NTNU – Trondheim
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SUBPRO
SUBSEA PRODUCTION AND PROCESSING

Novel Approaches to Online Process Optimization under Uncertainty

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Outline

1. Research question - Why? and How?
2. Part 1 - Optimal steady-state operation using transient measurements
3. Part 2 - Dynamic optimization - addressing computation time
4. Summary and future outlook

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Research Question - Why?

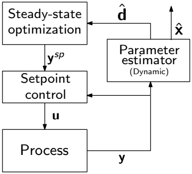
Why is traditional RTO not commonly used in industry?

- **Challenge 1** - Cost of developing the model (offline).
- **Challenge 2** - Model uncertainty, including wrong values of disturbances and parameters (online update).
- **Challenge 3** - Numerical robustness, including computational issues of solving optimization problems.
- **Challenge 4** - Frequent grade changes, which makes steady-state optimization less relevant.
- **Challenge 5** - Dynamic limitations, including infeasibility due to (dynamic) constraint violation.
- **Challenge 6** - Problem formulation - choosing the right formulation for the right problem.

- **Human factors** - Corporate culture and technical competence

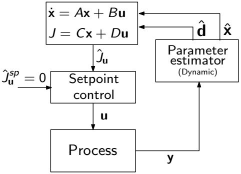
My Main Contributions - How?

Hybrid RTO



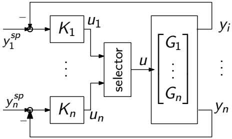
Challenge 2 & 5

Feedback RTO



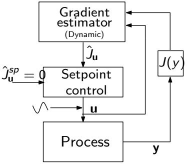
Challenge 2,3 & 5

Optimization using simple controllers



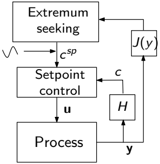
Challenge 1,2,3 & 5

Extremum seeking using transient measurements



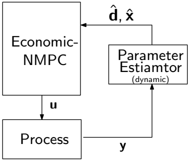
Challenge 1,2,3 & 5

Extremum seeking + Self-optimizing control



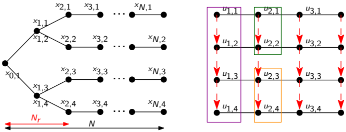
Challenge 1,2,3 & 5

economic MPC



Challenge 3,4& 5

Distributed multistage MPC using primal decomposition



Challenge 3, 4 & 5

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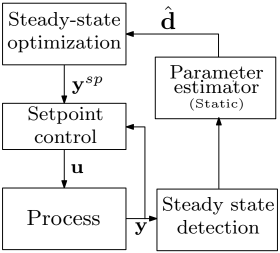
Conventional steady-state RTO

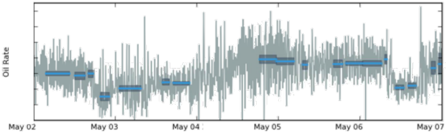
Two-step approach

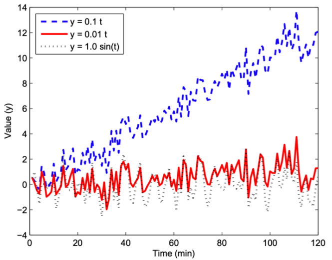


Chen & Joseph, *Ind. Eng. & Chem. Res* (1987)
Darby et al, *J. Proc. Control* (2011)

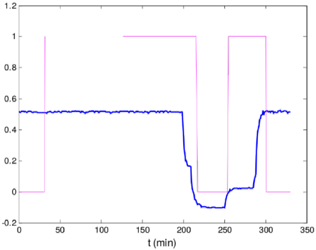
Steady-state wait time

Challenge 2 - Model uncertainty, including wrong values of disturbances and parameters (online update of the model).





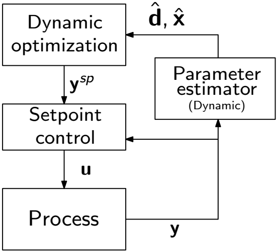
Source: Kelly, J.D. and Hedengren, J.D., 2013. A steady-state detection (SSD) algorithm to detect non-stationary drifts in processes. *Journal of Process Control*, *23*(3), pp.326-331.



Source: Câmara MM, Quelhas AD, Pinto JC. *Performance Evaluation of Real Industrial RTO Systems*. Processes. 2016, 4(4).

How to address steady-state wait time?

Obvious : Dynamic RTO



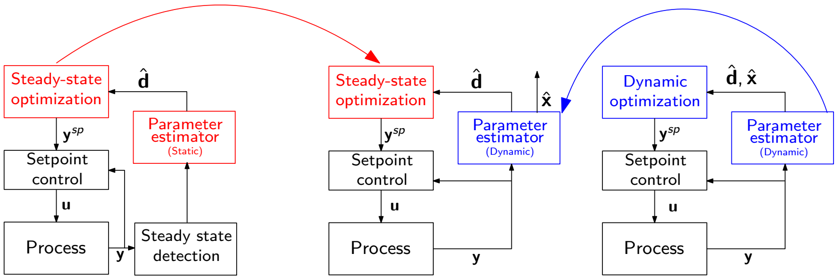
Dynamic RTO has problems - especially the optimization part (Challenge 3!)

Findeisen et al. *IFAC World Congress* (2004)

How to avoid steady-state wait time? - Hybrid RTO

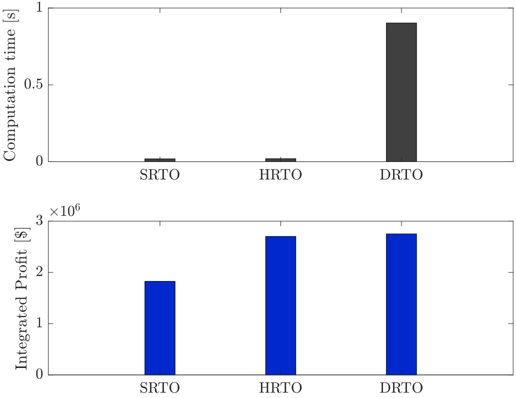
If the focus is steady-state optimization, dynamic terms are needed only for the model update.

- Dynamic models - Online model update
- Steady-state models - Optimization



Krishnamoorthy et al. *Comput. & Chem. Eng.* (2018)

Example: Hybrid RTO for oil production optimization

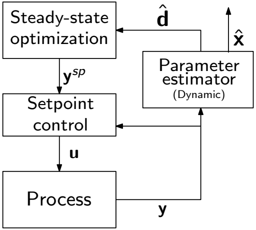


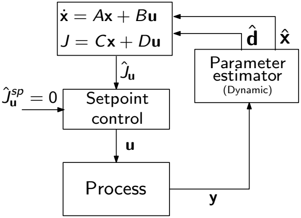
Krishnamoorthy et al. *(Submitted to ACC 2020)* - 6 gas-lifted wells with 20 differential states, 78 algebraic states and 6 inputs
Krishnamoorthy et al. *Comput. & Chem. Eng.* (2018)

Feedback RTO

Challenge 3 - Numerical robustness, including computational issues

Convert the Hybrid RTO problem into a **feedback control** problem





Krishnamoorthy et al. *Ind. Eng. Chem. Res* (2019)

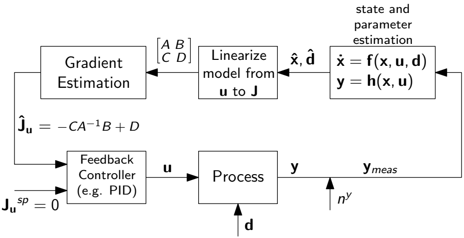
Novel gradient estimation using transient measurements

- **Step 1**- Linearize the dynamic model around $(\hat{x}, \hat{d})$

$$\begin{aligned} \dot{x} &= f(x,u,d) \\ J &= h(x,u,d) \end{aligned} \quad \Rightarrow \quad \begin{aligned} \dot{x} &= Ax + Bu \\ J &= Cx + Du \end{aligned}$$

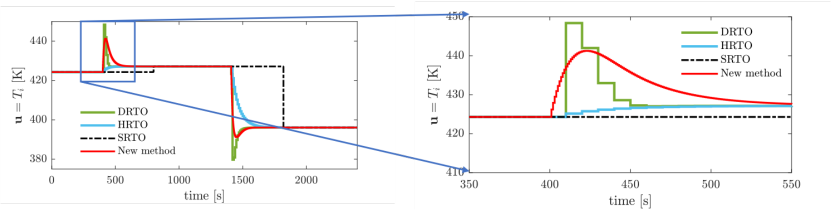
- **Step 2** - Set $\dot{x} = 0$ [†]

$$J = (\underbrace{-CA^{-1}B + D}_{J_u})u$$



Krishnamoorthy et al. *Ind. Eng. Chem. Res* (2019)
[†] Garcia & Morari, *AIChE J.* (1980), Bamberger & Isermann, *Automatica* (1978)

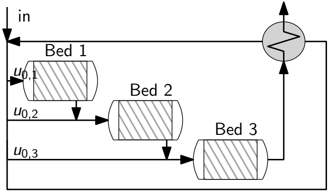
Comparison with other RTO approaches



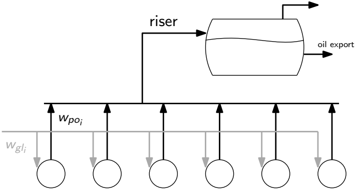
Krishnamoorthy et al. *Ind. Eng. Chem. Res.* (2019)
CSTR process from Economou et. al. (1986)

Other Case Examples

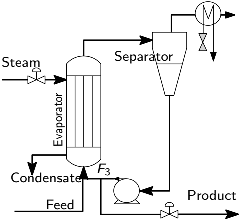
3-bed Ammonia reactor



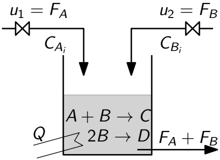
Gas-lift optimization



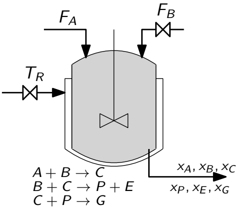
Evaporator process



Isothermal reactor



William-Otto reactor



Krishnamoorthy et al., *IFAC OOGP*, 2018

Krishnamoorthy et al., *Control Engineering Practice*, 2019

Bonnowitz et al.,*Computer Aided Chemical Engineering*, 2018

Krishnamoorthy & Skogestad, *Ind. Eng. Chem. Res*, 2019

Krishnamoorthy et al. *PSE Asia*, 2019

Krishnamoorthy & Skogestad, *Computer-Aided Chemical Engineering*, submitted

Do we always need a model to optimize?

Challenge 1 - Cost of developing the model

Example: Drive from A $\rightarrow$ B in shortest time

- CV: speed $\rightarrow$ 50km/h
- MV: gas pedal

Translate economic objectives into control objectives

What to control?

Control (in this order)

1. Active constraints $\mathbf{g}_{\mathbb{A}} \rightarrow 0$
2. Self-optimizing variable $\mathbf{c} = \mathbf{N}^{\mathsf{T}} \nabla_{\mathbf{u}} J \rightarrow 0$

$\mathbf{N}$ is chosen such that $\mathbf{N}^{\mathsf{T}} \nabla_{\mathbf{u}} \mathbf{g}_{\mathbb{A}} = 0$

- **Case 1: Fully constrained** - active constraint control
- **Case 2: Fully unconstrained** - control cost gradient to zero ($\mathbf{N} = I$)
- **Case 3: Partially constrained** - active constraint control + control linear gradient combination to zero
- **Case 4: Over constrained** - give up less important constraints

"...the ideal global self-optimizing variable is the gradient J_u."

– Halvorsen & Skogestad (1997)

But gradients are not measured!

Morari et al, *AIChE Journal* (1980)
Skogestad, *J. Proc. Control* (2000)
Krishnamoorthy & Skogestad, *Ind. Eng. Chem. res*, (2019)

Gradient Estimation

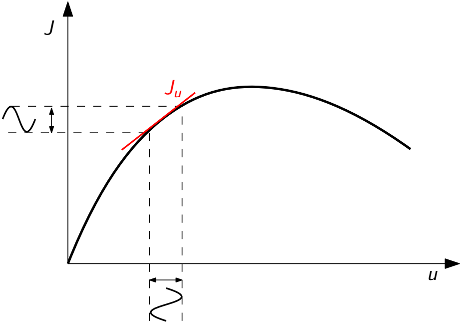
Model-free approaches

- Finite Difference
- Sinusoidal perturbation (Draper & Li, 1951)
- Least sqaures estimation (Hunnekens et al, 2014)
- Linear Identification ARX/ARMAX (Bamberger & Isermann, 1978)
- Nonlinear Identification NARX (Golden & Ydstie, 1989)
- Fixed dynamics (Krishnamoorthy & Skogestad)†
- Kalman filter (Gelbert et al, 2012)
- Polynomial fitted surfaces (Gao & Engell, 2005)
- Guassian Process regression (Ferriera et al, 2018/ Matias & Jaschke 2019)
- Multiple units (Srinivasan, 2007)

Model-based approaches

- Parameter estimation (Chen & Joseph, 1987 / Adetola & Guay 2007)
- Neighboring extremals (Gros et al, 2009)
- Nullspace method (Alstad & Skogestad, 2007)
- Feedback RTO (Krishnamoorthy et al, 2019)

Estimate cost gradient without model - Extremum seeking control



Prohibitively slow convergence!

- Dynamic plant assumed to be static map
- Dither signal $\approx 10\times$ slower than plant dynamics
- Integral gain $\approx 10\times$ slower than perturbations

Draper & Li, *ASME* (1951)
Krstic & Wang, *Automatica* (2000)

ARX-based Extremum seeking control

Identify local linear dynamic model, instead of local linear static model

- **Step 1** - Identify black-box ARX model

$$J(t) + a_1 J(t-1) + \cdots + a_{n_a} J(t-n_a)$$
$$= + b' u(t-1) + \cdots + b'_{n_b} u(t-n_b) + e(t)$$

$$\Rightarrow A_{poly}(q) J = B_{poly}(q) u$$

- **Step 2** - The steady-state part of the locally valid linear model is obtained with $q = 1$

$$J_u = A_{poly}^{-1} B_{poly} \qquad \Leftrightarrow \qquad -CA^{-1}B + D$$

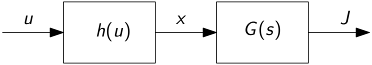
Bamberger & Isermann, *Automatica* (1979)
Garcia & Morari, *AIChE Journal* (1981)

ARX-based extremum seeking control

Robustness Issues with identifying $A_{poly}(q)J = B_{poly}(q)u$

- Too many parameters to estimate : $(n_a + n_b)$
- Neglected dynamics/unmodeled effects

Consider a Hammerstein class of systems



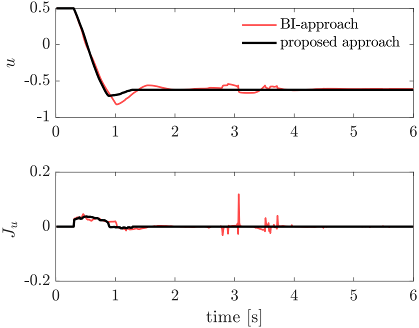
- Fix the nominal plant dynamics, in order to use transient measurements
- Estimate only the steady-state gradient

$$h(u)G_0(q) \approx J_u \underbrace{\left(\frac{b_1 q^{-1} + \cdots + b_{n_b} q^{-n_b}}{1 + a_1 q^{-1} + \cdots + a_{n_a} q^{-n_a}}\right)}_{\text{fixed nominal dynamics}} u$$

Illustrative Example

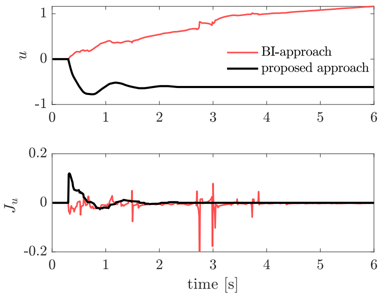
$$h(u) = d + \sum_{i=1}^{3} p_{2i-1}\sin(iu) + p_{2i}\cos(iu)$$

$$G_0(s) = \frac{1}{0.25s + 1}$$

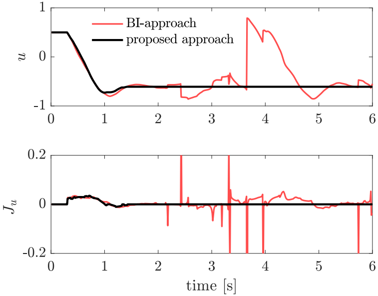


Illustrative Example - Unmodeled dynamics

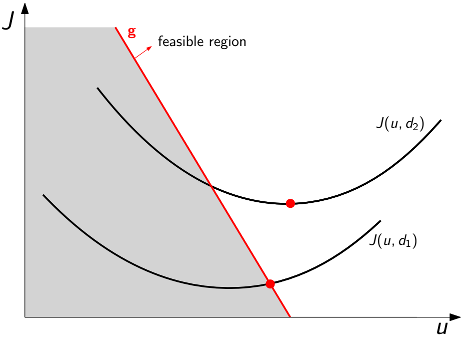
$$G(s) = G_0(s)\frac{-0.05s+1}{0.0001s+1}$$



$$G(s) = G_0(s)e^{-0.01s}$$



Change in active constraint regions



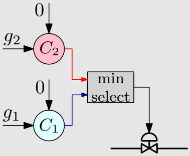
Optimal operation: Need to switch between regions using control system!

Switching between active constraints

- Output-to-output (CV-CV) switching - use selectors
- Input to output (CV-MV) switching - Pair MV that optimally saturates with the CV
- Input-to-input (MV-MV) switching - use split range control / valve position control [1]

Selector block

- Used when one input is used to switch between controlling several outputs.
- Each output has a separate controller and the selectors chooses which output to use.



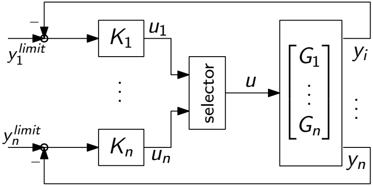
[1] Reyes-Lua et al, *Computer Aided Chemical Engineering*, 2018

CV-CV switching using selectors

For each MV

- at most 1 CV y_0 with setpoint control that may be given up
- n number of CV constraints y_i that may be optimally active

$$u = \min_{i\in[0,n]}(u_i) \quad \text{or} \quad u = \max_{i\in[0.n]}(u_i)$$



When to use min-selector? when to use max-selector? and when is it not feasible?

CV-CV switching using selectors

Introduce logic variable y_i^{lim} for CV constraints

$$y_i^{lim} = \begin{cases} 1, & \text{for max-constraint} \\ -1, & \text{for min-constraint} \end{cases}$$

Theorem

CV-CV switching is feasible only if

$$sgn(G_i)sgn(y_i^{lim}) = sgn(G_j)sgn(y_j^{lim}) \quad \forall i, j \in \{1, \dots, n\}$$

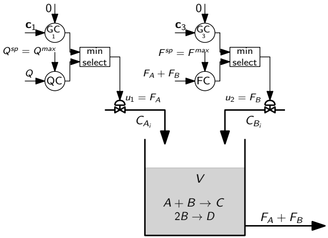
and,

$$sgn(G_i)sgn(y_i^{lim}) = 1 \Rightarrow \text{ min selector}$$
$$sgn(G_i)sgn(y_i^{lim}) = -1 \Rightarrow \text{ max selector}$$

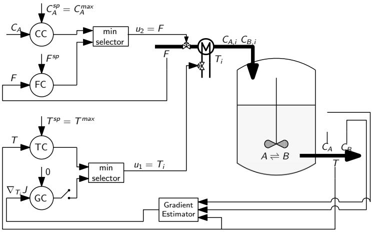
This ensures problem feasibility!

Case examples

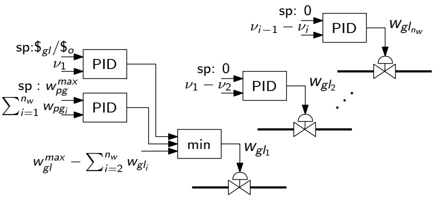
Isothermal CSTR



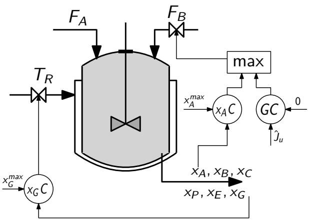
Exothermic Reactor



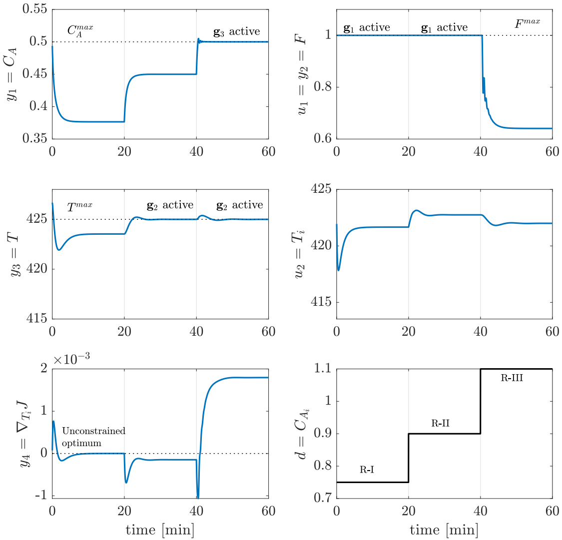
Gas-lift Optimization



William-Otto reactor

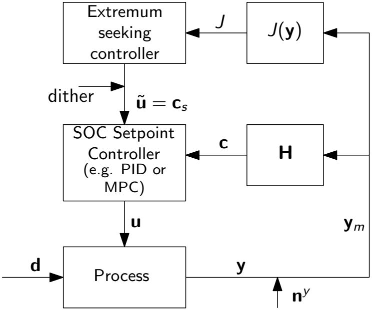


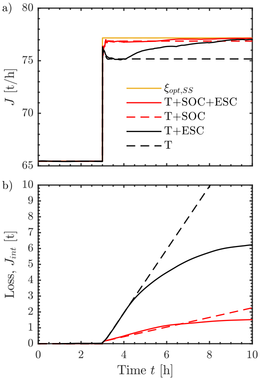
Illustrative example



Hierarchical combination of ESC and SOC

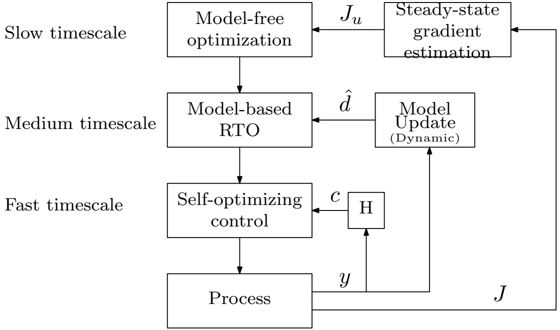
Optimal operation is when the plant gradient $\rightarrow 0$





Krishnamoorthy et al, *AIChE Annual Meeting* (2017)
Straus et al, *J. Proc. Control* (2019)

Hierarchical combination



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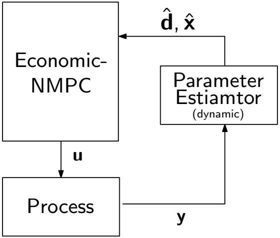
Dynamic RTO / economic MPC

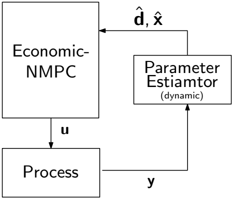
Challenge 4 - Frequent grade changes, which makes steady-state optimization less relevant.



Economic NMPC under uncertainty

Where to handle the uncertainty?

Externally

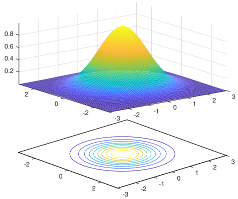


- Use external parameter estimator (observable)

$$\dot{\hat{\mathbf{d}}} = K_e(\mathbf{y} - h(\mathbf{x}, \mathbf{u}, \hat{\mathbf{d}}))$$

- Solve certainty-equivalence MPC with $\hat{\mathbf{d}}$

Internally



- Needs a-priori information (compact set / PDF)

$$\boldsymbol{p} \in \mathcal{P}$$

- Optimize under uncertainty

Economic NMPC under uncertainty

What is a good problem formulation?

- Easy to understand → operator confidence
- Low complexity → Maintenance and service by process engineers
- Not overly conservative → Management approval

Feedback!

"...optimize over a sequence of control laws, as done in Dynamic Programming, rather than over a sequence of control actions."

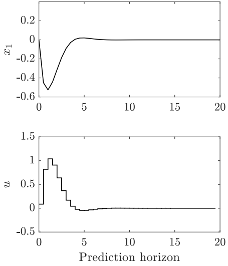
–*Mayne, Automatica (2014)*

Mayne, *Annual Reviews in Control* (2015)

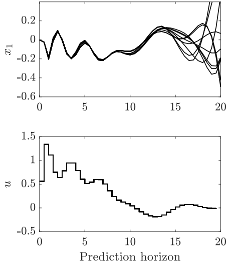
Solution strategies

	Open loop implementation	Closed loop implementation
Open loop optimization	Dynamic optimization	Standard MPC / Receding horizon control
Closed loop optimization	—	Multistage MPC /Feedback min-max MPC

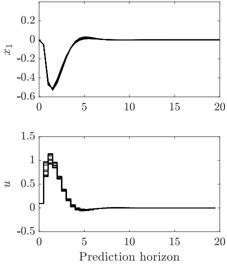
Perfect information



Open-loop optimization

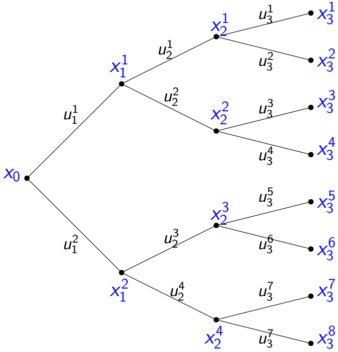


Closed-loop optimization



Multistage scenario MPC

One possible formulation: Multistage scenario MPC / Feedback min-max MPC



- Discrete scenario tree
- Introduces notion of feedback
- Different control trajectories
- Subject to non-anticipativity/causality constraints

Optimization over different control trajectories ≈ Optimization over control policies

(desirable vs. achievable)

Scokaert & Mayne, *IEEE Trans. Autom. Control* (1998)

Lucia et al, *J. Proc. Control* (2013)

Why Multistage MPC?

Certainty equivalence

$$\min_{\mathbf{x}_k,\mathbf{u}_k} \sum_{k=0}^{N-1} \ell(\mathbf{x}_k,\mathbf{u}_k)$$

s.t.

$$\begin{aligned}
\mathbf{x}_{k+1} &= \mathbf{f}(\mathbf{x}_k,\mathbf{u}_k,\boldsymbol{p}_k) && \forall k \in \mathcal{K}\\
\mathbf{g}(\mathbf{x}_k,\mathbf{u}_k,\boldsymbol{p}_k) &\leq 0 && \forall k \in \mathcal{K}\\
\mathbf{x}_0 &= \hat{\boldsymbol{x}}_t &&\\
\mathbf{x}_k \in \mathcal{X}, &\quad \mathbf{u}_k \in \mathcal{U} && \forall k \in \mathcal{K}
\end{aligned}$$

Multistage MPC

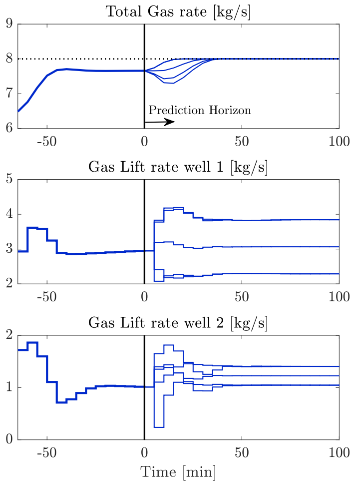
$$\min_{\mathbf{x}_{k,j},\mathbf{u}_{k,j}} \sum_{j=1}^{S} \omega_j \sum_{k=0}^{N-1} \ell(\mathbf{x}_{k,j},\mathbf{u}_{k,j})$$

s.t

$$\begin{aligned}
\mathbf{x}_{k+1,j} &= \mathbf{f}(\mathbf{x}_{k,j},\mathbf{u}_{k,j},\boldsymbol{p}_{k,j}) && \forall j \in \mathcal{S}, \forall k \in \mathcal{K}\\
\mathbf{g}(\mathbf{x}_{k,j},\mathbf{u}_{k,j},\boldsymbol{p}_{k,j}) &\leq 0 && \forall j \in \mathcal{S}, \forall k \in \mathcal{K}\\
\mathbf{x}_{0,j} &= \hat{\boldsymbol{x}}_t && \forall j \in \mathcal{S}\\
\mathbf{x}_{k,j} \in \mathcal{X}, &\quad \mathbf{u}_{k,j} \in \mathcal{U} && \forall j \in \mathcal{S}, \forall k \in \mathcal{K}\\
\sum_{j=1}^{S} \bar{\mathbf{E}}_j \mathbf{u}_j &= \mathbf{0} &&
\end{aligned}$$

Easier to implement and maintain

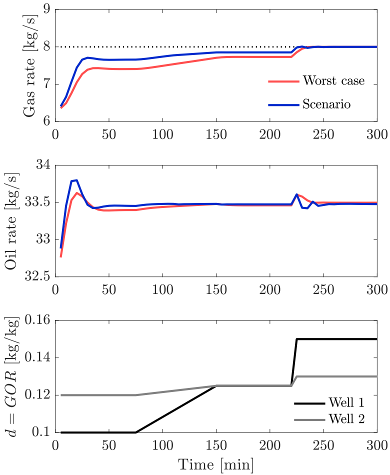
Why Multistage MPC?



Easier to understand the control actions

Krishnamoorthy et al, *Processes* (2016)

Why Multistage MPC?



Less conservative than worst-case MPC

Krishnamoorthy et al, *Processes* (2016)

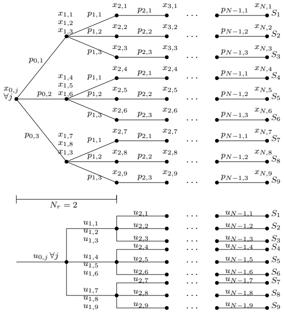
Multistage MPC - Major Drawback

Problem size explodes exponentially $S = M^N$

$$\min_{\mathbf{x}_{k,j},\mathbf{u}_{k,j}} \sum_{j=1}^{S} \omega_j \sum_{k=0}^{N-1} \ell(\mathbf{x}_{k,j},\mathbf{u}_{k,j})$$

s.t

$$\begin{aligned}
\mathbf{x}_{k+1,j} &= \mathbf{f}(\mathbf{x}_{k,j},\mathbf{u}_{k,j},\boldsymbol{p}_{k,j}) && \forall j \in \mathcal{S}, \forall k \in \mathcal{K} \\
\mathbf{g}(\mathbf{x}_{k,j},\mathbf{u}_{k,j},\boldsymbol{p}_{k,j}) &\leq 0 && \forall j \in \mathcal{S}, \forall k \in \mathcal{K} \\
\mathbf{x}_{0,j} &= \hat{\boldsymbol{x}}_t && \forall j \in \mathcal{S} \\
\mathbf{x}_{k,j} \in \mathcal{X}, &\quad \mathbf{u}_{k,j} \in \mathcal{U} && \forall j \in \mathcal{S}, \forall k \in \mathcal{K} \\
\sum_{j=1}^{S} \bar{\mathbf{E}}_j \mathbf{u}_j &= \mathbf{0}
\end{aligned}$$

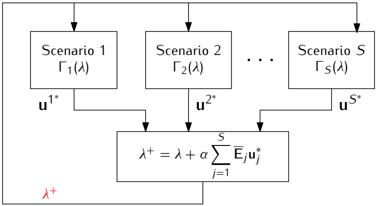


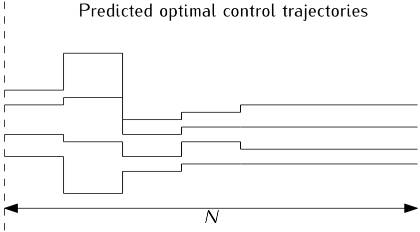
Non-anticipativity constraints enable closed-loop implementation

Lucia et al, *J. Proc. Control* (2013)

Scenario Decomposition using Dual Decomposition

- Relax Non-anticipativity constraints
- Non-anticipativity constraints are may not be feasible - (Duality gap) !

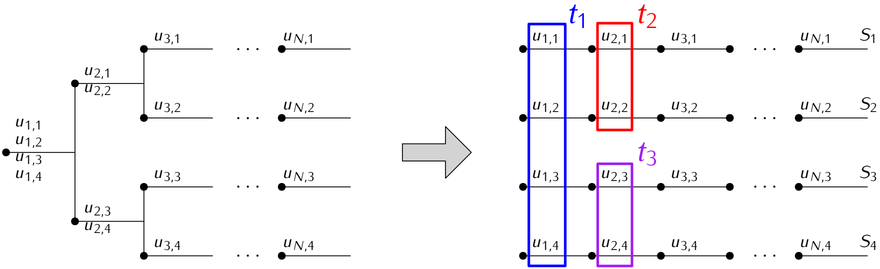




Closed-loop implementation fails!!

Marti et al, *Comput. & Chem. Eng* (2015)

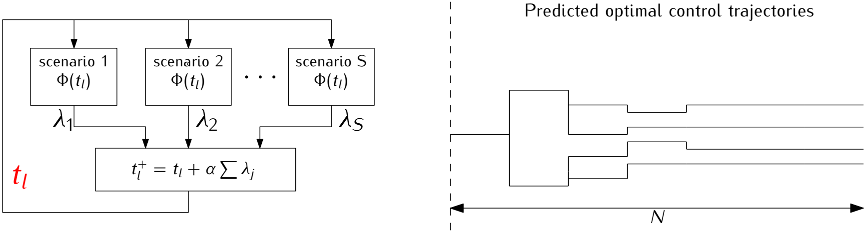
Scenario decomposition using Primal decomposition



Krishnamoorthy et al, *IFAC ADCHEM* (2018)
Krishnamoorthy et al, *J. Proc. Control* (2019)

Scenario Decomposition using Primal Decomposition

Non-anticipativity constraints are always feasible !



Closed-loop implementation OK!!

"Approximate solution now is better than accurate solution tomorrow!"

Krishnamoorthy et al, *J. Proc. Control* (2019)

Scenario Decomposition using Parametric Sensitivities

Reduce the number of NLPs needed to solve the distributed scenario MPC problem

$$\mathbf{w}_0^* = \arg\min_{\mathbf{w}} \quad \Phi(\mathbf{w}, \mathbf{p}_0)$$

s.t.

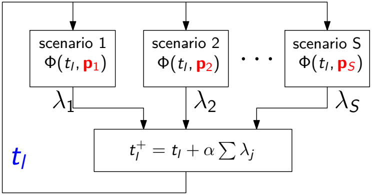
$$\mathbf{g}(\mathbf{w}, \mathbf{p}_0) \leq 0$$

$$\mathbf{w}_0^* + \Delta\mathbf{w}^* = \arg\min_{\mathbf{w}} \quad \Phi(\mathbf{w}, \mathbf{p}_0 + \Delta\mathbf{p})$$

s.t.

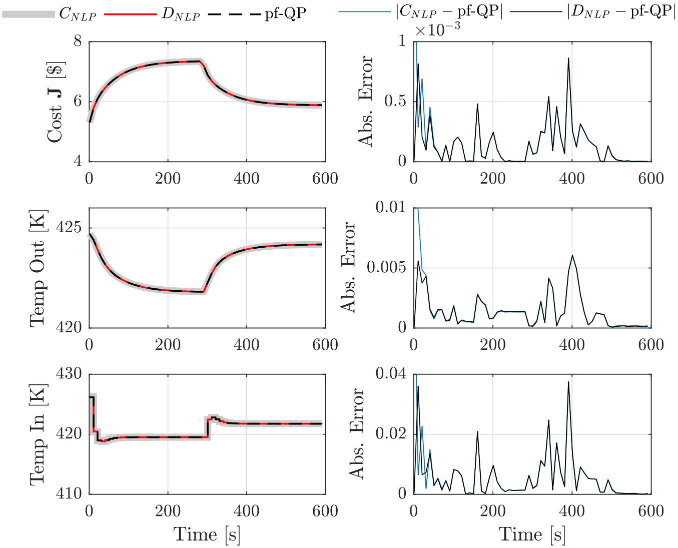
$$\mathbf{g}(\mathbf{w}, \mathbf{p}_0 + \Delta\mathbf{p}) \leq 0$$

Compute $\Delta\mathbf{w}^*$ using path-following predictor-corrector QP.



Krishnamoorthy et al, *IEEE Contr. Syst. Lett.* (2018)

Illustrative example



Outline

1 Research question - Why? and How?

2 Part 1 - Optimal steady-state operation using transient measurements

3 Part 2 - Dynamic optimization - addressing computation time

4 Summary and future outlook

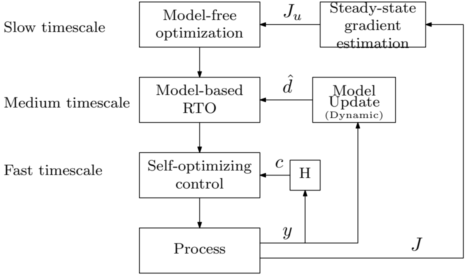
Future Outlook

Understand the uncertainty - Handle only those uncertainties that have an impact.

In the context of multistage NMPC,

- How to select the scenarios - use of data analytic tools
- Shrink the uncertainty set using data - Online scenario tree update

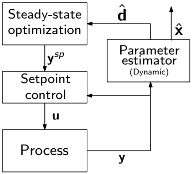
Different methods work in different timescales, can handle different kinds of uncertainty, and have different advantages and disadvantages!



Krishnamoorthy et al. *IFAC NMPC* (2018)
Krishnamoorthy et al. *ECC* (2019)

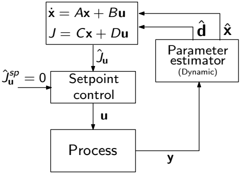
Summary - The Big Picture

Hybrid RTO



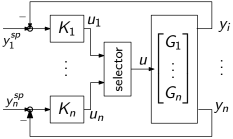
Challenge 2 & 5

Feedback RTO



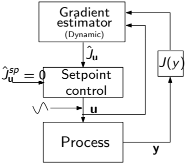
Challenge 2,3 & 5

Optimization using simple controllers



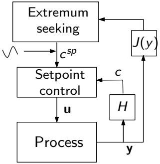
Challenge 1,2,3 & 5

Extremum seeking using transient measurements



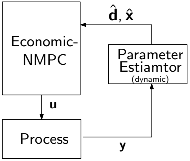
Challenge 1,2,3 & 5

Extremum seeking + Self-optimizing control



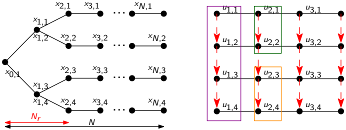
Challenge 1,2,3 & 5

economic MPC



Challenge 3,4& 5

Distributed multistage MPC using primal decomposition



Challenge 3, 4 & 5

Thank you!

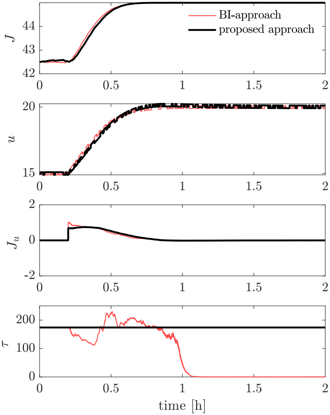
ARX-based extremum seeking control

$$G(s) = \frac{1}{(\tau s + 1)}$$

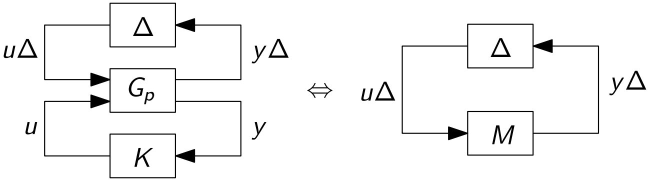
ARX model with $n_a = 1$ and $n_b = 1$

$$J(t) = -a_1 J(t-1) + b_1 u(t-1)$$

$$J_u = \frac{b_1}{1 + a_1}, \qquad \tau = \frac{-T_s}{ln(-a_1)}$$



Dynamic Extremum seeking control



$$P := \begin{bmatrix} 0 & 1 \\ w_I G_0 & G_0 \end{bmatrix}, \qquad M := w_I G_0 K (1 + G_0 K)^{-1}$$

$$w_I(j\omega) = \left| \frac{G_p(j\omega) - G_0(j\omega)}{G_0(j\omega)} \right| = \left| 1 - \frac{(-5j\omega + 1)}{(10j\omega + 1)} \right|$$

Dynamic Extremum seeking control - SISO case

SISO ARX model:

$$J(t) + a_1 J(t-1) + \cdots + a_{n_a} J(t-n_a) = J_u \left[b_1 u(t-1) + b_2 u(t-2) + \cdots + b_{n_b} u(t-n_b) \right]$$

$$\hat{\theta} = arg \ \min_{\theta} \|\psi - \Phi\theta\|_2^2$$

$$\theta = J_u$$

$$\psi = \begin{bmatrix} J(N) + a_1 J(N-1) + \cdots + a_{n_a} J(N-n_a) \\ J(N-1) + a_1 J(N-2) + \cdots + a_{n_a} J(N-1-n_a) \\ \vdots \\ J(n+1) + a_1 J(n+2) + \cdots + a_{n_a} J(n+1-n_a) \end{bmatrix}$$

$$\Phi = \begin{bmatrix} b_1 u(N-1) + \cdots + b_{n_b} u(N-n_b) \\ b_1 u(N-2) + \cdots + b_{n_b} u(N-1-n_b) \\ \vdots \\ b_1 u(n) + \cdots + b_{n_b} u(n-1-n_b) \end{bmatrix}$$

Dynamic Extremum seeking control - Multivariable case

MISO ARX model with n inputs

$$J(t) + a_1 J(t-1) + \cdots + a_{n_a} J(t-n_a) = \mathbf{B}_1 \mathbf{u}(t-1) + \cdots + \mathbf{B}_{n_b} \mathbf{u}(t-n_b)$$

with $\mathbf{B}_i \in \mathbb{R}^{1\times n}$

$$\begin{aligned} J(t) &+ a_1 J(t-1) + \cdots + a_{n_a} J(t-n_a) \\ &= \sum_{i=1}^{n} J_{u_i} \left[b_{1,i} u_i(t-1) + b_{2,i} u_i(t-2) + \cdots + b_{n_b,i} u_i(t-n_b) \right] \end{aligned}$$

$$\hat{\theta} = arg \ \min_{\theta} \|\psi - \Phi\theta\|_2^2$$

$$\theta = [J_{u_1}, J_{u_2}, \ldots J_{u_n}]^T, \ \psi \text{ - same as before}$$

$$\Phi = [\Phi_1, \Phi_2, \ldots \Phi_n]$$

Zhu, *Multivariable System Identification for Process Control*.

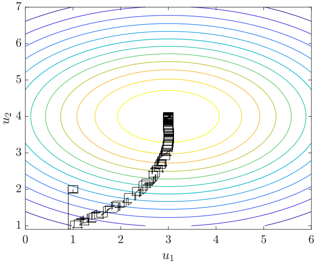
Lyung, *System Identification: Theory for the user*.

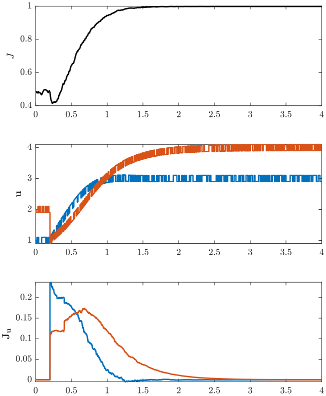
Dynamic Extremum seeking control - Multivariable case

Illustrative example - 2D Gaussian function

$$J(u_1, u_2) = exp\left(-\left(\frac{(u_1-3)^2}{8} + \frac{(u_2-4)^2}{18}\right)\right)$$

$$G(s) = \frac{1}{120s+1}$$





Linear gradient combination satisfies KKT conditions

For a steady-state optimization problem with $n_a < n_u$ active constraints $\mathbf{g}_{\mathbb{A}}(\mathbf{u}, \mathbf{d})$

$$\begin{aligned} \nabla_{\mathbf{u}} \mathcal{L}(\mathbf{u}, \mathbf{d}) =& \nabla_{\mathbf{u}} J(\mathbf{u}, \mathbf{d}) + \lambda_{\mathbb{A}}^T \nabla_{\mathbf{u}} \mathbf{g}_{\mathbb{A}}(\mathbf{u}, \mathbf{d}) = 0 \\ \Rightarrow& \nabla_{\mathbf{u}} J(\mathbf{u}, \mathbf{d}) = -\lambda_{\mathbb{A}}^T \nabla_{\mathbf{u}} \mathbf{g}_{\mathbb{A}}(\mathbf{u}, \mathbf{d}) \end{aligned}$$

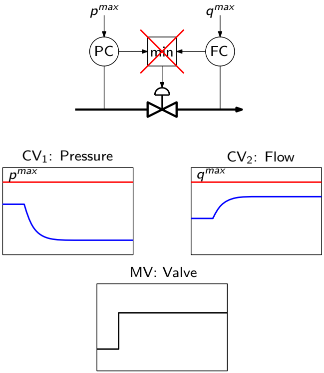
Pre-multiplying by $\mathbf{N}^\mathsf{T}$ gives

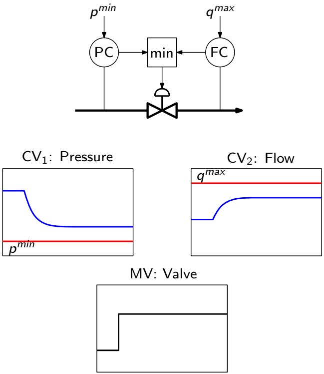
$$\mathbf{N}^\mathsf{T} \nabla_{\mathbf{u}} J(\mathbf{u}, \mathbf{d}) = -\mathbf{N}^\mathsf{T} \nabla_{\mathbf{u}} \mathbf{g}_{\mathbb{A}}(\mathbf{u}, \mathbf{d})^\mathsf{T} \lambda$$

Since $\mathbf{N}^\mathsf{T} \nabla_{\mathbf{u}} \mathbf{g}_{\mathbb{A}}(\mathbf{u}, \mathbf{d})^\mathsf{T} = 0$, $\Rightarrow$ $\mathbf{N}^\mathsf{T} \nabla_{\mathbf{u}} J(\mathbf{u}, \mathbf{d}) = 0$

Since $n_a < n_u$, $\nabla_{\mathbf{u}} \mathbf{g}_{\mathbb{A}}(\mathbf{u}, \mathbf{d})$ has full row rank and $\mathbf{N}$ is well defined.

CV-CV switching using selectors - Illustrative Example





When a CV constraint is active, all other CVs must be within its limit. To remain feasible,

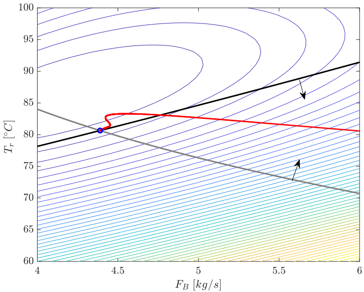
- when the MV changes, all the CVs must change in the same direction relative to their constraints
- if $sgn(G_i)sgn(y_i^{lim}) = 1$, then increasing u moves output y_i closer to its constraint.

$$u = \min(u_i) \Rightarrow \text{feasibility}$$

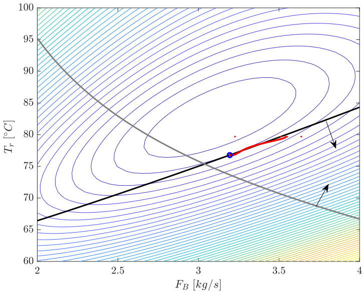
CV-CV switching: William-Otto reactor

2 CV constraints; $x_G \leq 0.08$kg/kg and $x_A \leq 0.12$kg/kg

$F_A > 1.5$

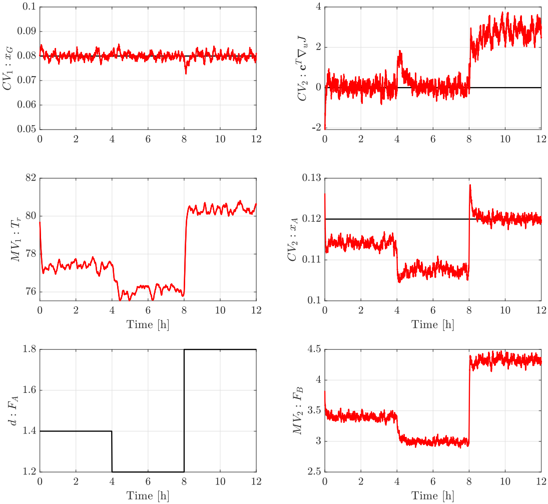


$F_A < 1.5$



CV-CV switching: William-Otto reactor

Use max-selector to switch between x_A^{max} and c



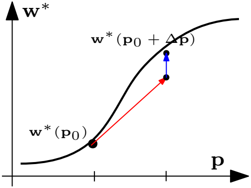
Parametric Optimization

$$\mathbf{w}_0^* = \arg\min_{\mathbf{w}} \quad \Phi(\mathbf{w}, \mathbf{p}_0)$$
s.t.
$$\mathbf{g}(\mathbf{w}, \mathbf{p}_0) \leq 0$$

$$\mathbf{w}_0^* + \Delta\mathbf{w}^* = \arg\min_{\mathbf{w}} \quad \Phi(\mathbf{w}, \mathbf{p}_0 + \Delta\mathbf{p})$$
s.t.
$$\mathbf{g}(\mathbf{w}, \mathbf{p}_0 + \Delta\mathbf{p}) \leq 0$$

- **Pure Predictor QP**

$$\Delta\mathbf{w}^* = \arg\min_{\Delta\mathbf{w}} \frac{1}{2}\Delta\mathbf{w}^T \nabla^2_{\mathbf{ww}}\mathcal{L}\Delta\mathbf{w} + \Delta\mathbf{w}^T \nabla_{\mathbf{wp}}\mathcal{L}\Delta\mathbf{p}$$
s.t
$$\nabla_{\mathbf{w}}\mathbf{g}^T\Delta\mathbf{w} + \nabla_{\mathbf{p}}\mathbf{g}^T\Delta\mathbf{p} \leq 0$$

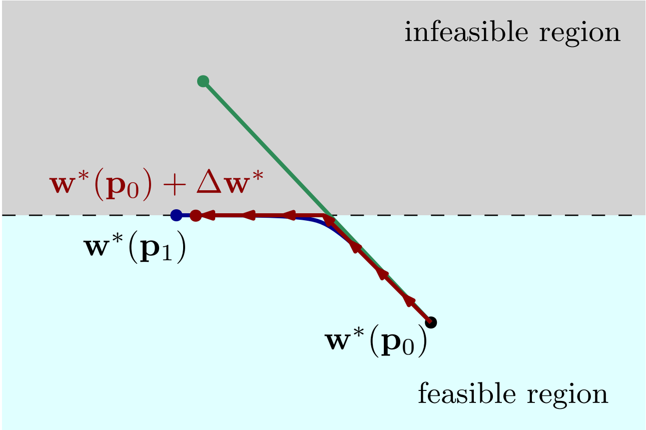


-**Predictor Corrector QP**

$$\Delta\mathbf{w}^* = \arg\min_{\Delta\mathbf{w}} \frac{1}{2}\Delta\mathbf{w}^T \nabla^2_{\mathbf{ww}}\mathcal{L}\Delta\mathbf{w} + \Delta\mathbf{w}^T \nabla_{\mathbf{wp}}\mathcal{L}\Delta\mathbf{p} + \nabla_{\mathbf{w}}\Phi^T\Delta\mathbf{w}$$
s.t
$$\nabla_{\mathbf{w}}\mathbf{g}^T\Delta\mathbf{w} + \nabla_{\mathbf{p}}\mathbf{g}^T\Delta\mathbf{p} \leq 0$$

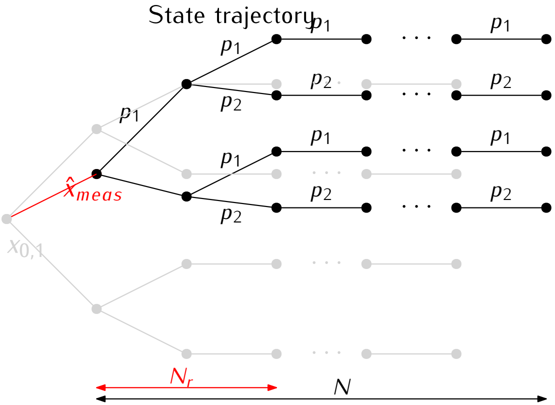
Suwartadi et al, *Processes* (2017)

Path-following predictor corrector QP



Suwartadi et al, *Processes* (2017)

Online update of the scenario tree



Feedback versus open-loop optimization

feedback optimization $\equiv$ open-loop optimization with closed-loop implementation

$$x_{k+1} = x_k + u_k + w_k$$

Open-loop optimization with $w_k = 0$

$$\mathbf{u}^0(x_0) = [-(8/13)x_0, -(3/13)x_0, -(1/13)x_0]$$
$$\mathbf{x}^0(x_0) = [x_0, (5/13)x_0, (2/13)x_0, (1/13)x_0]$$

Dynamic Programming

$$\boldsymbol{\mu} := (\mu_0(\cdot), \mu_1(\cdot), \mu_2(\cdot))$$

$$\mu_0 = -(8/13)x_0 \Rightarrow x^0(0) = x_0 \qquad u^0(0) = -(8/13)x_0$$
$$\mu_1 = -(3/5)x_1 \Rightarrow x^0(1) = (5/13)x_0 \qquad u^0(1) = -(3/13)x_0$$
$$\mu_2 = -(1/2)x_2 \Rightarrow x^0(2) = (2/13)x_0 \qquad u^0(2) = -(1/13)x_0$$
$$\Rightarrow x^0(3) = (1/13)x_0$$

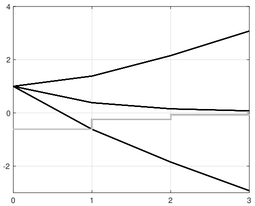
Section3.1.2 - Rawlings, Mayne, Diehl, *Model Predictive Control*, 2nd ed.

Feedback versus open-loop optimization

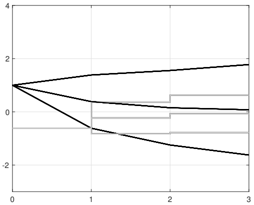
feedback optimization ≈ multistage optimization with closed-loop implementation

$$x_{k+1} = x_k + u_k + w_k \qquad w_k \in [-1, 1]$$

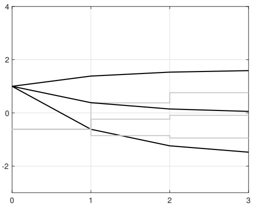
Open-loop



Feedback - DP



Feedback - multistage

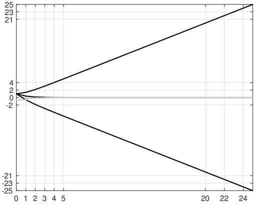


Section3.1.2 - Rawlings, Mayne, Diehl, *Model Predictive Control*, 2nd ed.

Feedback versus open-loop optimization

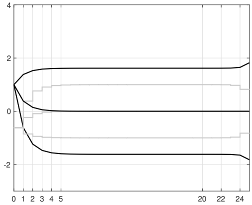
feedback optimization $\approx$ multistage optimization with closed-loop implementation

Open-loop



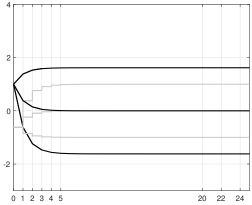
$|x_{w=-1}(N) - x_{w=1}(N)| = 2N$

Feedback - DP



$|x_{w=-1}(N) - x_{w=1}(N)| \to 3.24$
$N \to \infty$

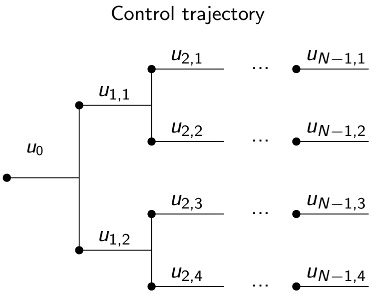
Feedback - multistage



$|x_{w=-1}(N) - x_{w=1}(N)| \to 3.24$
$N \to \infty$

Section3.1.2 - Rawlings, Mayne, Diehl, *Model Predictive Control*, 2nd ed.

Multistage MPC approximation of DP



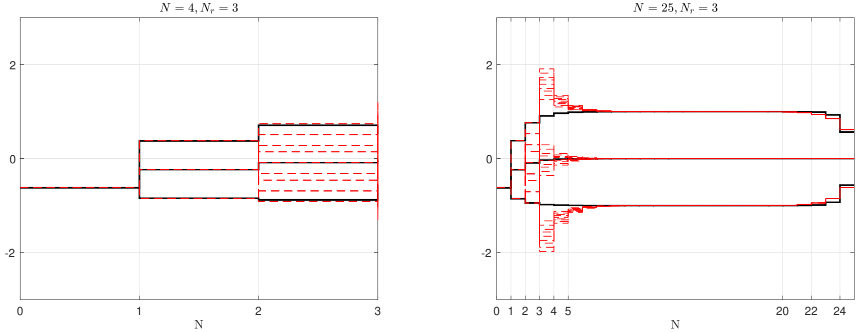
Ideal: Optimize over control policies

$$\boldsymbol{\mu} = (\mu_0, \mu_1, \mu_2 \ldots, \mu_{N-1})$$

Sampled control policy

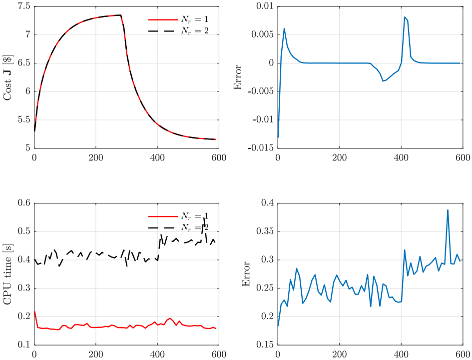
$$\begin{aligned}
\mu_0 &= \{u_0\} \\
\mu_1 &= \{u_{1,1}, u_{1,2}\} \\
\mu_2 &= \{u_{2,1}, u_{2,2}, u_{2,3}, u_{2,4}\} \\
&\vdots \\
\mu_{N-1} &= \{u_{N-1,1}, u_{N-1,2}, u_{N-1,3}, u_{N-1,4}\}
\end{aligned}$$

Multistage MPC approximation of DP



Price of robust horizon - what am I getting for the additional CPU time?

Simulation results from Chapter 9.



Partially deterministic approximation (Berstekas, 2019)

"When the problem is stochastic, one may consider an approximation to the ℓ-step lookahead, which is based on deterministic computations. This is a hybrid, partially deterministic approach, whereby at state x_k we allow for a stochastic disturbance at the current stage, but fix the future disturbances... up to the end of the lookahead horizon, to some typical values."

Stability properties of AHeNMPC

- Lyapunov stability framework

$$\ell(\mathbf{x}_0, \mathbf{u}_0) + \Delta\mathbf{x}_{\tilde{N}}^{\mathsf{T}} P \Delta\mathbf{x}_{\tilde{N}} - \ell(\mathbf{x}_{\tilde{N}}, \mathbf{u}_{\tilde{N}}) - \mathbf{f}(\mathbf{x}_{\tilde{N}}, \mathbf{u}_{\tilde{N}})^{\mathsf{T}} P \mathbf{f}(\mathbf{x}_{\tilde{N}}, \mathbf{u}_{\tilde{N}}) \leq 0$$

- For descent property, ℓ must be strictly dissipative $\Rightarrow$ $(\mathbf{x}_f, \mathbf{u}_f)$ unique global minima[2]
- if stage cost and dynamic model form strong duality $\Rightarrow$ dissipativity [3]
- Steady-state optimization problem has a strongly convex Lagrange function $\Rightarrow$ strong duality[4]
- Regularization of the stage cost $\Rightarrow$ strong convexity

$$\psi_{reg}(\mathbf{x}, \mathbf{u}) := \ell(\mathbf{x}, \mathbf{u}) + \frac{1}{2}\|(\mathbf{x} - \mathbf{x}_f, \mathbf{u} - \mathbf{u}_f)\|_{\hat{Q}}^2 + \lambda^{\mathsf{T}}(\mathbf{x} - \mathbf{f}(\mathbf{x}, \mathbf{u}))$$

- With $\hat{Q}$: $eig(\nabla^2\psi + \hat{Q}) > 0 \forall (x, u) \in \mathcal{X} \times \mathcal{U}$

[2] Faulwasser et al *Foundations and Trends in System and Control* (2018)
[3] Angeli et al, *IEEE Trans. Autom. Control*(2011)
[4] Huang et al, *J. Proc. Control* (2018)

Scenario-based MPC

OL Scenario-based MPC / Sampled average approximation/ Randomized MPC

$$\min_{\mathbf{x}_{k,j},\mathbf{u}_k} \sum_{j=1}^{S} \omega_j \sum_{k=0}^{N-1} \ell(\mathbf{x}_{k,j}, \mathbf{u}_k)$$

s.t

$$\begin{aligned}
\mathbf{x}_{k+1,j} &= \mathbf{f}(\mathbf{x}_{k,j}, \mathbf{u}_k, \boldsymbol{p}_j) && \forall j \in \mathcal{S}, \forall k \in \mathcal{K} \\
\mathbf{g}(\mathbf{x}_{k,j}, \mathbf{u}_k, \boldsymbol{p}_j) &\leq 0 && \forall j \in \mathcal{S}, \forall k \in \mathcal{K} \\
\mathbf{x}_{0,j} &= \hat{\boldsymbol{x}}_t && \forall j \in \mathcal{S} \\
\mathbf{x}_{k,j} \in \mathcal{X}, &\quad \mathbf{u}_k \in \mathcal{U} && \forall j \in \mathcal{S}, \forall k \in \mathcal{K}
\end{aligned}$$

Multistage scenario-based MPC / Feedback min-max MPC

$$\min_{\mathbf{x}_{k,j},\mathbf{u}_{k,j}} \sum_{j=1}^{S} \omega_j \sum_{k=0}^{N-1} \ell(\mathbf{x}_{k,j}, \mathbf{u}_{k,j})$$

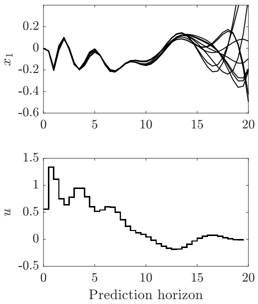
s.t

$$\begin{aligned}
\mathbf{x}_{k+1,j} &= \mathbf{f}(\mathbf{x}_{k,j}, \mathbf{u}_{k,j}, \boldsymbol{p}_{k,j}) && \forall j \in \mathcal{S}, \forall k \in \mathcal{K} \\
\mathbf{g}(\mathbf{x}_{k,j}, \mathbf{u}_{k,j}, \boldsymbol{p}_{k,j}) &\leq 0 && \forall j \in \mathcal{S}, \forall k \in \mathcal{K} \\
\mathbf{x}_{0,j} &= \hat{\boldsymbol{x}}_t && \forall j \in \mathcal{S} \\
\mathbf{x}_{k,j} \in \mathcal{X}, &\quad \mathbf{u}_{k,j} \in \mathcal{U} && \forall j \in \mathcal{S}, \forall k \in \mathcal{K} \\
\sum_{j=1}^{S} \bar{\mathbf{E}}_j \mathbf{u}_j &= \mathbf{0}
\end{aligned}$$

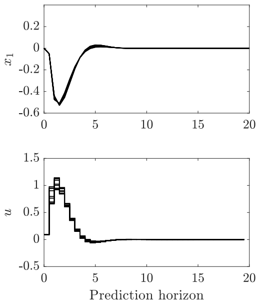
Schildbach et al, *Automatica* 2014
Fagiano et al, *IEEE Trans. Autom. Control* 2014
Campi & Garatti, *IEEE Trans. Autom. Control* (2012)

Scenario-based MPC

OL Scenario-based MPC / Sampled average approximation/ Randomized MPC



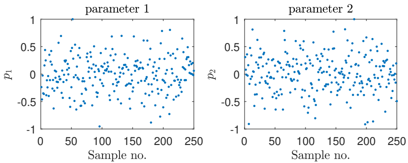
Multistage scenario-based MPC / Feedback min-max MPC

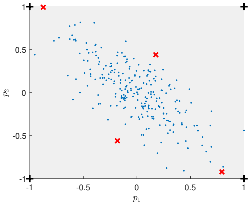


Data analytic tools to select the discrete scenarios

How to select the different scenarios?

- Aim: Seek robustness to variability
- Use data analytic tools to explain the variability





Uncover hidden data structures to select scenarios judiciously !

Krishnamoorthy et al, *IFAC NMPC* (2018)

Robust-adaptive framework - updating the uncertainty set

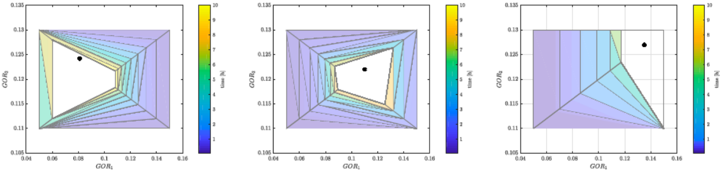
For time-invariant parameters, shrink the uncertainty set by updating the scenarios based on recursive Bayesian probability.

Recursive Bayes Theorem

Conditional probability for the j^{th} scenario being the true realization,

$$P_{k,j} = \frac{e^{-0.5\epsilon_{k,j}^T K \epsilon_{k,j}} P_{k-1,j}}{\sum_{m=1}^{S} e^{-0.5\epsilon_{k,m}^T K \epsilon_{k,m}} P_{k-1,m}}$$

$$W_{k,j} = \frac{P_{k,j}}{\sum_{m=1}^{S} P_{k,m}}$$



Krishnamoorthy et al, *ECC* (2019)

Real time optimization