Systematic design of advanced control structures

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Motivation and scope

**DV: disturbance variable \( (d) \)**
- Ambient temperature
- Raw materials
- Desired production

**CV: controlled variable \( (y) \)**
- Temperature
- Pressure
- Concentration

**MV: manipulated variable \( (u) \)**
- Valve opening
- Compressor rotational speed
Motivation and scope

Top-down analysis:
S1-S4: Identify steady-state optimal operation

Bottom-up analysis:
S5-S7: Design control structure
Motivation and scope

Bottom-up analysis:
S5: regulatory control layer
S6: supervisory control layer
S7: online optimization layer
Motivation and scope

Bottom-up analysis:
S5: regulatory control layer
S6: supervisory control layer
S7: online optimization layer

Keeps operation in the right active constraint region
Motivation and scope

Constraint region
«region in the disturbance space defined by which constraints are active within it»

S6: supervisory control layer

Motivation and scope

S6: supervisory control layer

Model predictive control

Advanced control structures
Active constraint switching with classical advanced control structures

Figure taken from www.transmittershop.com/blog/causes-solutions-annoying-noise-control-valves
Active constraint switching with classical advanced control structures

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Figure taken from www.transmittershop.com/blog/causes-solutions-annoying-noise-control-valves
Design procedure for active constraint switching with classical advanced control structures

A1 • Define control objectives, CV constraints and MV constraints
A2 • Organize constraints in priority list
A3 • Identify possible and relevant active constraint switches
A4 • Design control structure for optimal operation
A5 • Design control structure to handle active constraint switches
Design procedure for active constraint switching

**Case study:**
Mixing of air and MeOH
Design procedure for active constraint switching

Step A1: Define control objectives, CV constraints and MV constraints

Control objectives:
• Keep $y_1 = x_{\text{MeOH}} = 0.10 \leftarrow \text{ideal}$
• Keep $y_1 = x_{\text{MeOH}} > 0.08$
• Control $y_2 = m_{\text{tot}} \leftarrow \text{ideal}$
Design procedure for active constraint switching

Step A1: Define control objectives, CV constraints and MV constraints

Control objectives:
- Keep $y_1 = x_{MeOH} = 0.10$
- Keep $y_1 = x_{MeOH} > 0.08$
- Control $y_2 = m_{tot}$

<table>
<thead>
<tr>
<th>Variable</th>
<th>Units</th>
<th>Maximum</th>
<th>Nominal</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_1$</td>
<td>kmol/kmol</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>$y_2$</td>
<td>kg/h</td>
<td>26860</td>
<td>26860</td>
</tr>
<tr>
<td>$u_1$</td>
<td>kg/h</td>
<td>25800</td>
<td>23920</td>
</tr>
<tr>
<td>$u_2$</td>
<td>kg/h</td>
<td>25800</td>
<td>2940</td>
</tr>
</tbody>
</table>

$u_1$ is has a maximum value
Design procedure for active constraint switching

Step A2: Organize constraints in priority list
### Design procedure for active constraint switching

**Step A2: Organize constraints in priority list**

<table>
<thead>
<tr>
<th>Priority (P)</th>
<th>Constraints</th>
<th>Equations</th>
</tr>
</thead>
</table>
| (P1) Physical MV inequality constraints | Constraint on air flow \((u_1)\)  
Constraint on MeOH flow \((u_2)\)  
\[ \dot{m}_{\text{air}}^{\text{min}} \leq \dot{m}_{\text{air}} \leq \dot{m}_{\text{air}}^{\text{max}} \]  
\[ \dot{m}_{\text{MeOH}}^{\text{min}} \leq \dot{m}_{\text{MeOH}} \leq \dot{m}_{\text{MeOH}}^{\text{max}} \] |  |
| (P2) Critical CV inequality constraints | Constraint \((\text{max and min})\) on \(x_{\text{MeOH}}\) \((y_1)\)  
\[ x_{\text{MeOH}}^{\text{min}} \leq x_{\text{MeOH}} \leq x_{\text{MeOH}}^{\text{max}} \] |  |
| (P3) Less critical CV and MV constraints | Setpoint on \(x_{\text{MeOH}}\) \((y_1)\)  
\[ x_{\text{MeOH}} = x_{\text{MeOH}}^{sp} \] |  |
| (P4) Desired throughput | Setpoint on \(m_{\text{tot}}\) \((y_2)\)  
\[ \dot{m}_{\text{tot}} = \dot{m}_{\text{tot}}^{sp} \] |  |
| (P5) Self-optimizing variables | No unconstrained degrees of freedom |  |
Design procedure for active constraint switching

Step A3: Identify possible and relevant active constraint switches

- **Case 1: CV to CV constraint switching**
  One MV switching between two alternative CVs.
Design procedure for active constraint switching

Step A3: Identify possible and relevant active constraint switches

- **Case 2: MV to MV constraint switching**
  - More than one MV for one CV.

Split range control

Valve position control
Design procedure for active constraint switching

Step A3: Identify possible and relevant active constraint switches

- **Case 3: MV to CV constraint switching**
  MV controlling a CV that may saturate; no extra MVs

- MV that does not saturate
- High priority CV: always controlled
- Low priority CV:

Input saturation pairing rule
«an MV that is likely to saturate at steady-state should be paired with a CV that can be given up»
Design procedure for active constraint switching

Step A3: Identify possible and relevant active constraint switches

- **Case 3: MV to CV constraint switching**
  MV controlling a CV that may saturate; no extra MVs

**Following input saturation pairing rule**

**NOT following input saturation pairing rule**
Design procedure for active constraint switching

**Step A3: Identify possible and relevant active constraint switches**

- At nominal operation point all constraints are satisfied

- **Constraint switch:**
  - Reach maximum air flow ($u_1$)

  - Lose a degree of freedom (**case 3**)
    - Must give up controlling the constraint with the lowest priority (desired throughput)

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<tr>
<td>$y_1 = x_{MeOH}$</td>
<td>kmol/kmol</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>$y_2 = \dot{m}_{tot}$</td>
<td>kg/h</td>
<td>26860</td>
<td></td>
</tr>
<tr>
<td>$u_1 = \dot{m}_{air}$</td>
<td>kg/h</td>
<td>23920</td>
<td></td>
</tr>
<tr>
<td>$u_2 = \dot{m}_{MeOH}$</td>
<td>kg/h</td>
<td>2940</td>
<td></td>
</tr>
</tbody>
</table>

$$u_1 = \dot{m}_{air}^{max}$$

$\dot{m}_{air}^{max}$
Design procedure for active constraint switching

Step A4: Design control structure for optimal operation

Case A

Following input saturation pairing rule

Case B

NOT following input saturation pairing rule
Design procedure for active constraint switching

Step A4: Design control structure for optimal operation

Case A
Following input saturation pairing rule

Case B
NOT following input saturation pairing rule

Needs MV to CV switching
Design procedure for active constraint switching

Step A5: Design control structure to handle active constraint switches

Case B-SRC
Split range control + selector

Case B-VPC
Valve position control + selector
Design procedure for active constraint switching

Case study: Mixing of air and MeOH

High priority CV: concentration

Low priority CV (throughput)

MV1 is saturated: lost degree of freedom

MV2 is not saturated: It should be used to control the high priority CV
MV to MV constraint switching

Split range control

Valve position control

Different controllers with different setpoints
Classical split range control

Monogram of Instruments and Process Control prepared at Cornell, NY, in 1945

Classical split range control

\[ y^{sp} \rightarrow e \rightarrow C \rightarrow v \rightarrow SRC \rightarrow u_1, u_2 \rightarrow u_i \rightarrow G \rightarrow y \]

- \( v \): internal signal to split range block → limited physical meaning
- \( v^* \): split value
- \( u_i \): controller output → physical meaning
- \( \alpha_i \): gain from \( v \) to \( u_i \) → slope

![Diagram of classical split range control](image)
Classical split range control

- $v$: internal signal to split range block $\rightarrow$ limited physical meaning
- $v^*$: split value $\rightarrow$ degree of freedom
- $u_i$: controller output $\rightarrow$ physical meaning
- $\alpha_i$: gain from $v$ to $u_i$ $\rightarrow$ slope

\[ u_i = u_{i,0} + \alpha_i \ v \quad \forall i \in \{1, \ldots, N\} \]
Classical split range control

- $v$  : internal signal to split range block → limited physical meaning
- $v^*$ : split value → degree of freedom
- $u_i$ : controller output → physical meaning
- $\alpha_i$ : gain from $v$ to $u_i$ → slope

\[ u_i = u_{i,0} + \alpha_i \, v \quad \forall i \in \{1, \ldots, N\} \]
Design of split range control: select slopes

**Goal:** get desired loop gain at crossover frequency

\[ |g_C| \quad \omega_c = \frac{1}{\tau_C} \]

\[ C(s) = K_C \left(1 + \frac{1}{\tau_1 s}\right) \]

**Fast process**

Desired gain for \( u_i \)

\[ K_{I,i} = \alpha_i K_I \]

Common gain in \( C \)

\[ K_I = \frac{K_C}{\tau_I} \]

**Slow process**

Desired gain for \( u_i \)

\[ K_{C,i} = \alpha_i K_C \]

Common gain in \( C \)
Design of split range control: order of MVs

1. Define the desired operating point for every MV
2. Group the MVs according to the effect on the CV
3. Within each group, define order of use

- least expensive
- most used closer to the nominal operating point
- most expensive
- least used
Design of split range control

\[ y^{sp} = T^{sp} \]

\[ e \]

Standard split range controller

\[ G_p \]

\[ G_d \]

(Room)

\[ d = T^{amb} \]

\[ u_1 = u_{AC} \] : air conditioning (AC)

\[ u_2 = u_{CW} \] : cooling water (CW)

\[ u_3 = u_{HW} \] : heating water (HW)

\[ u_4 = u_{EH} \] : electrical heating (EH)
Classical split range control: a compromise

\[ C(s) = K_C \left(1 + \frac{1}{\tau IS}\right) \]

2 tuning parameters

\[ K_{C,i} = \alpha_i K_C \]

1 DOF

1 DOF, 2 tuning parameters
Generalized split range controller
Generalized split range controller

```
Preliminary step:
- Define order of use of MVs (j=1,…,N)
- Tune controllers

«Baton strategy» logic

k is the active input
- $C_k$ computes $u_k'$ (suggested value for $u_k$)
- If $u_k^{\min} < u_k' < u_k^{\max}$
  - Keep $u_k$ active and $u_k \leftarrow u_k'$
  - Keep remaining $u_i$ at limiting value
- else
  - Set $u_k = u_k^{\min}$ or $u_k < u_k^{\max}$, depending on the reached limit
  - New active input selected according to predefined sequence ($j = k-1$ or $j = k+1$)

The active input will decide when to switch and will remain active as long as it is not saturated.
```
Generalized split range controller

\[ u_1 = u_{AC} : \text{air conditioning (AC)} \]
\[ u_2 = u_{CW} : \text{cooling water (CW)} \]
\[ u_3 = u_{HW} : \text{heating water (HW)} \]
\[ u_4 = u_{EH} : \text{electrical heating (EH)} \]
Generalized vs standard split range controller

Ambient temperature ($d = T^{amb}$)

Room temperature ($y = T$)

Manipulated variables

$u_1$ (°C)

Time (min)
Generalized split range controller: initialization

How to start?

$$u'_k(t) = u^0_k + K_{C,k} \left( e(t) + \frac{1}{\tau_{I,k}} \int_{t_b}^{t} e(t) \right)$$

This suggested input was not being applied while input k was not in use

This accumulated error is not due to the previous actions of input k
Generalized split range controller: initialization

Resetting:

\[ u'_k(t) = u^0_k + K_{C,k} \left( e(t) + \frac{1}{\tau_{I,k}} \int_{t_b}^{t} e(t) \right) \]

\[ u_k(t_b) = u^0_k + K_{C,k} e(t_b) \]

Only use error when I receive the baton

Initial action proportional to error at \( t_b \)
Generalized split range controller: initialization

I was keeping track of the applied input

Back-calculation:
Generalized split range controller: initialization

Resetting: 
\[ u_k(t_b) = u_k^0 + K_{C,k} e(t_b) \]

Back-calculation:

Manipulated variables

- \( u_{AC} \)
- \( u_{CW} \)
- \( u_{HW} \)
- \( u_{EH} \)
Generalized split range controller vs MPC
Multiple controllers with different setpoints

Does this make sense at any point?
Multiple controllers with different setpoints

\[ J(u_k, \Delta y^{sp}) \]
Multiple controllers with different setpoints: Optimal setpoint deviation

Linear for $u$ and quadratic for $\Delta y$

$$J = p_{u_k} u_k + p_y (y - y^{sp})^2 + c$$

Inputs are a linear function of output

$$u_i = k_i \, y + u_{i,0}$$
Multiple controllers with different setpoints: Optimal setpoint deviation

Linear for $u$ and quadratic for $\Delta y$

$$J = p_{uk} u_k + p_y (y - y^{sp})^2 + c$$

Cost when using $u_k$ as input

$$J = p_{uk} k_x y + p_y (y - y^{sp})^2 + c_k + p_{uk} u_{k,0}$$

Inputs are a linear function of output

$$u_i = k_i y + u_{i,0}$$
Multiple controllers with different setpoints: Optimal setpoint deviation

Linear for $u$ and quadratic for $\Delta y$

$$J = p_{uk} u_k + p_y (y - y^{sp})^2 + c$$

Cost when using $u_k$ as input

$$J = p_{uk} k_k y + p_y (y - y^{sp})^2 + c_k + p_{uk} u_{k,0}$$

$$\frac{dJ}{dy} = 0$$

$$\Delta y^{sp*} = y^* - y^{sp} = -\frac{p_{uk} k_k}{2p_y}$$

Inputs are a linear function of output

$$u_i = k_i y + u_{i,0}$$

Optimal setpoint deviation minimizing cost
Multiple controllers with different setpoints: Case study

\[ Q_{AC} \] : air conditioning
\[ Q_{HW} \] : heating water
\[ Q_{EH} \] : electrical heating
Multiple controllers with different setpoints: Room T

Cost: linear for $u$ and quadratic for $\Delta y$

$$J_{\text{energy}} = \frac{p_{AC}Q_{AC}}{p_1 u_1} + \frac{p_{HW}Q_{HW}}{p_2 u_2} + \frac{p_{EH}Q_{EH}}{p_3 u_3} + \frac{p_T(T - T^{sp})^2}{p_v(y - y^{sp})^2} \quad [\$/s]$$

Inputs ($Q_i$) are a linear function of output (T)

$$0 = \alpha(T^{amb} - T) + Q_{HW} + Q_{EH} - Q_{AC} \quad [W]$$

Optimal setpoint deviation minimizing cost

$$\Delta y^{sp,1} = T_{AC}^{sp} - T^{sp} = +\frac{\alpha p_{AC}}{2p_T}$$

$$\Delta y^{sp,2} = T_{HW}^{sp} - T^{sp} = -\frac{\alpha p_{HW}}{2p_T}$$

$$\Delta y^{sp,3} = T_{EH}^{sp} - T^{sp} = -\frac{\alpha p_{el}}{2p_T}$$

$Q_{AC}$: air conditioning

$Q_{HW}$: heating water

$Q_{EH}$: electrical heating
Multiple controllers with different setpoints: Room T

- $Q_{AC}$: air conditioning
- $Q_{HW}$: heating water
- $Q_{EH}$: electrical heating

Optimal setpoint deviation minimizing cost

$$\Delta y^{\text{sp},1} = T_{AC}^{\text{sp}} - T^{\text{sp}} = \frac{\alpha p_{ac}}{2p_T}$$

$$\Delta y^{\text{sp},2} = T_{HW}^{\text{sp}} - T^{\text{sp}} = -\frac{\alpha p_{hw}}{2p_T}$$

$$\Delta y^{\text{sp},3} = T_{EH}^{\text{sp}} - T^{\text{sp}} = -\frac{\alpha p_{el}}{2p_T}$$
Multiple controllers with different setpoints: Room T

\[ d = T_{\text{amb}} \]

\[ y = T \]

\[ T^p \]

\[ T_{\text{amb}} \]

\[ T_{\text{sp}} \]

\[ T_{\text{HW}} \]

\[ T_{\text{EH}} \]

\[ y^{p,1} = y^{p} + \Delta y^{p,1} \]

\[ y^{p,2} = y^{p} + \Delta y^{p,2} \]

\[ y^{p,3} = y^{p} + \Delta y^{p,3} \]

\[ u_i = Q_i \]

\[ Q_{AC} \]

\[ Q_{HW} \]

\[ Q_{EH} \]

\[ 0 \]

\[ 2 \]

\[ 4 \]

\[ 0 \]

\[ 5 \]

\[ 10 \]

\[ 15 \]

\[ \text{time [h]} \]
Multiple controllers with different setpoints: Room T

\[ d = T_{amb} \]

\[ y = T \]

Lower accumulated cost with minimum setpoint deviation
Final comments

• Steady-state optimal operation may be easily achieved using PID-based control structures
  – Chapters 2, 3, 4: active constraint switching
  – Chapter 7: optimal setpoints

• Useful to systematically define control objectives, feasibility and tools
  – Priority list of constraints
  – Control structures available for each type of switch (CV-CV, MV-MV, MV-CV)

• Possible to improve performance of PID-based advanced control
  – Chapters 5, 6: design of split range controllers
  – Chapter 8: improved level control
One final comment

- The “gap” between theory and practice can be in both directions

Centrifugal governor used in steam engines in the 1780’s:
Proportionally controls fuel flow to maintain engine speed.

Theoretical investigation started about a century later.

Systematic design of advanced control structures

Thank you for your attention!