (15%)



(a) PI-controller:

$$q_2(t) = q_{20} + K_c(V(t)-V_0) + (K_c/\tau_I)\int(V(t)-V_0)dt$$

Advantage with I-action: No steady-state offset in V (but this may not be an important control objective as the main reason for having the tank is to have smooth changes in q2).
Disadvantage: More tuning parameters and may get cycling. Also, may actually want V to change (increase) when there is a larger flow (because then we have more capacity to get a smooth decrease in q2 when q1 drops).

(b) P-control: $q_2 = q_{20} + K_c(V-V_0)$

(i) 0.2 = 0.1 + 0.5 (V-1) -> V=1.2 m3
0.3 = 0.1 + 0.5 (V-1) -> V=1.4 m3
Comment: With a PI-controller the steady-state value of V is 1m3 in all cases.

(ii) Mass balance around tank

dm/dt = win – wout [kg/s]

where m = ρV, win = ρ1 q1, wout = ρ2 q2
Assume: constant (equal) density ρ [kg/m3]. Then:

dV/dt = q1 – q2

(iii) From the P-controller equation

dq2/dt = Kc dV/dt

Inserted into the material balance

(1/Kc) dq2/dt = q1-q2

Separate variables

dq2/(q1-q2) = Kc dt

Note that q1=0.3 (step in q1). Integrate from t=0 where q2=0.2:

ln (0.3-q2)/(0.3-0.2) = - Kc t

or

q2 = 0.3 – 0.1 exp(-Kc t)

so the time constant is

tau = 1/Kc = 1/0.5 = 2 min

Get

t [min]	0	1	2	5	∞
q2 [m3/min]	0.2	0.239	0.263	0.292	0.3

see the Figure on the next page

c)

i) $\underset{0.2}{q_2} = \overset{0.1}{q_{20}} + \overset{0.5}{K}(V - \overset{1\,m^3}{V_0}) \Rightarrow \underline{V = 1.2\,m^3}$

ii) Materialbalansen

Beholdning : $\underset{kg/m^3}{\rho} \cdot V$, Inn : $q_1 \cdot \rho_1$, Ut : $q_2 \cdot \rho_2$

Anta : $\rho = \rho_1$ (Blandetank) , $\rho_1 = \rho_2$

Får da $$\boxed{\frac{d}{dt}(\rho V) = \rho_1 q_1 - \rho_2 q_2 \Rightarrow \frac{dV}{dt} = q_1 - q_2}$$

iii) Fra ligningen for P-regulatoren fås
$$\frac{dq_2}{dt} = K\frac{dV}{dt}$$

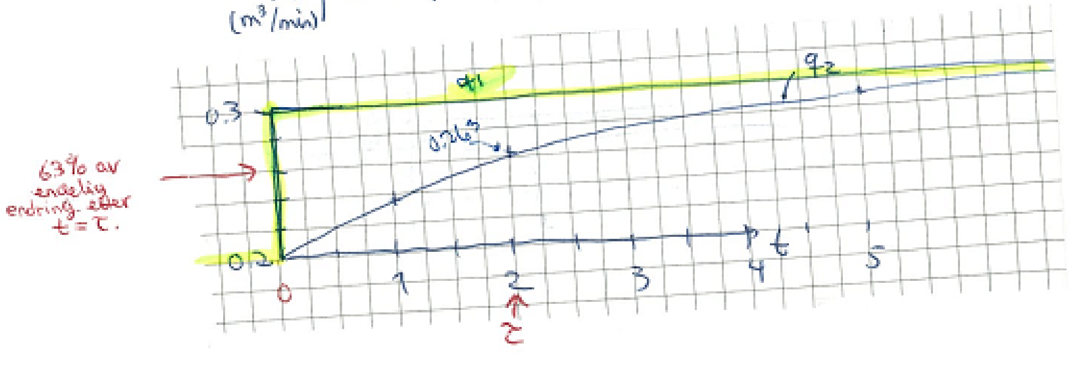
Innsatt i materialbalansen
$$\frac{1}{K}\frac{dq_2}{dt} = q_1 - q_2 \Rightarrow \boxed{\frac{dq_2}{\underset{0.3\to}{q_1} - q_2} = Kdt}$$

Integrerer fra startpunkt ved $t=0$ der $q_2 = 0.2$. $q_1 = 0.3$ (konst.)
$$\int_{0.2}^{q_2} \frac{dq_2}{0.3 - q_2} = \int_0^t Kdt \Rightarrow -\ln(0.3 - q_2)\Big|_{0.2}^{q_2} = Kt$$

$$\Rightarrow \ln\frac{0.3 - q_2}{0.1} = -Kt \Rightarrow \underline{q_2 = 0.3 - 0.1e^{-Kt}}$$

dvs. tidskonstanten
$$\boxed{\tau = 1/K} = 1/0.5 = \underline{\underline{2\,min}}$$

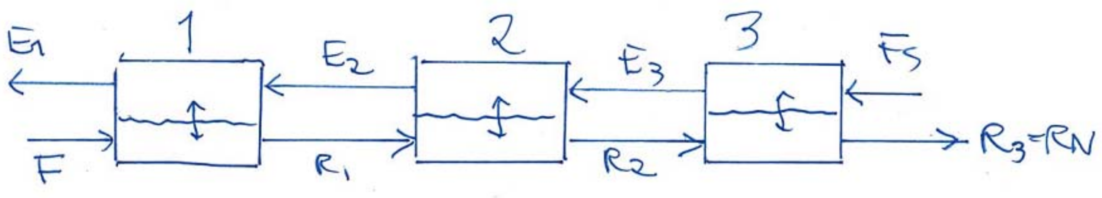
t (min)	0	1	2	5	∞
q_2 (m³/min)	0.2	0.239	0.263	0.292..	0.3



Problem 4 – Extraction (15%)

We have a given feed F (250 kg/min) with components A and C, and we want to reduce the amount of A using countercurrent extraction with pure component S. The desired products are R_N (main product) og E_1 (side product), see the figure.

(a) Make a flowsheet of the process for the case with three stages (N=3).



(b) From the liquid-liquid equilibrium diagram we read the compositions in red in the table.

Mass balances then give the flows.
Total: F + FS = RN + E1
A: 0.26*250 = 0.025 RN + 0.37 E1
C: 0.74*250 = 0.90 RN + 0.07 E1

From the last two balances: E1 = 162.6 kg, RN=192.9 kg
From total mass balance: Fs = 105.5 kg

Component/Stream	F (feed)	F_S (feed)	R_N (product)	E_1 (product)
Total (kg/min)	250	105.5 kg	192.9 kg	162.6 kg
Weight fraction A	0.26	0	0.025	0.37
Weight fraction C	0.74	0	0.90	0.07
Weight fraction S	0	1	0.075	0.56

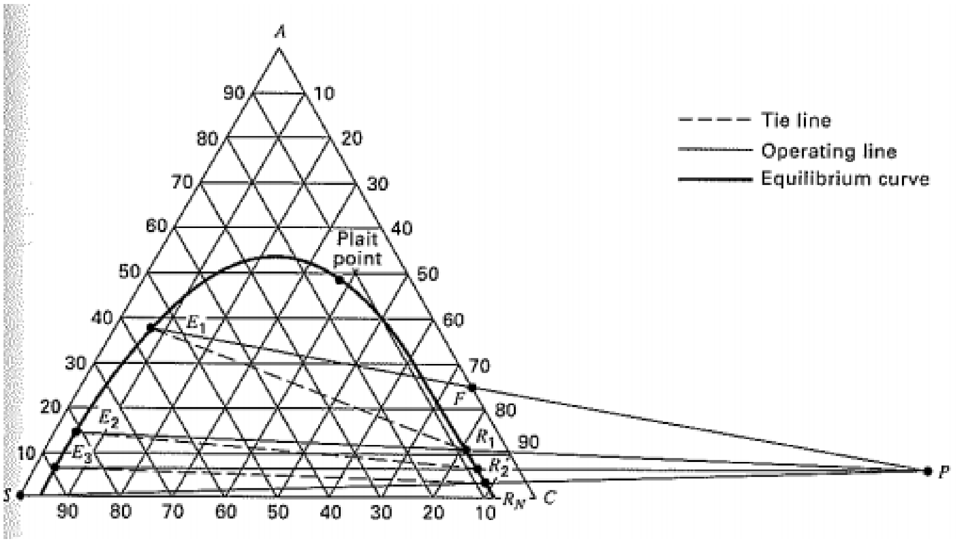
Note: Can alternatively use point M and level arm rule (which is the graphical mass balance). This is probably more accurate in this case since we use the diagram more directly.

Lever arm rule for Fs-M-R:
*Fs / F = F-M / Fs-M = 3.05 cm / 7.35 cm = 0.415 -> Fs = 0.415*250 = 103.7 kg*

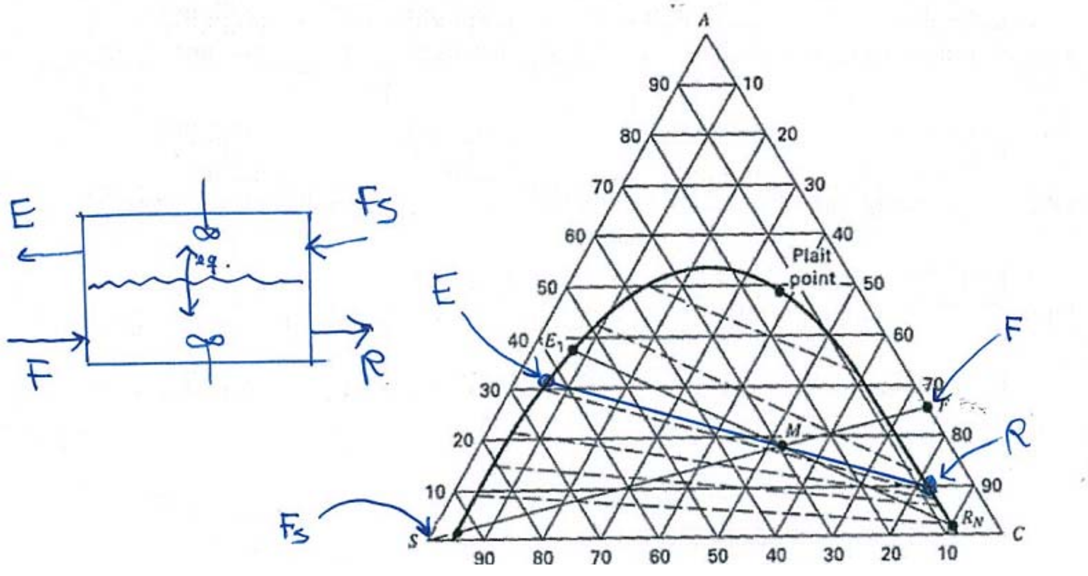
Lever arm rule for E1-M-RN:
E1 / RN = RN-M / E1-M = 3.75 cm/ 4.55cm = 0.824
And since E1+RN = M = 353.7 kg, we get, E1 = 159.8 kg and RN = 193.9 kg.

(c) Determine graphically the number of stages (N) required to get the desired products R_N og E_1.

The operating point (P in the figure below) is found as the crossing between the E1-F and FS-RN lines. The number of stages is then found by stepping between the equilibrium (on stages) and operating (between stages) points. We start from F and move to RN. Need about 2.8 stages (see below)



(d) One-stage extraction (N=1)



From the liquid-liquid diagram we red the compositions of the products E and R (given in red in the table below).

Mass balances then give:

Total: 250 + 105.5 = R + E
A: 0.26*250 = 0.1 R + 0.32 E

Get from these two: E = 133.9 kg, R = 221.6 kg

Component/Stream	F (feed)	F_S (feed)	R (product)	E (product)
Total (kg/min)	250	105.5 kg	221.6 kg	133.9 kg
Weight fraction A	0.26	0	0.1	0.32
Weight fraction C	0.74	0	0.82	0.05
Weight fraction S	0	1	0.08	0.63

Check using C-balance: In = 0.74*250 = 185 kg
Out = 0.82*221.6 + 0.05*133.9 = 188.4 kg (hm…some error)

Note: Could alternatively use point M and level arm rule.
Lever arm rule for E-M-R:
E / R = R-M / E-M = 3.05 cm/ 4.85cm = 0.629
And since E+R = M = 353.7 kg, we get, E = 136.6 kg and R = 217.1 kg.