Economic Model Predictive Control – Historical Perspective and Recent Developments and Industrial Examples

Public Trial Lecture

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Presentation outline

- Introduction and basic concepts
- Historical overview
- Recent developments
- Industrial examples
- Final thoughts

Introduction

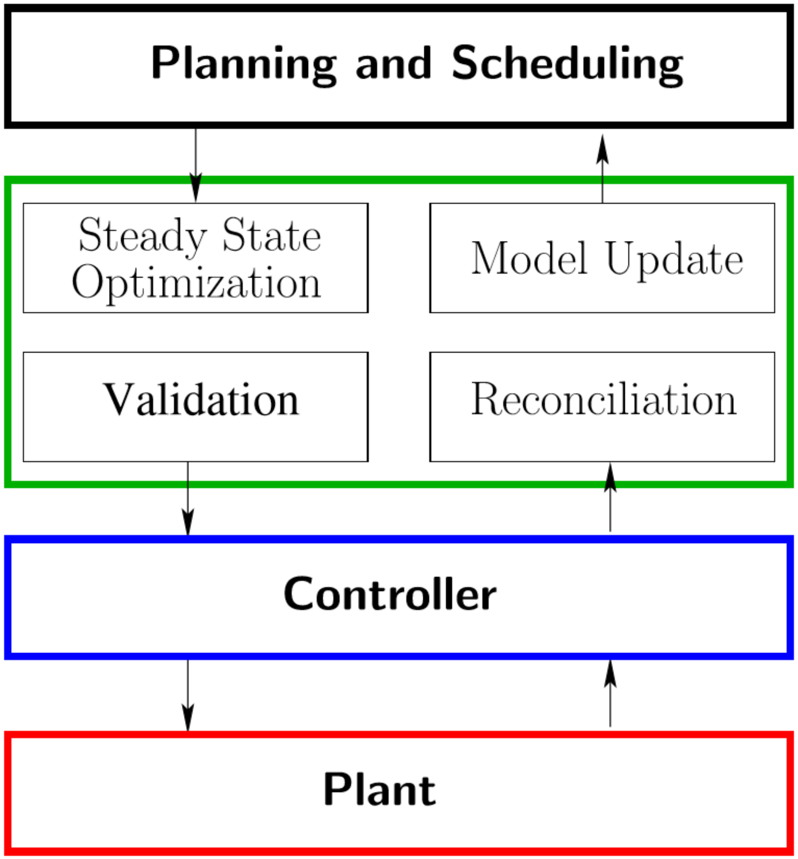
Plant operation objectives

- Maximize the economic operating value of the plant
- Achieve environment, health and safety targets (regulations)

Need tight integration between

- Plant management → Economics
- Process operation → Control

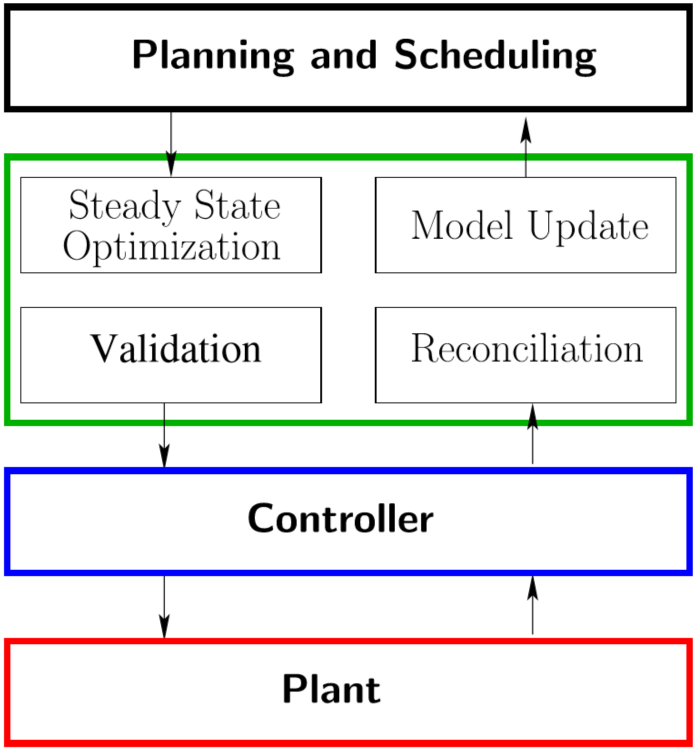
Traditional paradigm: Two layer structure



Angeli (2015), Engell (2007)

- Upper layer: steady state optimization - Real Time Optimization (RTO)
- RTO provides setpoints to a lower (dynamic) control layer
- Control layer follows setpoints
  - linear model predictive controllers (MPC) often employed

Traditional paradigm: Two layer structure



Advantages of two layers approach

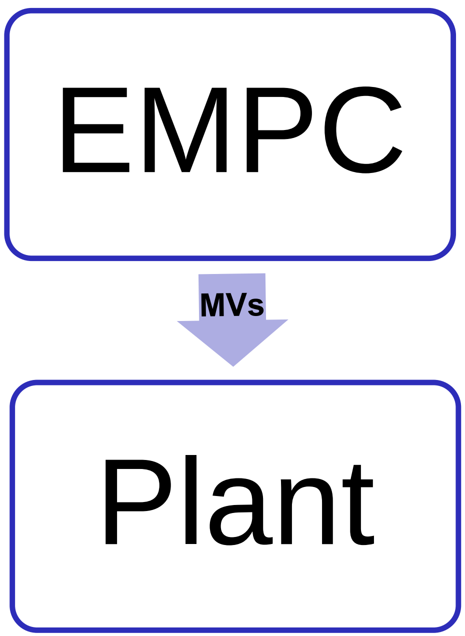
- Simpler sub-problems
- Reduced complexity
- Clear separation between economic and control objectives (based on time-scale separation)

But *every advantage is also a disadvantage**

- Delay in the optimization (need to wait for steady state)
- Time-scale separation may not hold

(*) Johan Cruyff

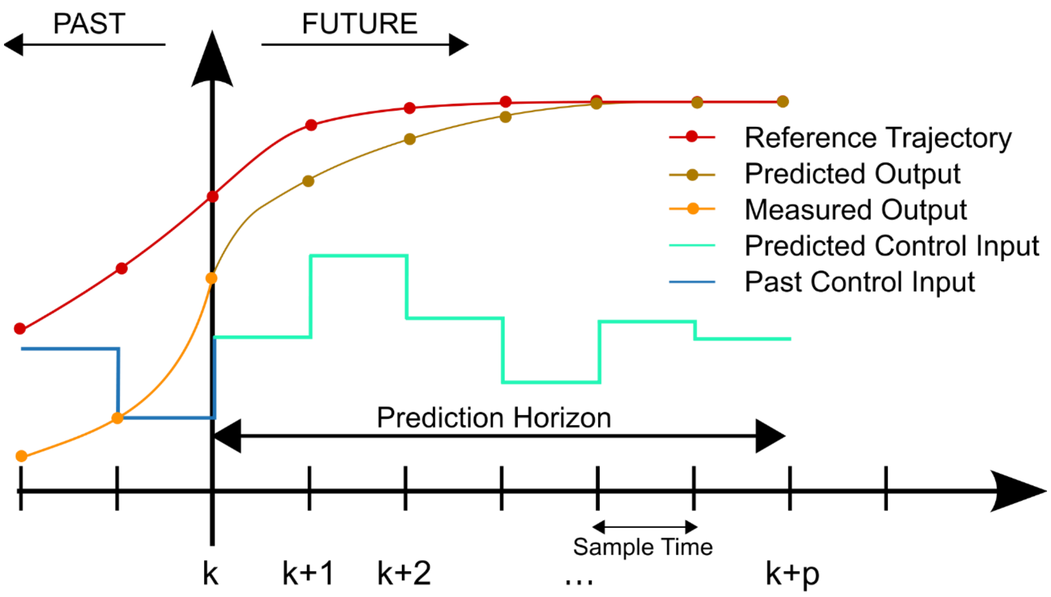
Current trend: Integrate control and economic optimization in one layer



Economic Model Predictive Control (EMPC)

- (Dynamic) optimization over a moving horizon of process economic performance
- Process constraints directly represented in the optimization problem
- Maximum freedom for optimization → better economic performance

Model Predictive Control (MPC)



$$\underset{u \in S(\Delta)}{\text{minimize}} \quad \int_0^{\tau_N} \left(|\tilde{x}(t)|^2_{Q_c} + |u(t)|^2_{R_c} \right) dt$$

$$\text{subject to} \quad \dot{\tilde{x}}(t) = f(\tilde{x}(t), u(t), 0)$$

$$\tilde{x}(0) = x(\tau_k)$$

$$g(\tilde{x}(t), u(t)) \leq 0, \quad \forall t \in [0, \tau_N)$$

Over 30 years of successful application in the industry

- Handles constrained multivariable processes

Successful application to large scale nonlinear processes(Seki 2001)

- Leveraged by great developments in the numerical solution strategies and increased computational power

Economic Model Predictive Control (EMPC)

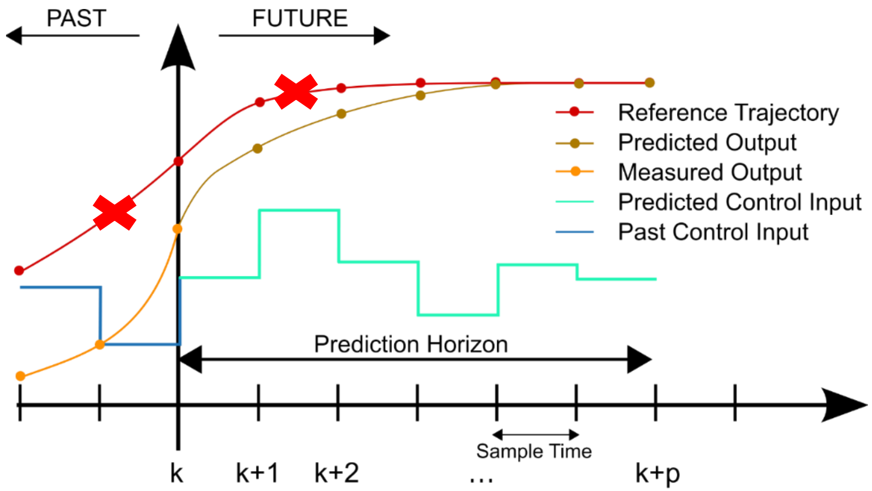
$$\underset{u \in S(\Delta)}{\text{minimize}} \quad \int_0^{\tau_N} l_e(\tilde{x}(t), u(t))\, dt$$

$$\text{subject to} \quad \dot{\tilde{x}}(t) = f(\tilde{x}(t), u(t), 0)$$

$$\tilde{x}(0) = x(\tau_k)$$

$$g(\tilde{x}(t), u(t)) \leq 0, \quad \forall t \in [0, \tau_N)$$

- Economic cost(profit)
- Dynamic model
- Initial condition
- State and input constraints



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Early beginning

- Application in the industry long before understanding it's theoretical properties ('Brave era')
  - ✓ First report of MPC application (Richalet et al. in 1976) on a Fluid catalytic cracking unit
  - ✓ Dynamic Matrix Control at Shell (Cutler and Ramaker, 1980)
- 3rd Generation MPC → Use of state space model, Kalman filters, hard/soft constraint handling, etc (**SMOC**: Shell Multivariable Optimizing control)

Early beginning

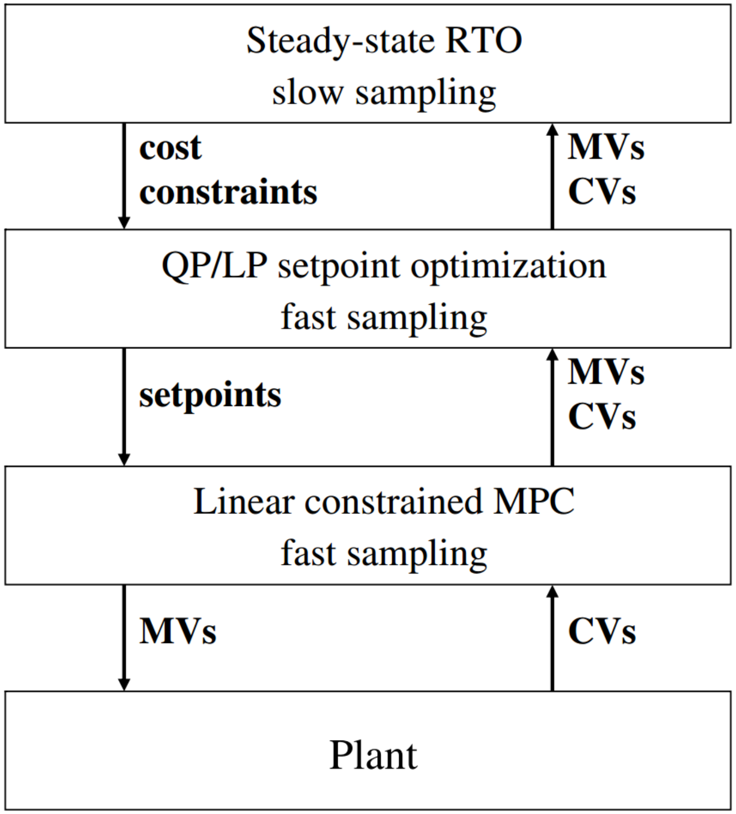
- Model predictive control initially used for multivariable process regulation (setpoint control)
- Smart practitioners, however, use/used the same framework for performance optimization:

Unreachable setpoint trick:

$$\text{minimize} \int_0^{\tau_N} \left(\left| y - y_{usp} \right|^2 + |u|^2 \right) dt$$

- y: variable we want to maximize/minimize
- y_{usp}: unreachable setpoint

Integrate steady-state optimization into MPC

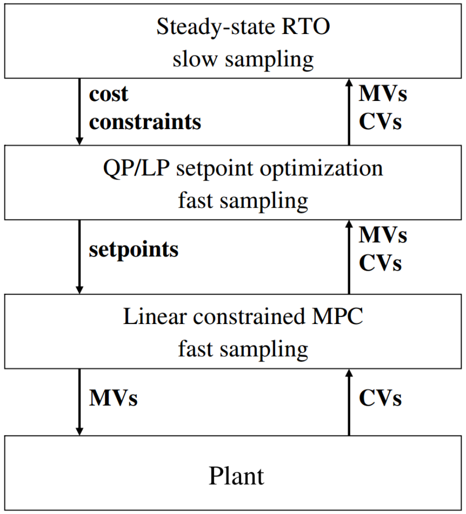


Intermediate optimization layer:

- ✓ Use info from RTO and lower MPC
- ✓ Computes setpoints for MPC → the best (dynamic) way to reache the RTO target

Very common in the industry (Morshedi (1985), Nath (2002))

Integrate steady-state optimization into MPC



$$\min_{y_{set}, u_{set}} [(y_{set} - y^*)^T C_y (y_{set} - y^*) + (u_{set} - u^*)^T C_u (u_{set} - u^*)$$
$$+ c_y (y_{set} - y^*) + c_u (u_{set} - u^*)]$$

$$\text{Subject to} \quad y_{set} = A_S u_{set} + d(k),$$
$$d(k) = d(k-1) + \Delta(k),$$
$$y_{min} \leqslant y_{set} \leqslant y_{max},$$
$$u_{min} \leqslant u_{set} \leqslant u_{max}$$

- Model constraints: from lower MPC
- Weights for cost function: linearized from RTO

Integration of nonlinear steady-state optimization in the linear MPC controller

$$\min_{\Delta u(k+i);\, i=0,\ldots,m-1} \sum_{j=1}^{p} \|W_1(y(k+j)-r)\|_2^2 + \sum_{i=0}^{m-1} \|W_2 \Delta u(k+i)\|_2^2$$
$$+ W_3 f_{\text{eco}}(u(k+m-1)) + \|W_4(u(k+m-1)$$
$$- u(k-1) - \Delta u(k))\|_2^2 + W_5[f_{\text{eco}}(u(k+m-1), y(k+\infty))$$
$$- f_{\text{eco}}(u(k), y'(k+\infty))]^2.$$

Add economic steady-state term to the cost function

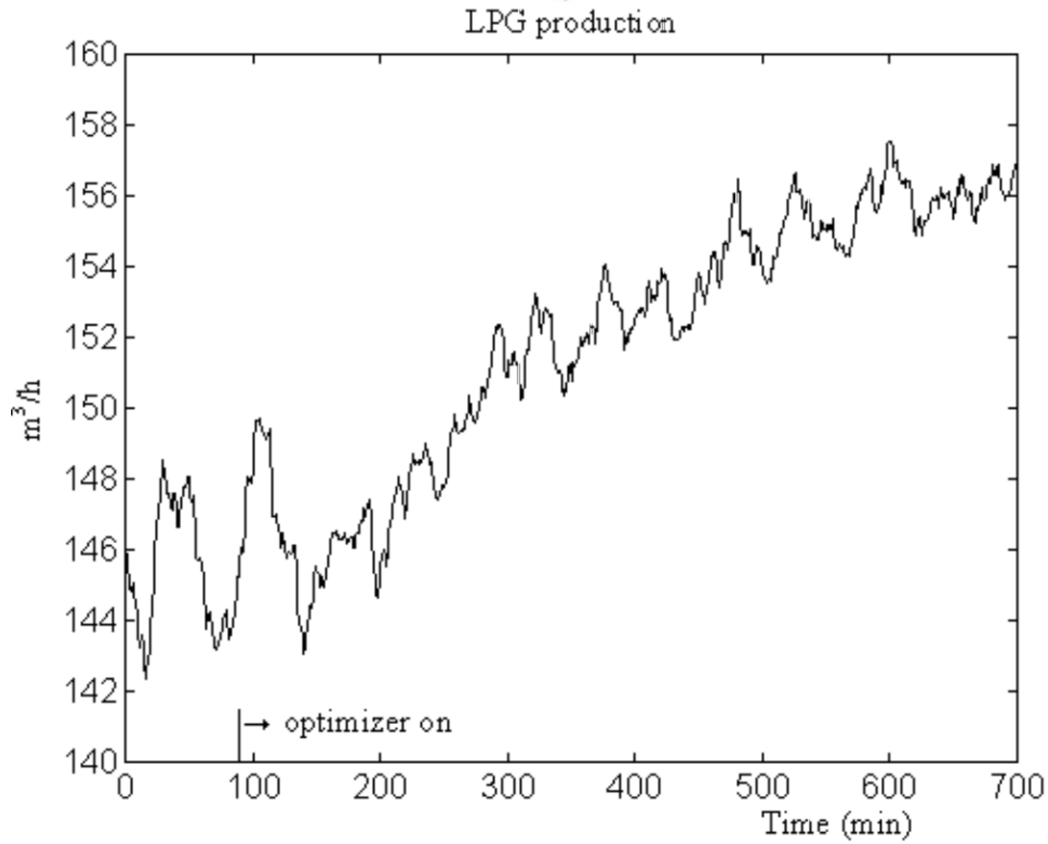
Economic objective f_{eco} computed using a nonlinear steady-state process model

Zanin et al. (2002)

Integration of nonlinear steady-state optimization in the linear MPC controller

Industrial implementation in a refinery by Petrobras.

Objective is maximize production of LPG in a FCC unit



Integrating real-time optimization into the model predictive Controller of the FCC system, Zanin et al. (2002)

Economic model predictive control

- Branded as 'Direct finite horizon optimizing control' (Engell, 2007)
  →Reported application to a Simulated Moving Bed (SMB) process
- Putting Nonlinear Model Predictive Control into Use (Foss & Schei, 2007)→ Several industrial applications
- 'Optimizing Process Economic Performance Using Model Predictive Control' (Rawlings, 2009)

Economic model predictive control is finally baptized

- Economic Model Predictive Control for Building Energy Systems (Ma, 2011)

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Presentation outline

Recent developments

- Infinite horizon EMPC
- Terminal cost/constraint EMPC
- Lyapunov based EMPC
- Closed-loop (economic) performance analysis

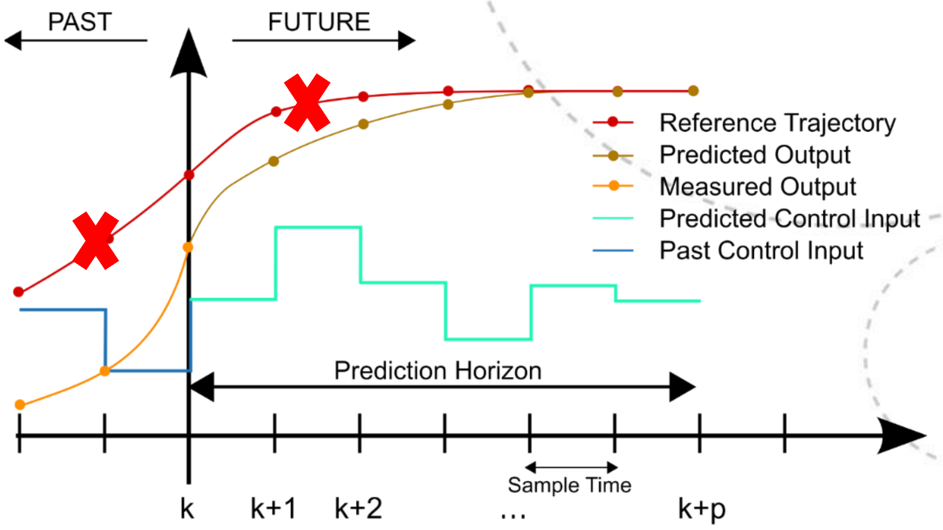
Economic Model Predictive Control (EMPC)

$$\underset{u \in S(\Delta)}{\text{minimize}} \quad \int_0^{\tau_N} l_e(\tilde{x}(t), u(t))\, dt$$

$$\text{subject to} \quad \dot{\tilde{x}}(t) = f(\tilde{x}(t), u(t), 0)$$

$$\tilde{x}(0) = x(\tau_k)$$

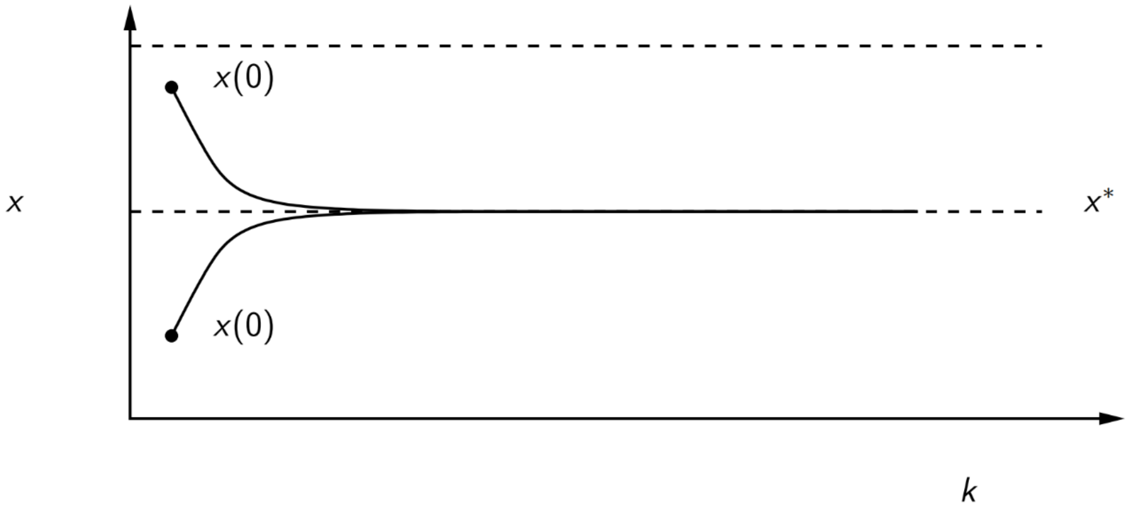
$$g(\tilde{x}(t), u(t)) \leq 0, \quad \forall\, t \in [0, \tau_N)$$

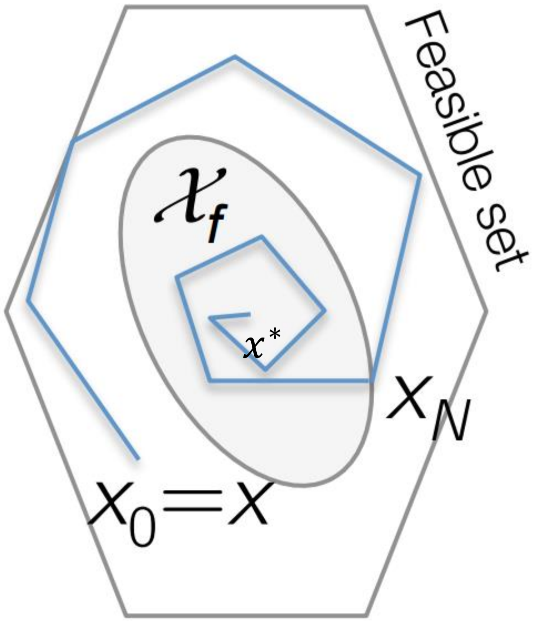


Replace the tracking cost function by an economic objective

There is no reference/setpoint to track

Recap: stability of tracking MPC





Nominal stability for finite horizon MPC

- Convergence to a desired equilibrium point
- Need to add terminal constraint set X_f and terminal cost $V_N(x_N)$ to original problem. X_f must be invariant with a local controller $k_f(x)$
- Optimal cost function is a Lyapunov function! → Monotonically decreasing

Challenges for the closed-loop stability analysis of EMPC

- There is no target to converge to
- Optimal cost is **not** a Lyapunov function for the closed-loop system
- Sequence of optimal costs is not monotone decreasing

Next we are going to see different EMPC formulations which tackle stability analysis in various ways

Infinite-horizon economic model predictive control

$$L_e(x(t), u(t)) = -\int_0^\infty e^{-\rho t} l_e(x(t), u(t))\, dt$$

$\rho>0$: discount factor

Commonly used in economic growth theory

Stability follows from Bellmann's optimality principle

But the problem is very hard to solve

Würth et al, *On the Numerical Solution of Discounted Economic NMPC on Infinite Horizons Dynamics and Control of Process Systems*, 2013.

Economic model predictive control with terminal constraints

$$\underset{u(0),u(1),\cdots,u(N-1)}{\text{minimize}} \quad \sum_{j=0}^{N-1} l_e(\tilde{x}(j), u(j)) + V_f(\tilde{x}(N))$$

$$\text{subject to} \quad \tilde{x}(j+1) = f_d(\tilde{x}(j), u(j), 0)$$

$$\tilde{x}(0) = x(k)$$

$$\tilde{x}(N) \in \mathbb{X}_f$$

$$(\tilde{x}(j), u(j)) \in \mathbb{Z}, \quad \forall j \in \mathbb{I}_{0:N-1}$$

V_f → final cost

X_f → terminal constraint set

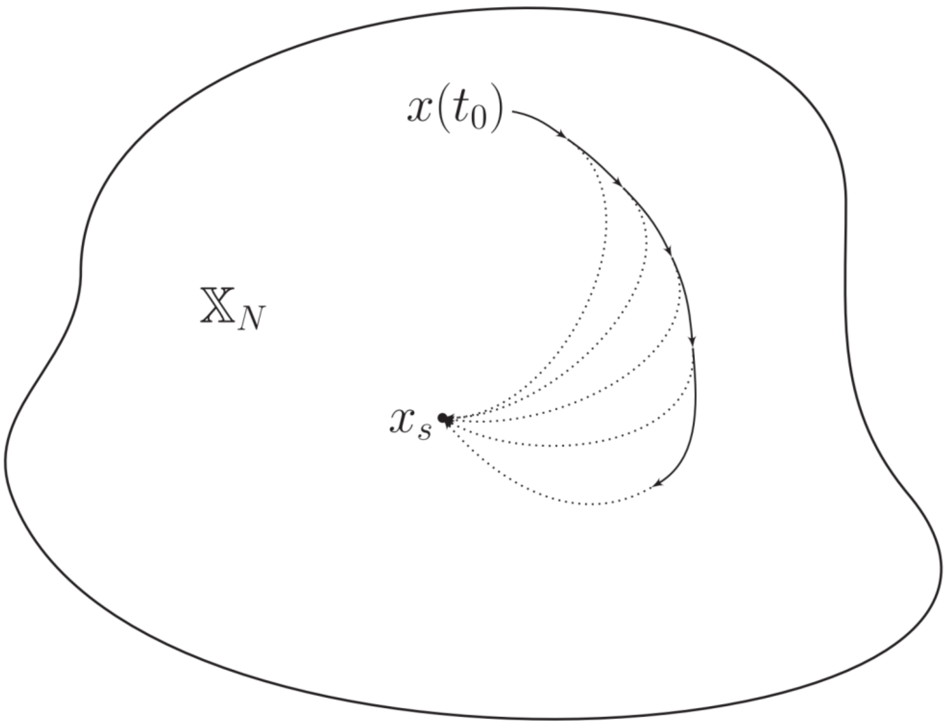
Commonly used:

$x(N) = x_s^*$

Optimal steady-state solution

M. Zanon, S. Gros, M. Diehl, *A Lyapunov function for periodic economic optimizing model predictive control*, 2013.

Example of trajectory of EMPC with terminal constraint ($x(N) = x_s$)



This formulation guarantees boundedness of trajectory

Not necessarily asymptotic stability

But the extra transients can be beneficial for economics

Ellis et al, *A tutorial review of economic model predictive control methods*, 2014.

Dissipativity

$$S(x(t_1)) \leq S(x(t_0)) + \int_{t_0}^{t_1} s(u(t), y(t))dt$$

S is a storage function

s is the supply rate

There can be no internal creation of energy; only internal **dissipation** of energy is possible.

Byrnes, and W. Lin, *Losslessness, feedback equivalence, and the global stabilization of discrete-time nonlinear systems*, 1994

Dissipativity

$$S(x(t_1)) \leq S(x(t_0)) + \int_{t_0}^{t_1} s(u(t), y(t))dt$$

For linear systems, this is equivalent to Positive Realness → Nyquist plot G(jw) always on RHP → Any negative feedback can stabilize the system

Byrnes, and W. Lin, *Losslessness, feedback equivalence, and the global stabilization of discrete-time nonlinear systems*, 1994

EMPC stability based on dissipativity

(1) Assume weak controllability

(2) Assume the closed loop under EMPC with terminal constraint is strictly dissipative with supply rate

- ✓ $s(x,u) = l_e(x,u) - l_e(x_s^*, u_s^*)$

Then optimal steady state x_s^* is asymptotically stable

D. Angeli et al, On average performance and stability of economic model predictive control, 2012

EMPC with Lyapunov-based constraints

Assume there exists a Lyapunov controller $u = k(x)$

- $V(x)$ → associated Lyapunov function
- Ω_ρ → stability region of the closed-loop under $k(x)$

A Lyapunov EMPC has two operating modes:

Mode 1: Ensures boundedness of state in $\Omega_{\rho_e} \subset \Omega_\rho$

Mode 2: Ensures convergence to the origin (x_s^*)

Heidarinejad et al., Economic model predictive control of nonlinear process systems using Lyapunov techniques, (2012)

EMPC with Lyapunov-based constraints

$$\underset{u \in S(\Delta)}{\text{minimize}} \quad L_e(\tilde{x}(t),\ u(t))$$

$$\text{subject to} \quad \dot{\tilde{x}}(t) = f(\tilde{x}(t), u(t), 0)$$

$$\tilde{x}(0) = x(\tau_k)$$

$$u(t) \in U,\ \forall t \in [0,\ \tau_N)$$

$$V(\tilde{x}(t)) \leq \rho_e, \quad \forall t \in [0, \tau_N)$$

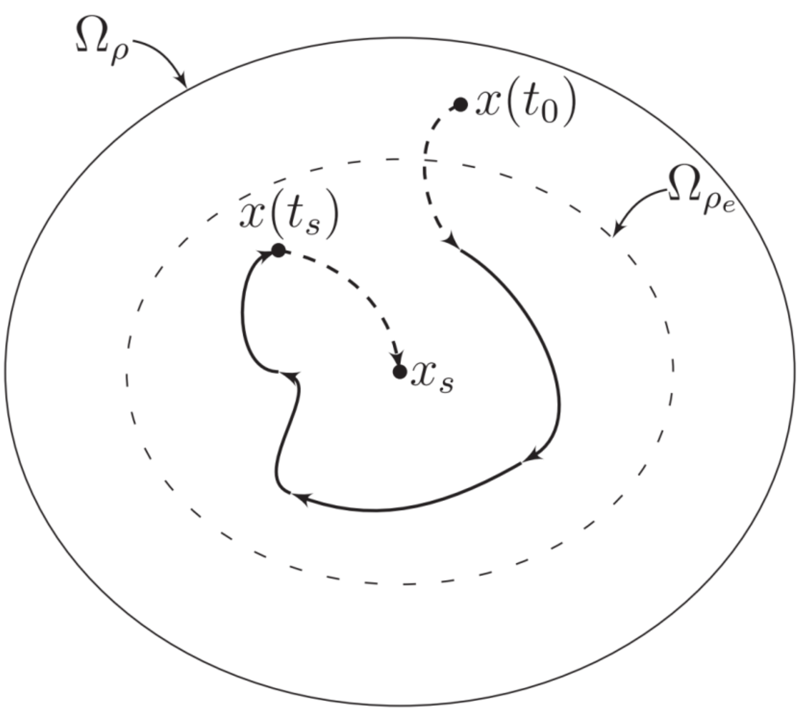
$$\text{if } V(x(\tau_k)) < \rho_e \quad \text{and} \quad t < t_s$$

Mode 1

$$\frac{\partial V}{\partial x} f(x(\tau_k), u(\tau_k), 0) \leq \frac{\partial V}{\partial x} f(x(\tau_k), k(x(\tau_k)), 0)$$

$$\text{if } V(x(\tau_k)) \geq \rho_e \quad \text{or} \quad t \geq t_s$$

Mode 2



Ellis & Christofides, *Economic Model Predictive Control with Time-Varying Objective Function for Nonlinear Process Systems*, 2014.

EMPC with Lyapunov-based constraints

✓ No need to modify economic cost

✓ Better feasibility and stability properties compared to end constraint EMPC

✓ Construction of controller $u = k(x)$ and corresponding Lyapunov function V(x) for general constrained nonlinear systems is hard!

Ellis & Christofides, *Economic Model Predictive Control with Time-Varying Objective Function for Nonlinear Process Systems*, 2014.

Closed-loop performance under EMPC

Two common methods to ensure performance:

- ✓ Use very large prediction horizon
- ✓ Use terminal constraint $x(N) = x_s^*$ (best steady state)

$$\limsup_{T\to\infty} \frac{\sum_{k=0}^{T} l_e(x(k), u(k))}{T+1} \leq l_e(x_s^*, u_s^*)$$

D. Angeli, et al, *On average performance and stability of economic model predictive control*, 2012.

Future directions

- ✓ Robustness → most of the results are based on nominal analysis
- ✓ Use of state-estimation → all EMPC schemes rely on state feedback
- ✓ Less conservative stability results → Only sufficient conditions so far

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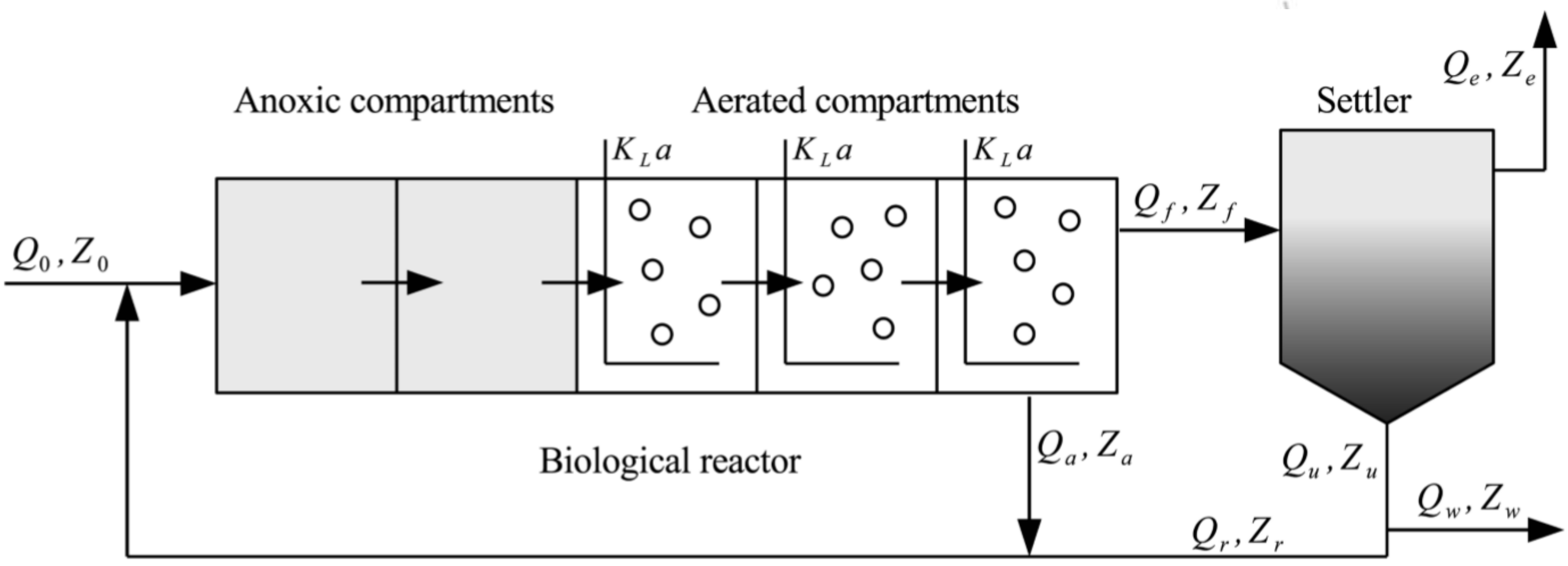
Economic Model Predictive Control of Wastewater Treatment Processes



- ✓ 145 differential states
- ✓ 2 manipulated inputs (Q_a and $K_L a$)
- ✓ 2 controlled variables ($S_{NO,2}$ and $S_{O,5}$)

Zeng & Liu (2014)

Objective function

$$\min_{u(\tau)\in S(\Delta)} \sum_{j=k}^{j=k+N} l(\tilde{x}(t_j|t_k), u(t_j|t_k))\,\mathrm{d}t + c(\tilde{x}(t_{k+N}), N_\mathrm{h})$$

$$\text{s.t.}\quad \dot{\tilde{x}}(t) = f(\tilde{x}(t)) + g(\tilde{x}(t))u(t)$$

$$\tilde{y}(t) = h(\tilde{x}(t))$$

$$\tilde{x}(t_k) = x(t_k)$$

$$u(t) \in \mathbb{U}$$

$$y(t) \in \mathbb{Y}$$

Stage cost: effluent quality+ operating cost

$$l(x(t_k), u(t_k)) = w_{\mathrm{EQ}}\,\mathrm{EQ}(t_k) + w_{\mathrm{OCI}}\mathrm{OCI}(t_k)$$

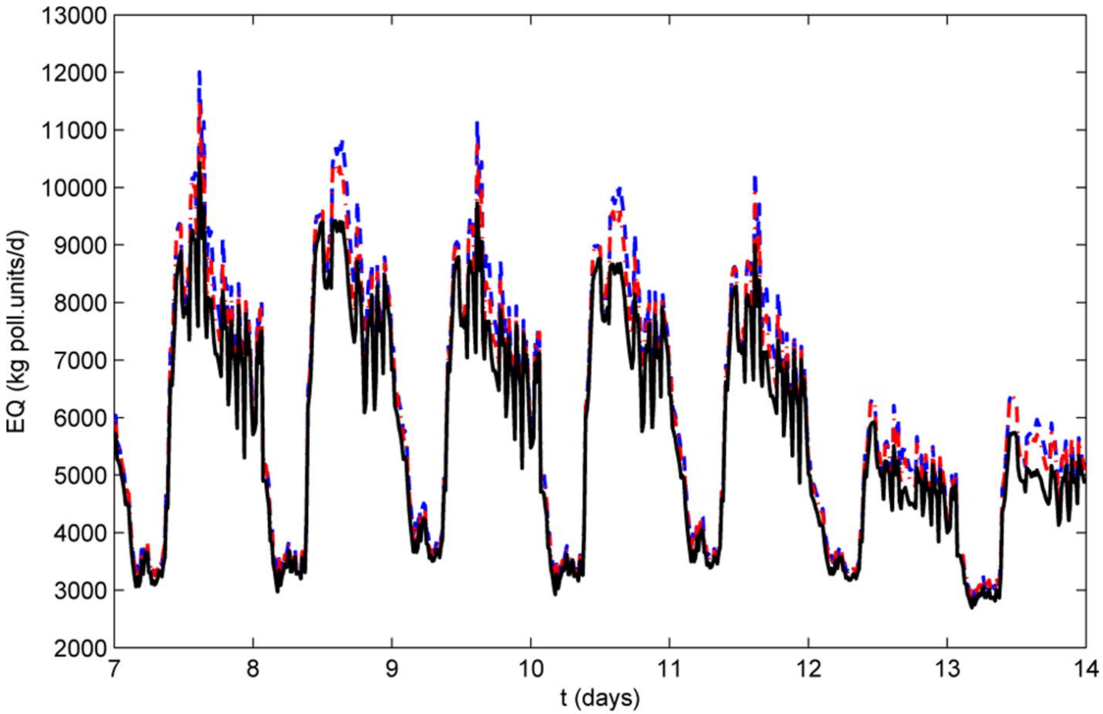
Zeng & Liu (2014)

Simulation results

Comparison with

- *PI control*
- *tracking MPC*

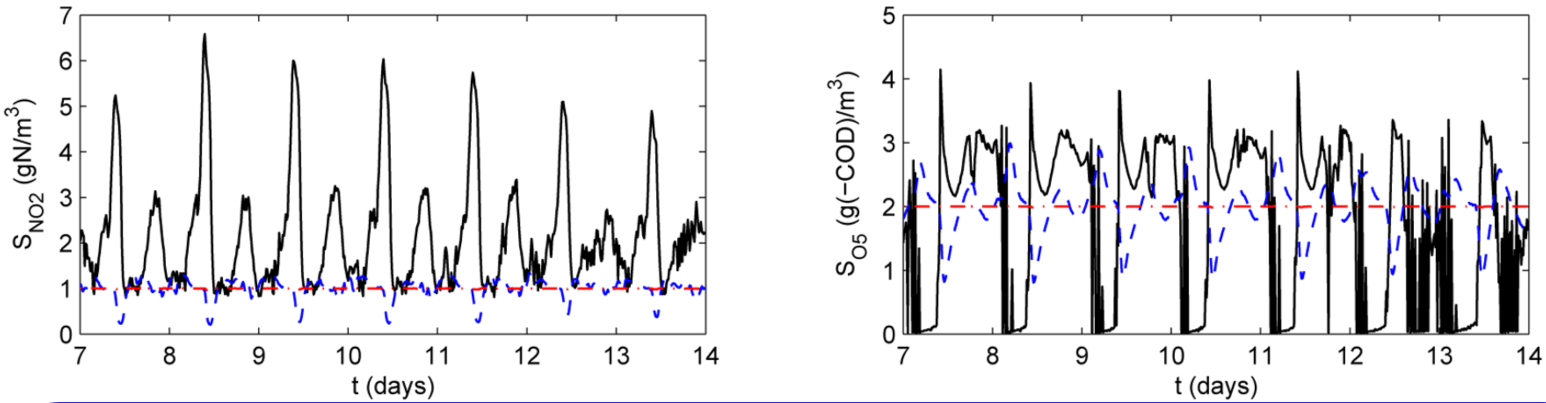
Performance comparison



7.4% Improvement over PI control and 5.8% over tracking MPC

Black: EMPC; Blue: PI; Red: tracking MPC

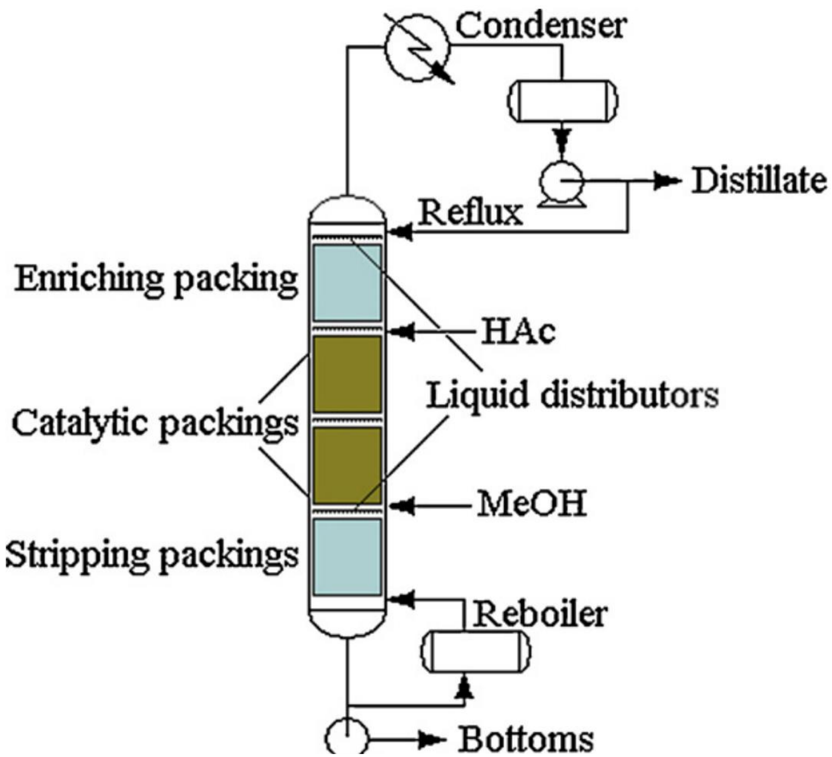
Process outputs (top) and manipulated variables (bottom)



- EMPC is not required to track setpoints
  - ✓ Great dynamic freedom for optimization
- PI and MPC may achieve similar performance by optimizing setpoints

Bl

Economic Model Predictive Control of a continuous catalytic distillation process



Reactants

- Acetic acid
- Methanol

Products

- Methyl acetate
- Water

Degrees of freedom

- Reboiler heat duty
- Reflux
- Inflow of reactants

581 differential states

Idris & Engell (2012)

Economic Model Predictive Control of a continuous catalytic distillation process

Profit={Product revenue − energy cost − cost of feeds}

$$\Psi(k) = \left(\dot{P}(k) \cdot C_P - \dot{H}(k) \cdot C_E - \sum_{j=1}^{N_f} \dot{R}_j(k) \cdot C_{R,j} \right)$$

Average quality constraint on the valuable product

$$\frac{\sum_{i=1}^{P} \text{Purity}_{\text{MeAc},k+i}}{P} \geq L_{\text{Purity}}^{\text{MeAc}}$$

Idris & Engell (2012)

Economic Model Predictive Control of a continuous catalytic distillation process

$$\Phi_{EOPC} = \sum_{b=1}^{R}\left(\sum_{j=1}^{M}\alpha_{b,j}\Delta u_b^2(k+j)\right) - \left(\sum_{i=1}^{P}\beta_i\left(\dot{P}(k+i)\cdot C_P - \dot{H}(k+i)\cdot C_E - \sum_{j=1}^{N_f}\dot{R}_j(k+i)\cdot C_{R,j}\right)\right)$$

Need quadratic regularization term for robustness

s.t.

$$x_{(i+1)} = f(x_i, z_i, u_i, i), i = k, \ldots, k+P$$

$$0 = g(x_i, z_i, u_i, i), i = k, \ldots, k+P$$

$$u_{min} \le u(i) \le u_{max}, i = k, \ldots, k+M$$

$$-\Delta u_{min} \le \Delta u(i) \le \Delta u_{max}, i = k, \ldots, k+M$$

$$u(i) = u(k+M), \forall i > k+M$$

Idris & Engell (2012)

Alternative I: tracking MPC with economic term

$$\Phi_{EoTC} = \sum_{n=1}^{N}\left(\sum_{i=1}^{P}\gamma_{n,i}(y_{n,ref}(k+i)-y_n(k+i))^2\right)+\sum_{b=1}^{R}\left(\sum_{j=1}^{M}\alpha_{b,j}\,\Delta u_b^2(k+j)\right)$$

$$-\left(\sum_{i=1}^{P}\beta_i\left(\dot{P}(k+i)\cdot C_P-\dot{H}(k+i)\cdot C_E-\sum_{j=1}^{N_f}\dot{R}_j(k+i)\cdot C_{R,j}\right)\right),$$

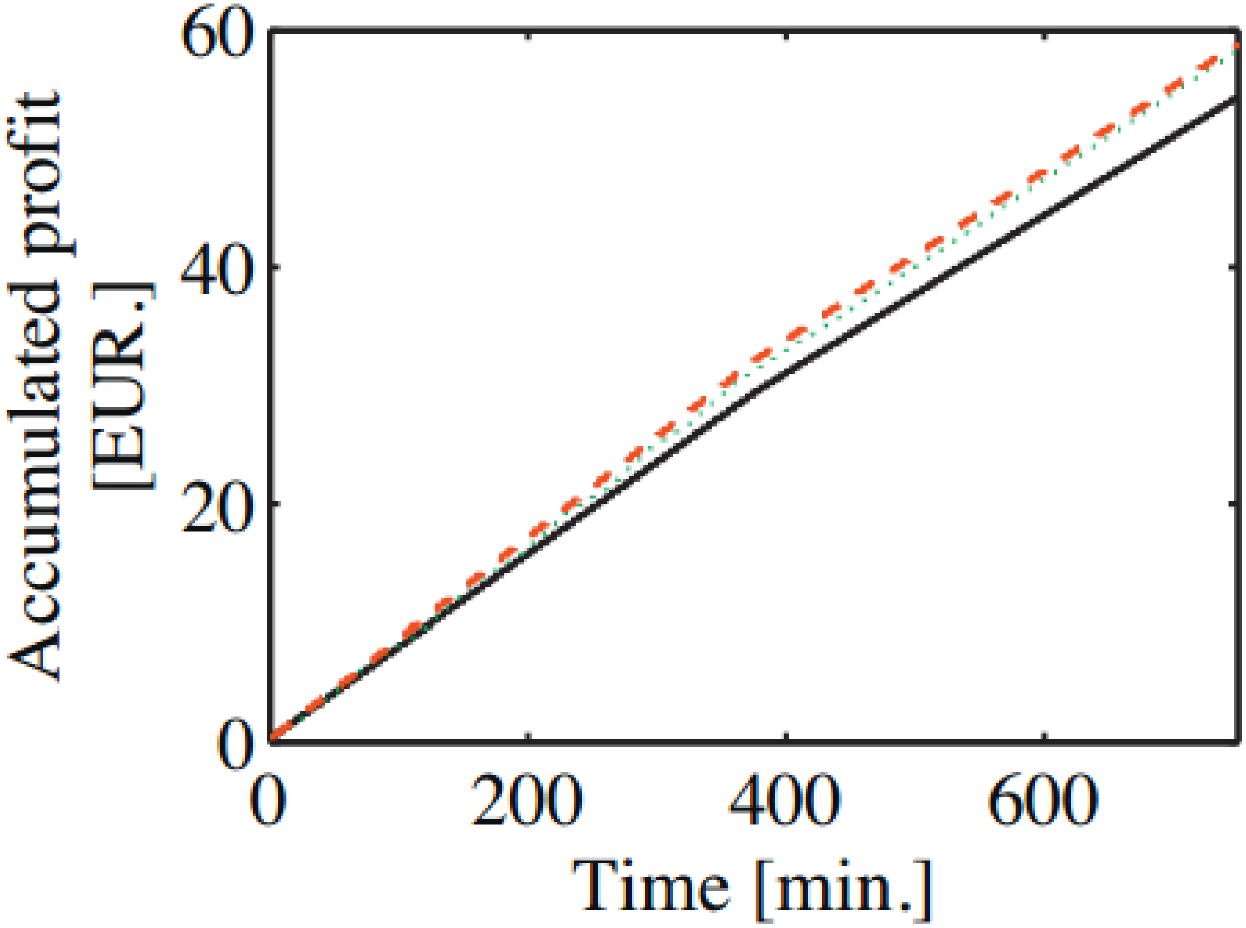
Idris & Engell (2012)

Alternative II: purely tracking MPC

$$\Phi_{EoTC} = \sum_{n=1}^{N} \left(\sum_{i=1}^{P} \gamma_{n,i} (y_{n,ref}(k+i) - y_n(k+i))^2 \right) + \sum_{b=1}^{R} \left(\sum_{j=1}^{M} \alpha_{b,j} \Delta u_b^2 (k+j) \right)$$

Idris & Engell (2012)

Cost comparison



8.2% improvement in profit over tracking MPC

Red: EMPC; Green: tracking MPC with economic term; ***Black****: pure tracking MPC*

Computational time

Method	Computational time
Tracking MPC	3.5-3.9min
Tracking MPC econ. term	4.1–5.0min
EMPC	15–20 min

Apparently, the economic cost function is very flat
→ NLP solver has problems converging

Final thoughts

We have seen an overview of a method that combines economic optimization and control in one layer

Economic Model Predictive Control

Makes sense if there is no time scale separation → time constant of process is comparable to that of the economics (e.g price variations)

Great research efforts in the latest years

Simulations suggest some economic benefit → not a lot of validation in practice

Limitations

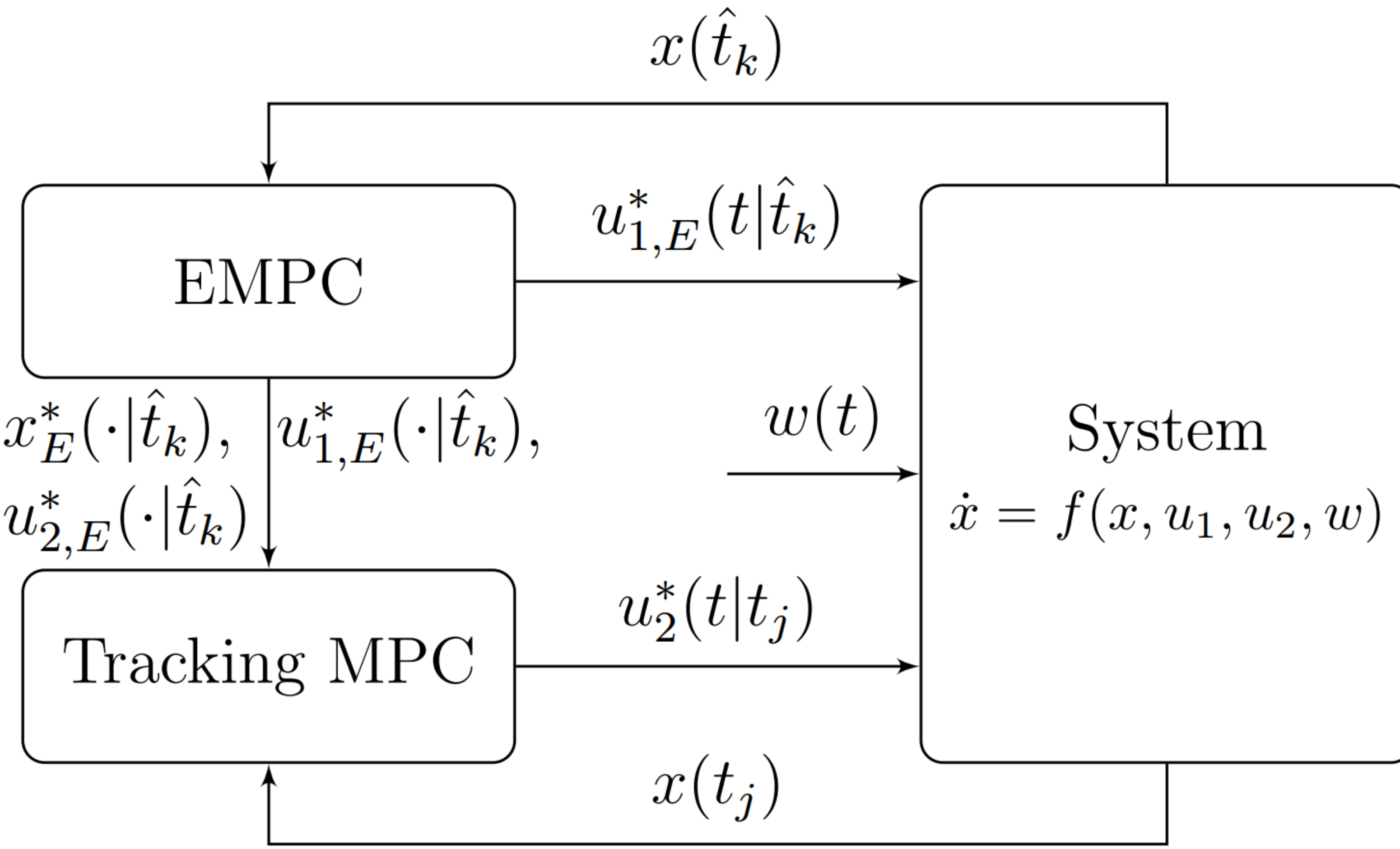
Reliability→ optimizer must converge or else...

Robustness → Some processes may be very hard to stabilize

Computational cost→ although it's becoming less of a problem due to improvements in computer power and solution approaches

Higher cost → implementation and maintenance
(Often good performance is achievable with simpler methods)

Alternative: Hierarchical EMPC approach



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Thank you!

The end