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MATHEMATICS AND COMPUTING MACHINERY SERIES No. 1

JENS G. BALCHEN

A Performance Index for Feedback Control Systems Based on the Fourier Transform of the Control Deviation

Norwegian Contribution No. 1
Trondheim 1958

A performance index for feedback control systems based on the Fourier transform of the control deviation.

Jens G. Balchen [x)]

Summary.

This paper discusses some commonly used performance indices for optimum adjustment of linear control systems.

A new index is developed which is simple to apply in conjunction with a graphical frequency response analysis. A method for introduction of additional constraints assuring proper system stability and damping is demonstrated.

When designing a feedback control system one is often faced with the problem: What is the best choice of the system parameters? A number of authors have discussed this problem, and for special cases there exists a variety theoretical methods and practical rules of thumb. The development of computing machines and simulators has made it possible to investigate a large number of different types of systems with the application of different optimum criteria and the results of such investigations have been published. [1] [2] [3] [4] [5]

Whether or not a comprehensive study of the optimum adjustment of a feedback control system is justified, is highly questionable. First of all it must be made clear what is meant by the optimum conditions and what this term embraces. A system can be adjusted according to a certain criterion for one particular kind of disturbance, while the same system will not show optimum performance when a different disturbance is applied. In general the system can only be adjusted to optimum performance for one type or one class of disturbance, and it has, therefore, become common practice to choose among a few standard types of disturbances. Although the actual disturbance known to occur in a system, is quite different, one often sees the step function disturbance used when optimum conditions are determined. The system obtained in such a manner will certainly not give optimum performance when the disturbance has a stochastic nature. The optimum adjustment will then have to be referred to the system behaviour in mean.

Further the price to be paid for an optimum design will often have to be taken into consideration. When doing so the final solution might be a compromise

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which to some extent makes a lengthy computation unjustified.

At last it should be pointed out that an optimum system should be realizable in a practical sense and not only exist on paper. It is therefore important to take into account non-linear effects and power limitations. An optimization based on the assumption of linearity will often be invalid in the practical case because one or more elements in the system reach saturation levels when the actual disturbance is applied, or a large static friction might cause permanent deviations or stable oscillations.

In spite of these reservations, however, it will be appreciated that the understanding gained by performing an analysis of the optimum adjustment of a control system under idealized conditions will become an aid in judging the performance of a nonideal system. Because the problems are so different, one must resort to idealized assumptions in order to get methods, which have some degree of generality.

2. Comments on common performance indices.

In the following a few comments will be given concerning the most frequently used performance indices which can be bases for an optimum adjustment of a feedback control system subject to a transient disturbance. Figure 1 shows the system to be studied. The two transferfunctions $H_1(s)$ and $H_2(s)$ can respectively represent the controller and the process in the actual system.

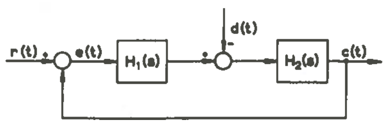

Fig. 1: Blockdiagram of elementary feedback control system.

The terms used are:

r (t), R (s) = reference, setpoint,

d (t), D (s) = disturbance,

c (t), C (s) = controlled quantity,

e (t), E (s) = deviation.

Most performance indices described in the literature are based on a study of the shape of the function e (t) in the range $0 < t < \infty$. The performance indices are usually chosen so that they take into account the magnitude of the deviation and the time during which the deviation exists. The system which gives the least value of the performance index is defined as the optimum. Usually performance indices are expressed as an integral over infinite time limits of some functions of the deviation e (t). Most commonly referred to are the following indices:

$$I_1 = \int_0^\infty e(t)\,dt \qquad 2.1$$

$$I_2 = \int_0^\infty |e(t)|\,dt \qquad 2.2$$

$$I_3 = \int_0^\infty e^2(t)\,dt \qquad 2.3$$

$$I_4 = \int_0^\infty t\,|e(t)|\,dt \qquad 2.4$$

$$I_5 = \int_0^\infty t e^2(t)\,dt \qquad 2.5$$

The figures 2 to 6 show how these integrals can be expressed as areas.

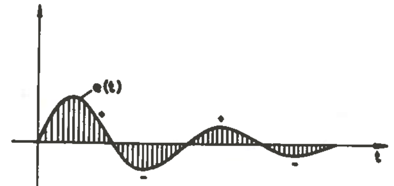

Fig. 2

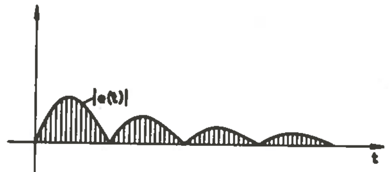

Fig. 3

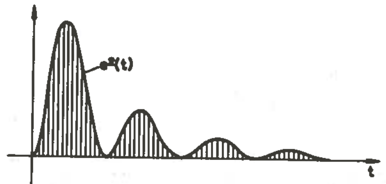

Fig. 4

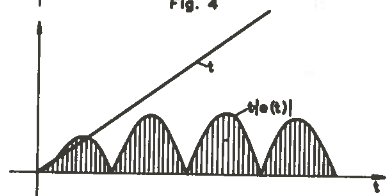

Fig. 5

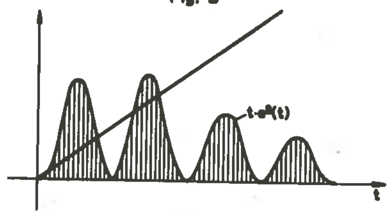

Fig. 6

Fig. 2, 3, 4, 5, 6: Illustration of different indices in terms of areas.

Three of these integrals can readily be solved when the Laplace transform $L(e(t)) = E(s)$ of the deviation is known. The expressions are:

$$I_1 = \int_0^\infty e(t)dt = \lim_{s \to o} \left(s \frac{E(s)}{s}\right) = E(o) \qquad 2.6$$

$$I_3 = \int_0^\infty e^2(t)dt = \lim_{s \to o} \left\{ \frac{1}{2\pi j} \int_{\gamma - j\infty}^{\gamma + j\infty} E(s-w)\ E(w)dw \right\} =$$

$$= \frac{1}{2\pi j} \int_{\gamma - j\infty}^{\gamma + j\infty} E(-w)\ E(w)dw \qquad 2.7$$

$$I_5 = \int_0^\infty t\, e^2(t)dt = \lim_{s \to o} \left\{ \frac{1}{2\pi j} \int_{\gamma - j\infty}^{\gamma + j\infty} E(s-w) \left(- \frac{dE(w)}{dw} \right) dw \right\} =$$

$$= - \frac{1}{2\pi j} \int_{\gamma - j\infty}^{\gamma + j\infty} E(-w) \left(\frac{dE(w)}{dw} \right) dw \qquad 2.8$$

The other integrals I_2 and I_4 can not be expressed directly from knowledge of $E(s)$. The integrals of equation 2.7 and 2.8 can easily be computed as the sum of the residues of the integrand when the poles of the function $E(-w)$ are known. According to the definition the optimum is achieved when the integral value is a minimum, and as the integral is a function of several adjustable parameters p_1, p_2 .... p_n, the minimizing procedure will become:

$$\frac{\partial I}{\partial p_1} = 0 \ , \ \frac{\partial I}{\partial p_2} = 0 \ , \ ..... \qquad 2.9$$

and so on for all the parameters.

An analytical determination of the optimum values of the parameters will usually be a very timeconsuming task. Only for the simplest types of systems can such calculations be performed with a reasonable effort, and therefore the practical solution, in most cases, will have to be found by means of a graphical minimization as shown in fig. 7.

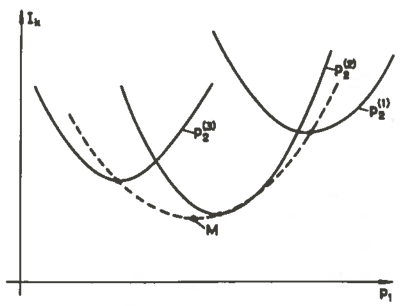

Fig. 7: Graphical minimization procedure.

The values of the parameters at the minimum point (M) will have to be determined by graphical interpolation.

The indices described above will certainly indicate different system characteristics and the choice of index will therefore largely be dependent on the significance of the magnitude and the duration of the deviation. Some comments on the most typical features of the five indices described might be appropriate.

$$\text{Index } I_1 = \int_0^\infty e(t)dt$$

The main feature of this index is that it is extremely simple to calculate. According to fig. 2, however, it will only be valid as a practical performance index provided that the deviation $e(t)$ does not have positive and negative oscillations. This index, therefore, is only useful in connection with constraints which take care of a suitable damping. Thereby the computation is no longer simple. Furthermore, this index as well as the others, is only valid when $e(\infty) = 0$.

$$\text{Index } I_2 = \int_0^\infty |e(t)| \cdot dt$$

This is often termed a logical index because it weighs the deviation according to its absolute value, thereby assuring a positive damping of the time function. However, a reasonable damping will not always be obtained when minimizing the index I_2. A very simple system containing only one adjustable parameter will demonstrate this fact.

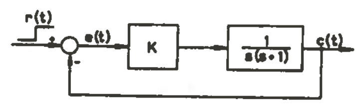

Fig. 8: Blockdiagram of elementary system with adjustable gain only.

Fig. 8 shows a blockdiagram of the system in which the adjustable parameter is the gain factor K. If the disturbance is assumed to be a step in the reference $(r(t))$, the deviation $(e(t))$ will become an exponentially damped oscillation as shown in fig. 9 for a small and a large value of K.

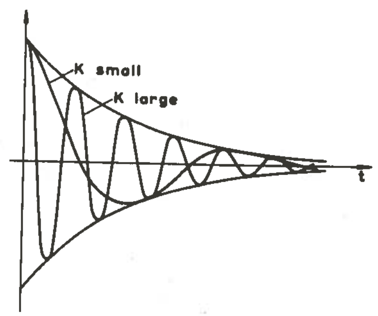

Fig. 9: Deviation response of system in fig. 8.

This response is characterizied by the location of the poles of the closed loop transferfunction. For actual values of K the poles will be located on a straight line parallel to the imaginary axis as shown in fig. 10.

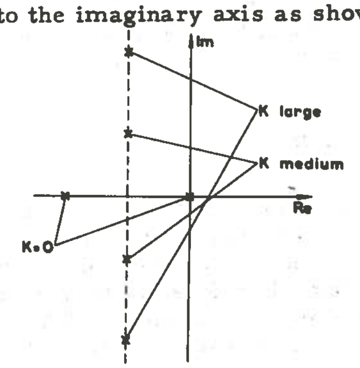

Fig. 10: Location of poles of system in fig. 8.

The result of this is that the absolute damping of the deviation is constant and does not decrease when K is increasing. The integral value I_2 thus approaches a limit equal to $\frac{2}{\pi} \cdot A$ when K is increasing. The factor A is the area under the exponential envelope of the time function.

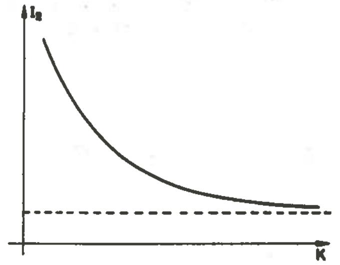

Fig. 11: Value of index I_2 for the system in fig. 8.

Fig. 11 shows the shape of the integral value as a function of K. Thus the index I_2 will indicate the optimum system for $K = \infty$. The obvious reason for this result is that the index I_2 does not contain any factor which takes care of the system stability. In this case, as well, it will be necessary to include in the optimum criterion an additional constraint which assures a sufficiant relative damping or stability margin.

$$\text{Index } I_3 = \int_0^\infty e^2(t)dt$$

The majority of authors discussing optimum adjustment of control systems have based their investigations on this index mainly because the integral value can be expressed analytically. A characterizing feature of this index is that it weighs large deviations heavier than small. The consequence of this is that a system adjusted according to this index will show slowly disappearing small oscillations while the first large deviation will be greatly reduced compared to what the index I_2 would have given. Some authors state that it is logical to weigh large deviations more heavily than small. This, however, should certainly be dependent on the type of system discussed. The index I_3 thus has the same disadvantage as the index I_2; it does not assure

stability for all types of systems. Thus additional constraints are necessary resulting in considerable difficulties.

$$\text{Index } I_4 = \int_0^\infty t \left| e(t) \right| dt$$

The consequence of including the time factor, t, in this index is that long lasting deviations will be suppressed due to their heavy weight. Neverthe-less this index is not quite effective in assuring system stability. Application of the index I_4 to the system in fig. 8. will show the same discrepancy as the previous indices because the factor $t\, e^{-a.t} \to 0$ when $t \to \infty$ where a = absolute damping factor. The main disadvantage of the index, however, is that computers or simulating equipment are necessary to determine the optimum setting.

$$\text{Index } I_5 = \int_0^\infty t.\, e^2(t)dt$$

This index has almost the same properties as the preceding index although it lends itself to analytical computation. Such computations will, how-ever, be quite timeconsuming as stated above.

The conclusion to be drawn from these rather superficial comments is that none of the simple performance indices give an adequate indication of system performance and therefore are not sufficient as bases for an optimum adjustment.

3. A performance index based on the Fourier transform of the deviation.

A number of authors have made simplifications and approximations to the indices described above so they could be applied to practical problems. Nevertheless, the computational work required still makes the methods quite impractical. It seems feasible to develope a simplified performance index which takes advantage of the very practical frequency response technique. The index, which will be developed has much in common with the index $I_2 = \int_0^\infty |e(t)| dt$, but it can be determined from the knowledge of the frequency response characteristics of the closed loop system. It is therefore applicable in case such characteristics are just known from experimental data.

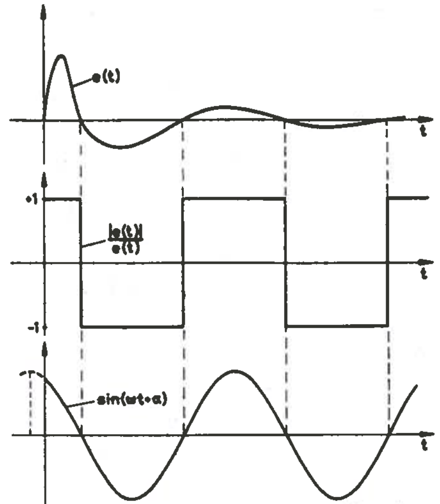

Fig. 12: Approximation of the function $\frac{|e(t)|}{e(t)}$ by a sine wave.

The two top sketches in fig. 12 illustrates how $|e(t)|$ can be thought of as the product $e(t) \cdot \frac{|e(t)|}{e(t)}$. In case the time function is oscillatory, the function $\frac{|e(t)|}{e(t)}$ will become a square wave of some kind with the magnitude ± 1. This square wave will have zero crossings at the same points as the original time function $e(t)$, but as the desired optimum time function always will be well damped, only a minor error would be introduced in the integral value by substituting the correct square wave by some square wave with equidistant zero crossings. Then by substituting the fundamental component (sinewave) for this artificial squarewave a second approximation to the original function is obtained. The frequency, ω, and the phase angle, α, of the sine wave which gives the best approximation is not known a priori, but can be determined by requiring that the integral

$$I_6(\omega,\alpha) = \int_0^\infty e(t)\sin(\omega t + \alpha)dt \qquad 3.1$$

be a maximum for the particular function $e(t)$ chosen. The first problem, therefore, is to determine the parameters, ω and α, which make $I_6(\omega,\alpha) = \max$. Two simple cases will illustrate this. First assume $e(t) \geq 0$ for all $t \geq 0$. Then it is easily seen that $I_6(\omega,\alpha) = \max$ when $\omega = 0$ and $\alpha = \frac{\pi}{2}$, i.e.

$$I_6(\omega,\alpha) = \int_0^\infty e(t).1\,dt = \int_0^\infty |e(t)|\,dt$$

If the deviation has the form

$$e(t) = e^{-at}.\ \sin(\beta t + \psi)$$

it follows that

$$I_6(\omega,\alpha) = \int_0^\infty e^{-at}.\ \sin(\beta t + \psi)\ .\ \sin(\omega t + \alpha)dt = \max \qquad 3.2$$

when $\omega = \beta$ and $\alpha = \psi$, since this condition gives an integrand which is positive for all $t \geq 0$.

The integral (3.1) can easily be developed when the Laplace transform of the deviation is known.

$$I_6(\omega,\alpha) = \int_0^\infty e(t)\frac{1}{2j}\left\{e^{j\omega t}.\ e^{j\alpha} - e^{-j\omega t}.\ e^{-j\alpha}\right\}dt =$$

$$= \frac{1}{2j}\left\{e^{j\alpha}.\int_0^\infty e(t)e^{j\omega t}\,dt - e^{-j\alpha}.\int_0^\infty e(t).e^{-j\omega t}\,dt\right\} \qquad 3.3$$

because

$$\mathcal{L}\left((e(t).e^{\pm j\omega t}\right) = E(s \mp j\omega) \qquad 3.4$$

and

$$\int_0^\infty g(t).\,dt = G(0)$$

it follows that

$$I_6(\omega,\alpha) = \frac{1}{2j}\left[e^{j\alpha}.\ E(-j\omega) - e^{-j\omega}.\ E(j\omega)\right] \qquad 3.5$$

This can be written

$$I_6(\omega,\alpha) = -\ |E(j\omega)|\ .\ \sin(\varphi - \alpha) \qquad 3.6$$

where φ is the phase angel of the vector $|E(j\omega)|$.
Equation 3.6 has its positive maximum when $\sin(\varphi - \alpha) = -1$ i.e. $\alpha = \frac{\pi}{2} + \varphi$ and at the value of ω which makes $|E(j\omega)| = \max$. The problem is, therefore, reduced to determining the maximum value of $|E(j\omega)|$ for different values of the adjustable parameters. The optimum adjustment of this system will thus be determined by the expression:

$$I_6 = \int_0^\infty e(t).\sin(\omega t + \alpha)dt = |E(j\omega)|_{max} = \min \qquad 3.7$$

By comparing three of the performance indices developed up to this point, some interesting relations are discovered:

$$I_1 = E(j\omega)_{j\omega = 0}$$

$$I_3 = \frac{1}{\pi}\int_0^\infty |E(j\omega)|^2 d\omega$$

$$I_6 = |E(j\omega)|_{max}$$

Furthermore it is seen that when $e(t) \geq 0$ for all $t \geq 0$:

$$I_1 = I_6 = I_2 \qquad 3.8$$

and when $e(t)$ assumes both positive and negative values:

$$I_1 < I_6 < I_2 \qquad 3.9$$

These properties are demonstrated in fig. 13 for a system with only adjustable parameter, K, and decreasing absolute damping with increasing K. By just minimizing the index, I_6, the system will become slightly more oscillatory than by minimizing I_2, but less oscillatory than by minimizing I_3. Physically the index, I_6, might be interpreted as the maximum energy of the deviation associated with any single frequency.

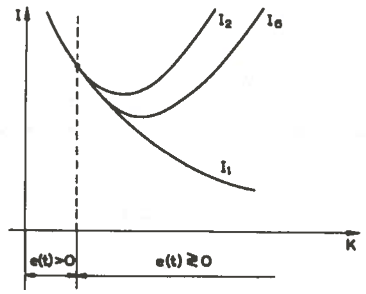

Fig. 13: Comparison of the values of I_1, I_2 and I_6.

4. Practical application of the performance index I_6.

The main advantage of applying the performance index, I_6, to optimum adjustment of control systems is that the method makes use of frequency response characteristics which usually are drawn anyway. In addition it bears a very close analogy to the resonance peak criterion commonly applied when designing servomechanisms.

A straight foreward procedure for determination of the index values for different settings of the system parameters can be outlined:

1. Amplitude- and phase characteristics versus frequency for the open loop transfer function $H_1(j\omega) \cdot H_2(j\omega)$ (fig. 1) are drawn for different values of the adjustable parameters.
2. This data are transferred to an amplitude - phase diagram equipped with constant magnitude loci for the quantity $|N(j\omega)| = |\frac{E}{R}(j\omega)|$. These loci are identical to the loci of a Nichols diagram when reversed.
3. The maximum value of the product $|D(j\omega)| \cdot |H_2(j\omega)| \cdot |N(j\omega)|$ is determined for different values of the parameters. These are the index values.
4. These index values are used when drawing curves of the type shown in fig. 7.

In general it is not necessary to draw the complete set of frequency response curves because only the maximum value of $|E(j\omega)|$ is required. A great help in determining the maximum product is achieved when the curves in the amplitude - phase diagram are drawn in terms of the quantity $|D(j\omega)| \cdot |H_2(j\omega)|$ instead of the frequency, ω, as usually is done. This procedure is demonstrated in fig. 16 by the following example.

Example 1.

A simple control loop containing a P.I.-controller and a process with three equal time constants is chosen to demonstrate the method. The system under consideration is shown in the block diagram in fig. 14.

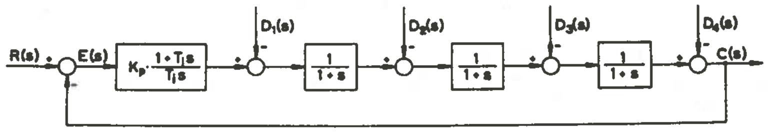

Fig. 14: Blockdiagram of system of example 1.

Disturbances can enter the system at four different points, but only the disturbance D_1 is considered in this example. The optimum adjustment of the controller parameters, K_p and T_i, is sought when the disturbance is a unit step function $D_1(s) = \frac{1}{s}$. Amplitude- and phase diagrams versus frequency for different values of the integral time, T_i, are shown in fig. 15. These curves are redrawn in the amplitude-phase-diagram in fig. 16 and the magnitude of the quantity $|D(j\omega)| \cdot \left|\frac{1}{1+j\omega}\right|^3$ in db is marked along the curves

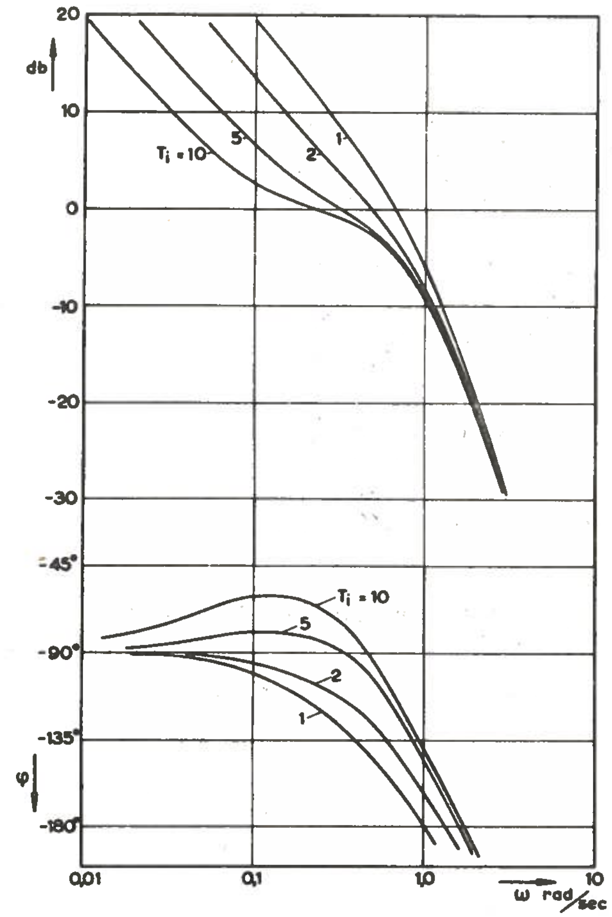

Fig. 15: Amplitude and phase characteristics of open loop system in fig. 14.

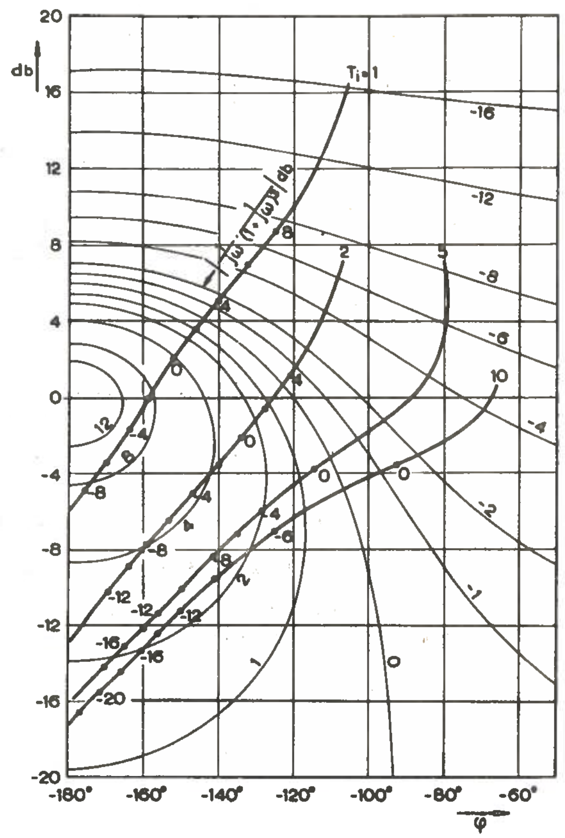

Fig. 16: Amplitude-phase loci of system in fig. 14.

The curves in fig. 16 are drawn for $K_p = 0$ (db). Other values of K_p can be considered by moving the N-loci in the vertical direction. These loci may be drawn separately on transparent paper. The resulting values of $|E(j\omega)|_{max}$ as a function of K_p are plotted in fig. 17 for four different values of T_i. As an aid in plotting these curves, straight line asymtotes describing $I_1 = E(0) = \frac{T_i}{K_p}$ are drawn in fig. 17. The minimum point of the four V-shaped curves are connected together with a fifth V-shaped curve whose minimum point determines the optimum setting of the system.

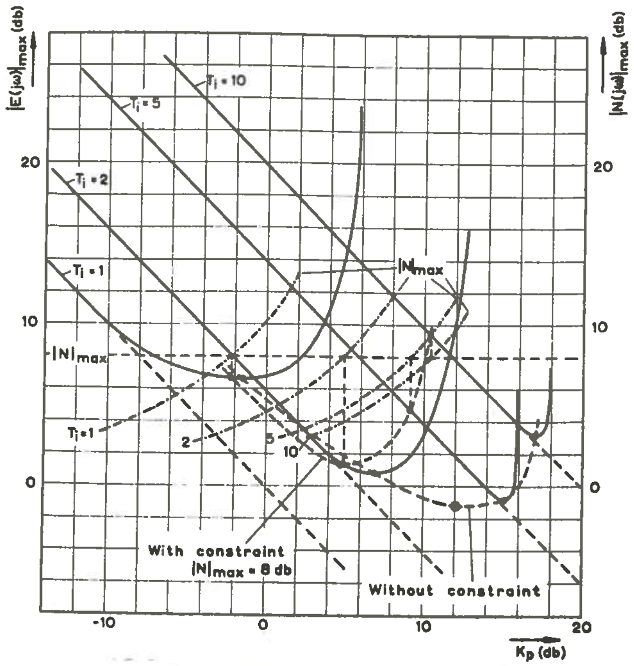

Fig. 17: Graphical minimization of index I_6 for system in fig. 14.

$y = \frac{1}{(s+1)^3}$
Balchen
$T_i = 3$
$K_p = 12db = 4$
SIMC: $g \approx \frac{e^{-1.5s}}{1.5s+1}$
$\tau_c = \theta = 1.5$
$T_i = 8\theta = 12$
$K_p = \frac{1}{k}\frac{1.5}{1.5 \cdot 2} = 0.5$

The result is $T_i \approx 3$ sec and $K_p = 12$db.

For the sake of comparison fig. 18 shows the equivalent V-shaped curves for the performance index $I_2 = \int_0^\infty |e(t)|\, dt$.

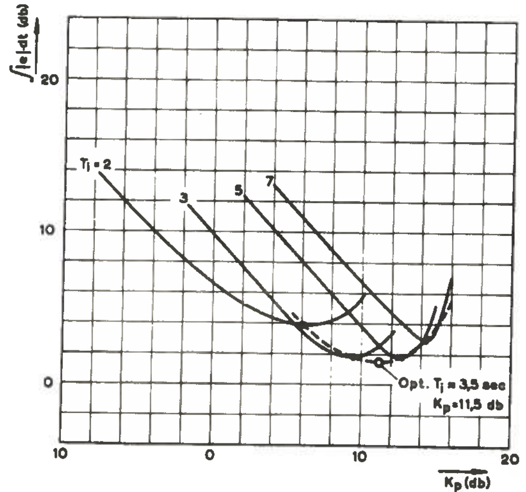

Fig. 18: Graphical minimization of index I_2 for system in fig. 14.

These curves have been obtained by means of an analog computer. The two indices give very closely the same result. The deviation as a function of time for this setting of the controller is shown in fig. 19. It will be seen at once that the system has a rather low degree of damping. As previously stated this is because the performance index does not take care of the relative stability. Obviously an additional constraint should be included to make the method useful. How this is done will be demonstrated in the next section.

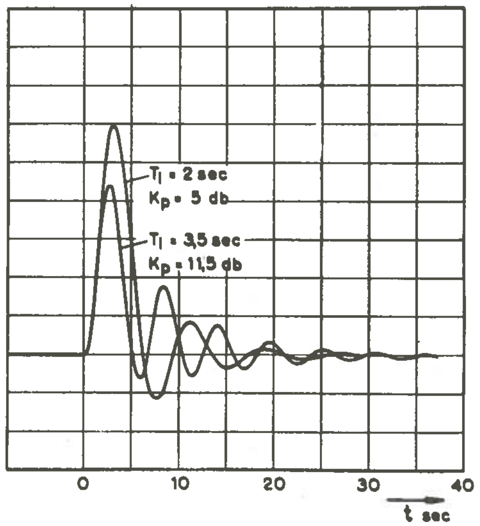

Fig. 19: Deviation response of system in fig. 14 for two different settings of the controler.

Example 2.

A control system for a waterturbine working on a long pipeline is chosen as the next example because the influence of the pipeline gives a rather interesting system. The transferfunction describing the relation between the controller output and the torque developed in the turbine is assumed to be of the form $\frac{1 - s}{1 + 0.5s}$ which gives a reasonable approximation to the actual system. A step change in power demand is considered as disturbance, and the problem is to determine the parameters of the controller which is of the ideal P. I. - type. A block diagram of this system is shown in fig. 20.

$y = \frac{1-s}{(1+0.5s)s}$
Balchen
$T_i = 5$
$K_p = -3.5dB = 1.5$
SIMC: $g \approx \frac{e^{-1.25s}}{s}$
$T_i = 10$
$K = \frac{1}{2.5} = 0.4$

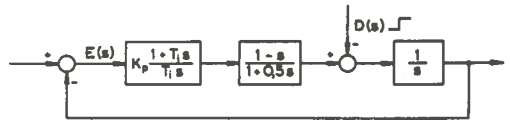

Fig. 20: Blockdiagram of turbine control system of example 2.

Following the same procedure as outlined in the preceding example, curves representing the index value as a function of the parameters are determined. Fig. 21 shows the result indicating an optimum setting at $T_i = 5$ sec and $K_p = -3{,}5$ db. Corresponding time functions for the turbine speed are shown in fig. 22.

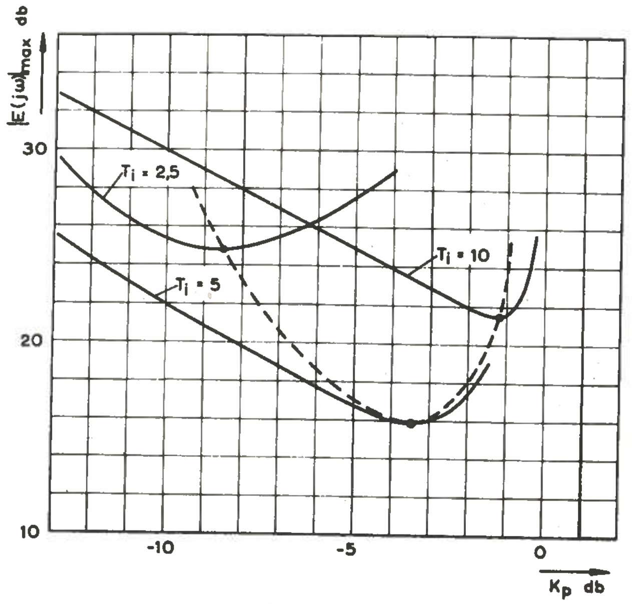

Fig. 21: Graphical minimization of index I_6 for system in fig. 20.

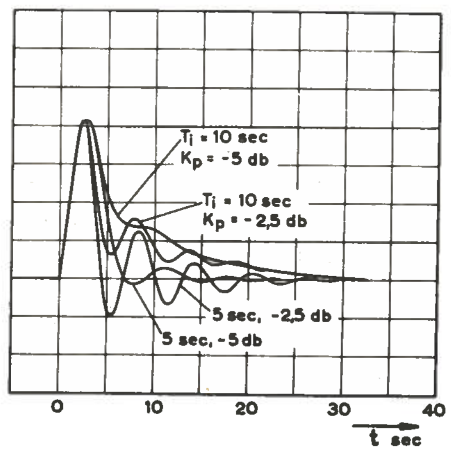

Fig. 22: Deviation response of system in fig. 20 for different settings of the controler.

5. Introduction of constraints to assure proper damping.

The necessity of introducing additional constraints have been pointed out above. As the shape of the deviation as a function of time is highly dependent on the type and location of the disturbance, it is generally of no use to present constraints on the basis of a damping factor. Of more general validity would be a specification of a stability margin in terms of the quantity $|N(j\omega)| = |\frac{E}{R}(j\omega)|$. This type of constraint is also much easier to apply. How much stability margin to be specified will certainly be dependent on the type of system, the damping required, and the variability of the process parameters. As far as damping is concerned, a value of $|N(j\omega)|_{max} = 6 - 8$db will usually be appropriate. In fig. 17 curves are drawn which determine the optimum adjustment of the system of example 1 when the two conditions $|E(j\omega)|_{max} =$ min and $|N(j\omega)|_{max} =$ 8db are satisfied simultaneously. The optimum setting of the system according to this criterion will be $T_i \approx 2$ and $K_p = 5$db. The corresponding time function is drawn in fig. 19.

Several other methods are possible for introducing constraints. Some authors have suggested to take into account not only the deviation but also its derivative in the performance index. This can be done by using the following index:

$$I_7 = \int_0^\infty (|e(t)| + \sigma|\dot{e}(t)|)dt \qquad 5.1$$

where σ is a factor dependent on the system bandwidth. Instead of the absolute value function, a square function could be used. The main disadvantage of this formulation is that it is difficult a priori to express a reasonable value of the factor σ. This drawback is eliminated by the following formulation:

$$I_8 = \int_0^\infty |e(t)|\,dt \cdot \int_0^\infty |\dot{e}(t)|\,dt \qquad 5.2$$

Expressed in terms of the Fourier transform approximation described previously, the last performance index would then get the form:

$$I_9 = |E(j\omega)|_{max} \cdot |j\omega\ E(j\omega)|_{max} \qquad 5.3$$

Or when expressed in logarithmic scale

$$I_9(db) = |E(j\omega)|_{max}(db) + |j\omega \cdot E(j\omega)|_{max}(db) \qquad 5.4$$

The last term of this index is easily determined following the procedure described in sec. 4. By minimizing I_9 a compromise is actually obtained between the optimum setting for the disturbance function itself and the optimum setting for the derivative of the disturbance function. Applying this procedure to the system of example 1 would give an optimum setting of T_i = 3 sec and K_p = 8 db.

Conclusion.

The performance index described makes it possible by graphical means to determine the optimum adjustment of linear control systems subject to transient disturbances of prescribed form. The system may be described by frequency response characteristics found experimentally and additional constraints assuring proper damping in all cases can easily be introduced. The performance index gives, by means of well known techniques, a quantitative measure of the influence of adjustable system parameters. However, if the shape of the disturbance is not specified, or if the system is not linear, this method and other equivalent methods are not applicable.

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