Quantitative methods for controlled variables selection

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Thesis outline

Ch. 1. Introduction

Ch. 2. Brief overview of control structure design and methods

Ch. 3. Convex formulations for optimal CV using MIQP

Ch. 4. Convex approximations for optimal CV with structured H

Ch. 5. Quantitative methods for regulatory layer selection

Ch. 6. Dynamic simulations with self-optimizing CV

Ch. 7. Conclusions and future work

Appendices A - E

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CV – Controlled Variables
MIQP - Mixed Integer Quadratic Programming

Presentation outline

- Plantwide control : Self optimizing control formulation for CV, c = Hy – Chapter 2
- Convex formulation for CV with full H - Chapter 3
  - Convex formulation
  - Globally optimal MIQP formulations
  - Case studies
- Convex approximation methods for CV with structured H – Chapter 4
  - Convex approximations
  - MIQP formulations for structured H with measurement subsets
  - Case studies
- Regulatory control layer selection – Chapter 5
  - Problem definition
  - Regulatory control layer selection with state drift minimization
  - Case studies
- Conclusions and Future work

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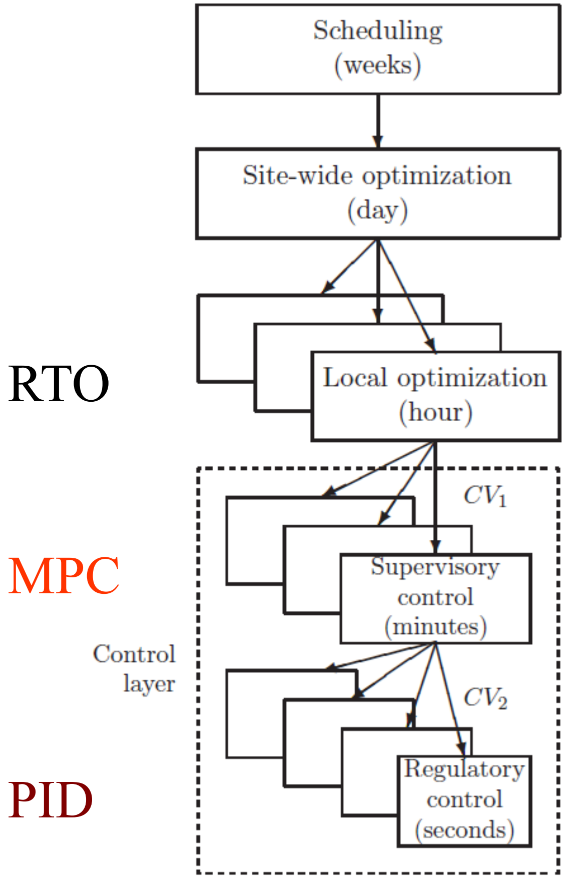
Plantwide control: Hierarchical decomposition



- Each layer operates at different time scales
- The decisions are cascaded from top to bottom
- Top layer provides set points to the bottom layer

- Scope of the thesis: Optimal operation constituting optimization layer and control layers
- Assumption: Economics are primarily decided by steady-state
- Focus is on the selection of controlled variables CV_1 and CV_2

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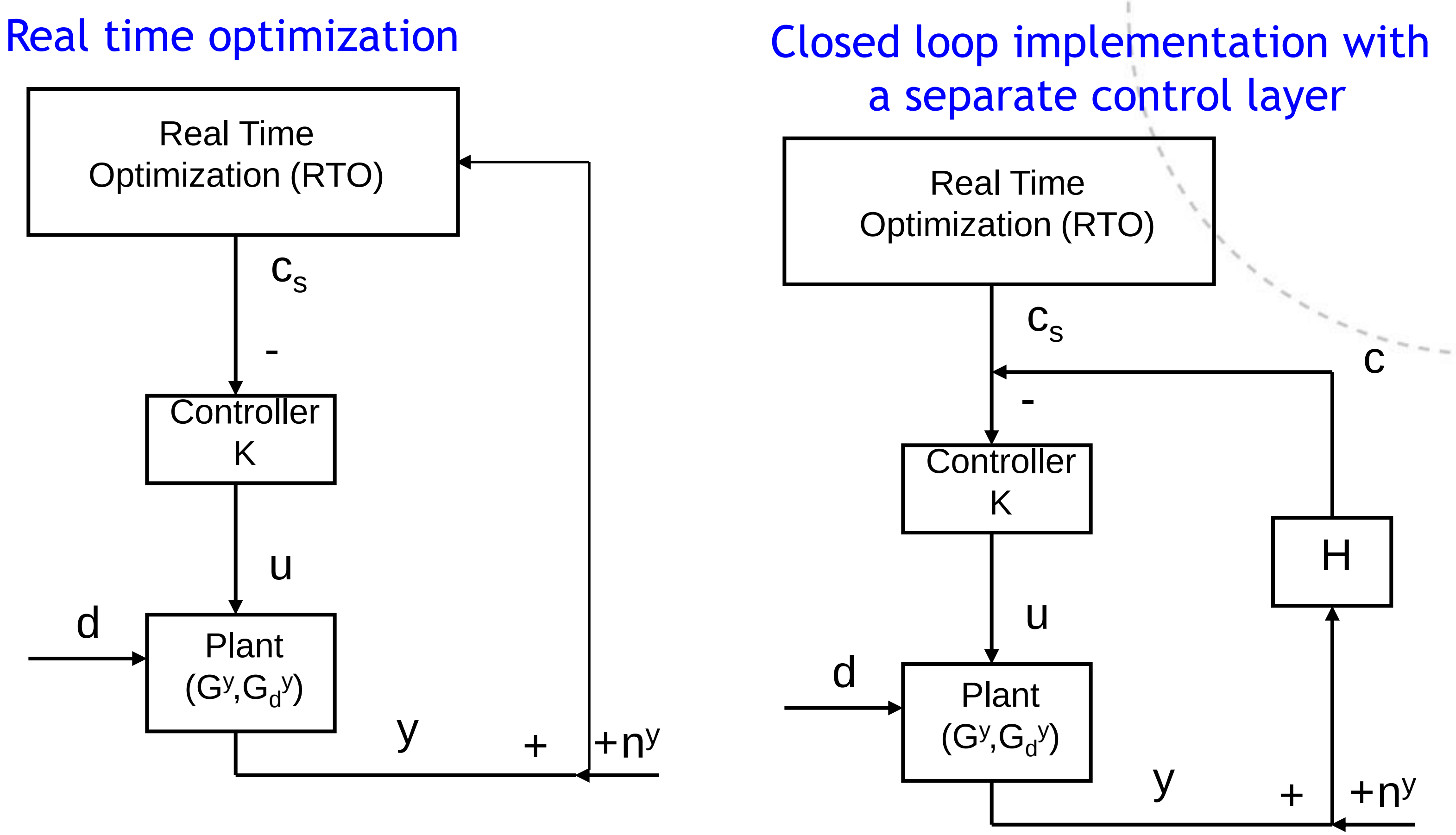
Optimal operation



Ref: Kassidas et al., 2000
Engell, 2007

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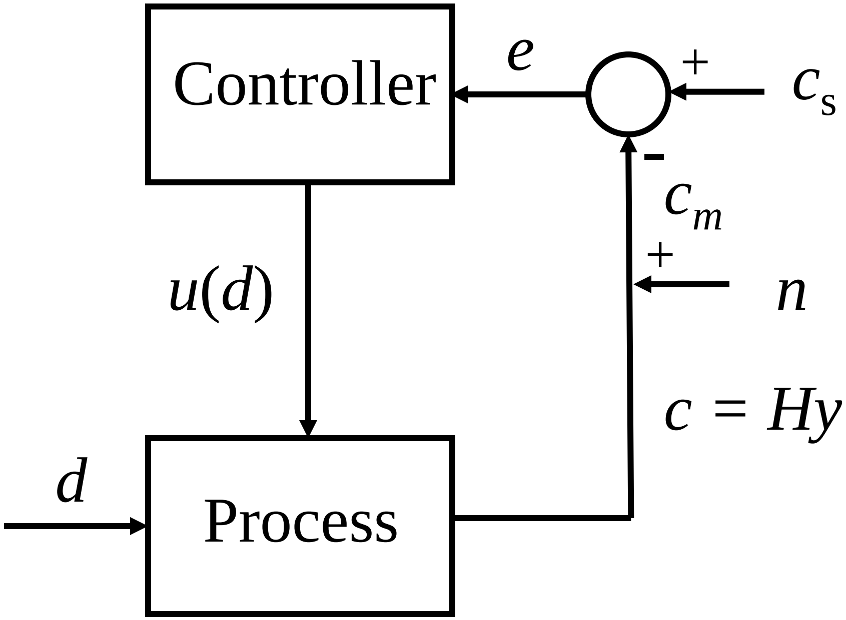
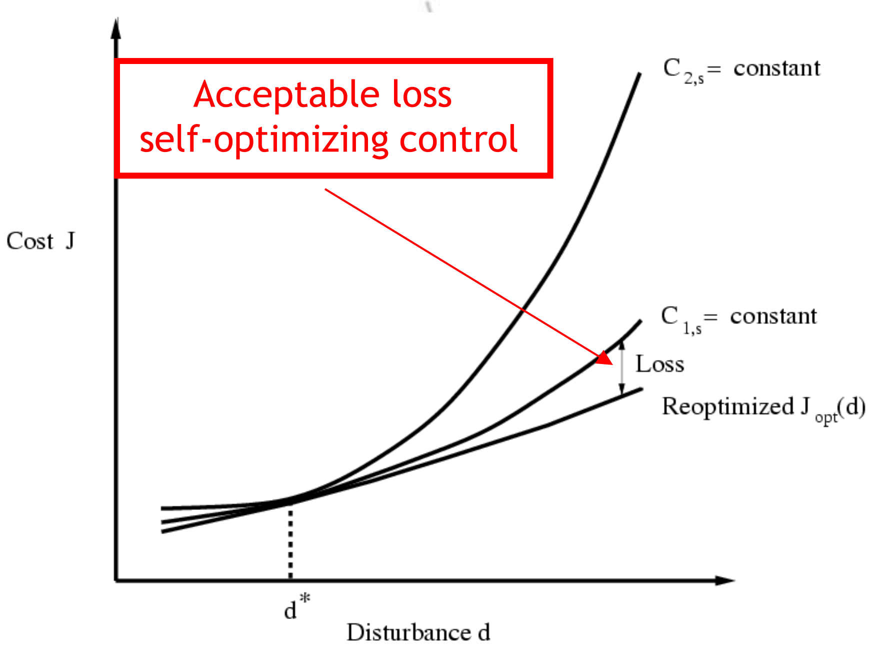
Self optimizing control





Self-optimizing control is said to occur when we can achieve an acceptable loss (in comparison with truly optimal operation) with constant setpoint values for the controlled variables without the need to reoptimize when disturbances occur.

Ref: Skogestad, JPC, 2000.

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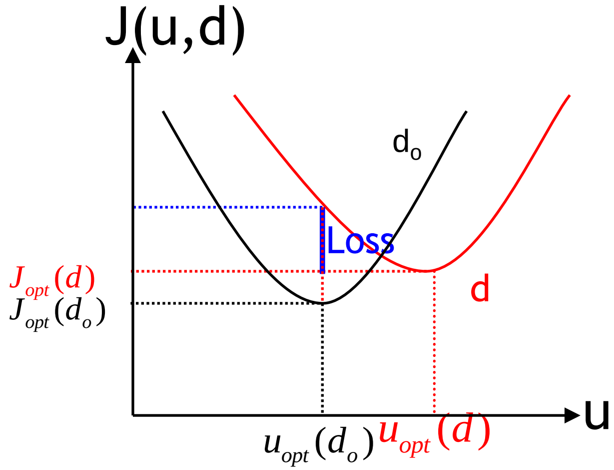
Problem Formulation, c = Hy

Assumptions:
(1) Active constraints are controlled
(2) Quadratic nature of J around $u_{opt}(d)$
(3) Active constraints remain same throughout the analysis

Optimal steady-state operation

$$\min_u J(u,d)$$



$$L = J(u,d) - J_{opt}(u_{opt}(d),d)$$

Real time optimization

$$J(u,d) = J(u_{opt}(d),d) + J_u(u-u_{opt}(d)) + \frac{1}{2}(u-u_{opt}(d))^T J_{uu}(u-u_{opt}(d)) + \zeta^3$$

$$L = \frac{1}{2}(u-u_{opt}(d))^T J_{uu}(u-u_{opt}(d))$$

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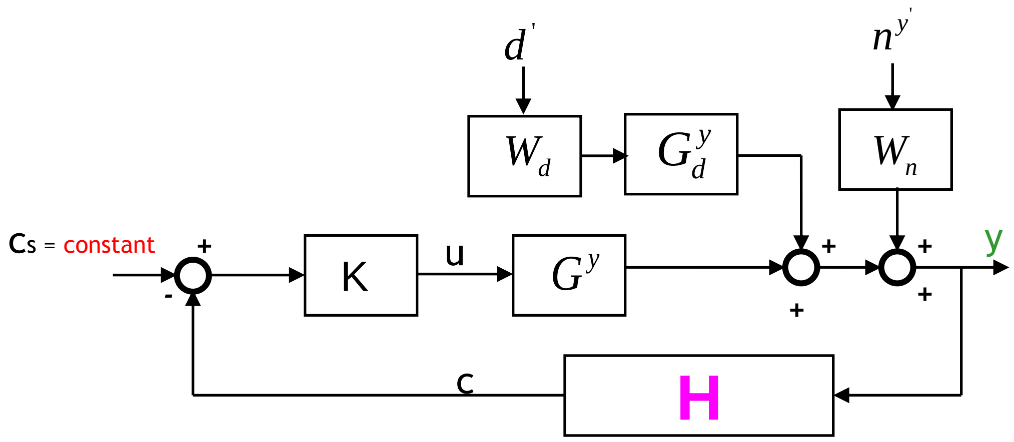
Problem Formulation, c = Hy



Loss $L = f(H, d', n^{y'})$

d´,ny´ as random variables

Controlled variables, $c = Hy$

$$L_{avg} = \left\| J_{uu}^{1/2} (HG^y)^{-1} HY \right\|_F^2 \quad \forall \left\| \begin{bmatrix} d' \\ n^{y'} \end{bmatrix} \right\| \in \mathbb{N}(0,1)$$

$$Y = [(G^y J_{uu}^{-1} J_{ud} - G_d^y) W_d \quad W_n]$$

Ref: Halvorsen et al. I&ECR, 2003
Kariwala et al. I&ECR, 2008

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Convex formulation (full H)

$$\min_H \left\| J_{uu}^{1/2} (HG^y)^{-1} HY \right\|_F$$

Seemingly
Non-convex
optimization problem

D : any non-singular matrix

$$H_1 = DH$$

$$(H_1G_y)^{-1}H_1 = (DHG_y)^{-1}DH = (HG_y)^{-1}D^{-1}DH = (HG_y)^{-1}H$$

Objective function unaffected by D.
So can choose HG^y freely.

H is made unique by adding a constraint as $HG^y = J_{uu}^{1/2}$

$$\min_H \|HY\|_F$$

subject to $HG^y = J_{uu}^{1/2}$

Problem is convex in decision matrix H

Full H
Convex
optimization problem

Global solution

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Ref: Alstad 2009

Vectorization

$$\min_{H} \|HY\|_F$$

subject to $HG^y = J_{uu}^{1/2}$

$$H = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1ny} \\ h_{21} & h_{22} & \cdots & h_{2ny} \\ \vdots & \vdots & \ddots & \vdots \\ h_{nu1} & h_{nu2} & \cdots & h_{nu*ny} \end{bmatrix}_{nu \times ny}$$

is vectorized along the rows of H to form

$$h_\delta = \begin{bmatrix} h_{11} \\ h_{12} \\ \vdots \\ h_{nu*ny} \end{bmatrix}_{(nu*ny)\times 1}$$

$$\min_{h_\delta} \quad h_\delta^T F_\delta X_\delta$$

$$st. \quad G_\delta^T X_\delta = J_\delta$$

$$F_\delta = Y_\delta Y_\delta^T$$

Problem is convex QP in decision vector h_δ

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Controlled variable selection

$$\min_{H} \left\| J_{uu}^{1/2} (HG^y)^{-1} HY \right\|_F \Leftrightarrow \min_{H} \|HY\|_F \quad \text{st.} \quad HG^y = J_{uu}^{1/2} \Leftrightarrow \min_{h_\delta} \; h_\delta^{\,T} F_\delta h_\delta \quad st. \quad G_\delta^{\,T} h_\delta = J_\delta$$

Optimization problem :

Minimize the average loss by selecting H and CVs as

(i) best individual measurements

(ii) best combinations of all measurements

(iii) best combinations with few measurements

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MIQP formulation (full H)

$$\begin{matrix} \sigma_1 & \sigma_2 & \cdots & \sigma_{ny} \end{matrix}$$

$$H = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1ny} \\ h_{21} & h_{22} & \cdots & h_{2ny} \\ \vdots & \vdots & \ddots & \vdots \\ h_{nu1} & h_{nu2} & \cdots & h_{nu*ny} \end{bmatrix}_{nu \times ny}$$

$$\sigma_i \in \{0,1\}$$
$$i = 1, 2, \cdots, ny$$

is vectorized along the rows of H to form

$$h_\delta = \begin{bmatrix} h_{11} \\ h_{12} \\ \vdots \\ h_{nu*ny} \end{bmatrix}_{(nu*ny)\times 1} \qquad \sigma_\delta = \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \vdots \\ \sigma_{ny} \end{bmatrix}_{ny \times 1}$$

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MIQP formulation

$$H = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1ny} \\ h_{21} & h_{22} & \cdots & h_{2ny} \\ \vdots & \vdots & \ddots & \vdots \\ h_{nu1} & h_{nu2} & \cdots & h_{nu*ny} \end{bmatrix}_{nu \times ny}$$

with columns labelled $\sigma_1 \quad \sigma_2 \quad \cdots \quad \sigma_{ny}$

Big-m method

$$\min_{x_\delta, \sigma_\delta} \quad h_\delta^T F_\delta h_\delta$$

$$st. \quad G_\delta^{y^T} h_\delta = J_\delta$$

$$P\sigma_\delta = n$$

$$\begin{bmatrix} -m \\ -m \\ \vdots \\ -m \end{bmatrix} \sigma_i \leq \begin{bmatrix} h_{1i} \\ h_{2i} \\ \vdots \\ h_{nui} \end{bmatrix} \leq \begin{bmatrix} m \\ m \\ \vdots \\ m \end{bmatrix} \sigma_i$$

$$\forall i = 1, 2, \cdots, ny$$

Selection of appropriate m is an iterative method and can increase the computational requirements

Indicator constraint method

$$\min_{x_\delta, \sigma_\delta} \quad h_\delta^T F_\delta h_\delta$$

$$st. \quad G_\delta^{y^T} h_\delta = J_\delta$$

$$P\sigma_\delta = n$$

Indicator constraints

$$\sigma_i = 0 \Rightarrow \begin{bmatrix} h_{1i} \\ h_{2i} \\ \vdots \\ h_{nui} \end{bmatrix} = \underline{0}_{n_u \times 1}$$

$$\forall i = 1, 2, \cdots, ny$$

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Case Study : Distillation Column

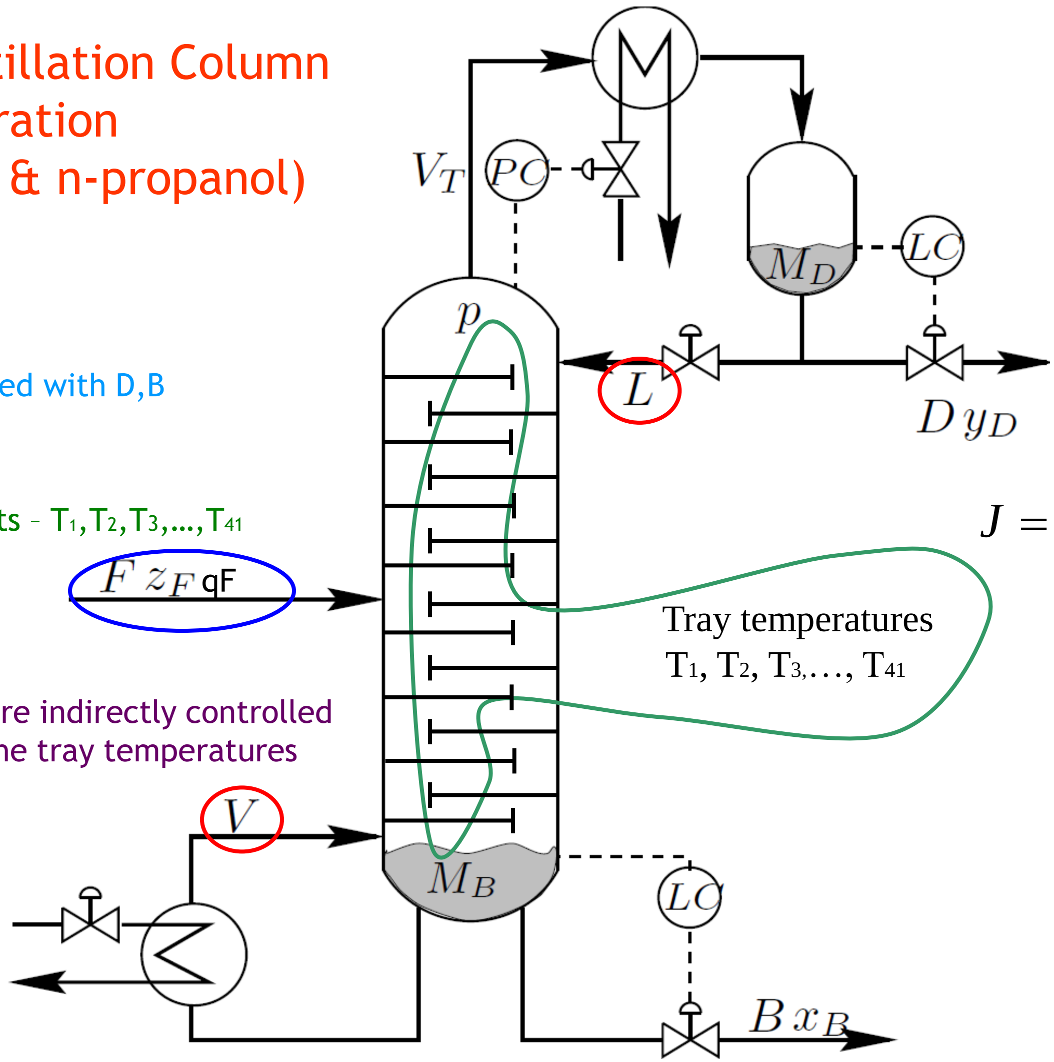
Binary Distillation Column
LV configuration
(methanol & n-propanol)

41 Trays

Level loops closed with D,B

2 MVs – L,V
41 Measurements – $T_1, T_2, T_3, \ldots, T_{41}$
3 DVs – F, ZF, qF

*Compositions are indirectly controlled by controlling the tray temperatures



$$J = \left(\frac{y_D - y_{D,s}}{y_{D,s}}\right)^2 + \left(\frac{x_B - x_{B,s}}{x_{B,s}}\right)^2$$

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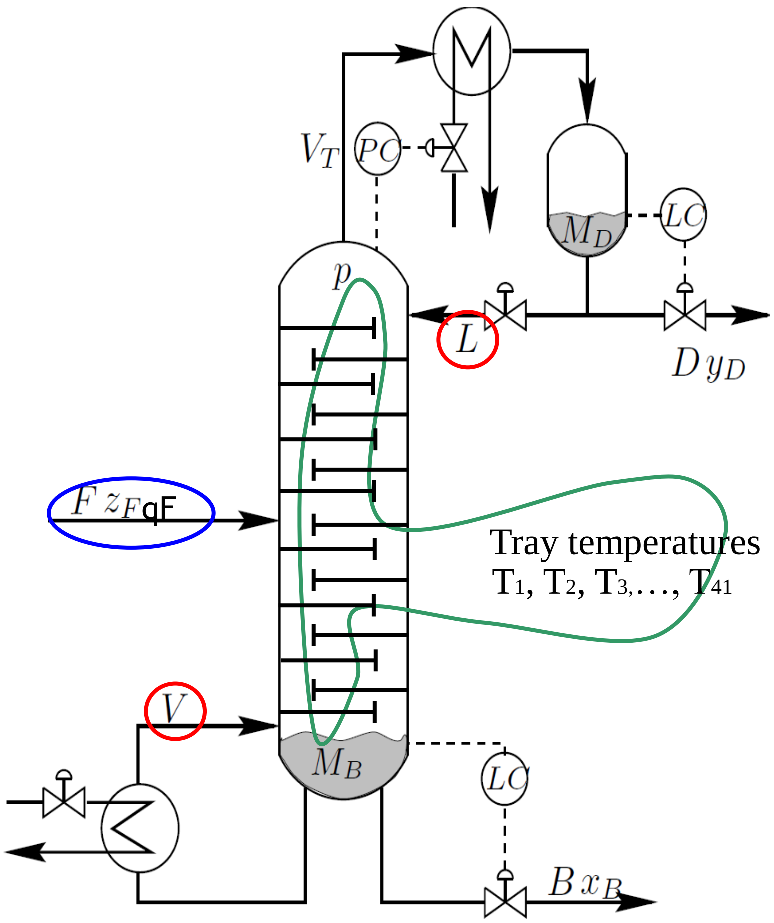
Distillation Column : Full H



Binary distillation column

c = Hy

$$c = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} \qquad y = \begin{bmatrix} T_1 \\ T_2 \\ \vdots \\ T_{41} \end{bmatrix}$$

$$c_1 = h_{11}T_1 + h_{12}T_2 + \cdots + h_{141}T_{41}$$
$$c_2 = h_{21}T_1 + h_{22}T_2 + \cdots + h_{241}T_{41}$$

$$H = \begin{bmatrix} h_{11}\, h_{12} \cdots h_{120} \cdots h_{130} \cdots\ h_{141} \\ h_{21}\, h_{22} \cdots h_{220} \cdots h_{230} \cdots h_{241} \end{bmatrix}$$

Find H that minimizes

$$L_{avg} = \left\| J_{uu}^{1/2} (HG^y)^{-1} HY \right\|_F$$

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Case Study : Distillation Column

$$L_{avg} = \frac{1}{2}\left\| (J_{uu}^{1/2}(HG^y)^{-1}HY) \right\|_F^2$$

$$Y = [FW_d \quad W_n]$$
$$F = G^y J_{uu}^{-1} J_{ud} - G_d^y$$

Data

$G^y \in \mathbb{R}^{41\times 2}; G_d^y \in \mathbb{R}^{41\times 3}; J_{uu} \in S_+^2; J_{ud} \in \mathbb{R}^{2\times 3}; W_d \in \mathbb{R}^{3\times 3}; W_n \in \mathbb{R}^{41\times 41}$

$$G^y = \begin{bmatrix} 10.83 & -10.96 \\ 15.36 & -15.55 \\ \vdots & \vdots \\ 13.01 & -12.81 \\ 8.76 & -8.62 \end{bmatrix}; G_d^y = \begin{bmatrix} 5.85 & 11.17 & 10.90 \\ 8.30 & 15.86 & 15.47 \\ \vdots & \vdots & \vdots \\ 5.85 & 13.10 & 12.90 \\ 3.94 & 8.82 & 8.68 \end{bmatrix};$$

$$J_{uu} = \begin{bmatrix} 3.88 & -3.88 \\ -3.89 & 3.90 \end{bmatrix}; J_{ud} = \begin{bmatrix} 1.96 & 3.96 & 3.88 \\ -1.97 & -3.97 & -3.89 \end{bmatrix};$$

$$W_d = \begin{bmatrix} 0.2 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}; Wn = diag(0.5 * ones(41,1))$$

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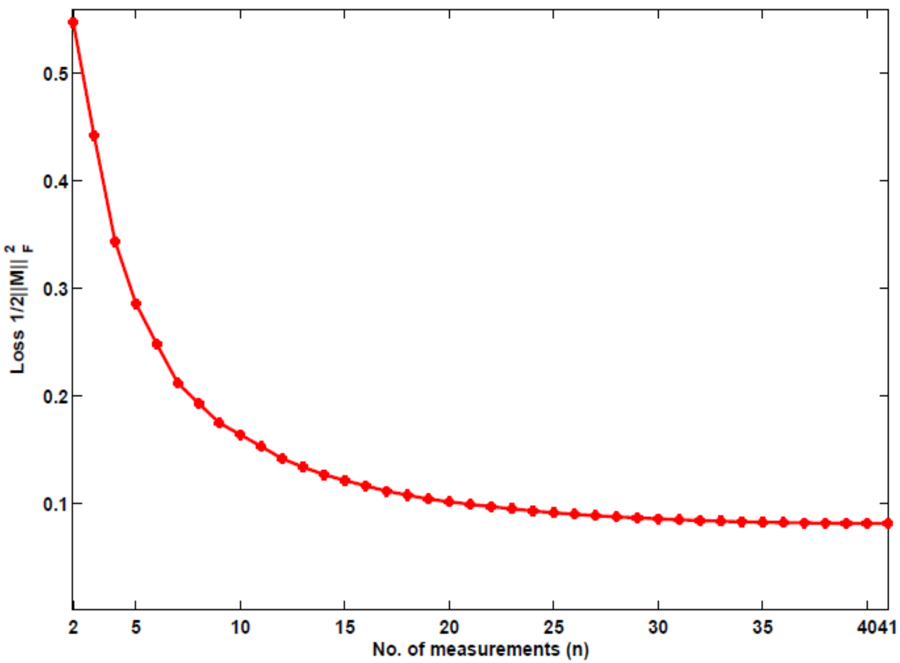
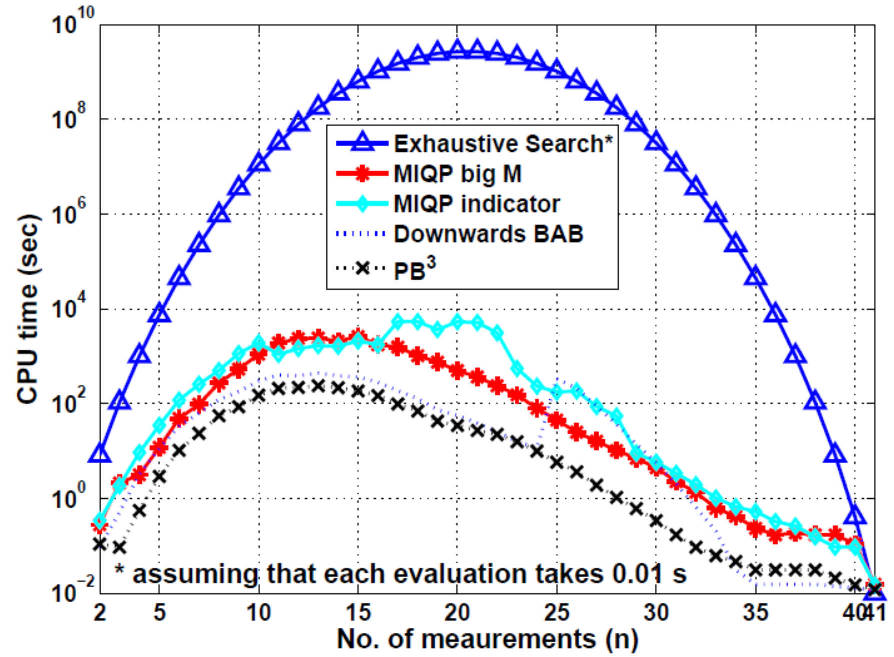
Distillation Column Full H : Result

No. Meas n	$\mathbf{c}'s$ as combinations of measurements	Loss $\frac{1}{2}\|\mathbf{M}\|_F^2$
2	$c_1 = T_{12}$ $c_2 = T_{30}$	0.5477
3	$c_1 = T_{12} + 0.0446T_{31}$ $c_2 = T_{30} + 1.0216T_{31}$	0.4425
4	$c_1 = 1.0316T_{11} + T_{12} + 0.0993T_{31}$ $c_2 = 0.0891T_{11} + T_{30} + 1.0263T_{31}$	0.3436
41	$c_1 = f(T_1, T_2, \ldots, T_{41})$ $c_2 = f(T_1, T_2, \ldots, T_{41})$	0.0813

Distillation Column Full H : Result





Comparison with customized Branch And Bound (BAB)*

- MIQP is computationally more intensive than Branch And Bound (BAB) methods
  (Note that computational time is not very important as control structure selection is an offline method)

- MIQP formulations are intuitive and easy to solve

* Kariwala and Cao, 2010

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Other case studies

- Toy example
  - 4 measurements, 2 inputs, 1 disturbance

- Evaporator system
  - 10 measurements, 2 inputs, 3 disturbances

- Kaibel distillation column
  - 71 measurements, 4 inputs, 7 disturbances

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Convex approximation methods for structured H

Structured H will have some zero elements in H

Example:
decentralized H
(block-diagonal H)

$$\mathbf{H}_{BD} = \begin{bmatrix} \mathbf{H}_1 & 0 & \cdots & 0 \\ 0 & \mathbf{H}_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{H}_{n_{iu}} \end{bmatrix}$$

triangular H

$$\mathbf{H}_{T} = \begin{bmatrix} \mathbf{H}_{11} & \mathbf{H}_{12} & \cdots & \mathbf{H}_{1n_{iu}} \\ 0 & \mathbf{H}_{22} & \cdots & \cdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{H}_{n_{iu}n_{iu}} \end{bmatrix}$$

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Convex approximations for Structured H

$$\min_H \left\| J_{uu}^{1/2} (HG^y)^{-1} HY \right\|_F$$

D : any non-singular matrix $H_1 = DH$ $(H_1 G_y)^{-1} H_1 = (DHG_y)^{-1} DH = (HG_y)^{-1} D^{-1} DH = (HG_y)^{-1} H$

For a structured H like

$$H_{BD} = \begin{bmatrix} H_1 & 0 & \cdots & 0 \\ 0 & H_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & H_{n_{iu}} \end{bmatrix} \text{ or } H_T = \begin{bmatrix} H_{11} & H_{12} & \cdots & H_{1n_{iu}} \\ 0 & H_{22} & \cdots & \cdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & H_{n_{iu}n_{iu}} \end{bmatrix}$$

only a block diagonal

$$D = \begin{bmatrix} D_1 & 0 & \cdots & 0 \\ 0 & D_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & D_{n_{iu}} \end{bmatrix}$$

or triangular

$$D = \begin{bmatrix} D_{11} & D_{12} & \cdots & D_{1n_{iu}} \\ 0 & D_{22} & \cdots & D_{2n_{iu}} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & D_{n_{iu}n_{iu}} \end{bmatrix}$$

preserves the structure in H and $H_1 = DH$ and the degrees of freedom in D is used to arrive at convex approximation methods

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CVs with structural constraints (structured H) : Convex upper bound (structured H)

Examples 1 :

Full H

$$H = \begin{bmatrix} h_{11} & h_{12} & h_{13} & h_{14} \\ h_{21} & h_{22} & h_{23} & h_{24} \end{bmatrix} \qquad D = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix} \qquad H_1 = DH$$

Decentralized H

$$H = \left[\begin{array}{cc|cc} h_{11} & h_{12} & 0 & 0 \\ \hline 0 & 0 & h_{23} & h_{24} \end{array}\right] \qquad D = \begin{bmatrix} d_{11} & 0 \\ 0 & d_{22} \end{bmatrix} \qquad H_1 = DH = \left[\begin{array}{cc|cc} d_{11}h_{11} & d_{11}h_{12} & 0 & 0 \\ \hline 0 & 0 & d_{22}h_{23} & d_{22}h_{24} \end{array}\right]$$

Traingular H

$$H = \begin{bmatrix} h_{11} & h_{12} & 0 & 0 \\ h_{21} & h_{22} & h_{23} & h_{24} \end{bmatrix} \qquad D = \begin{bmatrix} d_{11} & 0 \\ d_{21} & d_{22} \end{bmatrix} \qquad H_1 = DH$$

For structured H, less degrees of freedom in D result in convex upper bound

$$HG^y \neq J_{uu}^{1/2}$$

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Convex approximation methods for structured H

Convex approximation method 1:

matching elements in HGy to $J_{uu}^{1/2}$

$$\min_{\mathbf{h}_\delta,\beta_\delta} \mathbf{h}_\delta^T \mathbf{F}_\delta \mathbf{h}_\delta$$

s.t.

$$-b(1-\beta_l) \le (\mathbf{G}_\delta^{y^T} \mathbf{h}_\delta - \mathbf{j}_\delta)|_l \le b(1-\beta_l), \ \forall l = 1,2,\cdots, n_u n_u$$

$$n_u \le \sum_{l=1}^{n_u n_u} \beta_l \le n_{nz} \qquad \beta_l \in \{0,1\}$$

$$n_{u_k} \le \sum_{p=0}^{n_u-1} \sum_{j=\sum_k n_{u_{k-1}}+1}^{\sum_k n_{u_k}} \beta_{n_u p+j} \le n_{nz_k}, \ \forall k = 1,2,\ldots, \text{number of blocks}$$

$$\mathbf{h}_\delta(\text{ind}) = 0, \ \text{ind is for 0 in particular structure } \mathbf{H}$$

Convex approximation method 2:

Relaxing the equality constraint to inequality constraint

$$\min_{\mathbf{h}_\delta} \mathbf{h}_\delta^T \mathbf{F}_\delta \mathbf{h}_\delta$$

$$\text{s.t. } \mathbf{G}_\delta^{y^T} \mathbf{h}_\delta \le \mathbf{j}_\delta$$

$$\mathbf{h}_\delta(\text{ind}) = 0, \ \text{ind is for 0 in particular structure } \mathbf{H}$$

Controlled variable selection with structured H

Optimization problem :

Minimize the average loss by selecting a structured H and CVs as

(i) best individual measurements

(ii) best combinations of all measurements

(iii) best combinations with few measurements

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structured H with optimal measurement subsets

Convex approximation method 1:
matching elements of HG^y to $J_{uu}^{1/2}$

$$\min_{\mathbf{h}_\delta, \beta_\delta} \mathbf{h}_\delta^T \mathbf{F}_\delta \mathbf{h}_\delta \quad \text{s.t.} \tag{4.15a}$$

$$-b(1-\beta_l) \le (\mathbf{G}_\delta^{y^T} \mathbf{h}_\delta - \mathbf{j}_\delta)|_l \le b(1-\beta_l), \ \forall l = 1, 2, \cdots, n_u n_u$$

$$n_u \le \sum_{l=1}^{n_u n_u} \beta_l \le n_{nz} \qquad \beta_l \in \{0,1\}$$

$$n_{u_k} \le \sum_{p=0}^{n_u-1} \sum_{j=\sum_k n_{u_{k-1}}+1}^{\sum_k n_{u_k}} \beta_{n_u p + j} \le n_{nz_k}, \ \forall k = 1, 2, \ldots, \text{number of blocks} \tag{4.15b}$$

$$\mathbf{h}_\delta(\text{ind}) = 0, \ \text{ind is for 0 in particular structure } \mathbf{H} \tag{4.15c}$$

$$\mathbf{P}\boldsymbol{\sigma}_\delta = \mathbf{s}$$

$$\begin{bmatrix} -m \\ -m \\ \vdots \\ -m \end{bmatrix} \sigma_j \le \begin{bmatrix} h_{1j} \\ h_{2j} \\ \vdots \\ h_{n_u j} \end{bmatrix} \le \begin{bmatrix} m \\ m \\ \vdots \\ m \end{bmatrix} \sigma_j, \quad \forall j \in 1, 2, \cdots, n_y$$

Convex approximation method 2:
relaxing equality constraint to inequality constraint

$$\min_{\mathbf{h}_\delta} \mathbf{h}_\delta^T \mathbf{F}_\delta \mathbf{h}_\delta$$

$$\text{s.t. } \mathbf{G}_\delta^{y^T} \mathbf{h}_\delta \le \mathbf{j}_\delta$$

$$\mathbf{h}_\delta(\text{ind}) = 0, \ \text{ind is for 0 in particular structure } \mathbf{H}$$

$$\mathbf{P}\boldsymbol{\sigma}_\delta = \mathbf{s}$$

$$\begin{bmatrix} -m \\ -m \\ \vdots \\ -m \end{bmatrix} \sigma_j \le \begin{bmatrix} h_{1j} \\ h_{2j} \\ \vdots \\ h_{n_u j} \end{bmatrix} \le \begin{bmatrix} m \\ m \\ \vdots \\ m \end{bmatrix} \sigma_j, \quad \forall j \in 1, 2, \cdots, n_y$$

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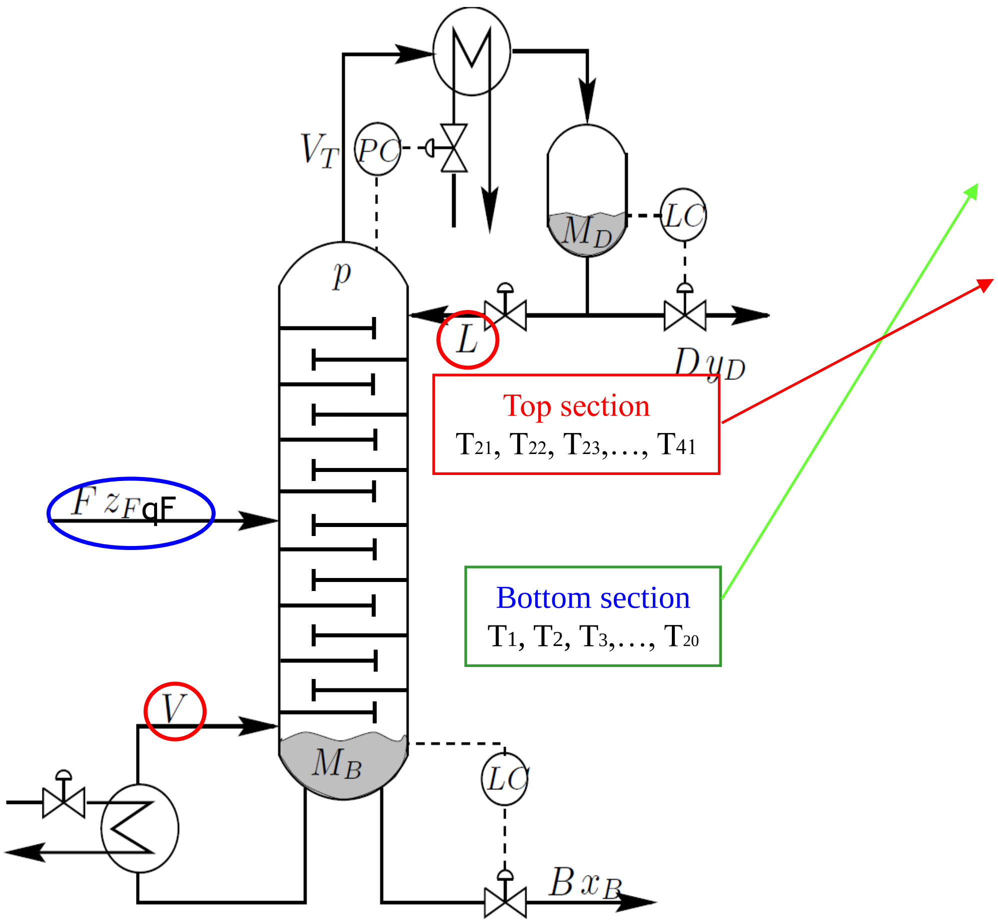
Distillation column : Decentralized H



Binary distillation column

$$c_1 = h_{11}T_1 + h_{12}T_2 + \cdots + h_{120}T_{20}$$

$$c_2 = h_{221}T_{21} + h_{222}T_{22} + \cdots + h_{241}T_{41}$$

$$H = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{120} & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & h_{221} & \cdots & h_{241} \end{bmatrix}$$

Decentralized structure

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Distillation Column : Results

Meas		Full H	Block diagonal H	
			Convex approximation method 1	Convex approximation method 2
2	CV	$c_1 = T_{12}$ $c_2 = T_{30}$	$c_1 = T_{12}$ $c_2 = T_{29}$	$c_1 = T_{12}$ $c_2 = T_{29}$
	Loss $\frac{1}{2}\|\mathbf{M}\|_F^2$	0.548	0.553*	0.553*
3	CV	$c_1 = -0.0369T_{12} + 0.6449T_{30} + 0.6572T_{31}$ $c_2 = -1.2500T_{12} + 0.2051T_{30} + 0.1537T_{31}$	$c_1 = 0.63T_{30} + 0.6229T_{31}$ $c_2 = 0.9675T_{12}$	$c_1 = 0.63T_{30} + 0.6229T_{31}$ $c_2 = 0.9675T_{12}$
	Loss $\frac{1}{2}\|\mathbf{M}\|_F^2$	0.443	0.443**	0.443**
4	CV	$c_1 = 0.01T_{11} - 0.0460T_{12} + 0.6450T_{30} + 0.6574T_{31}$ $c_2 = -0.6576T_{11} - 0.6548T_{12} + 0.2011T_{30} + 0.1413T_{31}$	$c_1 = 0.63T_{30} + 0.6229T_{31}$ $c_2 = -0.5151T_{11} - 0.5110T_{12}$	$c_1 = 0.63T_{30} + 0.6229T_{31}$ $c_2 = -0.5151T_{11} - 0.5110T_{12}$
	Loss $\frac{1}{2}\|\mathbf{M}\|_F^2$	0.344	0.344†	0.344†
41	CV	$c_1 = f(T_1, T_2, \ldots, T_{41})$ $c_2 = f(T_1, T_2, \ldots, T_{41})$	$c_1 = f(T_{21}, T_{22}, \ldots, T_{41})$ $c_2 = f(T_1, T_2, \ldots, T_{20})$	$c_1 = f(T_{21}, T_{22}, \ldots, T_{41})$ $c_2 = f(T_1, T_2, \ldots, T_{20})$
	Loss $\frac{1}{2}\|\mathbf{M}\|_F^2$	0.081	0.105†	0.127†

*clearly not optimal as the solutions must be same with CVs as individual measurements
ł small differences in the optimal solution in convex approximation methods 1 and 2 for triangular H and block diagonal H

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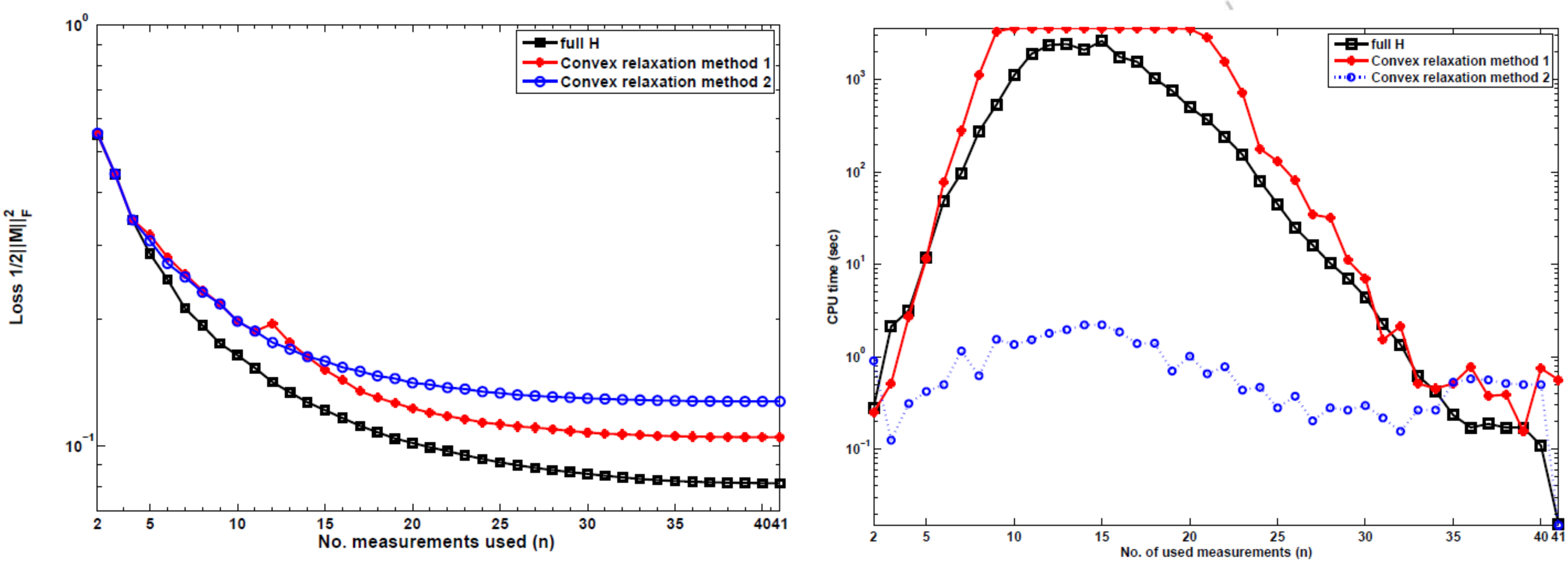
Decentralized H: Result



❖ The proposed methods are not exact (Loss should be same for H full and H disjoint for individual measurements)

❖ Proposed method provide good upper bounds for the distillation case

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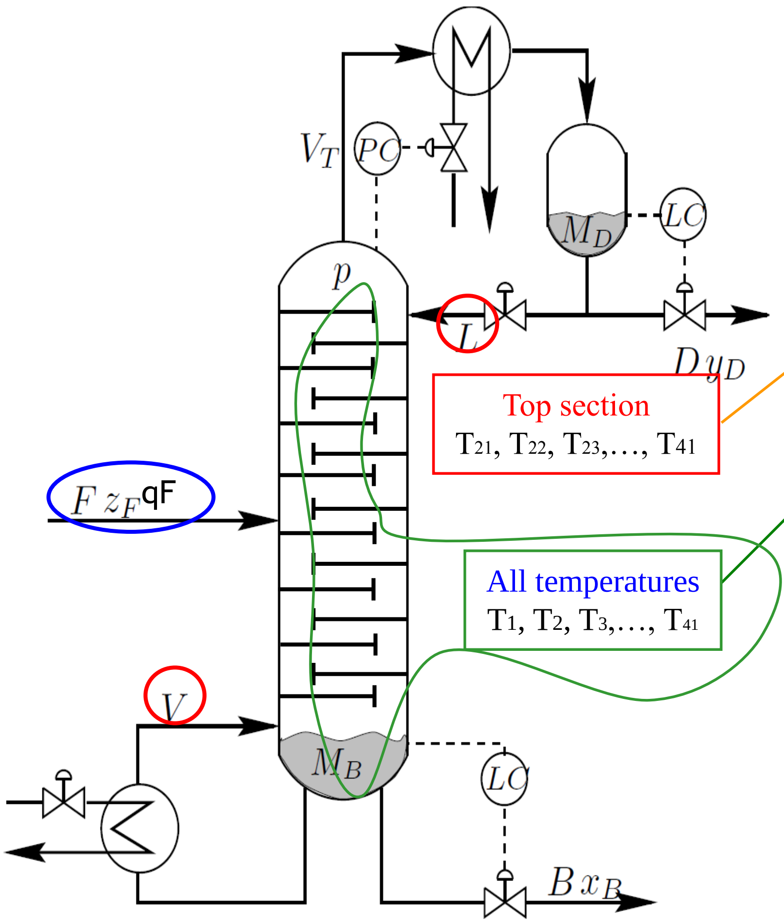
Distillation column : Triangular H



Binary distillation column

$$c_1 = h_{121}T_{21} + h_{122}T_{22} + \cdots + h_{141}T_{41}$$

$$c_2 = h_{21}T_1 + h_{22}T_2 + \cdots + h_{241}T_{41}$$

$$H = \begin{bmatrix} 0 & 0 \cdots & 0 & h_{121} & h_{122} \cdots & h_{141} \\ h_{21} & h_{22} \cdots & h_{220} & h_{221} & h_{222} \cdots & h_{241} \end{bmatrix}$$

Traingular structure

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Distillation Column : Results

Meas		Structure		
		Full **H**	Triangular **H**	
			Convex approximation method 1	Convex approximation method 2
2	CV	$c_1 = T_{12}$ $c_2 = T_{30}$	$c_1 = T_{12}$ $c_2 = T_{30}$	$c_1 = T_{12}$ $c_2 = T_{30}$
	Loss $\frac{1}{2}\lVert\mathbf{M}\rVert_F^2$	0.548	0.548	0.548
3	CV	$c_1 = -0.0369T_{12} + 0.6449T_{30} + 0.6572T_{31}$ $c_2 = -1.2500T_{12} + 0.2051T_{30} + 0.1537T_{31}$	$c_1 = T_{30} + 0.9898T_{31}$ $c_2 = T_{11} + 0.7365T_{30} + 0.7812T_{31}$	$c_1 = T_{30} + 0.9887T_{31}$ $c_2 = T_{11} + 0.7365T_{30} + 0.7812T_{31}$
	Loss $\frac{1}{2}\lVert\mathbf{M}\rVert_F^2$	0.443	0.464**	0.464**†
4	CV	$c_1 = 0.01T_{11} - 0.0460T_{12} + 0.6450T_{30} + 0.6574T_{31}$ $c_2 = -0.6576T_{11} - 0.6548T_{12} + 0.2011T_{30} + 0.1413T_{31}$	$c_1 = 0.6301T_{30} + 0.6237T_{31}$ $c_2 = -0.3463T_{10} - 0.3484T_{11} - 0.2390T_{30} - 0.2680T_{31}$	$c_1 = 0.6300T_{30} + 0.6229T_{31}$ $c_2 = -0.3463T_{10} - 0.3484T_{11} - 0.2390T_{30} - 0.2680T_{31}$
	Loss $\frac{1}{2}\lVert\mathbf{M}\rVert_F^2$	0.344	0.353**†	0.353**†
41	CV	$c_1 = f(T_1, T_2, \ldots, T_{41})$ $c_2 = f(T_1, T_2, \ldots, T_{41})$	$c_1 = f(T_21, T_2, \ldots, T_{41})$ $c_2 = f(T_1, T_2, \ldots, T_{41})$	$c_1 = f(T_{21}, T_{22}, \ldots, T_{41}$ $c_2 = f(T_1, T_2, \ldots, T_{41})$
	Loss $\frac{1}{2}\lVert\mathbf{M}\rVert_F^2$	0.081	0.094†	0.141†

**clearly not optimal as triangular H must at least be as good as H disjoint

ł small differences in the optimal solution in convex approximation methods 1 and 2 for triangular H and block diagonal H

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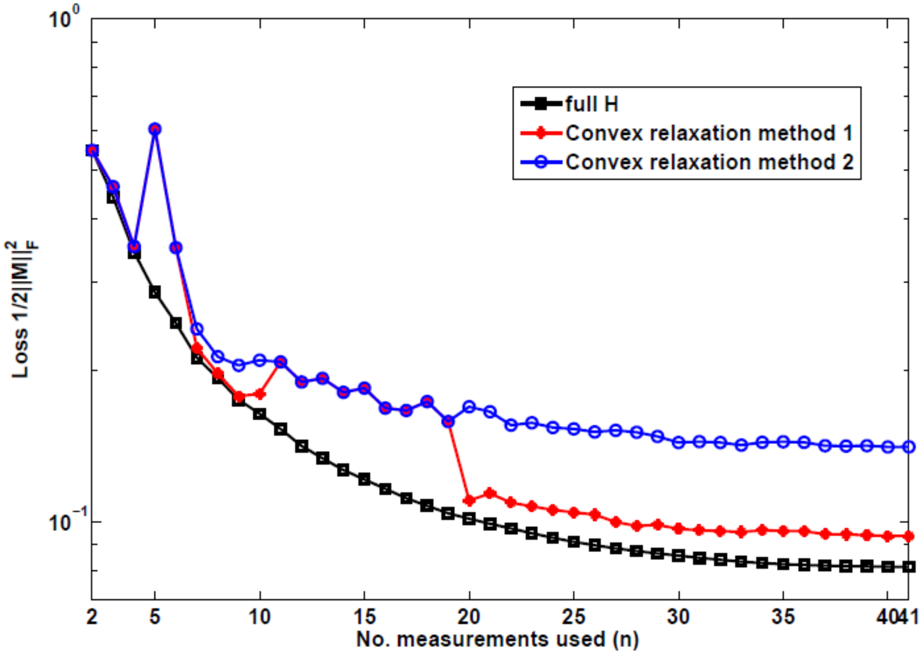
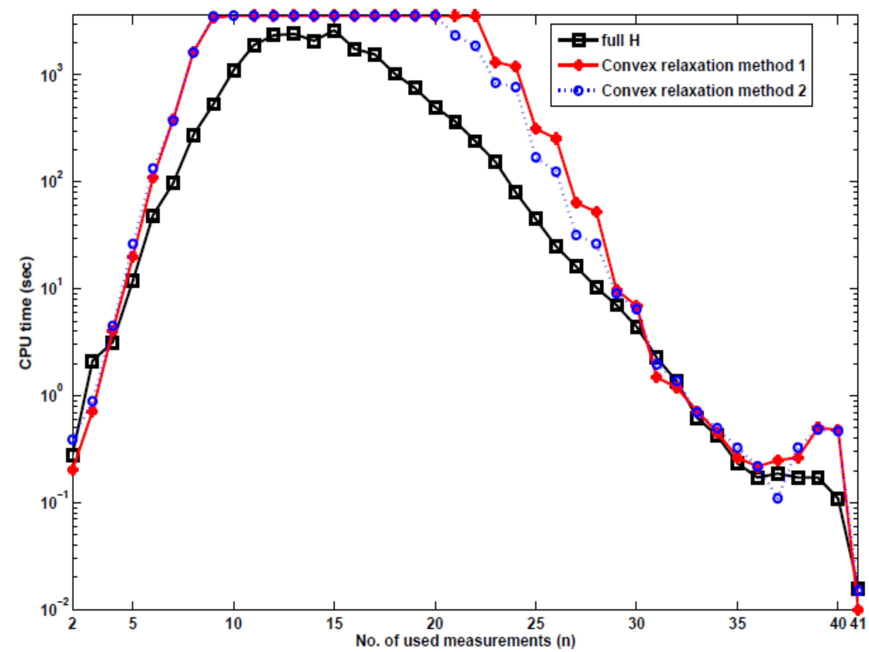
Triangular H: Result





The proposed methods are not exact (Loss should be same for full H, triangular H for individual measurements)

❖ Proposed method provide good upper bounds for the distillation case

❖ In convex approximation methods we are minimizing $\|HY\|_F$ and $\|HY\|_F$ smaller for n = 5 than n = 4, but the loss $\left\|J_{uu}^{1/2}(HG^y)^{-1}HY\right\|_F$ is higher for n = 5 than n = 4 and causes irregular behavior

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Presentation outline

- Plantwide control : Self optimizing control formulation for CV, c = Hy – Chapter 2
- Convex formulation for CV with full H - Chapter 3
  - Convex formulation
  - Globally optimal MIQP formulations
  - Case studies
- Convex approximation methods for CV with structured H – Chapter 4
  - Convex approximations
  - MIQP formulations for structured H with measurement subsets
  - Case studies
- **Regulatory control layer selection - Chapter 5**
  - **Problem definition**
  - **Regulatory control layer selection with state drift minimization**
  - **Case studies**
- Conclusions and Future work

CV – Controlled Variables
MIQP - Mixed Integer Quadratic Programming

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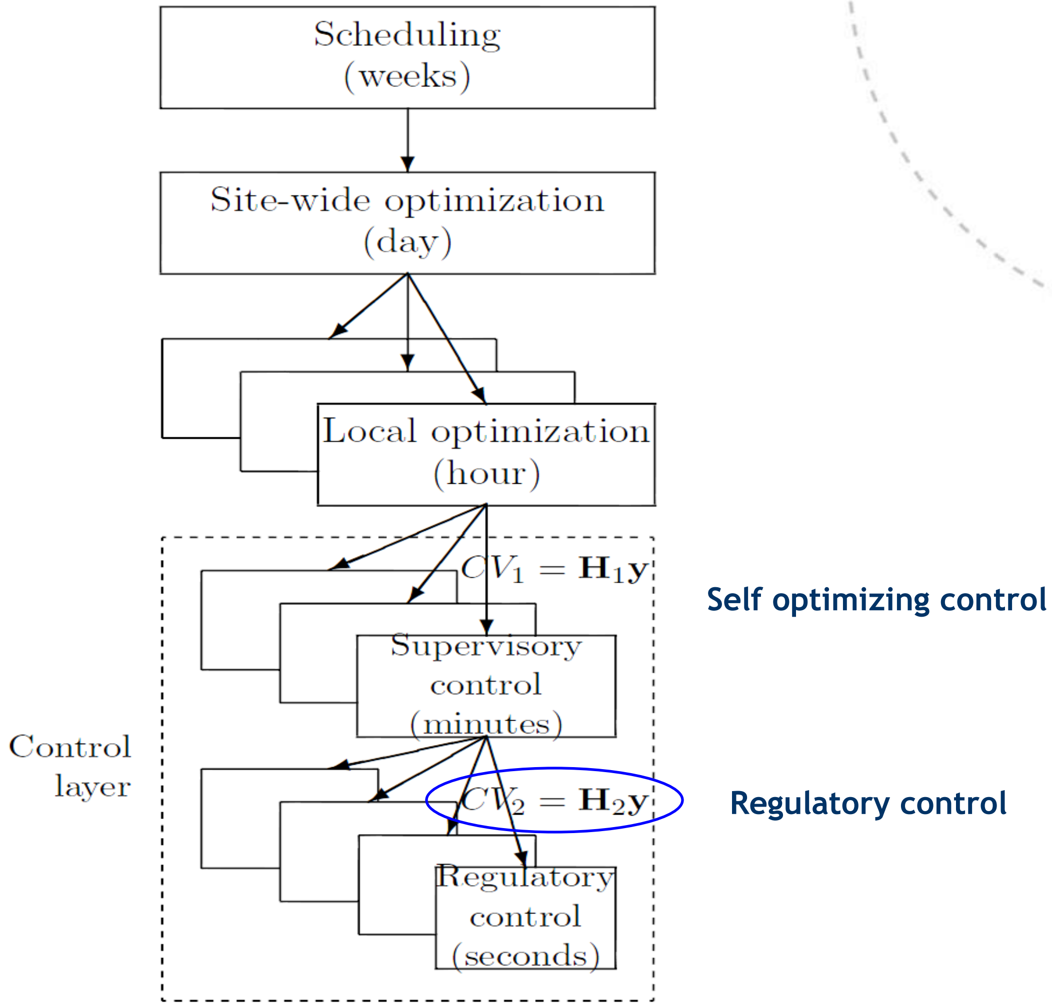
Control system hierarchy for plantwide control



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Regulatory control layer: Objectives

Regulatory layer should

(1) facilitate stable operation
regulate the process
operate the plant in a linear operating region

(2) be simple

(3) avoid control loop reconfiguration

How to quantify ?

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Regulatory control layer: Objectives

(1) Minimize state drift $J(\omega) = \|Wx(j\omega)\|_2^2$ W : state weighting matrix

(2) Simple: Close minimum number of loops

(3) Avoid control loop reconfiguration

Quantified the regulatory layer objectives

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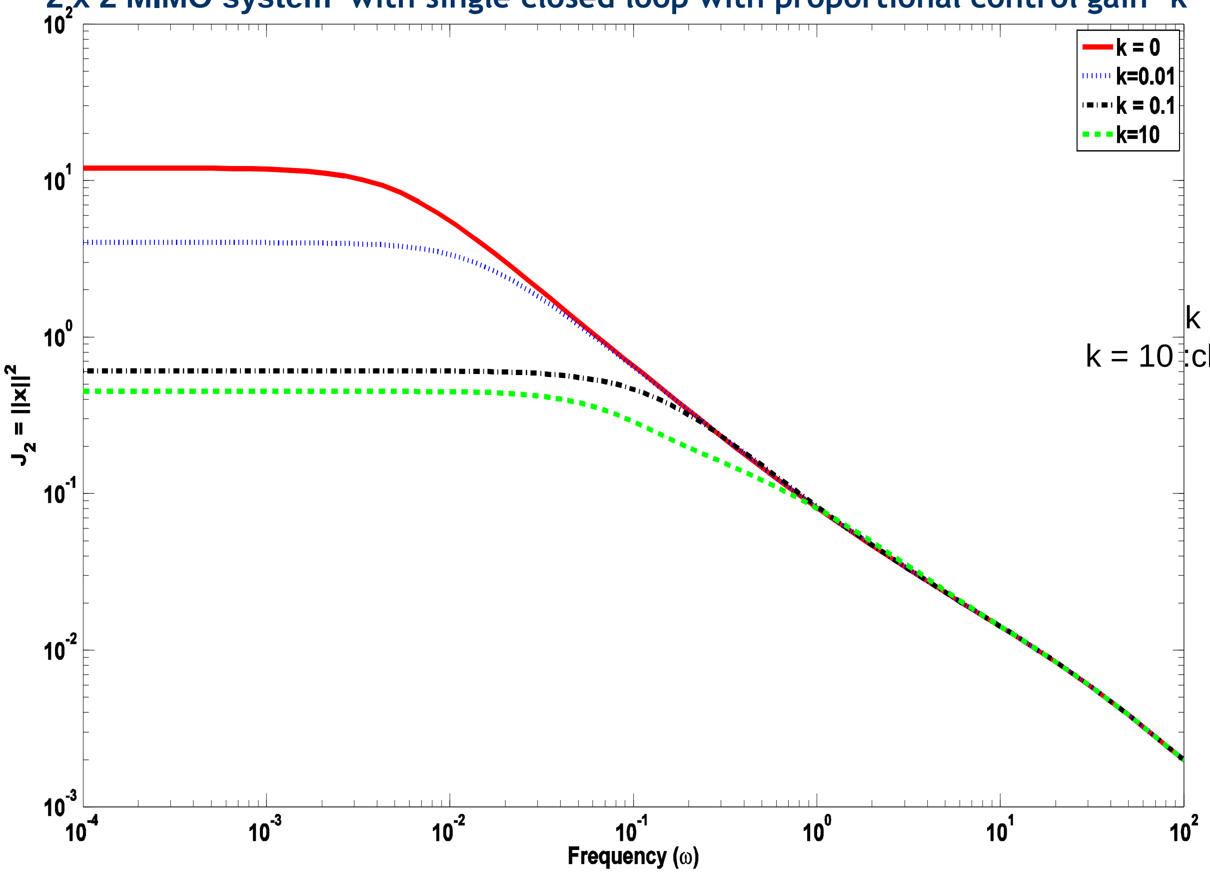
Regulatory control layer: Justification to use steady state analysis

Typical frequency dependancy plot

2 x 2 MIMO system with single closed loop with proportional control gain 'k'



k = 0 :open loop
k = 10 :close to perfect control

Steady state based state drift is
fairly good over a frequency bandwidth

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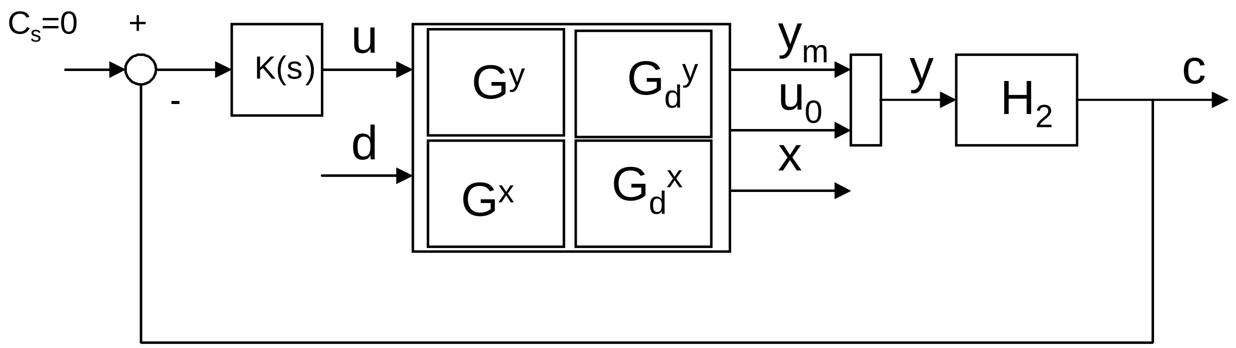
Regulatory control layer: Problem Formulation

$$L = J(u,d) - J_{opt}(u_{opt}(d),d)$$
$$= \|Wx\|_2^2 - \|Wx_{opt}(d)\|_2^2$$



$$L_{avg} = \left\| J_{2_{uu}}^{1/2} (H_2 G^y)^{-1} H_2 Y_2 \right\|_F^2$$

Loss is due to
(i) Varying disturbances
(ii) Implementation error in controlling c at set point c_s

$$Y_2 = [(G^y J_{2_{uu}}^{-1} J_{2_{ud}} - G_d^y) W_d \quad W_n]$$
$$= [F_2 W_d \quad W_n]$$

$$F_2 = \frac{\partial y^{opt}}{\partial d}$$

Ref: Halvorsen et al. I&ECR, 2003
Kariwala et al. I&ECR, 2008

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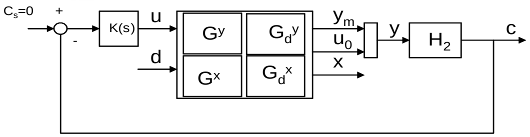

Problem formulation

$$c = H_2[y_m \quad u_0]$$

n_{ym} number of y_m
n_{u0} number of physical valves
n_c = number of CVs $=n_u$

Example

n_{ym} =4
n_{u0} =4
n_c = 2=n_u

$$H_2 = \begin{bmatrix} \overbrace{h_{11}\, h_{12} \cdots h_{14}}^{H_y} & \overbrace{h_{15}\, h_{16} \cdots h_{18}}^{H_u} \\ h_{21} h_{22} \cdots h_{24} & h_{25} h_{26} \cdots h_{28} \end{bmatrix}$$

P1. Close 0 loops : Select (n_c variables from u_0) or (0 variables from y_m) $\Rightarrow$ Pick n_c columns in H_u

P2. Close 1 loops : Select 1 variables from y_m $\Rightarrow$ Pick 1 column in H_y and n_c -1 columns in H_u

P3. Close 2 loops : Select 2 variables from y_m $\Rightarrow$ Pick 2 columns in H_y and n_c -2 columns in H_u

P4. Close k loops : Select k variables from y_m $\Rightarrow$ Pick k columns in H_y and n_c -k columns in H_u

P5. Close n_c loops : Select n_c variables from y_m $\Rightarrow$ Pick n_c columns in H_y and 0 columns in H_u

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MIQP formulation

$$\begin{matrix} \sigma_1 & \sigma_2 & \cdots & \sigma_{ny} \end{matrix}$$

$$H_2 = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1ny} \\ h_{21} & h_{22} & \cdots & h_{2ny} \\ \vdots & \vdots & \ddots & \vdots \\ h_{nu1} & h_{nu2} & \cdots & h_{nu*ny} \end{bmatrix}_{nu \times ny}$$

$$\sigma_i \in \{0,1\}$$
$$i = 1, 2, \cdots, ny$$

is vectorized along the rows of H to form

$$h_\delta = \begin{bmatrix} h_{11} \\ h_{12} \\ \vdots \\ h_{nu*ny} \end{bmatrix}_{(nu*ny)\times 1} \qquad \sigma_\delta = \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \vdots \\ \sigma_{ny} \end{bmatrix}_{ny\times 1}$$

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Regulatory layer selection: Solution approach

$$H = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1ny} \\ h_{21} & h_{22} & \cdots & h_{2ny} \\ \vdots & \vdots & \ddots & \vdots \\ h_{nu1} & h_{nu2} & \cdots & h_{nu*ny} \end{bmatrix}_{nu \times ny}$$

(columns labelled $\sigma_1 \quad \sigma_2 \quad \cdots \quad \sigma_{ny}$)

MIQP formulation

$$\min_{x_\delta, \sigma_\delta} \quad h_\delta^T F_\delta h_\delta$$

$$st. \quad G_\delta^{y^T} h_\delta = J_\delta$$

$$P\sigma_\delta = n$$

$$\begin{bmatrix} -m \\ -m \\ \vdots \\ -m \end{bmatrix} \sigma_i \leq \begin{bmatrix} h_{1i} \\ h_{2i} \\ \vdots \\ h_{nui} \end{bmatrix} \leq \begin{bmatrix} m \\ m \\ \vdots \\ m \end{bmatrix} \sigma_i$$

$$\forall i = 1, 2, \cdots, ny$$

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Case Study : Distillation Column

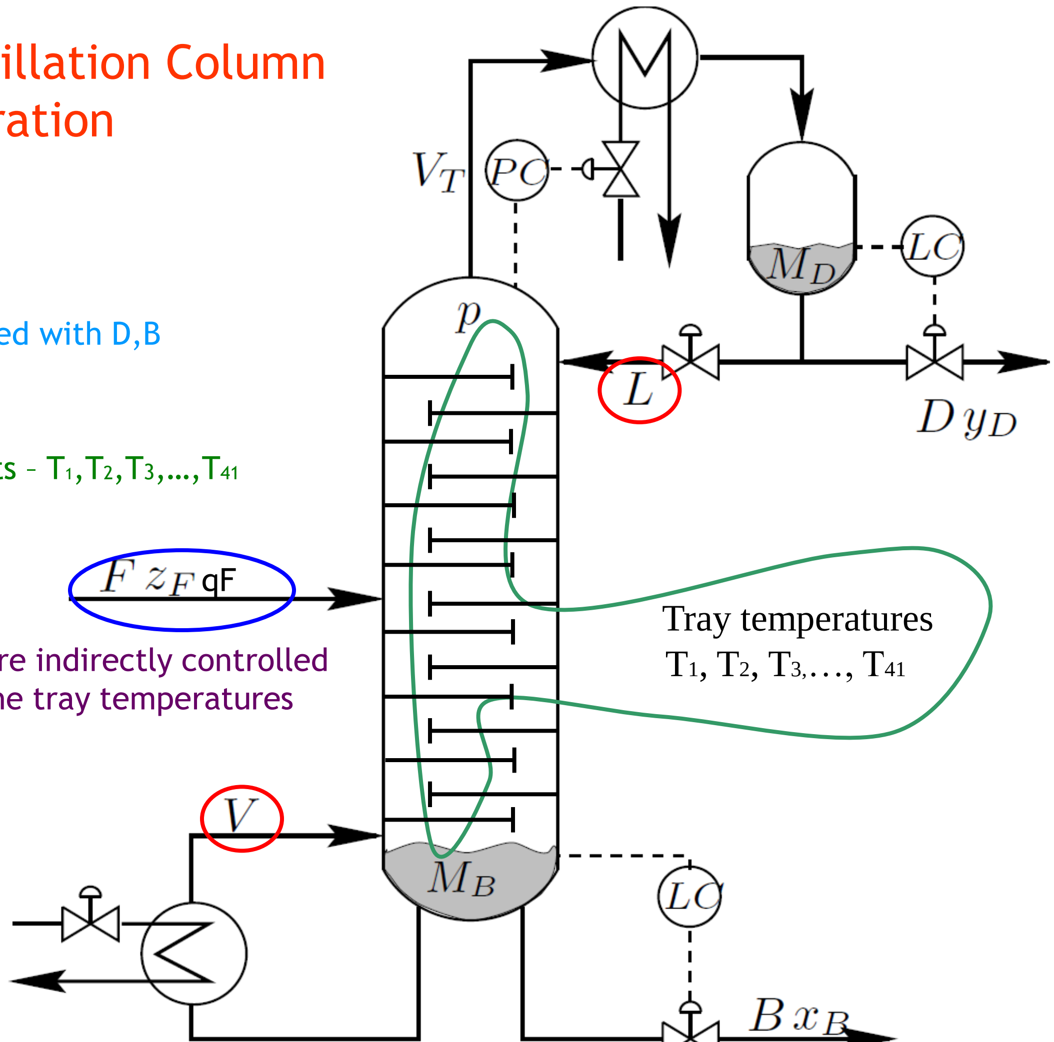
Binary Distillation Column
LV configuration

41 Trays

Level loops closed with D,B

2 MVs – L,V
41 Measurements – $T_1, T_2, T_3, \ldots, T_{41}$
3 DVs – F, ZF, qF

*Compositions are indirectly controlled by controlling the tray temperatures



$$J = \left\|W\Delta x\right\|_2^2$$

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Case Study : Distillation Column

$$L_{avg} = \left\| J_{2_{uu}}^{1/2} (H_2 G^y)^{-1} H_2 Y_2 \right\|_F^2 \qquad Y_2 = [(G^y J_{2_{uu}}^{-1} J_{2_{ud}} - G_d^y) W_d \quad W_n]$$

Data

$$G^y \in \mathbb{R}^{41\times 2}; G_d^y \in \mathbb{R}^{41\times 3}; J_{2_{uu}} \in S_+^2; J_{2_{ud}} \in \mathbb{R}^{2\times 3}; W_d \in \mathbb{R}^{3\times 3}; W_n \in \mathbb{R}^{41\times 41}$$

$$G^y = \begin{bmatrix} 10.83 & -10.96 \\ 15.36 & -15.55 \\ \vdots & \vdots \\ 13.01 & -12.81 \\ 8.76 & -8.62 \end{bmatrix}; G_d^y = \begin{bmatrix} 5.85 & 11.17 & 10.90 \\ 8.30 & 15.86 & 15.47 \\ \vdots & \vdots & \vdots \\ 5.85 & 13.10 & 12.90 \\ 3.94 & 8.82 & 8.68 \end{bmatrix}$$

$$W_d = \begin{bmatrix} 0.2 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}; Wn = diag(0.5 * ones(41,1))$$

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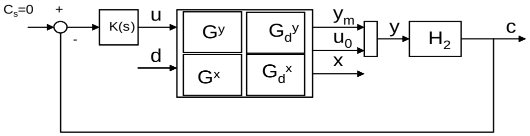
Regulatory control layer



CVs (c = H_2y) as individual measurements

$$c = H_2[y_m \quad u_0]$$

n_{ym} number of y_m
n_{u0} number of physical valves
n_c = number of CVs =n_u

n_{ym} =41
n_{u0} =4
n_c = 2=n_u

$$H_2 = \begin{bmatrix} \overbrace{h_{1,1}\, h_{1,2} \cdots h_{1,41}}^{H_y} & \overbrace{h_{1,42}\, h_{1,43}\, h_{1,44}\, h_{1,45}}^{H_u} \\ h_{2,1} h_{2,2} \cdots h_{2,41} & h_{2,42}\, h_{2,43}\, h_{2,44}\, h_{2,45} \end{bmatrix}$$

P1. Close 0 loops : Select (2 variables from u_0) or (0 variables from y_m) $\Rightarrow$ Pick 2 columns in H_u

P2. Close 1 loops : Select 1 variables from y_m $\Rightarrow$ Pick 1 column in H_y and 1 column in H_u

P3. Close 2 loops : Select 2 variables from y_m $\Rightarrow$ Pick 2 columns in H_y and 0 column in H_u

Total n_u+1 = 3 MIQP problems

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Regulatory control layer: Result

Table 5.1: Distillation column case study: the self optimizing variables $\mathbf{c}'s$ as combinations of $2, 3, 4, 5, 41$ measurements with their associated losses in state drift

No. of loops closed †	No. of meas. used	Optimal meas.	$c's$	Loss $(J - J_{opt}(\mathbf{d}))$ $(\frac{1}{2}\|\mathbf{M}_2\|_F^2)$	$J = \|\mathbf{W}\mathbf{x}\|_2^2$
0	2	$[V \quad B]$	$c_1 = V$ $c_2 = B$	109.669††	109.690
1	2	$[T_{18} \quad L]$	$c_1 = L$ $c_2 = T_{17}$	0.188	0.209
2	2	$[T_{15} \quad T_{27}]$	$c_1 = T_{15}$ $c_2 = T_{27}$	0.026	0.047
1	3	$[T_{15} \quad T_{26} \quad L]$	$c_1 = L$ $c_2 = 1.072T_{15} + T_{26}$	0.129*	0.150
2	3	$[T_{15} \quad T_{26} \quad T_{28}]$	$c_1 = T_{15} - 0.1352T_{28}$ $c_2 = T_{26} + 1.0008T_{28}$	0.020	0.040
1	4	$[T_{15} \quad T_{16} \quad T_{27} \quad L]$	$c_1 = L$ $c_2 = 0.6441T_{15} + 0.6803T_{16} + T_{27}$	0.126*	0.146
2	4	$[T_{14} \quad T_{16} \quad T_{26} \quad T_{28}]$	$c_1 = T_{14} - 6.1395T_{26} - 6.3356T_{28}$ $c_2 = T_{16} + 6.2462T_{26} + 6.2744T_{28}$	0.014	0.034
1	5	$[T_{15} \quad T_{16} \quad T_{26} \quad T_{27} \quad L]$	$c_1 = L$ $c_2 = 1.1926T_{15} + 1.1522T_{16} + 0.9836T_{26} + T_{27}$	0.123*	0.144
2	5	$[T_{14} \quad T_{16} \quad T_{26} \quad T_{27} \quad T_{28}]$	$c_1 = T_{14} - 4.9975T_{26} - 5.0717T_{27} - 4.9813T_{28}$ $c_2 = T_{16} + 5.1013T_{26} + 5.0847T_{27} + 4.9166T_{28}$	0.011	0.032
1	41	$[T_1, T_2, \ldots, T_{41}, \quad L, V, D, B]$	$c_1 = L$ $c_2 = f(T_1, T_2, \ldots, T_{41}, L, V, D, B)$	0.118*	0.138
2	41	$[T_1, T_2, \ldots, T_{41}]$	$c_1 = f(T_1, T_2, \ldots, T_{41})$ $c_2 = f(T_1, T_2, \ldots, T_{41})$	0.003	0.023

† In addition to two closed level loops
The loss is minimized to obtain $\mathbf{H}_2$
The optimal state drift $J_{opt}(\mathbf{d}) = 0.0204$
1 loop closed : 1 $\mathbf{c}$ from $\mathbf{y}_m$, 1 $\mathbf{c}$ from $\mathbf{u}_0$
2 loops closed: 2 $\mathbf{c}$ from $\mathbf{y}_m$
The loss is minimized to obtain $\mathbf{H}_2$
†† Such a high value is not physical, but it follows because our linear analysis is not appropriate when we close 0 loops
* used partial control idea to find optimal $\mathbf{H}_2$ in two step approach

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Regulatory control layer results

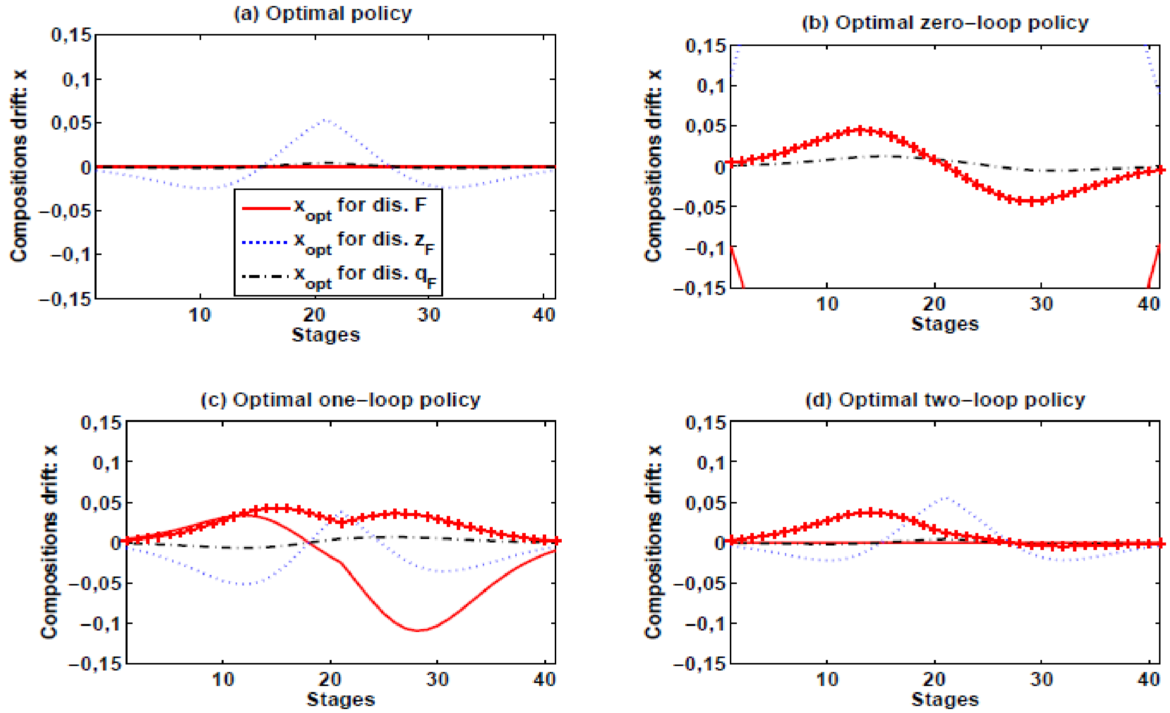

Figure 5.5: Distillation column state drift in the presence of disturbances F,z_F, q_F: (a) optimal policy (minimum achievable state drift), (b) optimal zero-loop policy, (c) optimal one-loop policy, (d) optimal two-loop policy. Effect of a measurement noise on state drift is shown with + in subplots (b),(c) and (d)

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Regulatory control layer result

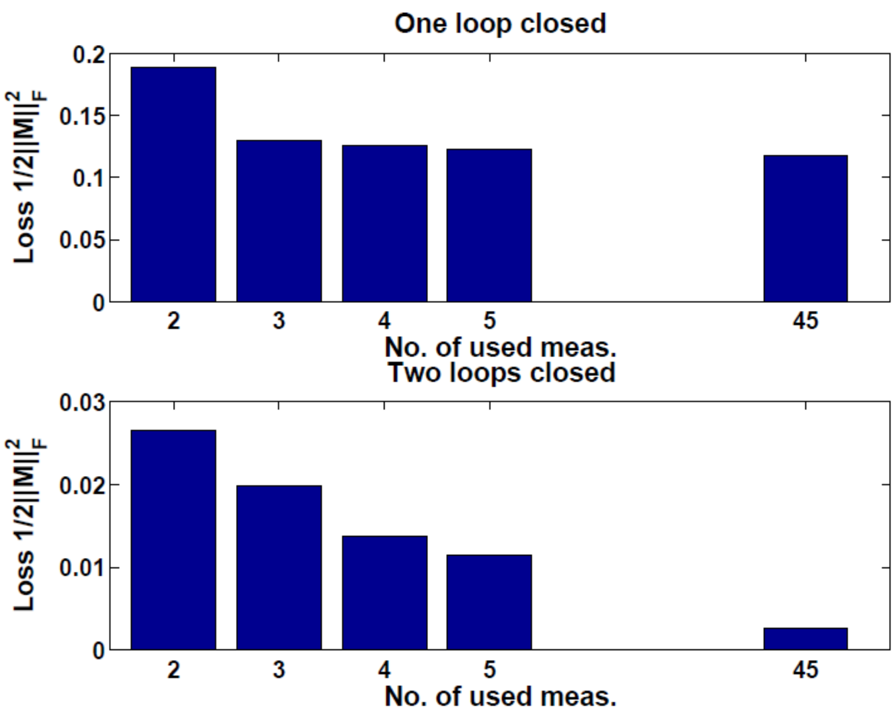

Figure 5.6: Distillation case study: The reduction in loss in state drift vs number of used measurements, top: loss with one loop closed, bottom : loss with two loops closed

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Presentation outline

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  - Convex formulation
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  - Case studies
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  - Problem definition
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  - Case studies
- **Conclusions and Future work**

CV – Controlled Variables
MIQP - Mixed Integer Quadratic Programming

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Conclusions and Future work

Concluding remakrs

- Controlled variables selection formulation in the self-optimizing control framework is presented
- Using steady state economics, the optimal controlled variables, c= Hy, are obtained as
  - optimal individual measurements
  - optimal combinations of 'n' measurements

  for full H using MIQP based formulations.
- Controlled variables c= Hy, are obtained with a structured H. The proposed convex approximation methods are not exact for structured H, but provide good upper bounds.
- Extended the self-optimizing control concepts to find regulatory layer control variables (CV_2) that minimize the state drift.

Future work:

- Robust optimal controlled varaible selection methods
- Fixed CV for all active constraint regions
- Economic optimal CV selection based on dynamics

Acknowledgements: GASSMAKS and Research Council of Norway

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4. Yelchuru, R., Skogestad, S., 2012. Convex formulations for optimal selection of controlled variables and measurements using Mixed Integer Quadratic Programming. Journal of Process Control, 22, 995-1007.

Chapter 4

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Chapter 5

8. Yelchuru, R., Skogestad, S., 2012. Regulatory layer selection through partial control. In: Nordic Process Control Workshop, Jan 25 - 27, Technical University of Denmark, Kgs Lyngby, Denmark.
9. Yelchuru, R., Skogestad, S., 2012. Quantitative methods for optimal regulatory layer selection. Accepted for ADCHEM 2012, Singapore.
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Thank You

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