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DYNAMIC MATRIX CONTROL — A COMPUTER CONTROL ALGORITHM

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ABSTRACT

The Dynamic Matrix Control (DMC) Algorithm is a control technology that has been used successfully in process computer applications in Shell for the last six years. The general development of the DMC Algorithm to incorporate feedforward and multivariable control is covered in this paper. The DMC Algorithm evolved from a technique of representing process dynamics with a set of numerical coefficients. The numerical technique, in conjunction with a least square formulation to minimize the integral of the error/time curve, make it possible to solve complex control problems on a digital computer which are not solvable with traditional PID control concepts. The incorporation of the process dynamics into the synthesis of the design of the DMC, make it possible to maintain an awareness of deadtime and unusual dynamic behavior.

DYNAMIC MATRIX CONTROL DEVELOPMENT

The Dynamic Matrix Control (DMC) Algorithm is a control technology that has been used successfully in process computer applications in Shell for the last six years. The general development of the DMC Algorithm to incorporate feedforward and multivariable control will be covered in this paper. A subsequent paper will expand the scope of the algorithm to address the problem of constrained multivariable control. The need for the expanded algorithm evolved from the application to a catalytic cracking unit of a non-linear steady state optimization which characteristically drives the process to a number of constraints.

The DMC Algorithm evolved from a technique of representing process dynamics with a set of numerical coefficients. The numerical technique, in conjunction with a least square formulation to minimize the integral of the error/time curve, make it possible to solve complex control problems on a digital computer which are not solvable with traditional PID control concepts. The incorporation of the process dynamics into the synthesis of the design of the DMC, make it possible to maintain an awareness of deadtime and unusual dynamic behavior. An awareness of deadtime alone prevents the controller from overcompensating which can only be obtained in the PID controller by suppressing the integral action with the corresponding degradation of the control. Examples of unusual dynamic response to a step change are illustrated in Figure 1. The first curve is characteristic of a system which is out

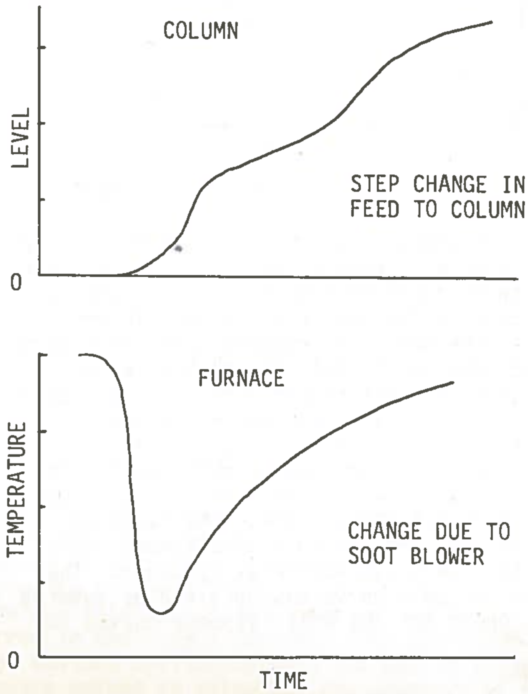

FIG. 1 UNUSUAL DYNAMIC BEHAVIOR

of material balance. The second curve is the response of a furnace transfer temperature to a soot blowing operation on a large preheater. Note in both illustrations, the response curve cannot be adequately described by a first or second order differential equation typically used in control analysis.

Any system which can be described or approximated by a system of linear differential equations can utilize the Dynamic Matrix Control technique which is based upon the numerical representation of the system dynamics. Two properties of linear systems makes the numerical representation possible. The first of these principles is the preservation of the scale factor. It is illustrated in Figure 2 where the response of the output variable O is shown for a change in the input variable I. The solid line represents the response of the output variable to a unit change in the input variable and the dashed line illustrates the response for a two-unit change in the input variable. Note the response of the two-unit change has twice the amplitude of the one-unit change. For a linear system, the response of the output variable for any size change in the input variable may be obtained by multiplying the scalar value of the input variable times

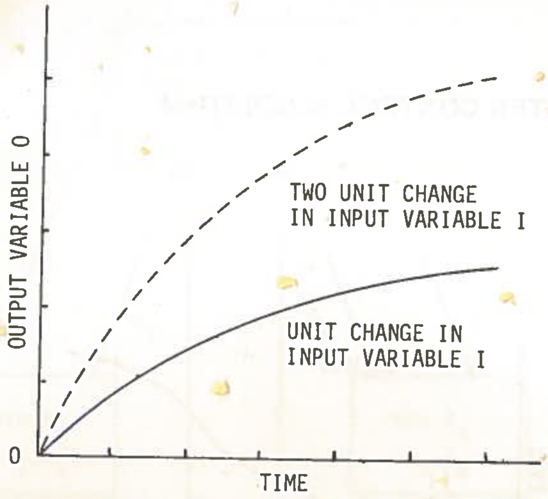

FIG. 2 PRESERVATION OF SCALE FACTOR

the unit response curve for the output variable. Further note on Figure 2 that the unit response curve can be approximated by a set of numbers if the curve is broken into discrete intervals of time. The two-unit response curve in Figure 2 can be obtained by multiplying the set of numbers for the unit response by 2. The second characteristic of a linear system is the principle of superposition. This principle is illustrated in Figure 3 where the response of the output variable is shown for a unit change in two input variables. Also, the response of the output variable to a simultaneous unit change in both input variables is shown. The response for this curve was obtained by summing the responses for the unit response curves for the input variables. Mathematically these concepts are given by:

$$
\begin{aligned}
\delta O_1 &= a_1 \, \Delta I_1 + b_1 \, \Delta I_2 + \ldots\ldots \\
\delta O_2 &= a_2 \, \Delta I_1 + b_2 \, \Delta I_2 + \ldots\ldots \\
\delta O_3 &= a_3 \, \Delta I_1 + b_3 \, \Delta I_2 + \ldots\ldots \qquad (1) \\
&\;\;\vdots \\
\delta O_i &= a_i \, \Delta I_1 + b_i \, \Delta I_2 + \ldots\ldots
\end{aligned}
$$

where the δO_i are the changes in the output variable from its initial value to its value at time interval i and the ΔI_j are the changes in the input variables from their initial value at time equal to zero. The a_i and b_i are the numerical coefficients referred to in the preceding paragraphs. Figure 4 illustrates the response of the outlet temperature of a preheat furnace to a step change in the fuel to the furnace and to a step change in the inlet temperature of the feed to the furnace.

The fuel coefficients shown in Figure 4 are an illustrative example of the a_i and the inlet temperature coefficients are illustrative of the b_i. The input variables can be manipulated control variables or measured disturbances. For example, in the furnace control problem to be described in this paper, the fuel is a manipulated input variable, the inlet feed temperature is a measured disturbance, and the controlled variable is the furnace transfer temperature.

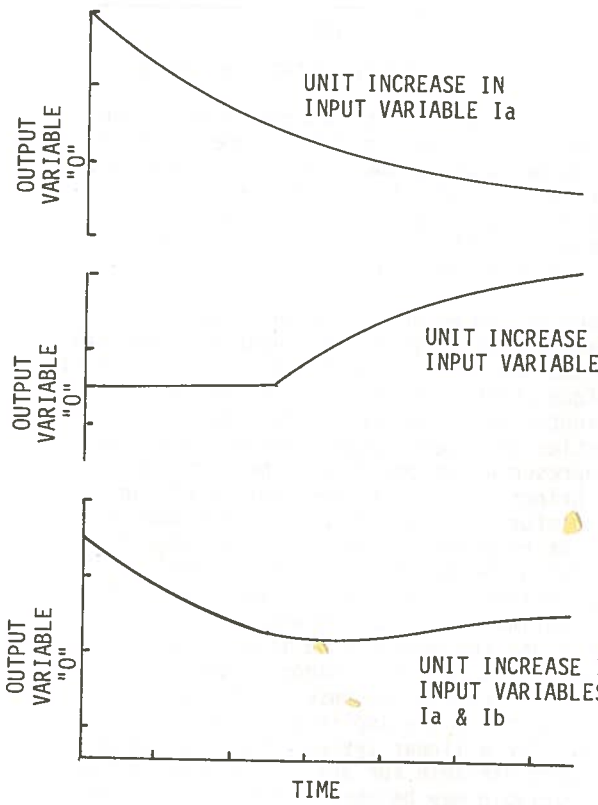

FIGURE 3 PRINCIPLE OF SUPERPOSITION

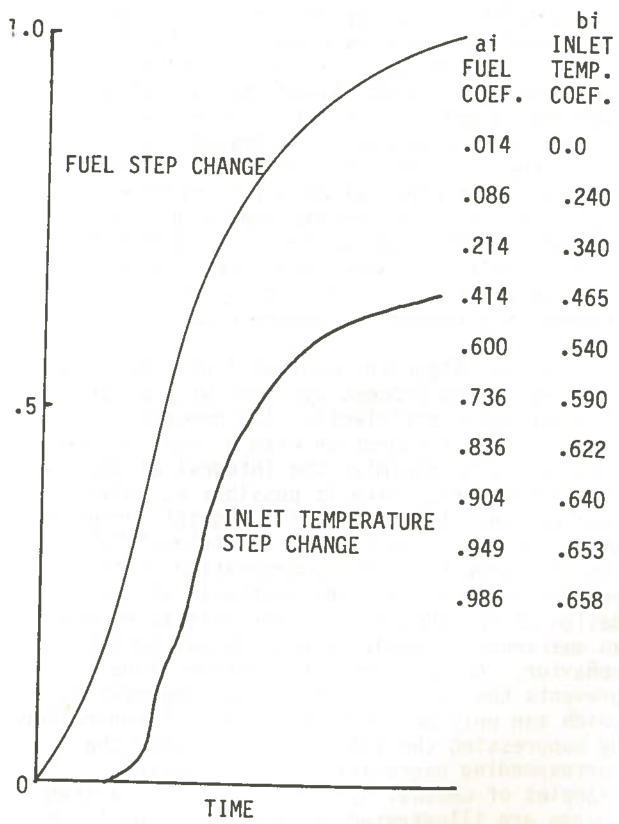

FIG. 4 FURNACE RESPONSE, FUEL, TEMPERATURE

The change in the controlled variable from time equal to zero to some future time that will result from changes in the manipulated input can be represented by the following equations:

$$\delta O_1 = a_1 \, \Delta I_1^1 \qquad (2)$$

$$\delta O_2 = a_2 \, \Delta I_1^1 + a_1 \, \Delta I_1^2$$

$$\delta O_3 = a_3 \, \Delta I_1^1 + a_2 \, \Delta I_1^2 + a_1 \, \Delta I_1^3 + \ldots$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

$$\delta O_i = a_i \, \Delta I_1^1 + a_{i-1} \, \Delta I_1^2 + a_{i-2} \, \Delta I_1^3 + \ldots$$

where movement of manipulated variable I_1 is considered for three intervals of time into the future with the superscripts on I_1 representing the time interval. The $\Delta^2_1 I$ is the step change in the manipulated input variable from its value at the end of the first time interval and the beginning of the second. Similarly $\Delta^3_1 I$ is the change in the manipulated input variable from its value at the end of the second time interval and the beginning of the third. Further, note the same set of coefficients is used for each column. The coefficients in each column are shifted down one time interval for each successive column to correspond with the first time interval which the future inputs can impact on the output variable.

FEED FORWARD DMC

Feedforward control is accomplished by moving the measured disturbance input variables in equation set (1) to the left hand side. For example, if input I_2 is a disturbance input, equation set (1) becomes:

$$\delta O_1 + b_1 \, \Delta I_2 = a_1 \, \Delta I_1$$

$$\delta O_2 + b_2 \, \Delta I_2 = a_2 \, \Delta I_1$$

$$\delta O_3 + b_3 \, \Delta I_2 = a_3 \, \Delta I_1$$

$$\delta O_4 + b_4 \, \Delta I_2 = a_4 \, \Delta I_1 \qquad (3)$$

$$\delta O_5 + b_5 \, \Delta I_2 = a_5 \, \Delta I_1$$

$$\delta O_6 + b_6 \, \Delta I_2 = a_6 \, \Delta I_1$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$\delta O_i + b_i \, \Delta I_2 = a_i \, \Delta I_1$$

Combining equation set (2) with set (3) yields the general form of the equations used to do feedforward predictive control of a variable O.

$$\delta O_1 + b_1 \, \Delta I_2 = a_1 \, \Delta I_1^1$$

$$\delta O_2 + b_2 \, \Delta I_2 = a_2 \, \Delta I_1^1 + a_1 \, \Delta I_1^2$$

$$\delta O_3 + b_3 \, \Delta I_2 = a_3 \, \Delta I_1^1 + a_2 \, \Delta I_1^2 + a_1 \, \Delta I_1^3$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

$$\delta O_i + b_i \, \Delta I_2 = a_i \, \Delta I_1^1 + a_{i-1} \, \Delta I_1^2 + a_{i-2} \, \Delta I_1^3$$

The desired response of the system is determined by subtracting the predicted response of the system from the setpoint, which is determined from the past history of inputs to the system. With the desired output response δO_i known, the measured input disturbances ΔI_2 known, and the numerical coefficients a_i and b_i known, all the information is available to solve equation set (4) for the set of time dependent moves in the manipulated input variable.

DMC SOLUTION TECHNIQUE

The set of equations is over-determined which prevents direct solution, but can be solved using a least square criterion. Such a solution produces a projected set of moves in the manipulated variable that minimizes the error in the output variable from its setpoint. The obvious difficulty with the use of the least square method to calculate the movement of the manipulated variable is the unconstrained nature of the solution. The method without constraint will yield very large changes in the manipulated variable that would not be physically realizable.

One technique for suppressing the change in the manipulated variable is to multiply by a number greater than one, the main diagonal elements of the square matrix that evolves from the least square formulation. The effectiveness of such a multiplier is illustrated in Figure 5 where a square wave change in setpoint was made. The unconstrained least square reduction in the error resulted in a total change in the absolute value of the manipulated input variable of 37.57 taken over 10 intervals of time. With a multiplier of 1.005 the total change was 4.91 and with 1.010 the change was 3.42. The multiplier effectively adds another row to the original data for the input variable for each interval in which it is allowed to move. All elements in the row are zero except for the specific input variable which has a coefficient related to the size of the multiplier. As can be seen from Figure 5, the suppression of the manipulated input variable moved by an order of magnitude did not significantly impair the reduction in the projected error.

The matrix of coefficients which describe the dynamics of the system is the basis for

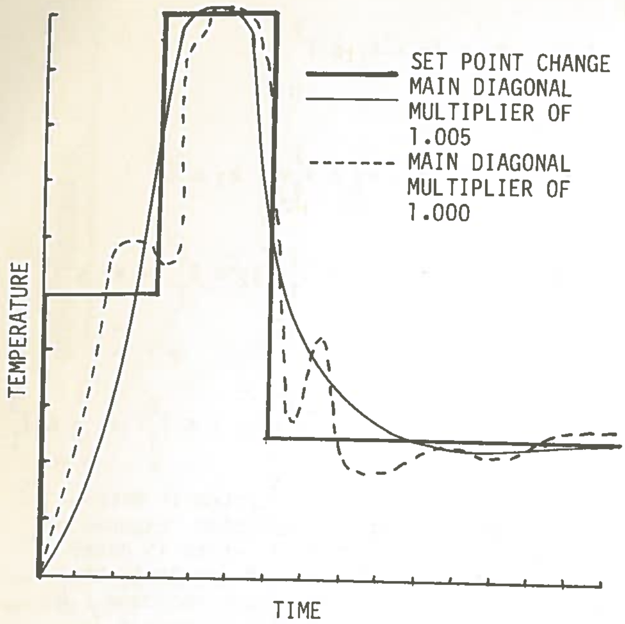

FIG. 5 MOVE SUPPRESSION FUEL TO FURNACE

the DMC Algorithm. For the furnace control problem the response of the output variable was considered for 30 intervals of time and the movement of the fuel gas was considered for 10 intervals. Thirty intervals of time represents about 4 1/2 time constants for the response of the outlet temperature to a change in the fuel. At the tenth time interval, the outlet temperature has three time constants to settle from the last change in the fuel. This choice of time intervals results in a matrix with 10 columns and 30 rows. To initialize the algorithm, the measured outlet temperature is stored into the 30 element vector that represents predicted values of the output variable. This assumes the system is at steady state, but is not a necessary criterion. An error is then calculated from the projected value of the output variable and the setpoint for the 30 intervals of time. This vector of errors becomes the right hand side for the 10 by 30 matrix. The least square solution of this set of equations yields the best set of fuel moves to eliminate the projected errors for 30 time intervals. The projected set of fuel moves is used to calculate the outlet temperature change for the forthcoming 30 intervals of time, and the temperature changes are then added into the 30 element vector for the predicted value of the dependent variable. The first fuel move is implemented and the entire vector of predicted output variable values is shifted forward one interval of time. At the start of the next interval of time the predicted value of the output variable is compared with the measured value. The error in the projection is used to adjust all 30 values in the predicted output variable vector. This adjustment in the prediction provides the feedback to compensate for unmeasured disturbances and errors in the dynamic prediction. At the next interval the set of errors between the setpoint and the predicted values of the output variable is used to solve for another set of 10 fuel moves. The nine remaining fuel moves from the previous calculations are summed with newly calculated fuel moves and the current fuel move is implemented. The pattern is repeated at each successive interval of time.

Feedforward control is implemented by measuring the change in the feed inlet temperature between time intervals, multiplying by the numerical coefficient for the effect of the inlet temperature on the outlet, and summing this response of the outlet temperature into the vector of predicted values for the output variable.

The description of the technique for the DMC was given in some detail to foster understanding. The actual calculations involved in the technique are at least two orders of magnitude less than would be required by the outlined procedure. The first simplification is to recognize that the matrix of coefficients representing the dynamics are fixed and only the right hand side changes from one time interval to the next. Furthermore, the square matrix that results from the formulation of the least square procedure does not change, which also means the inverse matrix does not change. The errors between the projected vector of the output variable and the setpoint appear in the caluclation of the right hand side for the least square formulation. The right hand side of the formulation is the transpose matrix of the matrix given in equation set (2) times the vector of projected errors in the output variable from its setpoint. Consequently, the inverse matrix times the transpose of the original matrix times the error vector gives the projected set of moves in the manipulated input variable. Since the inverse matrix and the transpose are also constant, the calculation is reduced to the multiplication of a constant matrix times the projected error vector. The preceding description of the DMC calculations can be expressed conveniently in the following matrix notation:

$$- (A^T A)^{-1} \; A^T \; \vec{E} = \vec{I} \qquad (5)$$

where the A matrix is the matrix of coefficients describing the dynamics [the a_i of equation set (2)], E is the vector of projected errors, and the I is the vector of projected moves in the input variable. The $(A^T A)^{-1} A^T$ is the constant matrix referred to above. As a further simplification it can be shown that only the first row of the constant matrix and the error vector are needed to solve the control problem. This calculation yields the control move to make at the present interval of time. The other control moves in the input variable are imbedded in the first row of the constant matrix, in the successive updating of the error vector, and by the coupling of time intervals from adding in the projected response from the fuel moves in each interval of time. Since the calculation of $(A^T A)^{-1} A^T$ can be done offline in the furnace control discussed earlier, the actual real time calculations for the DMC Algorithm are reduced to the multiplication of the 30 elements in the first row of the constant matrix times the projected error vector.

A comparison can be made between Figures 6, 7, and 8 to illustrate the merit of the DMC control relative to conventional control techniques. Displayed in each of these figures is the response of the outlet temperature of the preheat furnace to a disturbance in the inlet feed temperature. Each figure represents real data taken off the operating unit. In Figure 6 the performance of the units standard analog temperature controller is illustrated. The temperature controller is cascaded to a flow controller on the fuel and the temperature controller's settings were established after years of experience by the operating personnel. In Figure 7, the temperature control is being accomplished via the computer using a PID algorithm with lead/lag compensation on the feed inlet temperature. The computer output is directed to the flow controller on the fuel. In Figure 8, the DMC is on the temperature control with feedforward control of the inlet temperature. The fuel is on flow control which is reset by the computer. As can be seen from this comparison, the DMC is substantially better than either of the conventional methods of control. A further example which displays the robustness of the DMC is shown in Figure 9. The set point for the outlet temperature of the furnace in this case was being reset by another controller within the computer. The DMC algorithm effectively handled a large temperature disturbance while having a load change imposed on it by the outer control loop of the temperature controller. Fuel moves to handle the disturbance are also shown.

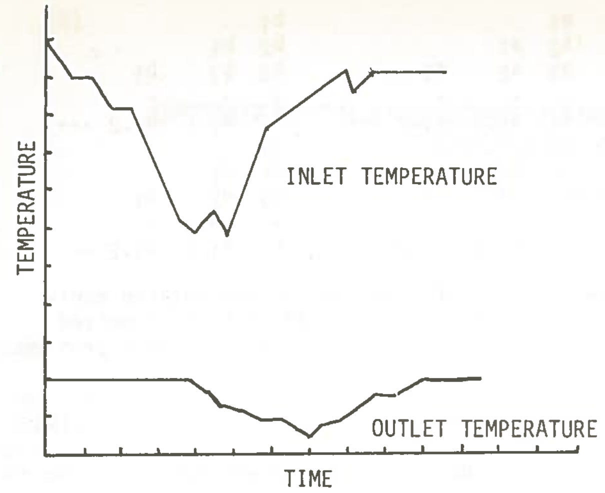

FIG. 6 CONVENTIONAL ANALOG CONTROL OF FURNACE

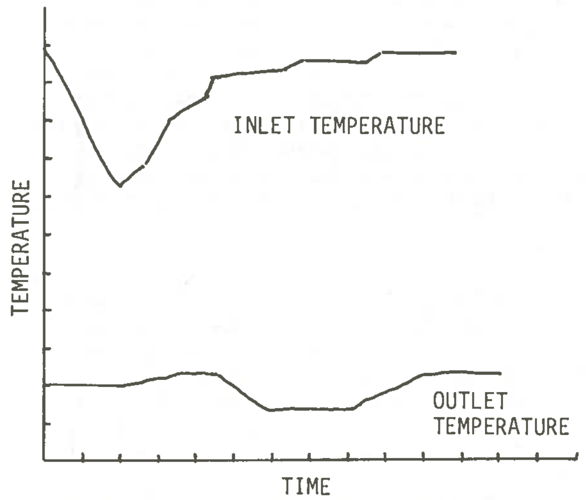

FIG. 7 COMPUTER PID/FEED FORWARD CONTROL OF FURNACE

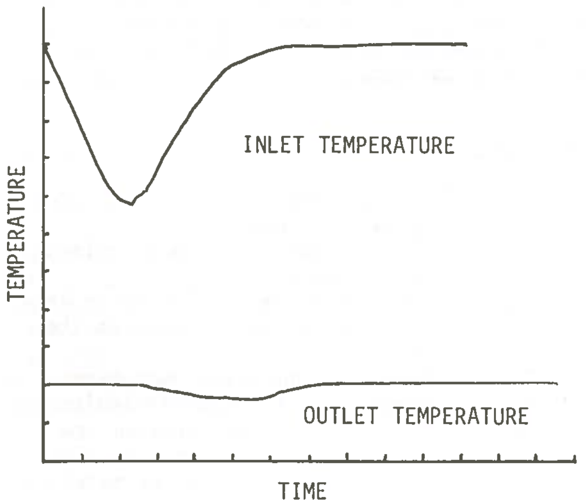

FIG. 8 DYNAMIC MATRIX CONTROL OF FURNACE

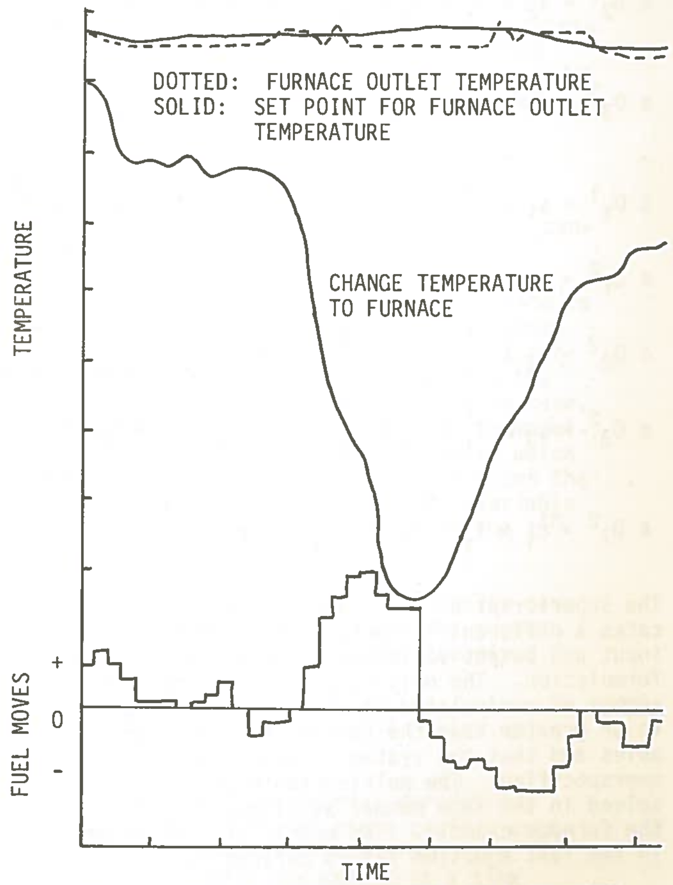

FIG. 9 DYNAMIC MATRIX CONTROL OF A FURNACE

MULTIVARIABLE DMC

In equation set (1), both input variables may be manipulated. In fact, any number of manipulated input variables that influence the output variable may be included, e.g.

$$
\begin{aligned}
\delta O_1 &= a_1 \Delta I_1^1 + b_1 \Delta I_2^1 \qquad (6)\\
\delta O_2 &= a_2 \Delta I_1^1 + a_1 \Delta I_1^2 + b_2 \Delta I_2^1 + b_1 \Delta I_2^2\\
\delta O_3 &= a_3 \Delta I_1^1 + a_2 \Delta I_1^2 + \ldots b_3 \Delta I_2^1 + b_2 \Delta I_2^2\\
&\cdot \qquad \cdot \qquad \cdot \qquad \cdot \qquad \cdot\\
\delta O_i &= a_i \Delta I_1^1 + a_{i-1} \Delta I_1^2 + \ldots b_i \Delta I_2^1 + b_{i-1} \Delta I_2^2
\end{aligned}
$$

where the subscript on I represents a different variable, the superscript on I indicates the time period in which the input is changed, and the a_i, and b_i are the numerical coefficients associated with I_1, I_2, respectively.

To incorporate the control of more than one output variable in the DMC Algorithm, the matrix of coefficients is expanded below to illustrate the case of two manipulated inputs and two outputs:

$$
\begin{aligned}
\delta O_1^1 &= a_1 \Delta I_1^1 + b_1 \Delta I_2^1 \qquad (7)\\
\delta O_2^1 &= a_2 \Delta I_1^1 + a_1 \Delta I_1^2 + b_2 \Delta I_2^1 + b_1 \Delta I_2^2\\
\delta O_3^1 &= a_3 \Delta I_1^1 + a_2 \Delta I_1^2 + b_3 \Delta I_2^1 + b_2 \Delta I_2^2\\
&\cdot \qquad \cdot \qquad \cdot \qquad \cdot \qquad \cdot\\
\delta O_i^1 &= a_i \Delta I_1^1 + a_{i-1} \Delta I_1^2 + \ldots b_i \Delta I_2^1 + b_{i-1} \Delta I_2^2\\
\delta O_1^2 &= c_1 \Delta I_1^1 + d_1 \Delta I_2^1\\
\delta O_2^2 &= c_2 \Delta I_1^1 + c_1 \Delta I_1^2 + d_2 \Delta I_2^1 + d_1 \Delta I_2^2\\
\delta O_3^2 &= c_3 \Delta I_1^1 + c_2 \Delta I_1^2 + d_3 \Delta I_2^1 + d_2 \Delta I_2^2\\
&\cdot \qquad \cdot \qquad \cdot \qquad \cdot \qquad \cdot\\
\delta O_i^2 &= c_i \Delta I_1^1 + c_{i-1} \Delta I_1^2 + \ldots d_i \Delta I_2^1 + d_{i-1} \Delta I_2^2
\end{aligned}
$$

The superscript on the output variables indicates a different variable. The number of input and output variables is unlimited in this formulation. The only constraint is that the number of manipulated input variables be equal to or greater than the number of output variables and that the system of equations be overspecified. The multivariable problem is solved in the same manner as illustrated for the furnace example. The matrix of coefficient in the last equation set is defined as:

$$
A = \begin{array}{cccccccc}
a_1 & & & & b_1 & & & \\
a_2 & a_1 & & & b_2 & b_1 & & \\
a_3 & a_2 & a_1 & & b_3 & b_2 & b_1 & \\
\cdot & \cdot & \cdot & & \cdot & \cdot & \cdot & \\
a_i & a_{i-1} & a_{i-2} & \cdots & b_i & b_{i-1} & b_{i-2} & \cdots\\
c_1 & & & & d_1 & & & \\
c_2 & c_1 & & & d_2 & d_1 & & \\
c_3 & c_2 & c_1 & & d_3 & d_2 & d_1 & \\
\cdot & \cdot & \cdot & & \cdot & \cdot & \cdot & \\
c_i & c_{i-1} & c_{i-2} & \cdots & d_i & d_{i-1} & d_{i-2} & \cdots
\end{array} \qquad (8)
$$

The vector I of moves in the manipulated variables for the multivariable problem is solved the same way as with the single variable problem and is given by:

$$- (A^T A)^{-1} \; A^T \; \vec{E} \; = \; \vec{I} \qquad (9)$$

where E is the vector of project errors in the outputs.

The first application in Shell of the multivariable control was on a catalytic cracking unit. The algorithm has worked well except under conditions when one or more of the manipulated variables became constrained as a result of being integrated with a real time optimization of the process. The next paper will describe the optimization of the catalytic cracking unit and the enhancements to the algorithm which allow the projected moves in the input variables to be constrained in time.

SUMMARY

The DMC algorithm described in this paper has permitted the on-line computer solution of multivariable control problems. Dynamic data taken from the process can be directly inserted to configure the controller. The controller calculates moves for the present time and a number of future time periods. This feature always for the correct compensation for processes with dead time. Feedback has been incorporated into the control algorithm to compensate for unmeasured disturbances and dynamic model errors. Feedforward is simply configured into the controller by the addition of the dynamic data for measured disturbances. This technique has been successfully applied to control problems within Shell for the last six years.

NOMENCLATURE

a_i, b_i, c_i, d_i =	Numerical Coefficients Describing Process Dynamics
A =	Matrix of Coefficients Describing Process Dynamics
E =	Vector of Projected Errors in Time
I =	Vector of Projected Moves in the Input Variable
Δ Ij =	Changes in the Input Variables
δ O_i =	Changes in the Output Variables