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Design of plantwide control systems with focus on maximizing throughput

Thesis for the degree of philosophiae doctor

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Abstract

This thesis discusses plantwide control configuration with focus on maximizing throughput. The most important plantwide control issue is to maintain the mass balances in the plant. The inventory control system must be consistent, which means that the mass balances are satisfied. Self-consistency is usually required, meaning that the steady-state balances are maintained with the local inventory loops only. We propose the *self-consistency rule* to evaluate consistency of an inventory control system.

In many cases, economic optimal operation is the same as maximum plant throughput, which corresponds to maximum flow through the bottleneck(s). This insight may greatly simplify implementation of optimal operation, without the need for dynamic optimization based on a detailed model of the entire plant.

Throughput maximization requires tight bottleneck control. In the simplest case when the bottleneck is fixed to one unit, maximum throughput can be realized with single-loop control. The throughput manipulator should then be located at the bottleneck unit. This gives a short effective delay in the control loop. Effective delay determines the necessary back off from constraints to ensure feasible operation. Back off implies a reduction in throughput and an unrecoverable economic loss and should therefore be minimized. We obtain a rough estimate of the necessary back off based on controllability analysis.

In some cases it is not desirable to locate the throughput manipulator at the bottleneck. To reduce the effective time delay in the control loop from the throughput manipulator to the bottleneck unit, dynamic degrees of freedom, like most inventories, can be used to reduce the effective time delay.

In larger plants there may be several independent feeds, crossovers and splits that should all be utilized to obtain maximum throughput. The proposed *coordinator MPC* both identifies the bottlenecks and implements the optimal policy. A key idea in the coordinator MPC is to decompose the plantwide control problem by estimating the remaining capacity for each unit using models and constraint in the local MPC applications. The coordinator MPC is demonstrated by dynamic simulation and by implementation on a large-scale gas processing plant.

Acknowledgements

First of all, I would like to acknowledge my supervisor, professor Sigurd Skogestad, for his enthusiasm, interest of the work and for always having time to question and (often long) discussions. His knowledge and his ability to see both finer details and the bigger picture in this work have been very valuable.

Second, I would like to thank my co-supervisor dr.ing Stig Strand for guidance, especially on MPC and dynamic optimization issues, and his focus on progression.

Gassco are gratefully acknowledged for financing this work. I would also thank my employer Statoil/StatoilHydro for the ability to do this thesis and finishing it. Former and present colleagues in Process Control group at the StatoilHydro Research Centre in Trondheim are greatly acknowledged. They have always been willing to discuss parts of my work.

In addition I would like to thank Svein Olav Hauger and Marius Støre Govatsmark at Cybernetica for always have time to discussions. Svein Olav's knowledge of the Kårstø plant and MPC implementation issues has been very helpful for me. Marius has been very helpful with D-SPICE simulations and has an impressive endurance on solving simulator problems.

The personnel at the Kårstø Gas Plant are greatly acknowledged for their cooperation with coordinator MPC implementation. In this work, Kjetil Meyer and Roar Sørensen have been very valuable for the implementation.

I would like to thank former and present colleagues at the Process Control group at the Department of Chemical Engineering for discussions, solving computer issues and good times. I would like to thank Kari Unneland, Marius Støre Govatsmark and Ramprasad Yelchuru for proof-reading parts of the thesis. I would like to thanks friends and family for support. My friend Åshild has been very supporting through many, many emails. It helped me keeping the spirit!

Finally, I would like to thank my husband Gunnar and our child Marita for being supportive and patient and for making me understands the real perspectives of life.

Elvira Marie B. Aske

Contents

Chapter 1

Introduction

The purpose of this chapter is to motivate the research, to define the scope and place it in a wider perspective. The contributions and publications arising from this thesis are listed.

1.1 Motivation and focus

Optimal economic operation of processes is important, especially in mature industries where it is difficult to maintain competitive advantages. In some cases, steady-state considerations may be sufficient to track the economic operation point. In other cases, where the important economic disturbances are frequent compared to the plant response time, dynamic considerations to track the optimum is preferable. Some dynamic economic disturbances that most likely call for dynamic optimization are feed flow, feed quality, energy supplies and product specifications (Strand, 1991). To decide whether a dynamic or steady-state process model should be used, the dynamics of the plant and the disturbances must be considered.

In practice, the control and optimization is organized in a hierarchical structure (or layer) (e.g. Findeisen *et al.* 1980; Skogestad and Postlethwaite 2005). Each layer acts at different time intervals (time scale separation) and a typical control hierarchy is displayed in Figure 1.1.

This thesis discusses the control layer, that is, the regulatory control and supervisory control. In addition, implementation of maximum throughput (local optimization) in the control layer is discussed. The stabilizing regulatory control typically includes single-loop PID controllers. Supervisory control (or advanced control) should keep the plant at its target values and model predictive control (MPC) has become the unifying tool with many applications (Qin and Badgwell,

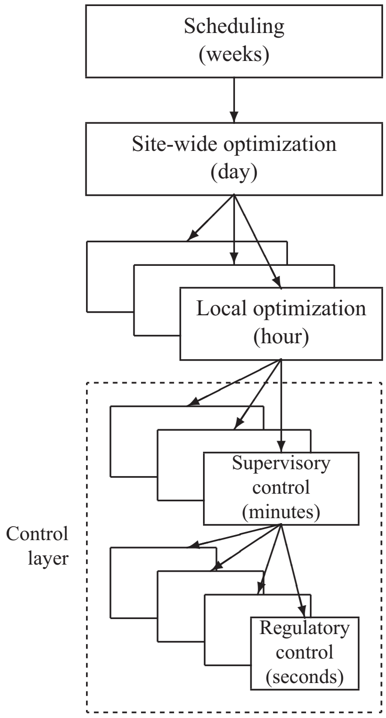


Figure 1.1: Typical control system hierarchy in chemical plants (Skogestad and Postlethwaite, 2005, p.387).

2003) and has replaced previous complex systems with selectors, decouplers, feedforward control and logic.

Engell (2007) gives a review of how to realize optimal process operation by feedback control with direct optimization control, that is, optimization of a online economic cost criterion over finite horizon. Optimal operation can be implemented by conventional feedback control if a self-optimizing control structure is found. This is called *self-optimizing control* where acceptable operation is achieved under all conditions with constant set points for the controlled variables (Skogestad, 2000*a*; Morari *et al.*, 1980). Today, model based economic optimization has become common, and several real-time optimization (RTO) applications based on detailed nonlinear steady-state models are reported (Marlin and Hrymak, 1997). However, there are several challenges regarding (steady-state) RTO. To mention some of these challenges, an RTO requires highly predictive and robust models. Steady-state detection and data reconciliation are necessary to detect current operation point and to update models and this is not a straight forward task (Forbes *et al.*, 2006; Marlin and Hrymak, 1997).

In particular, for plants that are seldom in steady-state, dynamic optimization is more suitable, which may be realized using dynamic RTO (DRTO) or nonlinear model predictive controller (MPC) with an economic objective, e.g. Kadam *et al.*

(2007); Engell (2007); BenAmor *et al.* (2004); Tosukhowong *et al.* (2004); Diehl *et al.* (2002).

In many cases, we can assume that optimal economic operation is the same as maximizing plant throughput, subject to achieving feasible operation (satisfying operational constraints in all units) with the available feeds. This corresponds to a constrained operation mode (Maarleveld and Rijnsdorp, 1970) with maximum flow through the bottleneck(s). Note that the overall feed rate (or more generally the throughput) affects all units in the plant. For this reason, the throughput is usually not used as a degree of freedom for control of any individual unit, but must be set at the plant-wide level. The throughput manipulators are decided at the design stage and cannot easily be moved later because this requires reconfiguration of the inventory loops to ensure self-consistency (Chapter 2). Plant operation depends on its control structure design and plantwide control related to that design for complete chemical plants (Skogestad, 2004). The focus in this thesis is the control configuration design for throughput maximization.

The economic importance of throughput and the resulting earnings from improved control is stated by Bauer and Craig (2008). They performed a web-based survey by over 60 industrial experts in advanced process control (APC) on the economic assessment of process control. From the survey they found that in particular *throughput* and quality were the important profit factors: *"Both suppliers and users regard an increase in throughput and therefore production as the main profit contributor of process control. Several respondents estimate that the throughput increase lies between 5% and 10%."*.

In this thesis, dynamic optimization is approached by using linear MPC under the assumption of the economic optimum is at maximum throughput (Chapter 5 and 6). Since the objective function is simplified to a linear and constrained function, approaching dynamic optimization by linear MPC is suitable. In the simplest cases, the regulatory control layer can realize throughput maximization (Chapter 3 and 4).

1.2 Thesis overview

The thesis is composed of six independent articles, five of them in the main part of the thesis as chapters and one already published conference paper in the appendix. Some of the chapters have their own appendices. The thesis has a common bibliography. The chapters are written as independent articles, so background material is in some cases repeated. At the end of the thesis, there is a concluding chapter.

The starting point for this research was that the optimum operating policy in many cases is the same as maximum throughput that can be realized with a coordinator MPC (Chapter 5). The location of the throughput manipulator is crucial when it comes to the required back off in the maximum throughput case. The effect the throughput manipulator location has on the required back off and its effect on the bottleneck unit was studied next (Chapter 3). The inventory control configuration is (partly) derived from the placement of the throughput manipulator, and a clear rule for a self-consistent inventory control structure was developed as it was not reported in the open literature (Chapter 2). Another path that arose from tight bottleneck control was the idea to include dynamic degrees of freedom (hold-up volumes) to obtain tighter bottleneck control (Chapter 4). Finally, through my employer, StatoilHydro, I got the possibility to implement the coordinator MPC in practice at a gas processing plant (Chapter 6). A short summary of the contents of the thesis is given next.

In **Chapter 2: Self-consistent inventory control**, we define consistency and self-consistency for an inventory control system. Consistency means that the (steady-state) mass balances are fulfilled and self-consistency means that the mass balances in the individual units are satisfied by the local inventory loops. This leads to the proposed self-consistency rule. The proposed rule is demonstrated on several examples, including units in series, recycle systems and closed systems. Specific rules that deal with the inventory control system are developed from the self-consistency rule.

In **Chapter 3: Throughput maximization requires tight bottleneck control**, we derive under which conditions maximum throughput is an optimal economic operation policy. We discuss back off in a general setting and for the special case for maximum throughput. We consider the case with a fixed bottleneck where a single-loop controller can realize maximum throughput. Further, the location of the throughput manipulator is discussed, where the effective time delay from the throughput manipulator to the bottleneck is important. The location of throughput manipulators is illustrated through examples. Possible improvements to reduce back off and hence increase the throughput are listed.

Chapter 4: Dynamic degrees of freedom for tighter bottleneck control, extend the ideas from Chapter 3 to include dynamic degrees of freedom to reduce the effective delay from the throughput manipulator to the bottleneck. The control structure single-loop with ratio control is proposed to include dynamic degrees of freedom for cases with fixed bottleneck. A multivariable controller like MPC that uses inventory set points as manipulated variables can also be used. Both control structures are demonstrated with an example. The required inventory size is estimated for the case with single-loop with ratio control structure.

In **Chapter 5: Coordinator MPC for maximizing plant throughput**, we consider the case where the bottlenecks may move, with parallel flows that give rise to multiple bottlenecks and with crossover flows as extra degrees of freedom. We present a coordinator MPC that solves the maximum throughput problem dynamically. The plantwide control problem is decomposed by estimating the capacity to each unit, that is, the feed rate each unit is able to receive within feasible operation. The coordinator MPC is demonstrated with a case study.

In **Chapter 6: Implementation of a coordinator MPC for maximizing throughput at a large-scale gas plant**, the industrial implementation of a coordinator MPC (Chapter 5) at the Kårstø gas plant is described. This includes design, modelling and tuning of the coordinator MPC, in addition to the plantwide decomposition by the remaining capacity estimate. Experiences from implementation and test runs are reported.

Chapter 7: Conclusions and directions for further work sums up and concludes the thesis, together with proposals for further work.

Appendix A: Implementation of MPC on a deethanizer at Kårstø gas plant discusses implementation of MPC on a deethanizer column located at the Kårstø gas plant. The appendix contains basic information about MPC design, dynamic modelling and tuning. The MPC software, SEPTIC*, is described briefly. The SEPTIC MPC tool is used in other parts of the thesis (Chapter 5 and 6) and the Appendix is therefore included for completeness.

1.3 Main contributions

The main contributions of the thesis are:

- Plantwide decomposition by estimating the remaining capacity in each unit. An important parameter for the maximum throughput case is the maximum flow for the individual (local) units. This can be obtained by using the models and constraint in the local MPC applications. This decomposes the plant significantly, leading to a much smaller plantwide control problem.
- The idea of using a "decentralized" coordinator MPC to maximize throughput. Throughput manipulators strongly affect several units and are therefore left as "unused" degree of freedom to be set at the plant-wide level. The coordinator manipulates on feed rates, splits and crossover (throughput manipulators) to maximize the plant throughput subject to feasible operation.

*Statoil Estimation and Prediction Tool for Identification and Control

The remaining capacity estimate for each unit is constraints in the coordinator MPC.

- The self-consistency rule and the explanation of a self-consistent inventory control system. Consistency is a very important property of inventory control that must be fulfilled. An experienced engineer can usually immediately say if a proposed inventory control system is workable. However, for a student or newcomer to the field it is not obvious, and even for an experienced engineer there may be cases where the experience and intuition fails. Therefore, we find the self-consistency rule useful together with the illustrative examples.
- Single-loop with ratio control as an alternative structure to obtain tight bottleneck control. With a fixed bottleneck and with a long effective delay from the throughput manipulator to the bottleneck, tight bottleneck control can still be obtained by using dynamic degrees of freedom. Single-loop with ratio control use inventories upstream the bottleneck by adding bias to the inventory controller outputs, whereas the throughput manipulator (e.g. feed rate) controls the bottleneck flow rate. This structure makes it possible to obtain tight bottleneck control without moving the throughput manipulator or reconfiguring the inventory loops.

1.4 Publications

The following is a complete list of the publications written during the work contained in this thesis. This includes submitted, accepted and published work.

Chapter 2

Aske, E.M.B. and Skogestad, S. Self-consistent inventory control. *Ind. Eng. Chem. Res.*, Submitted.

Chapter 3

Aske, E.M.B, Skogestad,S. and Strand, S. Throughput maximization by improved bottleneck control. *8th International Symposium on Dynamics and Control of Process Systems (DYCOPS)*. Vol. 1, June 6-8 2007, Cancun, Mexico. pp 63-68.

Chapter 4

Aske, E.M.B. and Skogestad, S. Dynamic degrees of freedom for tighter bottleneck control. *Comput. Chem. Eng.*, Submitted.

Aske, E.M.B. and Skogestad, S. Dynamic degrees of freedom for tighter bottleneck control. *10th International Symposium on Process Systems Engineering*, August 16-20, 2009, Salvador-Bahia, Brazil. Submitted.

Chapter 5

Aske, E.M.B., Strand S. and Skogestad, S. Coordinator MPC with focus on maximizing throughput, In: *Proc. PSE-ESCAPE Symposium*, (W. Marquardt and C. Pantelides, Eds.), July 10-13 2006, Garmisch-Partenkirchen, Germany. Published by Elsevier, ISBN 0-444-52969-1 978-0-444-52969-5, Vol. 21B, pp. 1203-1208.

Aske, E.M.B., Strand, S. and Skogestad, S. Coordinator MPC for maximization of plant throughput. *AIChE Annual Meeting*, San Francisco, USA, Nov. 2006, Abstract and Presentation 330b.

Aske, E.M.B., Strand, S. and Skogestad,S. Coordinator MPC for maximizing plant throughput. *Comput. Chem. Eng.* **32**, 195-204 (2008).

Chapter 6

Aske, E.M.B., Strand, S. and Skogestad, S. Implementation of Coordinator MPC on a Large-Scale Gas Plant. *AIChE Annual Meeting*, Philadelphia, USA, Nov. 2008, Abstract and Presentation 409g.

Aske, E.M.B., Strand, S. and Skogestad, S. Industrial implementation of a coordinator MPC for maximizing throughput at a large-scale gas plant. *International Symposium on Advanced Control of Chemical Processes*, July 12-15, 2009, Istanbul, Turkey. Submitted.

Appendix A

Aske, E.M.B., Strand, S. and Skogestad, S. Implementation of MPC on a deethanizer at Kårstø gas plant. In: *16th IFAC World Congress*, Prague, Czech Republic, July 2005, paper We-M06-TO/2. CD-rom published by International Federation of Automatic Control.

Chapter 2

Self-consistent inventory control

Submitted to Ind. Eng. Chem. Res.

Inventory or material balance control is an important part of process control. A requirement is that the inventory control system is *consistent* meaning that the steady-state mass balances (total, component and phase) for the individual units and the overall plant are satisfied. In addition, *self-consistency* is a desired property, meaning that the mass balances are satisfied locally with local inventory loops only. In practice, if a control structure is inconsistent, then at least one control valve will become fully open (or in rare cases closed) and cannot attain its set point. The main result of this paper is a *self-consistency rule* for evaluating the consistency of inventory control systems.

2.1 Introduction

One of the more elusive parts of process control education is inventory or material balance control. An engineer with some experience can usually immediately say if a proposed inventory control system is workable. However, for a student or newcomer to the field it is not obvious, and even for an experienced engineer there may be cases where experience and intuition are not sufficient. The objective of this paper is to present concise results on inventory control, relate to previous work, tie up loose ends, and to provide some good illustrative examples. The main result (self-consistency rule) can be regarded as obvious, but nevertheless we have not seen it presented in this way before.

The main result is a simple rule to check whether an inventory control system is *consistent*. Here, consistency means that the mass balances for the entire plant are satisfied (Price and Georgakis, 1993). In addition, we usually want the inventory control system to be *self-consistent*. Self-consistency means that, in addition to plantwide consistency, the mass balance for each unit is satisfied by itself (locally), without the need to rely on control loops outside the unit. Consistency is a required property, because the mass balances must be satisfied in a plant, whereas self-consistency is a desired property of an inventory control system. In practice, an inconsistent control structure will lead to a situation with a fully open or closed control valve and the associated control loop cannot fulfill or attain the control set point.

In most plants, we want the inventory control system to use simple PID controllers and be part of the basic (regulatory) control layer. This is because it is generally desirable to separate the tasks of regulatory (stabilizing) control and supervisory (economic) control. From this it follows that the structure of the inventory control system is usually difficult to change later.

The importance of consistency of inventory control structures is often overlooked. Our work is partly inspired by the many examples of Kida, who has given industrial courses in Japan on control structures for many years. In a personal communication (Kida, 2008) he states that *"most process engineers, and even academic people, do not understand the serious problem of inconsistency of plantwide control configurations. When writing a paper, you have to clearly explain this point and make them convinced at the very outset. Otherwise they will not listen to or read through your detailed statements, but skip them all"*.

A very good early reference on inventory control in a plantwide setting is Buckley (1964). He states that material balance control must be in the direction of flow downstream a given flow and opposite the direction of flow upstream a given flow. Price and Georgakis (1993); Price *et al.* (1994) extended this and state that the inventory control must "radiate" outwards from the point of a given flow (throughput manipulator). As shown in this paper, all these statements are a consequence of requiring the inventory control system to be self-consistent.

Downs (1992) provides a very good discussion of material balance control in a plantwide control environment, with many clarifying examples. However, it is somewhat difficult for the reader to find a general rule or method that can be applied to new cases.

Luyben *et al.* (1997) propose a mainly heuristic design procedure for plantwide control. Luyben *et al.* procedure consist of, among others, "*Step 6. Control inventories (pressures and levels) and fix a flow in every recycle loop*". Possible limitations of this guideline are discussed in the present paper. Another guideline of Luyben *et al.* (1997) is to *"ensure that the overall component balances for each chemical species can be satisfied either through reaction or exit streams by accounting for the component's composition or inventory at some point in the process"*. As discussed later, this guideline is a bit limited because entrance (feed) streams is not considered.

Specific guidelines for designing inventory control structures are presented by

Georgakis and coauthors (Price and Georgakis, 1993; Price *et al.*, 1994). They propose a set of heuristic guidelines for inventory control design in a plantwide environment and also discuss consistency. The authors also state the importance of a self-consistent inventory control structure: *"Self-consistency appears to be the single most important characteristic governing the impact of the inventory control structure on system performance*".

As already mentioned, Fujio Kida from JGC Corporation in Japan has developed a lot of teaching material (Kida, 2008) and written several papers (e.g. Kida, 2004) on inventory control. Unfortunately, the work is published in Japanese only, but nevertheless it is clear that there are many detailed rules and some require detailed calculations. Our objective is to derive, if possible, a single rule for evaluating the consistency of inventory control system that applies to all cases and that only requires structural information.

The organization of the paper is as follows. First, we define self-consistent inventory control in Section 2.2. The main result in this paper is the self-consistency rule presented in Section 2.3. Thereafter, the rule is used to discuss consistency of flow networks in Section 2.4, which also discusses more specific rules that can be derived from the general self-consistency rule. Several examples in terms of inventory control are given in Section 2.5, before the paper is concluded in Section 2.6. Note that the present paper focuses on analysis of a given control structure. The design of the inventory control system, which in particular is related to the placement of the throughput manipulator, is discussed in more detail in a separate paper (Chapter 3).

Remark on notation: In this paper, when a flow is left unused or with a flow controller (FC), then this indicates that this is a *given* flow. By the term ”given flow” we mean that the flow is *not* used for inventory control but rather given by conditions outside the inventory control system. For example, a ”given flow” can be

1. a throughput manipulator (TPM),
2. a flow that comes from another part of the plant (disturbance for our part),
3. a fixed flow
4. a flow that is used for other control tasks (eg., control of composition or temperature).

2.2 Definition of self-consistent inventory control

The dynamic mass balance for total or component mass in any unit or process section can be written (e.g. Downs, 1992):

Inflow + Generation - Outflow - Consumption = Change in inventory

To keep the inventory within bounds, the change in inventory must be within bounds, and over a long time (at steady-state) the change in inventory must be zero. Thus, there must be a balance between the In-terms (inflow + generation) and Out-terms (outflow + consumption). However, without control this is not necessarily satisfied. The main objective of the inventory control system is to "stabilize" or provide "self-regulation" of all inventories such that the mass balances are satisfied. This leads to the self-consistency rule, which is the main result in this paper, but let us first define some terms.

Definition 2.1. ***Consistency***. *An inventory control system is said to be* ***consistent*** *if the steady-state mass balances (total, components and phases) are satisfied for any part of the process, including the individual units and the overall plant.*

Remark. The use of mass balances for a phase may seem odd, and is discussed in more detail in the next section.

Since the mass balance must be satisfied for the overall plant, it follows that a consistent inventory control system must be *"able to propagate a production rate change throughout the process and in particular if such a change produces changes in the flow rates of major feed and product streams"* (Price and Georgakis, 1993).

Definition 2.2. ***Self-regulation***. *Self-regulation is when an acceptable variation in the output variable is achieved without the need for additional control when disturbances occur.*

Note that the above definition of consistency allows for "long loops" (not local loops) where, for example, the feed rate controls the inventory at the other end of the process (as illustrated in Figure 2.4). This is often undesirable and self-consistency is when the steady-state mass balances are satisfied also locally. More precisely, we propose the following definition:

Definition 2.3. ***Self-consistency***. *A consistent inventory control system is said to be* ***self-consistent*** *if there is* local *"self-regulation" of all inventories. This means that for each unit the* local *inventory control loops by themselves are sufficient to achieve steady-state mass balance consistency for that unit.*

Remark 1 "Self-regulation" here refers to the response of the process with its inventory control system in operation. If self-regulation is achieved without active control then this is referred to as "true" self-regulation.

Remark 2 The term "*local* inventory control loops" means that no control loops involving manipulated variables outside the unit are needed for inventory control of the unit (see Figure 2.4 for a system that does not satisfy this requirement).

Remark 3 The definitions require that the "steady-state mass balances" are satisfied. We are here referring to the *desired* steady-state, because an inconsistent inventory control system may give a steady-state which is not the desired one. For example, a component with no specified exit will eventually have to exit somewhere but this may not be a desired operation point.

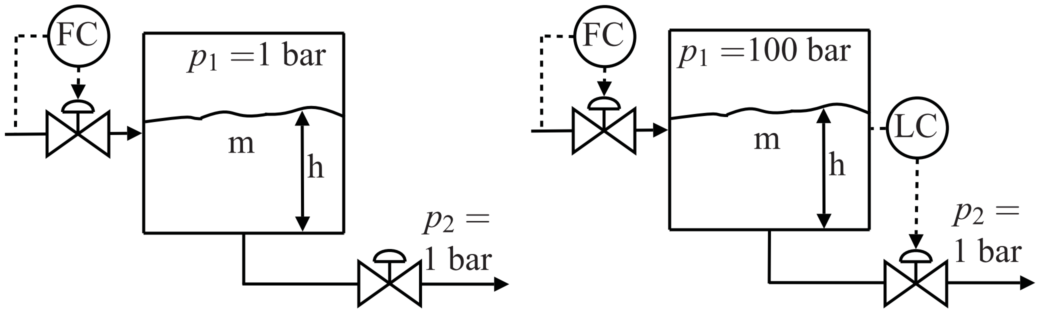
Example 2.1. ***Self-regulation.*** *"Self-regulation" may or may not require "active" control, as mentioned in Remark 1. As an example, consider regulation of liquid inventory (m) in a tank; see Figure 2.1(a). The outflow is given by a valve equation*

$$\dot{m}_{out} = C_v f(z)\sqrt{\Delta p \cdot \rho} \quad \text{[kg/s]}$$

where z is valve position. The pressure drop over the valve is

$$\Delta p = p_1 - p_2 + \rho g h$$

where h is the liquid level, which is proportional to the mass inventory, e.g., $m = h\rho A$ for a tank with constant cross section area A. If the pressure drop Δp depends mainly on the liquid level h, then the inventory m is self-regulated. This is the case in Figure 2.1(a) where $p_1 = p_2$ so $\Delta p = \rho g h$ and the entire pressure drop over the valve is caused by the liquid level. Thus, $\dot{m}_{out} \sim \sqrt{h}$, which means that without control a doubling of the flow $\dot{m}_{out}$ will result an a four times larger liquid level (h). If this change is acceptable, then we have self-regulation. In other cases, it may be necessary to use "active" control to get sufficient self-regulation of the inventory. Specifically: In Figure 2.1(b), $p_1 - p_2 = 99$ bar so the relative pressure contribution from the liquid level (ρgh) is much too small to provide acceptable self-regulation. For example, for a large tank of water with $h = 10$ m, the contribution from the level is only about 1 % ($\rho gh \approx 1000$ kg/m$^3 \cdot$ 10 m/s^2 $\cdot$ 10 m $= 10^5$ N/m^2 = 1 bar). In this case "active" control is required, where the level controller (LC) adjusts the valve position z, see Figure 2.1(b).



(a) Self-regulation is possible without "active" control.

(b) "Self-regulation" requires level control.

Figure 2.1: Self-regulation of inventory in a tank with a given feed rate.

2.3 Self-consistency rule

As a direct consequence (implication) of the statements in Section 2.2, we propose the following rule to check if an inventory control system is self-consistent.

Rule 2.1. ***"Self-consistency rule":*** *Self-consistency (local "self-regulation" of all inventories) requires that*

1. *The total inventory (mass) of any part of the process (unit) must be "self-regulated" by its in- or outflows, which implies that at least one flow in or out of any part of the process (unit) must depend on the inventory inside that part of the process (unit).*

2. *For systems with several components, the inventory of each component of any part of the process must be "self-regulated" by its in- or outflows or by chemical reaction.*

3. *For systems with several phases, the inventory of each phase of any part of the process must be "self-regulated" by its in- or outflows or by phase transition.*

Remark 1 The above requirement must be satisfied for "any part of the process". In practice, it is sufficient to consider the individual units plus the overall process.

Remark 2 A flow that depends on the inventory inside a part of the process, is often said to be on "inventory control". Inventory control usually involves a level controller (LC) (liquid) or pressure controller (PC) (gas and in some cases liquid), but it may also be a temperature controller (TC), composition controller (CC) or even no control ("true" self-regulation, e.g. with a constant valve opening). Obviously, a flow controller (FC) can not be used for inventory control because flow is not a measure of inventory.

Remark 3 It is possible to extend the "self-regulation" rule to energy inventory, but this is not done here. We also doubt if such an extension is very useful, because in most cases the energy balance will maintain itself by "true" self-regulation (without control), for example because a warmer inflow in a tank leads to a warmer outflow.

Proof of self-consistency rule.

1. A boundary (control volume) may be defined for any part of the process. Let m [kg] denote the inventory inside the control volume and let $\dot{m}_{\text{in}}$ and $\dot{m}_{\text{out}}$ [kg/s] denote in- and outflows. Then the (total) mass balance is

$$\frac{dm}{dt} = \sum \dot{m}_{\text{in}} - \sum \dot{m}_{\text{out}} \quad \text{[kg/s]}$$

   If all terms are independent of the inventory m, then this is an integrating process where m will drift out of bounds ($\frac{dm}{dt} \neq 0$ at steady-state) when there is a disturbance in one of the terms (e.g. $\dot{m}_{\text{in}}$, $\dot{m}_{\text{out}}$). To stabilize the inventory we must have "self-regulation" where $\dot{m}_{\text{in}}$ or $\dot{m}_{\text{out}}$ depends on the inventory (m), such that m is kept within given bounds in spite of disturbances. More precisely, $\dot{m}_{\text{in}}$ must decrease when m increases or $\dot{m}_{\text{out}}$ must increase when m increases, such that m is kept within given bounds in spite of disturbances.

2. Similarly, let n_A [mol A] denote the inventory of component A inside the control volume and let $\dot{n}_{A,\text{in}}$ and $\dot{n}_{A,\text{out}}$ [mol A/s] denote the in- and outflows. The mass balance for component A is

$$\frac{dn_A}{dt} = \sum \dot{n}_{A,\text{in}} - \sum \dot{n}_{A,\text{out}} + G_A \quad \text{[mol A/s]}$$

where G_A is the net amount generated by chemical reaction. To stabilize the inventory we must have "self-regulation" where $\dot{n}_{A,\text{in}}$, $\dot{n}_{A,\text{out}}$ or G_A depend on n_A such that n_A is kept within given bounds in spite of disturbances.

An example where the inventory n_A is self-regulated because of the reaction term G_A is the irreversible reaction $A + B \rightarrow P$, where B is in excess and A is the limiting reactant. In this case, an increase in inflow of A ($\dot{n}_{A,\text{in}}$) will be consumed by the chemical reaction.

3. The rule for the individual phase follows by simply defining the control volume as the parts of the process that contain a given phase P and applying the mass balance to this control volume. Let m^P [kg] denote the inventory of the given phase inside the control volume and let $\dot{m}^P{}_{\text{in}}$ and $\dot{m}^P{}_{\text{out}}$ [kg/s] denote the in- and outflows. The mass balance for a given phase is then

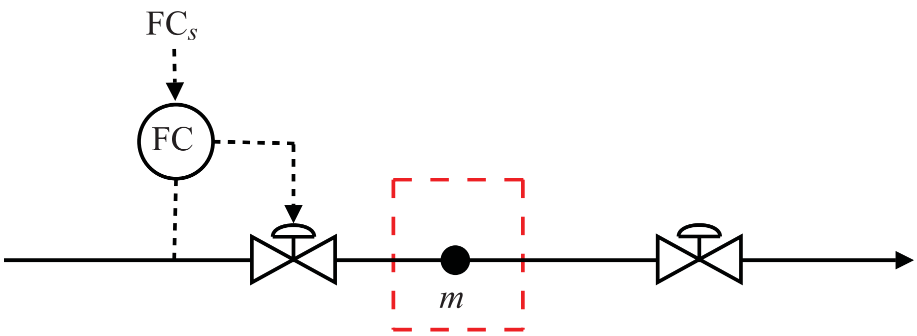
$$\frac{dm^P}{dt} = \sum \dot{m}^P_{\text{in}} - \sum \dot{m}^P_{\text{out}} + G^P \quad \text{[kg/s]}$$

where G^P is the net phase transition over the phase boundary. To stabilize the inventory we must have "self-regulation" where $\dot{m}^P_{\text{in}}$, $\dot{m}^P_{\text{out}}$ or G^P depends on the inventory (m^P) such that m^P is kept within given bounds in spite of disturbances.

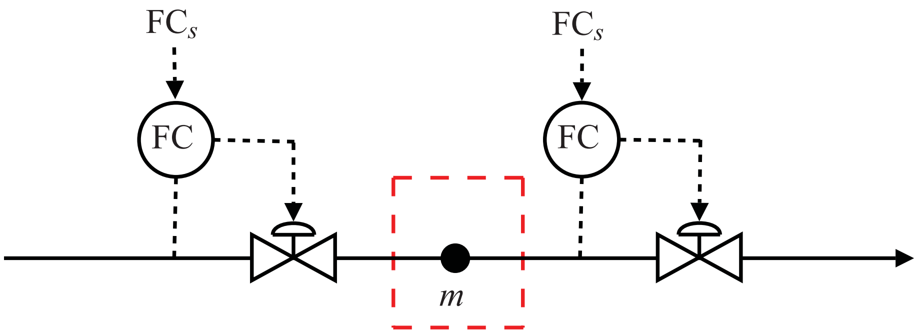
An example where we need to consider individual phases is a flash tank where a two-phase feed is separated into gas and liquid.

□

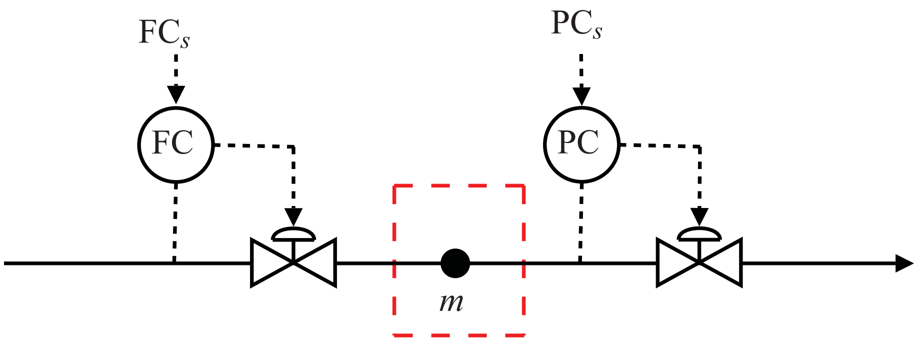
Example 2.2. ***Stream with two valves.*** *To demonstrate the self-consistency rule on a very simple example, consider a single stream with two valves; see Figure 2.2(a). There is only a single (small) hold-up m in this simple process (illustrated by the big dot), so consistency and self-consistency are here the same. The pressure p depends directly on the inventory m (for a liquid the dependency is very strong; for an ideal gas it is* $p = \frac{mRT}{V}$*). Thus, self-regulation of inventory is the same as self-regulation of pressure. To apply the self-consistency rule, we define a control volume (dotted box) as shown in Figure 2.2 and note that the inflow is on flow control in all four cases, that is, the inflow is independent of the*



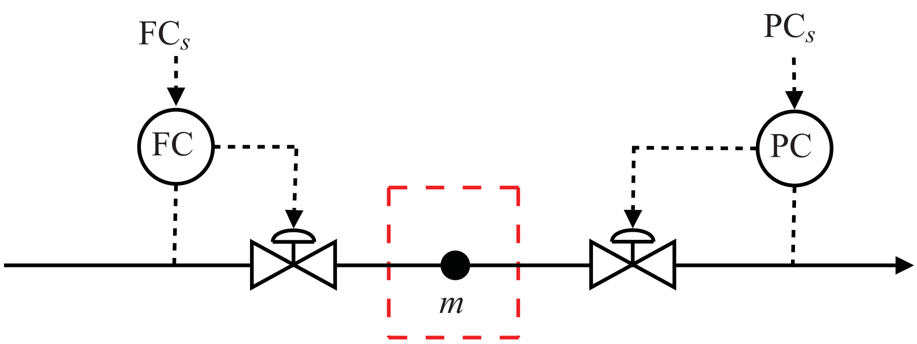
(a) OK (consistent control structure since outflow depends on inventory m).



(b) Not consistent control structure since outflow is given.



(c) OK (consistent control structure since outflow depends on inventory m).



(d) Not consistent control structure since outflow does not depend correctly on inventory m.

Figure 2.2: Four different control structures with two valves and given inflow. Note: For the flow controllers (FC) it does no matter whether the valve is downstream (as shown above) or upstream of the flow measurement.

inventory m. Thus, according to Rule 2.1, to have consistency (self-regulation), the outflow must depend on the pressure p (inventory m) and more specifically the outflow must increase when p increases.

Four different control structures are displayed in Figure 2.2. According to Rule 2.1, the structure in Figure 2.2(a) is consistent since the outflow increases when the pressure p (inventory m) increases. Thus, we have "true" self-regulation with no need for active control.

The control structure in Figure 2.2(b) is not consistent because the outflow is independent on the inventory m. Even if the set points for the two flow controllers were set equal, any error in the actual flow would lead to an imbalance, which would lead to accumulation or depletion of mass and the inventory would not be self-regulated.

The structure in Figure 2.2(c) is consistent because the outflow increases when the pressure (inventory m) increases.

Finally, the control structure in Figure 2.2(d) is not consistent because the outflow depends on the inventory m (and pressure) in the wrong (opposite) manner. To understand this, consider a decrease in inflow, which will lead to a decreased pressure in the control volume. A lower differential pressure over the pressure-controlled valve leads to a smaller flow through the valve and the pressure at the downstream measuring point will decrease, leading the pressure controller to open *the valve. The result is a further pressure decrease in the control volume, so the pressure controller is actually working in the wrong direction. The opening of the pressure-controlled valve will also affect the flow-controlled valve and, depending on the set point of the controllers, either the flow-controlled valve or the pressure-controlled valve will move to fully open. The other pressure-controlled valve or flow-controlled valve will continue to control pressure or flow. It should also be noted that the pressure control loop is in the direction* opposite *to flow, which is not correct when the inflow is given (see further discussion in Section 2.4.1).*

This is confirmed by dynamic simulations of the simple configuration in Figure 2.2(d) using the flowsheet simulator Aspen HYSYS®(see Figure 2.3):

10% increase in FC set point: *The FC saturates at fully open and the PC maintains its set point (Figures 2.3(a) and 2.3(b)).*

10% decrease in FC set point: *The FC maintains its set point and the PC saturates at fully open (Figures 2.3(c) and 2.3(d)).*

5% increase in PC set point: *The FC maintains its set point and the PC saturates at fully open (Figures 2.3(e) and 2.3(f)).*

5% decrease in PC set point: *The FC saturates at fully open and the PC maintains its set point (Figures 2.3(g) and 2.3(h)).*

In all cases the system is assumed to be at steady-state initially.

A remark about the sign of the controllers: Overall, the controller and the plant should give a negative feedback loop:

1. **Flow control**. Opening a valve always increases the flow (positive gain), so a flow controller is always "reverse acting" (with a negative feedback sign).
2. **Level and pressure control**. The controller sign depends on the location of the valve relative to the inventory (level or pressure). If control is in the direction of flow (with the inventory measurement for level or pressure upstream the valve) then the controller must be "direct acting" (positive feedback sign), if control is in opposite direction of flow then it must be "reverse acting".

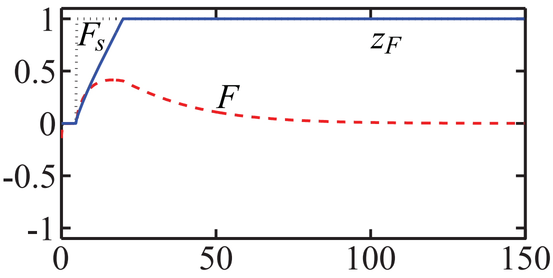
These remarks were used when deciding the controller tunings in Figure 2.3.

Example 2.3. ***Units in series.*** *To understand the difference between the terms consistency (Definition 2.1) and self-consistency (Definition 2.3), consider inventory control of the series process in Figure 2.4. The control structure is* ***consistent*** *and is able to propagate a production rate change to a change in the feed rate. However, the in- and outflows for the last unit (dashed box) do not depend directly on the inventory inside the unit and the control volume is therefore* ***not self-consistent*** *according to the "self-consistency rule"(Rule 2.1). This can also be seen because the inventory controllers are not in the direction opposite to flow as they should be for a self-consistent process with a given product rate (see also Section 2.4.1). To make the structure consistent we have in Figure 2.4 introduced a "long loop" where the inflow to the first unit is used to control the inventory in the last unit.*

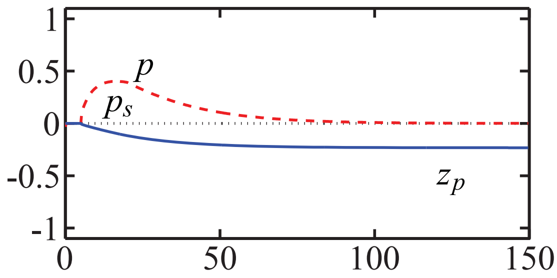
Example 2.4. ***Phase transition****. In some cases, phase transition needs to be considered for self-consistency. Consider Figure 2.5 where the inflow F is given. Thus, according to Rule 2.1, to have consistency the outflow must depend on the inventory in the tank.*

In Figure 2.5(a), the inflow is a single phase (liquid) and the outflow from the single-phase tank is split in two liquid streams (L_1 and L_2). There is one inventory, so for self-consistency, one of the outflows must be on inventory control whereas the other outflow can be flow controlled. This follows because the adjustable split introduces an extra degree of freedom, but the number of inventories that need to be controlled is unchanged.

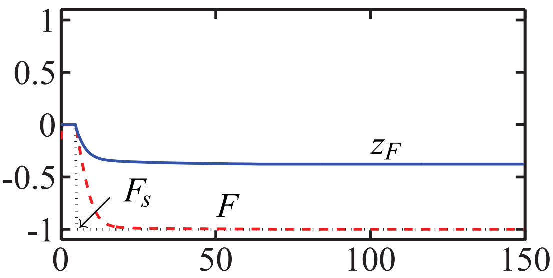
In Figure 2.5(b) the inflow is two-phase (liquid and vapor) and there are two inventories (liquid and vapor) that needs to be regulated. To have a consistent inventory control structure, both the outflows (vapor and liquid) must be used for inventory control. In Figure 2.5(b) this is illustrated by the LC (liquid inventory)



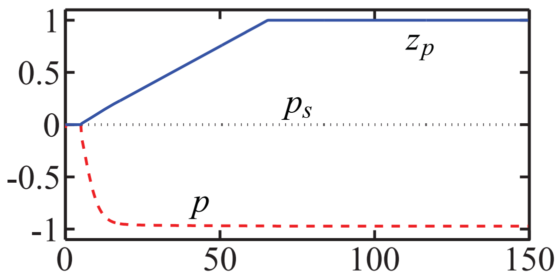
(a) Increase FC set point: FC values.



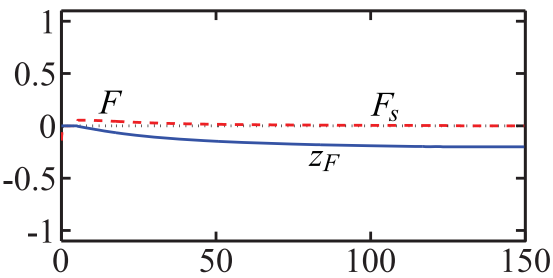
(b) Increase FC set point: PC values.



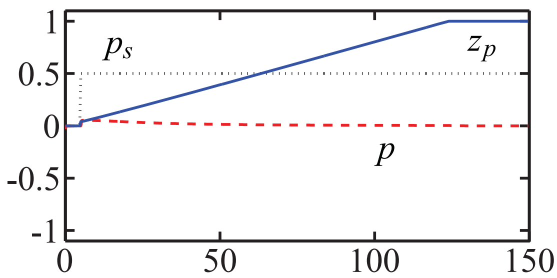
(c) Decrease FC set point: FC values.



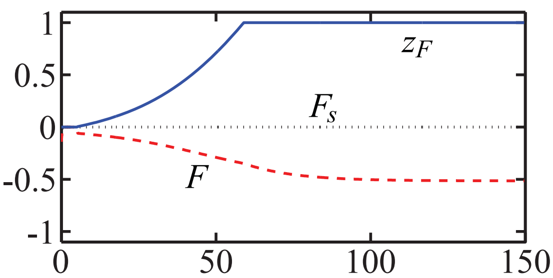
(d) Decrease FC set point: PC values.



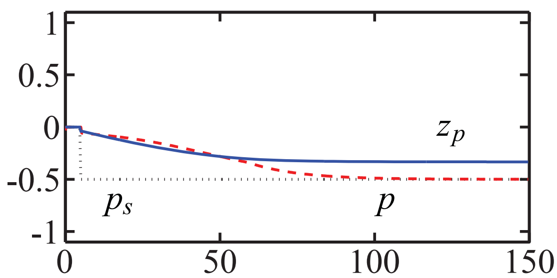
(e) Increase PC set point: FC values.



(f) Increase PC set point: PC values.



(g) Decrease PC set point: FC values.



(h) Decrease PC set point: PC values.

Figure 2.3: Dynamic simulations of the simple configuration in Figure 2.2(d). Left column: Flow controller. Right column: Pressure controller. In all cases, one of the valves moves to fully open.

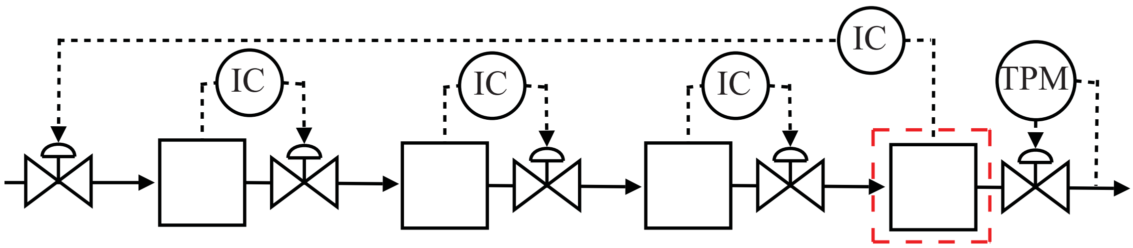
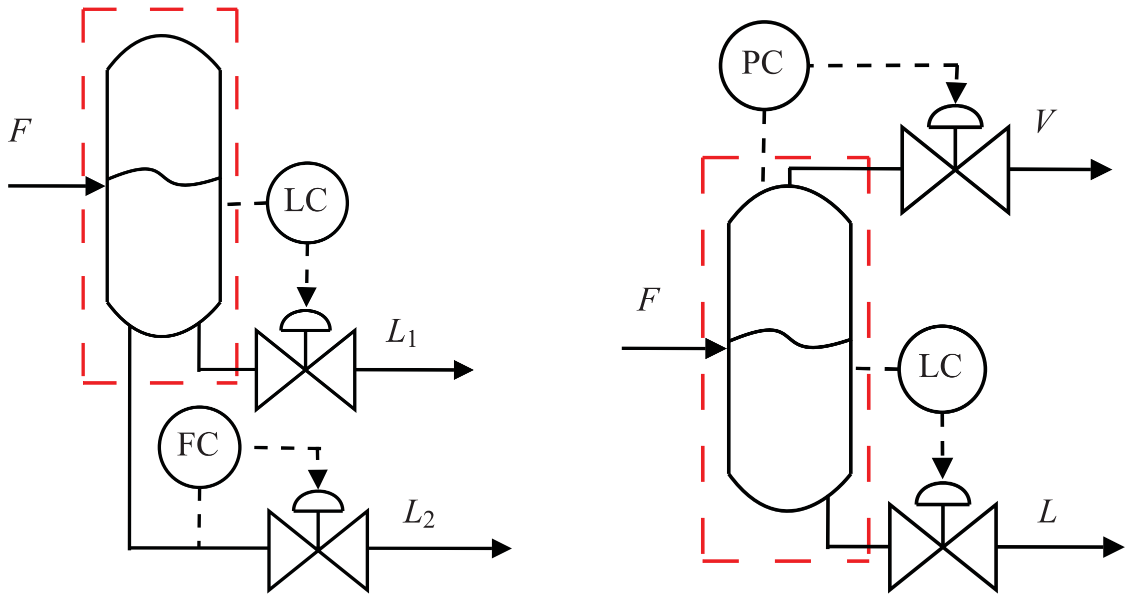


Figure 2.4: Consistent, but not self-consistent inventory control structure.



(a) Single-phase tank: Adjustable split.

(b) Two-phase tank: Split indirectly given by inventory control.

Figure 2.5: Self-consistent inventory control of split with one and two phases.

and PC (vapor inventory). In this case, the split does not actually give an extra degree of freedom because the split is indirectly determined by the feed quality (fraction of vapor).

2.4 Specific rules and consistency of flow networks

In a flow network there is at least one degree of freedom, called the throughput manipulator (TPM), which sets the network flow. More generally, *a TPM is a degree of freedom that affects the network flow and which is not directly or indirectly determined by the control of the individual units, including their inventory control* (see Chapter 3). Typically, a given flow (e.g. flow controller with an adjustable set point) is a TPM. As discussed in more detail below, the location of the TPM is very important. In particular, if the flow network has no splits or junctions, then for a given placement of the TPM, there is only one *self-consistent* inventory control system.

However, at splits (e.g. multiple products) or junctions (e.g. multiple feeds), there are several possibilities. At a split or junction, a common choice is to use the largest flow for inventory control (Luyben *et al.*, 1997). For example, with a given feed, the largest product stream may be used for inventory control with the flow rates of the smaller product streams used for quality control. Similarly, with a given production rate, the largest feed rate is often used for inventory control and the smaller feed flows are set in ratio relative to this, with the ratio set point possibly used for quality control.

The objective is now to apply the self-consistency rule to analyze inventory control structures for real processes (flow networks). We consider three network classes:

1. Units in series
2. Recycle systems
3. Closed systems

A series network may have splits, provided the flow is still in the same direction. Note that each single-phase split introduces one extra degree of freedom (the split ratio; see Figure 2.5). A recycle system contains one or more splits that are (partly) fed back to the system. A closed system has total recycle with no feeds or products.

2.4.1 Units in series ("radiating rule")

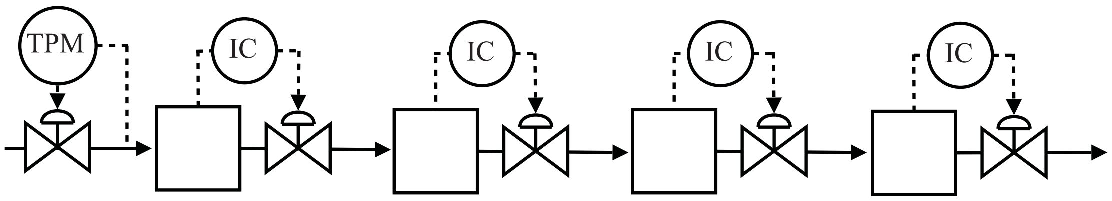
As mentioned above, if there are no splits or junctions, the location of the throughput manipulator determines the self-consistent inventory control structure. Specifically, a direct consequence of the self-consistency rule is

- *Inventory control must be in direction of flow downstream the location of a given flow (TPM).*
- *Inventory control must be in direction opposite to flow upstream the location of a given flow (TPM).*

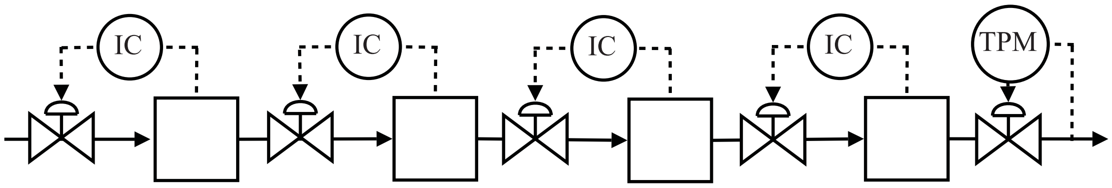
More generally, we have:

Rule 2.2. ***Radiation rule*** *(Price and Georgakis, 1993): A self-consistent inventory control structure must be radiating around the location of a given flow (TPM).*

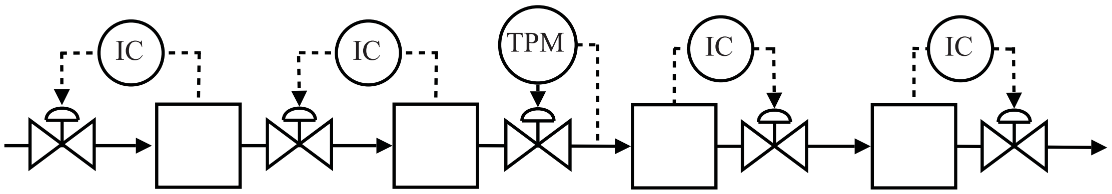
These rules are further illustrated in Figure 2.6.



(a) TPM at inlet (feed): Inventory control in direction of flow.



(b) TPM at outlet (on-demand): Inventory control in direction opposite to flow



(c) General case with TPM inside the plant: Radiating inventory control

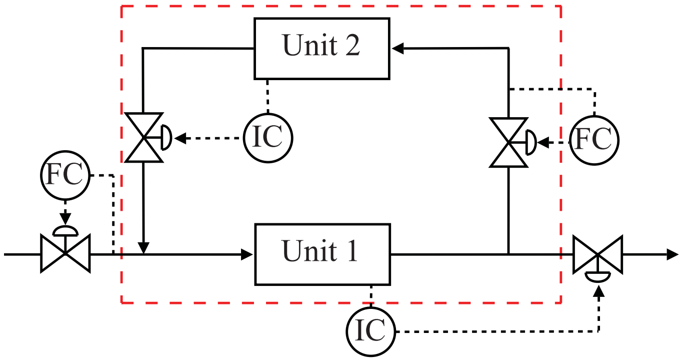
Figure 2.6: Self-consistency requires a radiating inventory control around a given flow (TPM).

2.4.2 Recycle systems

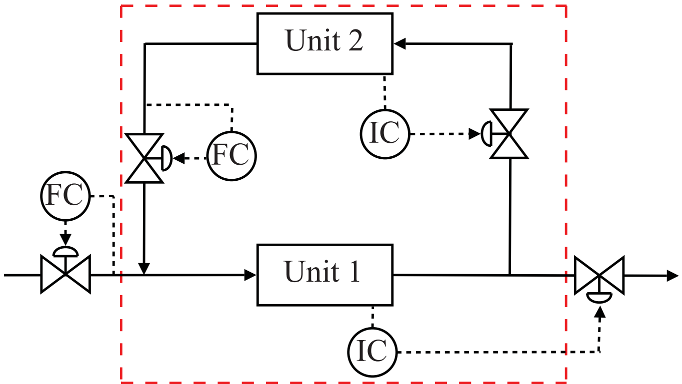
A recycle system usually has an adjustable split, which (but not always) *"introduces an extra degree of freedom for control of the network flow"* (Kida, 2008). On the other hand, the requirement of self-consistency imposes limitations. As an example, consider the simple single-phase recycle example with given feed and an adjustable split in Figure 2.7 (there is a pump or compressor in the recycle loop which is not shown). Figures 2.7(a) and 2.7(b) have consistent inventory control structures, because the outflows from units 1 and 2 depend on the inventory inside each unit. In both cases one flow in the recycle loop is given (flow controlled with an adjustable set point that may be used for other purposes than inventory control). Note that *the inventory control in the recycle loop can be either in direction of flow* (Figure 2.7(a)) or *direction opposite to flow* (Figure 2.7(b)), because the flow rate can be given at any location in the recycle loop.

In Figure 2.7(c) the inventory loops for units 1 and 2 are paired opposite. This structure is not self-consistent because the inventory of unit 2 is not "self-regulated by its in- or outflows" and thus violates Rule 2.1. In addition, the inventory control of unit 2 requires that the other inventory loop is closed, and thus violates Definition 2.3.

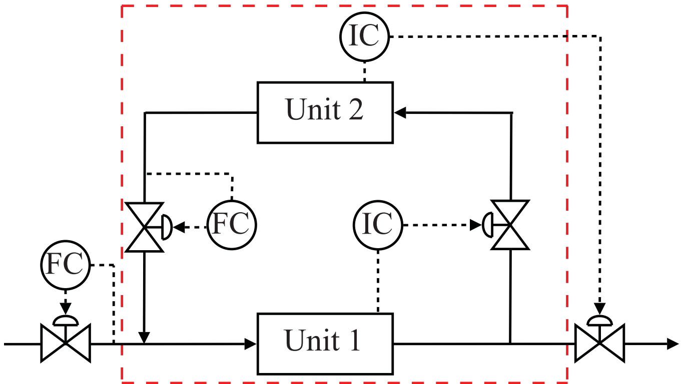
Finally, Figure 2.7(d) is obviously not consistent since both the feed rate and



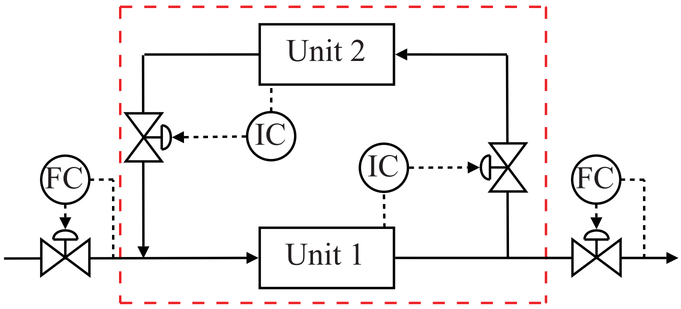
(a) Self-consistent inventory control.



(b) Self-consistent inventory control.



(c) Consistent, but not self-consistent inventory control.



(d) Not consistent inventory control.

Figure 2.7: Inventory control of simple recycle process with given feed.

the product rate are given. In particular, the inflow and outflow to the dotted box do not depend on the inventory inside this part of the process, which violates Rule 2.1.

Remark. This simple example seems to prove the rule that "*one flow rate somewhere in the recycle loop should be flow controlled*" (Luyben, 1993*c*). This rule follows because there is an extra degree of freedom introduced by the split, but the number of inventories that need to be controlled are unchanged. However, first one should note that the set point of the flow controller is a degree of freedom which may be used for other purposes, for example , control of composition. Second, a "counter-example" is provided by the self-consistent reactor-separator-recycle process in Figure 2.11(a). In this case, the split is not actually an extra degree of freedom because the split is indirectly determined by the feed composition to the separator (distillation column), as discussed in Example 2.4.

2.4.3 Closed systems

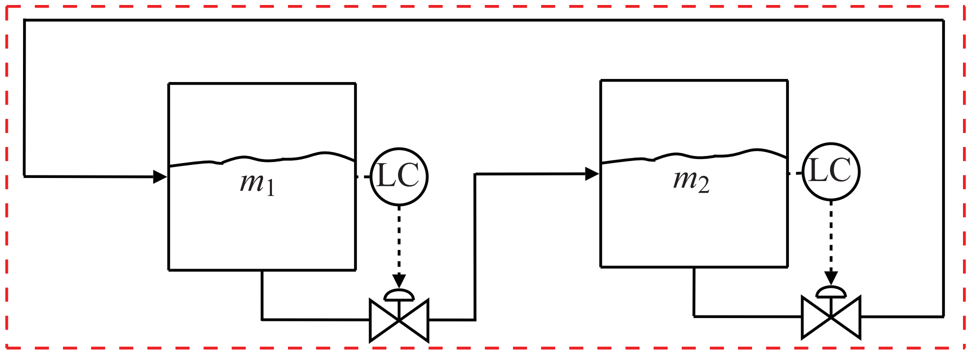
Closed systems require particular attention. It is clear from the total mass balance that the total inventory of a closed system cannot be self-regulated since there are no in- or out streams. Thus, our previously derived rule (Rule 2.1) does not really apply. As an example, consider a closed system with two inventories. In Figure 2.8(a) we attempt to control both inventories, but the two loops will "fight each other" and will drift to a solution with either a fully open or fully closed valve. For example, a (feasible) solution is to have zero flow in the cycle. The problem is that the flow is not set anywhere in the loop. To get a consistent inventory control structure, *one must let one of the inventories be uncontrolled*, as shown in Figures 2.8(b) and 2.8(c). *The corresponding unused degree of freedom (flow) sets the flow rate ("load", throughput) of the closed system.*

To be able to use our self-consistency rule (Rule 2.1) for closed systems there are two alternative "fixes":

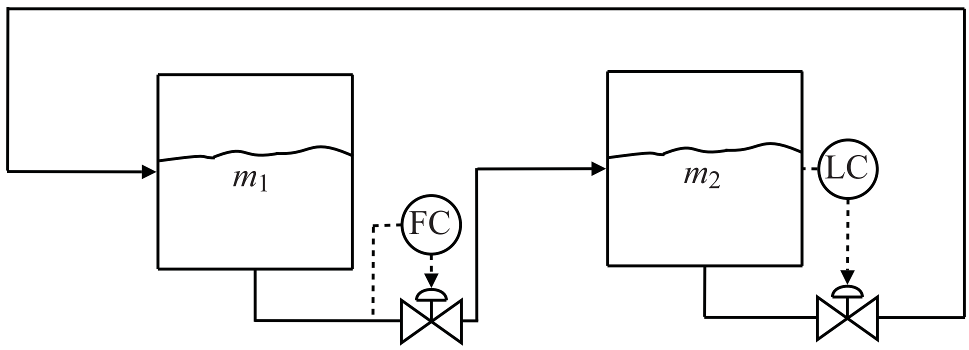
1. Let the total inventory be uncontrolled (*not* self-regulated), which is how such systems are usually operated in practice. Typically the largest single inventory is uncontrolled. However, the remaining inventories must be self-regulated, as usual, to have self-consistency of the inventory control system.

2. Introduce a "dummy" stream that keeps the total inventory constant. This corresponds to allowing for filling (charging) or emptying the system. In practice, this stream may be a make-up stream line that refills or empties the largest inventory, e.g. on a daily or monthly basis.

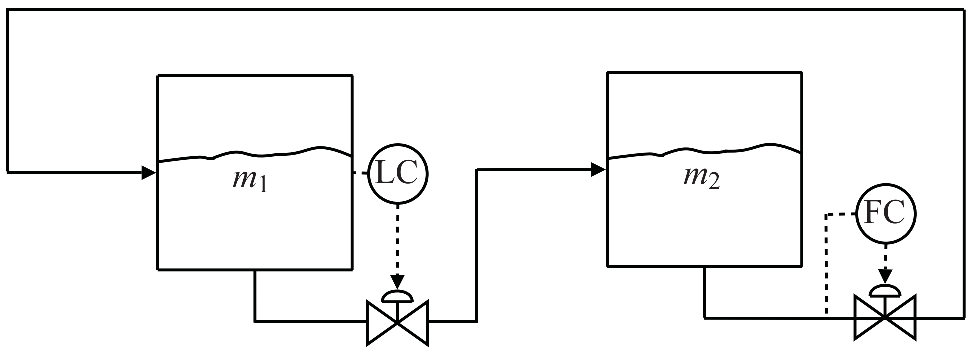
Both approaches allow for disturbances, such as leaks or supply. The inventory control system can then be analyzed using the normal self-consistency rule (Rule 2.1). Figure 2.8(a) is clearly not allowed by Fix 1 as the total inventory is



(a) Not consistent (because there is no uncontrolled inventory).



(b) Self-consistent (inventory m_1 is uncontrolled).



(c) Self-consistent (inventory m_2 is uncontrolled).

Figure 2.8: Inventory control for closed system.

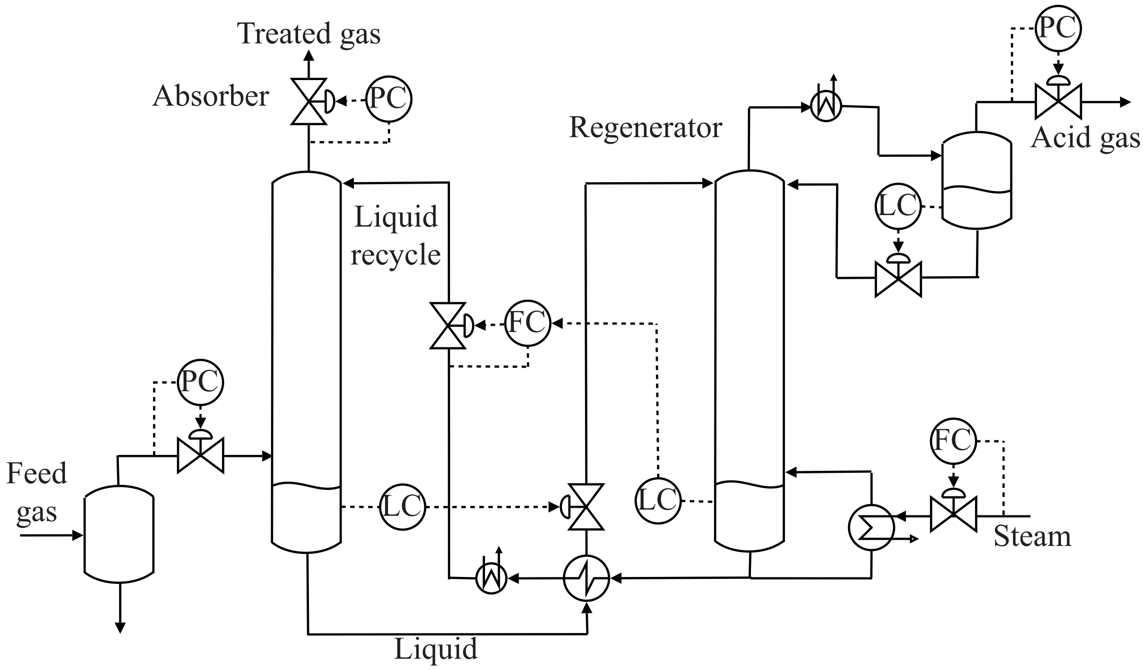


Figure 2.9: Absorber and regenerator example: Not consistent liquid inventory control.

not left uncontrolled. Figure 2.8(a) is also not consistent by Fix 2, since for self-consistency the dummy stream must be used for inventory control instead of one of the two flows in the recycle loop.

Example 2.5. ***Absorber-regenerator example.*** *In this example, the consistency rule (Rule 2.1) is used for an individual phase (liquid), which forms a closed system. Consider the absorber and regenerator example in Figure 2.9 (Kida, 2008) where a component (e.g. CO_2) is removed from a gas by absorption. The inlet gas flow (feed) is indirectly given because there is a pressure control in the direction of flow at the inlet. The gas outlet flows are on pressure control in the direction of flow and thus depend on the gas holdup in the plant. Therefore the gas-phase inventory control is consistent. However, the liquid flows between the absorber and regenerator make up a "closed system" (expect for minor losses). There is a flow controller for the recycled liquid, but its set point is given by the inventory in the regenerator, hence all inventories in the closed system are on inventory control, which violates the rule just derived. To get a consistent inventory control structure, we must break the level-flow cascade loop and let the inventory in the bottom of the regenerator remain uncontrolled.*

2.4.4 Summary of specific rules

In the literature there are many rules that deal with the inventory control structure. In addition to the radiating rule, some useful rules that can be developed from the self-consistency rule (Rule 2.1) are:

1. *All systems must have at least one given flow (throughput manipulator).*

   Proof. Assume there is no throughput manipulator. Then all flows must be on inventory control, which will not result in a unique solution. For example, zero flow will be an allowed solution. □

2. *Component balance rule (Downs, 1992, p. 414): Each component, whether important or insignificant, must have its inventory controlled within each unit operation and within the whole process.* Luyben *et al.* (1998, p. 56) refers to this as "Downs drill".

   Proof. This comes from the requirement of component self-consistency (Rule 2.1). □

3. *A stream cannot be flow controlled more than once, that is, a structure with two flow controllers on the same stream is not consistent.*

   Proof. Make a control volume with the two flow-controlled streams as in- and outflows. Then neither the inflow nor the outflow depends on the control volume and the inventory is not self-regulated. This is demonstrated in Figure 2.2(b). □

4. *Price and Georgakis (1993, p.2699): If a change in the throughput manipulator does not result in a change in the main feed flow, then the control structure is inconsistent.*

   Proof. This follows from the requirement of satisfying the steady-state mass balances. □

5. *Generalized from Price and Georgakis (1993, p.2699): A self-consistent inventory control structure must use the feed or the product (or both) for inventory control.*

   Proof. This follows from the steady-state mass balance. This is also discussed in Section 2.4.1 and a clear illustration of this statement is found in Figure 2.6. □

6. *For closed systems: One inventory must be left uncontrolled and one flow in the closed system must be used to set the load.*

Proof. This follows from that all systems must have at least one given flow to be unique. To be able to set the load for a closed system, one inventory must be uncontrolled. □

The rules are summarized by the proposed procedure for inventory control system design in Table 2.1, which is inspired by the inventory control guidelines in Price *et al.* (1994).

1	Choose the location of the throughput manipulator
2	Identify inventories that need to be controlled including: a) Total mass b) Components c) Individual phases
3	Identify manipulators suitable for adjusting each inventory
4	Design a self-consistent radiation inventory control system that controls all the identified inventories. This means: a) Inventory control in direction of flow downstream the throughput manipulator b) Inventory control in direction opposite to flow upstream the throughput manipulator
5	At junctions or splits a decision has to be made on which flow to use for inventory control. Typically, the largest flow is used, or both streams are changed such that their ratio is held constant (often the ratio is set by a slower outer composition loop).
6	Recycles require special consideration. Make a block (control volume) around the entire section and make sure that there is self-consistency for total mass, (individual) components and phases (if relevant).
7	Assign control loops for any process external flow that remain uncontrolled. Typically, "extra" feed rates are put on ratio control with the ratio set point being set by an outer composition loop.

Table 2.1: Proposed guidelines for design of self-consistent inventory control system. In case of doubt consult the general self-consistency rule (Rule 2.1).

2.5 Examples

In this section we apply the self-consistency rule to some examples from the academic literature.

2.5.1 Distillation column with DB-configuration

An example of a recycle system is a distillation column. As seen from Figure 2.10, a distillation column has one split in the condenser (V_T splits into L and D) and one split in the reboiler (L_B splits into B and V). In both cases one of the streams is recycled to the column (L and V, respectively). The two splits introduce two degrees of freedom and this gives rise to many possible inventory control structures ("configurations"), as has been discussed widely in the literature (e.g., see Skogestad (2007) for a summary of this discussion).

Figure 2.10 displays the DB-configuration, which uses reflux L and boilup V for inventory control (condenser and reboiler level control), such that the flows of D and B remain as degrees of freedom for other purposes. The DB-configuration has earlier been labeled "impossible", "unacceptable" or "infeasible" by distillation experts (e.g. Perry and Chilton 1973, p.22-123; Shinskey 1984, p.154). This inventory control system also violates Luybens rule of "fixing a flow in the recycle loop" and it is indeed true that this inventory control system is not self-consistent. To see this, consider the dashed box in Figure 2.10 where we note that none of the flows in or out of the column (F, D and B) depend on the inventory inside the column. However, an inconsistent inventory control system can usually be made consistent by adding control loops and the DB-configuration is workable (and consistent) provided one closes at least one extra loop, for example by using D to control a temperature inside the column (Finco *et al.*, 1989; Skogestad *et al.*, 1990). Thus, labeling the DB-configuration as "impossible" is wrong. In summary, the DB-configuration is not self-consistent, but it can be made consistent by adding a temperature (or composition) control loop.

Remark 1 An example of a self-consistent inventory control structure for distillation is the common LV-configuration, where the two level loops have been interchanged such that D and B are used for level control and L and V remain as degrees of freedom (e.g. on flow control). In the LV-configuration, inventory is controlled in the direction of flow, as expected since the feed is given.

Remark 2 An additional inventory issue for distillation columns is related to the split between light and heavy components (component inventory). One may regard the column as a "tank" with light component in the upper part and heavy in the lower part. Thus, one is not really free to set the split between D and B and to avoid a "drifting" composition profile (with possible "breakthrough" of light component in the bottom or of heavy component in the top), one must in practice close a quality (e.g., temperature or pressure) loop to achieve component self-consistency (Skogestad, 2007). For example, for the LV-configuration one may use the boilup V to control a temperature inside the column. This consideration about controlling the column profile also applies to the DB-configuration. Thus, in practice, the DB-configuration requires closing *two* quality loops to maintain mass and component balances. This means that both D and B must be used for quality control for the DB-configuration, rather than only one (L or V) for the LV-configuration.

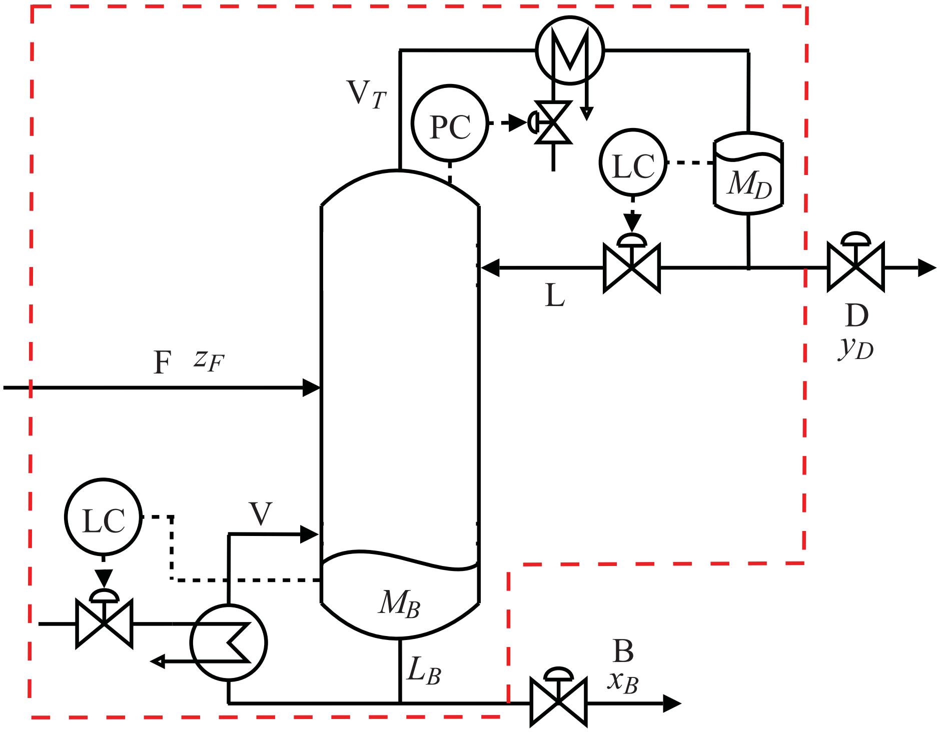


Figure 2.10: Example of inconsistent inventory control at recycle process: Distillation column with DB-configuration.

2.5.2 Reactor-separator-recycle example with one reactant

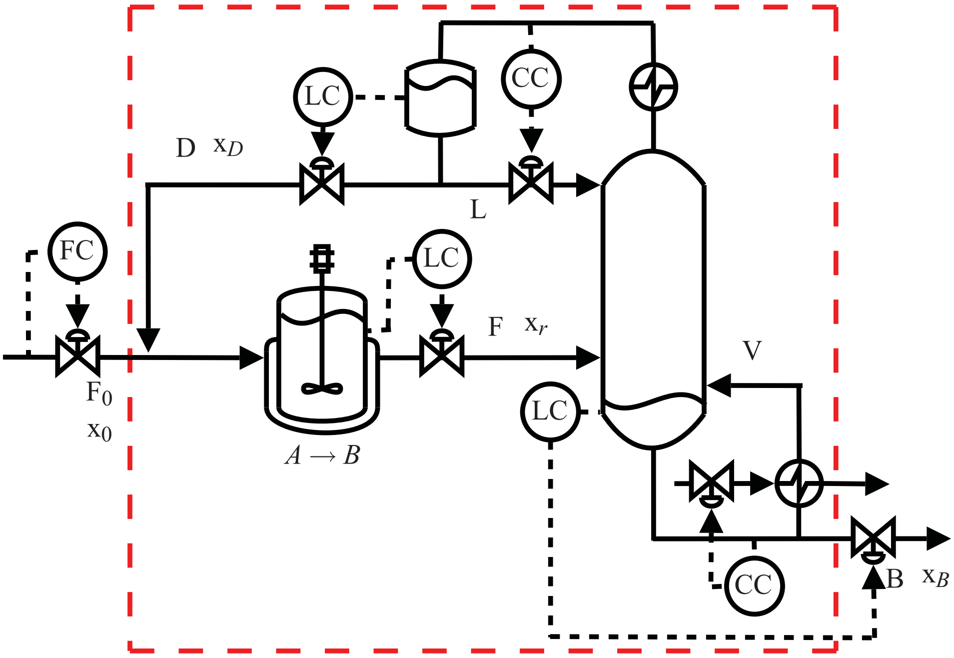
A common recycle example from the academic literature is the reactor-separator-recycle system in Figure 2.11. The system has a continuous stirred-tank reactor (CSTR) with an irreversible, isothermal, first order reaction $A \rightarrow B$, followed by separation (distillation) and recycle of the unreacted feed component back to the reactor (e.g. Luyben 1993*a*,*b*; Price and Georgakis 1993; Larsson *et al.* 2003).

The feed (F_0) is pure reactant A and the component mass balances become

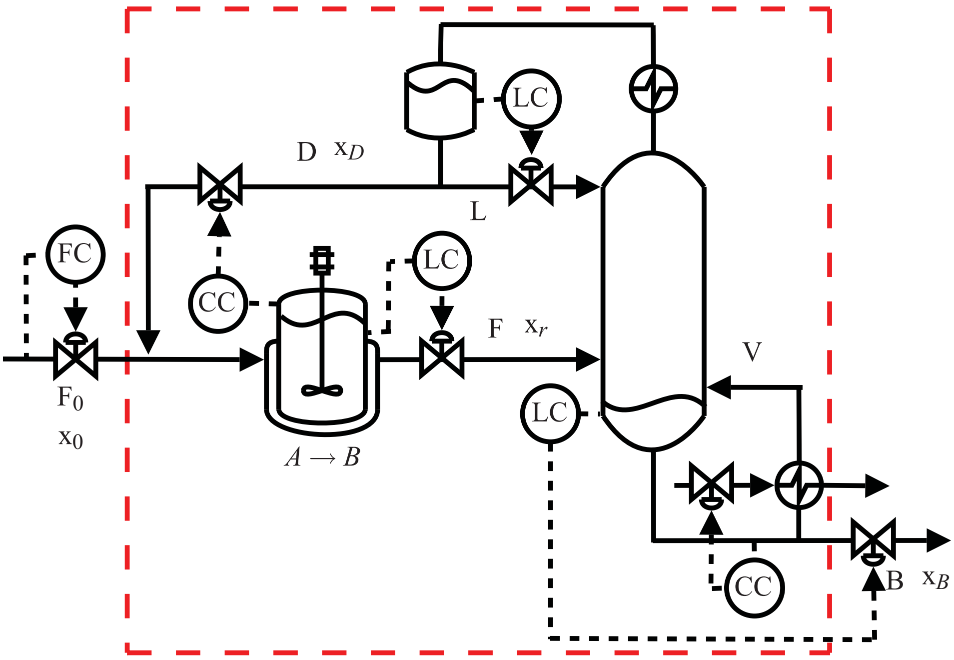
$$\text{Component A:} \quad F_0 = \underbrace{k(T) \cdot x_{r,A} \cdot V}_{-G_A = G_B} + B \cdot x_{B,A}$$

$$\text{Component B:} \quad \underbrace{k(T) \cdot x_{r,A} \cdot V}_{G_B} = B \cdot x_{B,B}$$

where x is the mole fraction, V is the reactor volume and $k(T)$ is the reaction rate constant. Note that $B = F_0$ [mol/s] at steady-state. Component A enters the process in the feed stream and its consumption in the reactor increases with the amount of A. The inventory of component A is therefore expected to be self-regulated by the reaction. Component B is produced in the reactor (G_B) and exits the process in stream B. Component B is not self-regulated by the reaction (because the reaction



(a) Conventional: Self-consistent inventory control structure.



(b) Composition control of reactor composition: Not consistent for component A.

Figure 2.11: Reactor-separator-recycle process with one reactant (A).

rate is independent of the amount of B) and thus requires a controller to adjust its inventory.

Two different control structures for the reactor-separator-recycle process are displayed in Figure 2.11. Both have given feed (F_0) and inventory control is the direction of flow. Thus, both of them are self-consistent in total mass, because the outflow B from the process depends on the inventory inside the process (indicated by the dashed control volume) (Rule 2.1). Since the outflow B mainly consists of component B, this implies that both structures are also consistent (self-regulated) with respect to the inventory of component B. The difference between the two structures is related to the control of component A. The "conventional" structure in Figure 2.11(a) uses the LV-configuration for the distillation column where the reflux (L) controls the composition in the recycle (distillate) D. The structure in Figure 2.11(b) uses the DV-configuration for the column where the reactor composition $x_{r,A}$ is controlled instead of the recycle (distillate) composition.

As already mentioned, the inventory of component A is expected to be self-regulated by the reaction $A \rightarrow B$, so one would expect both structures to be consistent with respect to component A. In fact, both structures would be consistent if one *removed* the composition loop in the recycle loop (thus, setting reflux L in Figure 2.11(a) and setting recycle D in Figure 2.11(b)). With the composition loop closed, the "conventional" structure in Figure 2.11(a) remains consistent, but not the structure with control of reactor composition in Figure 2.11(b). The reason for the inconsistency is that control of reactor composition eliminated the self-regulation by reaction: The amount of A that reacts is given by $-G_A = G_B = k(T)x_{r,A}V$ and with given $x_{r,A}$ (because of the controller), T and V there is no self-regulation. The inconsistency of this control structure is pointed out by e.g. Downs (1992) and Luyben (1994).

Remark 1 The control structures in Figure 2.11 would both be self-consistent without closing the composition loop (CC) in the recycle part of the process, that is, with (a) L given or (b) D given. The reason for closing these composition loops is therefore not for consistent inventory control but rather for other (economic) reasons (Larsson *et al.*, 2003). The interesting point to note, is that cosing an extra loop can in some cases make the system inconsistent (Figure 2.11(b)).

Remark 2 Luyben (1994) has proposed to make the system in Figure 2.11(b) consistent by introducing an adjustable reactor volume, but this is not a good solution, because we always want to use the maximum reactor volume for economic reasons (energy saving) (Larsson *et al.*, 2003).

Remark 3 The inventory of component A is expected to be self-regulated by the reaction $A \rightarrow B$. More precisely, the amount that reacts is $-G_A = kx_{r,A}V$ and the composition $x_{r,A}$ will "self-regulate" such that at steady-state $F_0 \approx -G_A$, that is, $x_{r,A} \approx F_0/(kV)$.

Remark 4 We already noted that setting $x_{r,A}$ (Figure 2.11(b)) breaks this self-regulation and makes the system inconsistent. A related problem is when the reactor volume V is too small relative to the feed F_0, such that the required $x_{r,A}$ exceeds 1, which is impossible. In practice, if we increase the feed rate F_0 and approach this situation, we will experience "snow-balling" (Luyben, 1993*c*) where the recycle D becomes very large, and also the boilup V becomes very large. Eventually, V may reach its maximum value, and we loose composition control and we will get "break-through" of A in the bottom product. Snow-balling is therefore a result of a too small reactor.

Remark 5 Consider the same process (Figure 2.11), but assume that the fresh feed (F_0) contains an inert component I in addition to the reactant A. If I is more volatile than component B, then component I will be recycled back to the reactor and will accumulate in the process. None of the inventory control systems in Figure 2.11 are consistent for the inert I.To make the system self-consistent for the inert, a purge stream must be introduced where part of stream D is taken out as a by-product.

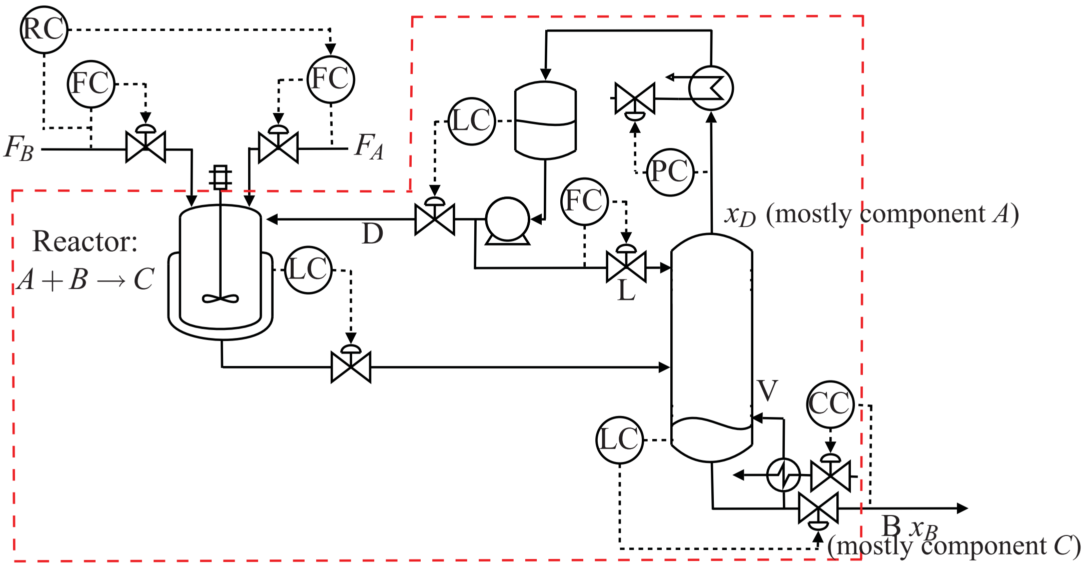
2.5.3 Reactor-separator-recycle process with two reactants

Another well studied recycle example is a reactor-separator-recycle process where two reactants A and B reacts according to the reaction $A+B \rightarrow C$ (e.g. Tyreus and Luyben, 1993). Component B is the limiting reactant as the recycle D contains mostly component A. Two different control structures are displayed in Figure 2.12. In both cases the distillate flow D (recycle of A) is used to control the condenser level (main inventory of A).

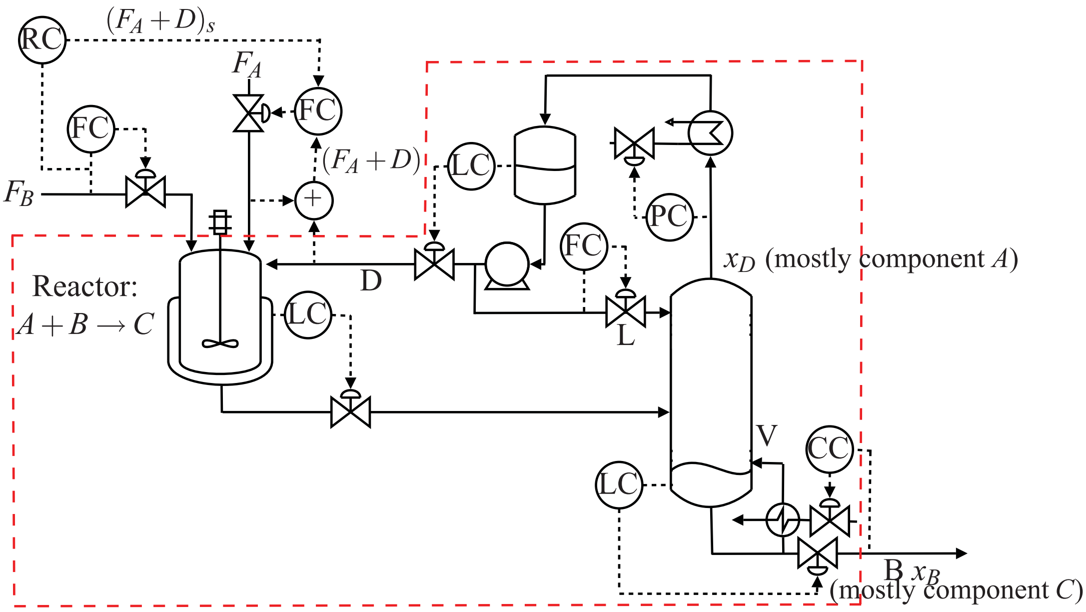
In Figure 2.12(a), both fresh reactant feeds (F_A and F_B) are flow controlled into the reactor, where reactant A is set in ratio to reactant B such that $F_A/F_B = 1$. This control strategy is not consistent because the two feeds is not independent and one of them needs to be dependent of the inventory inside since it is not possible to feed exactly the stoichiometric ratio of the two reactants (Luyben *et al.*, 1998, p.37). Any imbalance will over time lead to a situation where the recycle of A either goes towards zero or towards infinity.

To get a consistent inventory control structure, the first requirement is that one of the feed rates (F_A or F_B) must be dependent on what happens inside the process, such that we at steady-state can achieve $F_A = F_B$. One solution is to set F_B (the limiting reactant) and adjust F_A such that the desired excess of A is achieved, resulting in the self-consistent control structure in Figure 2.12(b). Here F_A depends on the inventory of A as reflected by the recycle flow D by keeping the reactor feed ratio $(F_A+D)/F_B$ constant at a given value (larger than 1 to make B the limiting reactant). The structure is consistent for all components: C has an outlet in the bottom of the column; B is self-regulated by reaction because it it the limiting reactant, and the feed of A depends on the inventory of A.

There exist also other consistent inventory control structures, e.g. see Luyben



(a) Inconsistent structure with both reactant flows given



(b) Self-consistent structure where feed of reactant A depends on inventory of A (as reflected by D)

Figure 2.12: Reactor-recycle system with two reactants ($A+B$).

et al. (1998, Figure 2.11(b)), but these seem to be more complicated than the one proposed in Figure 2.12(b). For example, one could keep the recycle D constant and use F_A to control the condenser level (main inventory of A), but the dynamics for this "long level" loop are not favorable and this consistent structure is not self-consistent.

2.6 Conclusion

Consistency is a required property since the mass balances must be satisfied for the individual units and the overall plant. An inventory control system can be evaluated whether it is self-consistent (local "self-regulation" of all inventories) by using the self-consistency rule (Rule 2.1). The self-consistency rule follows from the mass balance that must be satisfied for the total mass, component and individual phases.

A direct consequence of the self-consistency rule is the "radiation rule" (Price and Georgakis, 1993), which states that the inventory control structure must be radiating around the location of a given flow. Other useful rules that can be developed from the self-consistency rule, is that all system must have at least one given flow (throughput manipulator). Thus, for closed systems, one inventory (preferable the largest) must be left uncontrolled.

Luyben provides the rule to "fix a flow in each recycle". If we interpret the term "fix a flow" to mean "do not use a flow for inventory control", then this rule follows from the requirement of self-consistency provided the recycle loop contains a split that introduced an extra degree of freedom (see Section 2.4.2). If no degree of freedom is introduced by the recycle, as is in the case if we have a separator or flash where the split is (indirectly) given by the feed properties, then this rule is not a requirement, e.g. see Figure 2.11(a), where all the flows in the recycle loop are on inventory control.

Chapter 3

Throughput maximization requires tight bottleneck control

Based on paper presented at
8th International Symposium on Dynamics and Control of Process Systems (DYCOPS) 2007, June 6-8, Cancun, Mexico

With sufficiently high product prices and the feed is available, it is shown that maximum throughput is an optimal economic operation policy. This paper discusses the maximum throughput case, which is characterized by the existence of a *bottleneck* and the need for *back off* from active constraints to ensure feasibility. To implement maximum throughput, maximum flow in the bottleneck(s) must be realized. Obtaining tight bottleneck control in practice requires that the throughput manipulator is located close to the bottleneck (short effective delay). If the throughput manipulator is located close enough compared to the disturbance time constant, automatic control can reduce the back off significantly. Poor control of the bottleneck, including any deviation or back off, implies a reduction in throughput and an unrecoverable economic loss.

3.1 Introduction

In general, real-time optimization (RTO) based on a detailed process model may be used to find the optimal operation conditions of a plant, including identifying the optimal active constraints and computing the optimal set point for the unconstrained variables. However, in many cases, prices and market conditions are such that optimal operation is the same as maximizing plant throughput. Hence, the problem formulation can be simplified, and RTO based on a detailed nonlinear process model is not needed.

Maximum throughput in a network is a common problem in several settings (e.g. Phillips *et al.*, 1976; Ahuja *et al.*, 1993). From network theory, the *max-flow min-cut* theorem states that the maximum throughput in a plant (network) is limited by the ”bottleneck” of the network. In order to maximize the throughput, the flow through the bottleneck should be at its maximum flow. In particular, if the actual flow at the bottleneck is not at its maximum at any given time, then this gives a loss in production which can never be recovered (sometimes referred to as a ”lost opportunity”).

To implement maximum throughput there are three important issues: 1) locate the bottleneck unit(s), 2) implement maximum throughput in the bottleneck unit and 3) minimize the back off from active constraints in the bottleneck unit. To locate the bottleneck in the first place, there are several opportunities. The most common is simply to increase the flow rate during operation (online) until feasible operation is no longer possible. Alternatively, the location can be estimated using a commercial flowsheet simulator or plant data. Litzen and Bravo (1999) discuss how to estimate the capacity for process units and find the bottleneck(s) for debottlenecking (design) purposes (steady-state). A third approach is to use the models that are implemented in the model predictive controllers (MPC) to estimate the available capacity for each unit on-line (dynamically) (Aske *et al.*, 2008).

Maximizing throughput requires manipulation of the *throughput manipulator* (TPM). This is usually the feed rate (Price *et al.*, 1994), but it can more generally be anywhere in the plant. Usually the location of the TPM is determined by the original design of the control system for the plant, and cannot be easily changed because it requires reconfiguration of the inventory loops to ensure a self-consistent inventory control system (Chapter 2). If one is free to place the TPM(s), then two considerations may come into account. First, one must consider its effect on the inventory control structure, including propagation of disturbances, dynamic lags, process time constants and interactions (Luyben, 1999). A second consideration, which is based on economics, is to locate the TPM such that tight control of the bottleneck unit is possible. Skogestad (2004) propose to set the production rate at the bottleneck.

Price and coauthors (Price and Georgakis, 1993; Price *et al.*, 1994) propose a plantwide design structure using a tiered framework with throughput, inventory and product quality controls. They discuss the importance of proper selection of the TPM and their general recommendation is to select an internal process flow as the TPM because: 1) ”*they impede the propagation of disturbances through the system*” and 2) “*internal flows have a substantial chance of more rapidly affecting a throughput change*”. On the other hand, Cheng *et al.* (2002) claim the opposite; the TPM should be a feed or product flow, and internal flows should be avoided from a dynamic interaction point of view. Price *et al.* (1994) also mentioned on

TPM location that "*some plants have a single processing unit which is markedly more difficult to control than the others. Selecting a flow very close to that unit as the throughput manipulator will help minimize or control the variation affecting the unit and so should make it easier to control*." Moore and Percell (1995) evaluated control alternatives by simulation on a three-unit module and concluded that *"the plant is capable of the highest production rate with the widest variation in feed composition when the production rate is set at the column feed, which is immediately before the process bottleneck"*. However, there are no attempts trying to explain the results from the simulation study. Luyben *et al.* (1997) propose a heuristic design procedure for plantwide control. In the procedure, the authors recommend locating the TPM so it provides a smooth and stable production rate transitions and reject disturbances. However, all these approaches lack an economic evaluation of the TPM selection; whereas Larsson and Skogestad (2000) point out that the economics is a key factor for the placement of the TPM. They suggest that for a plant running at maximum capacity, the production rate should be set at the bottleneck, which is usually inside the plant.

From a literature search and based on our own industrial experience, it seems like the feed valves (or more general the throughput manipulator) is very rarely used in practice for closed-loop control, in spite of its great importance on the plant economics in cases where maximum throughput is optimal. The reason is probably the large effect the feed rate has on the operation of the entire plant, but the result may be a loss in economic performance. The main goal of this paper is to discuss the importance of using the throughput (often the feed rate) for closed-loop control.

When operating at maximum throughput, the plant is at the limit to infeasibility. For this reason, a "safety factor" or "back off" is required to achieve feasible operation under presence of disturbances, uncertainties, measurement error and other sources for imperfect control (Narraway and Perkins, 1993; Govatsmark and Skogestad, 2005). More precisely, the back off is the distance between the active constraint and the actual average value (set point). The necessary back off can generally be reduced by improving the control of the bottleneck unit, for example, by retuning the control system to reduce the dynamic variation. The idea is that improved control requires a smaller back off or, in short, "squeeze and shift" (squeeze the variance - and shift the set point closer to the constraints) (e.g. Richalet *et al.*, 1978; Richalet, 2007).

This paper addresses the maximized throughput case, and starts by considering the case under which considerations this is optimal (Section 3.2). In Section 3.3, back off is defined and reasons for why back off is needed together with its influence on the economics is discussed. The location of the throughput manipulator is discussed in Section 3.4, whereas in Section 3.5 the characteristics of maximum throughput are treated. By using controllability analysis, an estimate of minimum back off is given in Section 3.6 with a more detail description is given in Appendix 3.A. In Section 3.7 we discuss actions to reduce back off, followed by a discussion in Section 3.8 before we conclude in Section 3.9.

3.2 Optimal operation (steady-state)

In this section, we discuss under which considerations, maximum throughput is economically optimal.

3.2.1 Modes of optimal operation

Mathematically, steady-state optimal operation is to minimize the cost J (or maximize the profit -J), subject to satisfying given specifications and model equations ($f = 0$) and given operational constraints ($g \leq 0$):

$$\begin{aligned} &\min_u J(x,u,d) \\ \text{s. t. } &f(x,u,d) = 0 \\ &g(x,u,d) \leq 0 \end{aligned} \tag{3.1}$$

Here are u the degrees of freedom (manipulated variables including the feed rates F_i), d the disturbances and x the (dependent) state variables.

A typical profit function is

$$-J = \sum_j p_{P_j} \cdot Pj - \sum_i p_{F_i} \cdot F_i - \sum_k p_{Q_k} \cdot Q_k \tag{3.2}$$

where P_j are product flows, F_i the feed flows, Q_k are utility duties (heating, cooling, power), and p (with subscript) denote the prices of the corresponding flow and utility. Let F be a measure of the throughput in the plant. Depending on market conditions, a process has two main modes in terms of optimal operation:

Mode 1. *Given throughput (F given). The economic optimum is then usually the same as optimal efficiency, that is, to minimize utility (energy) consumption for the given throughput.*

This mode of operation typically occurs when the feed rate is given (or limited) or the product rate is given (or limited, for example, by market conditions), and the optimization problem (3.1) is modified by adding a set of constraints on the feed rate, $F_i = F_{i0}$.

Mode 2. *Feed is available and the throughput F is a degree of freedom. We here have two cases:*

(a) **Maximum throughput**. This mode of operation, which is the main focus of this paper, occurs when product prizes are sufficiently high and feed is available. We then have that the cost can be written $J = \text{-}pF$ where $p > 0$ (see (3.6) below). Optimal economic operation then corresponds to *maximizing* the throughput F, subject to achieving feasible operation and this does not depend on cost data. The optimum is *constrained* with respect to the throughput, and we have $dJ/dF_i < 0$ where the feed rates F_i are degrees of freedom.

(b) **Optimized throughput**. This mode of operation occurs when feed is available, but it is not optimal to go all the way to maximum throughput because the efficiency drops as the throughput increases. For example, increased throughput may be possible by increasing the purge rate, but this result in less efficient operation because of loss of valuable components. The optimum is *unconstrained* with respect to the feed rates F_i and we have $dJ/dF_i = 0$. Thus, increasing F_i above its optimal value is feasible, but gives a higher cost J.

3.2.2 Maximum throughput (Mode 2a)

We here want to show that when product prices are high compared to feed and utility costs, optimal operation of the plant is the same as maximizing throughput (Mode 2a). Let F be a measure of the throughput in the plant, and assume that all feed flows are set in proportion to F,

$$F_i = k_{F,i}F \tag{3.3}$$

Then, under the assumption of constant efficiency in all units (independent of throughput) and assuming that all intensive (property) variables are constant, all extensive variables (flows and heat duties) in the plant will scale with the throughput F (e.g. Skogestad, 1991). In particular, we have that

$$P_j = k_{P,j}F \qquad Q_k = k_{Q,k}F \tag{3.4}$$

where the gains $k_{P,j}$ and $k_{Q,k}$ are constants. Note from (3.4) that the gains may be obtained from nominal (denoted 0) mass balance data:

$$k_{P,j} = P_{j0}/F_0 \qquad k_{F,i} = F_{i0}/F_0 \qquad k_{Q,k} = Q_{k0}/F_0 \tag{3.5}$$

Substituting (3.3) and (3.4) into (3.2) gives

$$(\text{-}J) = \left(\sum_j p_{P_j} \cdot k_{P,j} - \sum_i p_{F_i} \cdot k_{F,i} - \sum_k p_{Q_k} \cdot k_{Q,k}\right) F = pF \tag{3.6}$$

where p is the operational profit per unit of feed F processed. From the above derivation, p is a constant for the case with constant efficiencies. We assume $p > 0$ such that we have a meaningful case where the products are worth more than the feed stocks and utilities. Then, from (3.6) it is clear that maximizing the profit ($-J$) is equivalent to maximizing the (plant) throughput F. However, F cannot go to infinity, because the operational constraints ($g \leq 0$) related to achieving feasible operation (indirectly) impose a maximum value for F.

In practice, the gains $k_{P,j}$, $k_{F,i}$ and $k_{Q,k}$ are not constant, because the efficiency of the plant changes. Usually, operation becomes less efficient and p in (3.6) decreases when F increases. Nevertheless, as long as p remains positive, we have that $d(-J)/dF = p > 0$ is nonzero, and we have a constrained optimum with respect to the throughput F. From (3.6) we see that p will remain positive if the product prices $p_{P,j}$ are sufficiently high compared to the prices of feeds and utilities.

If the efficiency drops, for example because $k_{Q,k}$ increases and $k_{P,j}$ decreases when the feed rate is increased, then p in (3.6) may become negative. Then there is no bottleneck and Mode 2b (optimized throughput) is optimal. This mode of operation is common for recycle systems. For example, this applies to the ammonia synthesis problem (Araújo and Skogestad, 2008).

3.3 Back off

Back off is a general concept that applies to operation close to any "hard" output constraint (not only to bottleneck operation). In this section we present a general discussion of back off.

Arkun and Stephanopoulos (1980) discussed moving away from the nominal optimal operation point to ensure feasible operation when there are disturbances. Narraway and Perkins (1993) discussed this in more detail and introduced the term "back off" to describe the distance from the active constraint that is required to accommodate the effects of disturbances.

3.3.1 Definition of back off

We use the following definition of back off (also see Figure 3.1):

Definition 3.1. ***Back off***. *The (chosen) back off is the distance between the (optimal) active constraint value ($y_{constraint}$) and its set point (y_s) (actual steady-state operation point),*

$$Back\ off = b = |y_{constraint} - y_s|, \tag{3.7}$$

which is needed to obtain feasible operation in spite of:

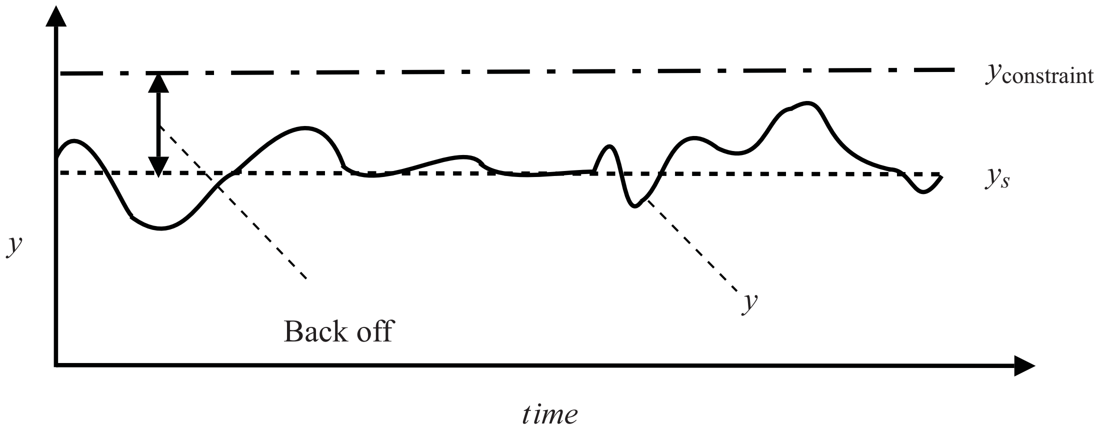


Figure 3.1: Illustration of back off, $b = |y_{\text{constraint}} - y_s|$

1. *Dynamic variations in the variable y caused by imperfect control (due to disturbances, model errors, effective delays and other sources of imperfect control).*

2. *Measurement errors.*

Remark 1 Here we assume integral action, such that on average $y_s = \bar{y}$ where

$$\bar{y} = \lim_{T\to\infty} \frac{1}{T} \int_0^T y(t)dt$$

In this case, only the steady-state measurement error (bias) is of importance, and not its dynamic variation (noise).

Remark 2 Back off was defined by Govatsmark and Skogestad (2005, eq. 20) as the difference between the actual set points and some reference values for the set points:

$$b = c_s - c_{s,ref}$$

where c_s is the actual set point and $c_{s,ref}$ is some reference value for the set point which depends on the method for set point computation (e.g. nominal, robust, on-line feasibility correction). Definition 3.1 coincides with their definition.

3.3.2 Required back off

Back off is needed to avoid constraints violation, and the required back off b depends on whether the active constrained variable y is an input or an output.

Output constraints

Generally, back off is *always* required for output constraints. Let us first distinguish between two constraint types:

- *Hard constraint:* Constraint cannot be violated at any time.
- *Steady-state (average) constraint:* Constraint must be satisfied at steady-state average, but dynamic violation is acceptable.

Safety constraints, like pressure and temperature limitations, are usually hard constraints. An example of a steady-state constraint is the composition of the overhead product from a distillation column which goes to a storage tank where mixing takes place. Another example may be emissions from a plant which often are in terms of hourly or daily averages.

For a steady-state (average) constraint, integral action is sufficient to ensure that $\bar{y} = y_{\text{constraint}} = y_s$ (on average) and no back off is required for dynamic variations caused by imperfect control. However, back off is required to account for possible steady-state measurement errors (bias).

In summary, we have:

- **Hard output constraints:** Required back off is sum of expected dynamic variation and steady-state measurement error (bias).
- **Steady-state (average) output constraint:** Required back off is equal to the steady-state measurement error (bias).

Note that there in addition may be maximum limits (hard constraints) on the allowed dynamic variation even for steady-state (average) constraints.

If no constraint violation is allowed, then the worst-case variation gives the required back off b together with the measurement error. However, in many cases a small constraint violation for a short-time is acceptable and therefore the worst-case variation may be too strict to determine the required back off. In practice, for stochastic signals, one needs to specify an acceptable likelihood for constraint violations. For example, the likelihood is 99.7% that the signal variation remains within ± 3 times standard deviation (σ), or 95% that the variations are within $\pm 2\sigma$ (for normal distribution). In this paper, we consider the worst-case variation and do not include probability for constraint violation.

Input constraints

Inputs have no associated control error. However, for cases where the input constraint does not correspond to a physical (hard) constraint, we must introduce back off to guard against steady-state measurements errors. For example, there may be a constraint on the allowed flow that goes to the effluent.

For *hard* input constraint, there is normally no need to introduce back off, because we may simply set the input at its constrained value (it cannot be violated even if we want to). There is one exception and this is when the input variable is optimally saturated and is used for (dynamic) control. For example, the cooling rate to a reactor, which optimally should be at maximum, may be needed to stabilize the reactor if the desired operating point is unstable. In other cases, the input may be needed for dynamic control to obtain tight control of an important output variable.

In summary, we have:

- **Hard input constraint:** No back off is normally required.
- **Steady-state (average) input constraint:** Required back off is equal to the steady-state measurement error (bias).

3.3.3 Reducing effect of back off on economics

Any back off from an active constraint will results in an economic loss and should be as small as possible. There are in principle two ways of reducing the economic penalty caused by back off:

1. "Squeeze and shift" (e.g. Richalet, 2007): By improved control one can reduce ("squeeze") the variation and "shift" the set point towards the constraint to reduce back off. Also improved measurements that reduces the measurements variation will reduce the required back off.
2. "Move variation to variables where the economic loss is small": In many cases one can reconfigure the control system (single-loop control) or change the control weights (multivariable control) to obtain tighter control of economically important variables. In practice, this means:
   (a) Move variation to variables without hard constraints
   (b) Move variation to variables where a back off has a small economic effect. For example, this may be quantified by the Lagrange multiplier (shadow prices) (e.g. Edgar *et al.*, 2001).

Mathematically, for a constrained optimization problem, the economic loss caused by back off from an active constraint is represented by the Lagrange multiplier λ

$$\text{Loss} = \frac{\partial(\text{-}J^*)}{\partial c} \cdot \Delta c = \lambda \cdot b \tag{3.8}$$

where -J^* is the optimal value of the profit, c is the active constraint variable with back off $b = \Delta c$, and λ is the Lagrange multiplier.

At the end, selecting the back off is a trade-off between the improved profit resulting from a small back off and the cost of reducing the back off (e.g. by improved measurements or improved control).

3.4 Throughput manipulator

In this section, we discuss and define the term throughput manipulator. The structure of the inventory control system depends mainly on where in the process the *throughput manipulator*, see Figure 3.2 (Buckley, 1964; Price and Georgakis, 1993):

1. **Feed as TPM (given feed)**: inventory control system in the direction of flow (conventional approach).

2. **Product as TPM (”on-demand”)**: inventory control system opposite to flow.

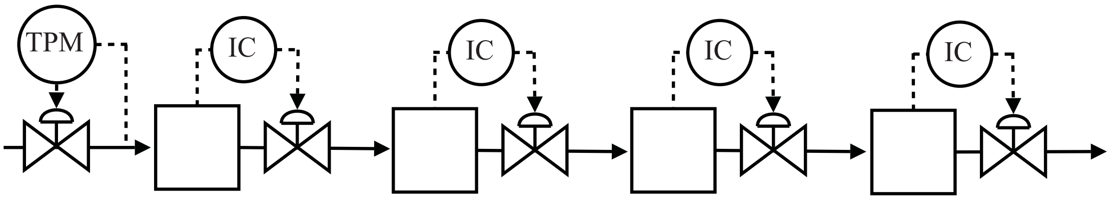
3. **TPM inside plant (general case)**: radiating inventory control.

These rules follow from the requirement of a self-consistent inventory control system, as discussed in detail in Chapter 2.

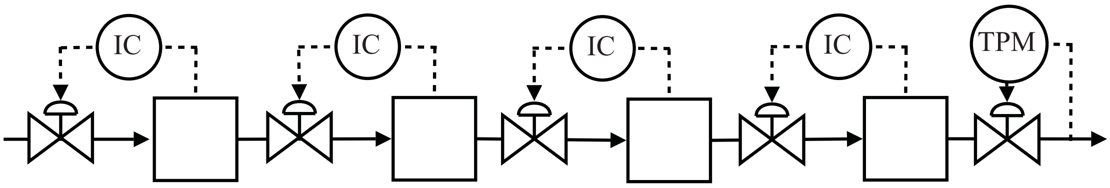
In terms of location of the TPM, Scheme 1 (Figure 3.2(a)) is the natural choice for Mode 1 with given feed rate, Scheme 2 (Figure 3.2(b)) is the natural choice for Mode 1 with given product rate, whereas Scheme 3 (Figure 3.2(c)) is usually the best choice for Modes 2a and 2b (feed rate is degree of freedom) where the optimal throughput is determined by some conditions internally in the plant.

In the above discussion, we have used the term “throughput manipulator” (TPM) without defining it. The term was introduced by Price and Georgakis (1993), but they did not give a clear definition. From the discussions of Price and coauthors (Price and Georgakis, 1993; Price *et al.*, 1994) on throughput manipulator, it is implicitly understood that a plant has only one throughput manipulator, which is related to the main feed stream. This is reasonable in most cases, because if a plant has several feeds, then these are usually set in proportion to each other, for example, based on the reaction stoichiometric. This was also used in (3.3) and (3.4), were we assumed that all flows and utilities are set in proportion to the throughput F.

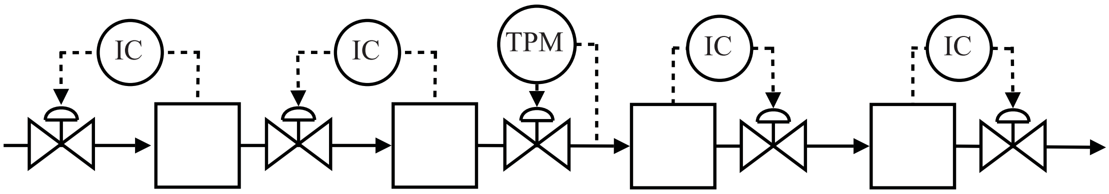
However, there are cases that are not quite as simple. First, some plants may have several similar or alternative feeds that do not need to be set in proportion to each other. Thus, fixing one feed rate does not indirectly determine the value of



(a) Scheme 1: Throughput manipulator at feed, inventory control in the direction of flow



(b) Scheme 2: Throughput manipulator at product, inventory control in the direction opposite to flow



(c) Scheme 3: Throughput manipulator inside plant, radiating inventory control

Figure 3.2: Basic schemes for inventory control. IC stands for inventory control and are typically a level controller (liquid) or a pressure controller (gas).

the others. Second, plants with parallel trains must have at least one TPM for each train. There may also be parallel trains inside the process, and the corresponding split may be viewed as a throughput manipulator. In addition, plants with parallel trains may have crossover flows, which also affect the throughput and may be viewed as throughput manipulators. To account for this, we propose the following general definition:

Definition 3.2. ***Throughput manipulator (TPM).*** *A throughput manipulator is a degree of freedom that affects the network flows (normally including feed and product flows), and which is not indirectly determined by other process requirements.*

Thus, a TPM is an “extra” degree of freedom, which is not needed for the control of individual units, but that can be used to set or optimize the network flows. Splits and crossovers can be viewed as throughput manipulators but they do not necessarily affect both the feed and the product flows. For example, if there is a split and the parallel processes are combined further downstream, the split factor will affect neither the feed nor the product flow. In Definition 3.2, “other process requirements” are often related to satisfying the component material balances, as discussed in the following examples.

Example 3.1. *Consider a process with two feeds, F_A of pure component A and F_B of pure component B, where the reaction $A+B \rightarrow P$ (product) takes place. Normally, in order to avoid losses, the feeds should be stoichiometric. Thus, we need $F_A = F_B$ at steady-state, which indirectly removes one degree of freedom, so the process has only one TPM.*

Example 3.2. *Consider the same process as in Example 3.1 with three feeds F_A, F_B and F_{AB}, where the latter consist of a mixture of A and B. The stoichiometry imposes one constraint, but otherwise the optimal ratio between these feeds is determined by plantwide economic arguments, and not by process requirements. Thus, according to Definition 3.2, this process has two TPMs. For example, the TPMs could be F_A and F_{AB}, with F_B adjusted to satisfy the stoichiometry.*

Example 3.3. *Consider a process with two feeds, F_A with pure component A and F_{AI} with A plus some inert I. The reaction $A \rightarrow P$ (product) takes place. This process has two TPMs because the (optimal) amount of the two feeds is determined by plantwide consideration.*

Example 3.4. *Consider a process with two feeds; F_A contains pure A and F_B contains pure B. The reactions $A \rightarrow P+X$ and $B \rightarrow P+Y$ take place, where P is the main product, and X and Y are byproducts. This process has two TPMs, because the ratio F_A/F_B is not given by other process requirements.*

In summary, we see from these examples that even quite simple processes can have more than one TPM. In addition to these examples, we have the more obvious cases of multiple TPMs, such as a process with parallel trains and crossovers.

3.5 Characteristics of the maximum throughput case

We have shown that maximum throughput is often the economically optimal mode of operation. In this section, we want to identify the main characteristics of the maximum throughput case.

3.5.1 Bottleneck

The *max-flow min-cut* theorem (Ford and Fulkerson, 1962, p.11) from linear network theory states that: “for any network the maximal flow value from source to sink is equal to the minimal cut capacity of all cuts separating source and sink”. In simple terms, the theorem states that the maximum flow in a network is dictated by its bottleneck. To study bottlenecks in more detail, we need to define some terms.

Definition 3.3. ***Maximum flow (capacity) of a unit***. *The maximum flow (capacity) of a unit is the maximum feed rate the unit can accept subject to achieving feasible operation.*

Mathematically, this corresponds to solving the maximum flow problem (3.1) with $(\text{-}J) = F_{\max,i}$, where $F_{\max,i}$ is the maximum feed for the unit i and u_i are the degrees of freedom for unit i. This means to find the maximum value of $F_{\max,i}$ that satisfies the constraints $f_i = 0$ and $g_i \leq 0$ for the unit.

Definition 3.4. ***Maximum throughput of a plant***. *Let the throughput F be the (weighted) sum of all the feed flows. The maximum throughput $F_{\max}$ of a plant is the maximum network flow that a plant accept subject to achieving feasible operation.*

In the optimization problem, implied by Definition 3.4, all degrees of freedom (all F_i's) should be used to maximize the throughput, subject to achieving feasible operation (satisfying the constraints).

Definition 3.5. ***Bottleneck***. *A unit is a bottleneck if maximum throughput (maximum network flow for the system) is obtained by operating this unit at maximum flow (see Definition 3.3).*

Definition 3.6. ***Bottleneck constraints***. *The active constraints in the bottleneck unit are called the bottleneck constraints.*

The term ”unit” in Definitions 3.5 and 3.6 needs some attention. For a simple process, where the process units are in series, a ”unit” is the same as a single process unit. However, for integrated processes, one may need to consider a combined system of integrated units as a ”unit”. For example, for a chemical reactor with recycle, the combined ”unit” may be the system of units consisting of the reactor, separator and recycle unit (e.g. compressor or pump). This is because the maximum flow to the combined system is not necessarily determined by the maximum flow in an individual unit. For example, if the chemical reactor is too small such that the conversion is too small (and thus in practice is a bottleneck); then this will result in increased recycle of unconverted reactant (also known as the “snowball effect”), which eventually will overload the separator, the compressor or pump. Thus, it will appear that one of these units is the bottleneck, whereas it is really the entire reactor system, and the reactor in particular, which is the problem in terms of capacity.

In Definition 3.5, note that if a flow inside a unit is at its maximum, this does not necessarily mean that the unit is a bottleneck. The unit is only a bottleneck if it operates at maximum feed rate according to Definition 3.3. For example, the heat flow in a distillation column (the unit) may optimally be at its maximum, because overpurification of the “cheap” product is optimal in order to recover more of the valuable product. This does not mean that the column is a bottleneck, because it is possible, by reducing the overpurification, to increase the feed rate to the column. Only when *all* degrees of freedom are used to satisfy active constraints, do we have a bottleneck.

Note that in Definition 3.6, the active constraints in a bottleneck unit do not need to be flows or even extensive variables. For example, for the distillation column just mentioned, as the feed rate is increased, one will eventually reach the purity constraint on the ”cheap” product, and if there are no remaining unconstrained degrees of freedom, the distillation column becomes the bottleneck unit. The active purity constraints on the products together with the maximum heat flow constraint then comprise the “bottleneck constraints”.

3.5.2 Back off

Back off is generally required to guarantee feasibility when operating at active constraints (except for hard input constraints), as discussed in Section 3.3. We here discuss the implication of this. As we reach the bottleneck (and encounter a new active constraint), the throughput manipulator (e.g. feed rate) is the only remaining unconstrained input. To operate at the bottleneck, the throughput manipulator must be used as a degree of freedom to control this new active constraint. Based on the discussion in Section 3.3, we have the following cases:

1. The new bottleneck constraint is an output variable. The result in terms of control is “obvious”: the TPM controls this output at the active constraint (with back off included).

2. The new bottleneck constraint is an input constraint. Here we have two cases:

    (a) The input variable is not used for control. Then the input is simply set at its constraint (no back off for hard input constraints).

    (b) The input variable is already used for control of a constrained output variable. There are two possibilities, depending on which back off is most costly:

        i. The TPM takes over the lost task. However, we usually have to increase the back off on this output, because of poorer dynamic control, since the TPM is generally located farther away from the output constraint than the saturated input.

        ii. Alternatively, we can let the original loop be unchanged, but we must then introduce an additional a back off on the input to encounter for dynamic variations. The TPM is then used to keep the input in desired operation range.

3.5.3 Summary of characteristics of maximum throughput case

From the discussion above we derive the following useful insights (rules) for the TPM in the maximum throughput case (Mode 2a):

Rule 3.1. *All plants have at least one throughput manipulator and at maximum throughput the network must have at least one bottleneck unit.*

Rule 3.2. *Additional independent feeds and flows splits may give additional TPMs (see Definition 3.2) and additional bottlenecks. The idea of "minimal cut" from network theory may be used to identify the location of the corresponding bottleneck units.*

Further, for tight control of the bottleneck unit and to minimize loss the following insights (rules) are stated for the maximum throughput case:

Rule 3.3. *The throughput manipulator(s) (TPM) is the steady-state degree of freedom for control of the bottleneck unit(s). Typically, the TPM is used to control one of the bottleneck constraints (Definition 3.6). The TPM should therefore be located so that controllability of the bottleneck unit is good (Skogestad, 2004).*

Rule 3.4. *Bottleneck unit: focus on tight control on the bottleneck constraint with the most costly back off in terms of loss in throughput.*

The last rule follows because any deviation from optimal operation in the bottleneck unit due to poor control (including any deviation or back off from the bottleneck constraints) implies a loss in throughput which can never be recovered (Section 3.3.3).

3.5.4 Moving bottlenecks

In the simplest maximum throughput case, the bottleneck is fixed and known and we can use single-loop control (Skogestad, 2004), where the TPM controls the constraint variable in the bottleneck unit.

If the bottleneck moves in the plant, then single-loop control requires reassignment of loops. Reassignment will involve the loop from TPM to the bottleneck (Rule 3.3), as well as the inventory loops needed to ensure self-consistency in the plant (Chapter 2). In addition, the moving bottleneck(s) itself needs to be identified.

For moving bottlenecks, a better approach in most cases is to use multivariable control were also input and output constraints can be included directly in the problem formulation (e.g. MPC). A case study using MPC for maximizing throughput with moving bottlenecks is described in Aske *et al.* (2008). In this case study, the capacity of the individual units is obtained using the models in the local (units) MPC. The main TPMs are located at the feed (conventional inventory control, Figure 3.2(a)), but there are additional degrees of freedom (splits and crossovers) to manipulate the throughput.

3.6 Obtaining (estimate) the back off

If we have a maximum throughput situation (Mode 2a) and the bottleneck has been correctly identified, then operation is optimal, except for the economic loss associated with the back off from active constraints. Back off is usually most costly in the bottleneck unit. It is important to know (or estimate) the expected back off in order to quantify the possible benefits of moving the TPM (changing the inventory control system), adding dynamic degrees of freedom (Chapter 4), changing or retuning the supervisory control system etc.

In the following we consider the case with a single input (TPM) that controls an active output constraint (y) in the bottleneck unit. A back off is then required to account for dynamic variations caused by imperfect control.

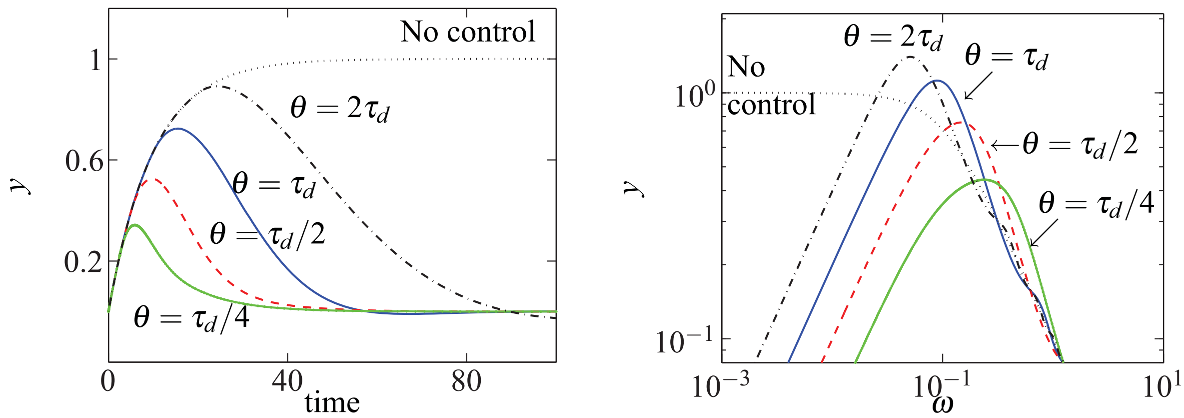
The magnitude of the back off for the dynamic control error should be obtained based on information about the disturbances and the expected control performance. Mathematically, this is given by the worst-case control error (variation) in terms of the "∞-norm" (maximum deviation). In the time domain the dynamic control error (and hence the minimum back off) is given by:

$$b_{\min} = \max_{d,\Delta} \|y(t) - y_s\|_\infty \tag{3.9}$$

where d and Δ denotes disturbance and uncertainty, respectively. The optimal (minimal) back off b is equal to the expected dynamic variation in the controlled variable y. In practice, determining the expected dynamic variation is difficult. However, the point here is not to estimate the minimum back off exactly, but to obtain a rough estimate. The simple method is based on controllability analysis.

3.6.1 Model-based approach (controllability analysis)

Without control, we assume here that the effect of the disturbance on the output (in this case a bottleneck constraint variable) is given by a first-order response with steady-state gain k_d ($= |\Delta y|/|\Delta d|$) and the time constant τ_d. Without control, the



(a) Step response in d at $t = 0$

(b) Corresponding frequency response of Sg_d where $S = (1 - gc)^{-1}$

Figure 3.3: PI-control of first-order disturbance: Effect of effective delay θ. $y = g(s)u + g_d(s)d$. Process: $g(s) = k\frac{e^{-\theta s}}{\tau_1 s+1}$, $\tau_1 = 10$. Disturbance: $g_d = \frac{1}{\tau_d s+1}$, $\tau_d = 10$. Controller: $c(s) = K_c\frac{\tau_I s+1}{\tau_I s}$ where $K_c = \frac{1}{k}\frac{\tau_1}{\tau_c+\theta}$ and $\tau_c = \theta$.

required minimum back off is then $b_{\min} = k_d|d_0|$, where $|d_0|$ is the magnitude of the disturbance. To counteract the effect of the disturbance using feedback control, and thus be able to reduce the back off, the control system needs response with a closed-loop time constant τ_c less than about τ_d. The main “enemy” of feedback control, which limits the achievable τ_c, is the time delay θ. In practice, most processes do not have a “pure” time delay, but they have an “effective” time delay θ_{eff}, which can be estimated from the dynamic model, for example, using the “half rule” of Skogestad (2003).

A simple example of a PI-controlled process with a first-order disturbance is illustrated in Figure 3.3: We see from Figure 3.3(a) that when the delay θ is equal to about τ_d or larger, then there is no significant improvement for a step disturbance. In fact, if we look at sinusoidal disturbances (Figure 3.3(b)), significant improvement in the maximum peak (which determines the necessary back off) is obtained by requiring $\theta \leq \tau_d/4$. A more realistic process with five units is given in Example 3.5.

Example 3.5. ***Minimum back off for different TPM locations.*** *Consider a process with 5 units in series and a fixed bottleneck which is located at the outlet of the last unit (Figure 3.4). The objective is to maximize the throughput using single-loop control in spite of disturbances d_1 to d_5. The disturbances are of equal magnitude, but d_1 is located closest to the bottleneck and has therefore the major effect on the bottleneck. Consider three locations of the TPM:*

- ***A:*** *the conventional approach where the TPM is located at the feed,*

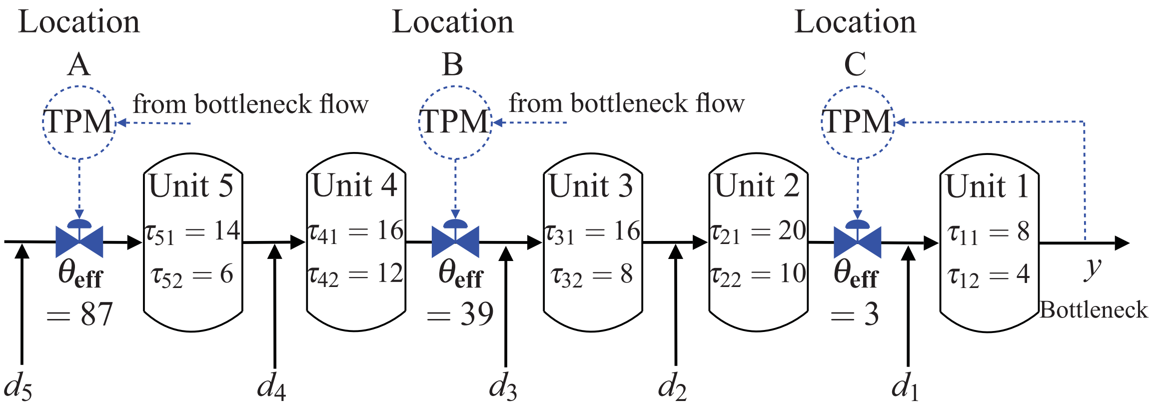


Figure 3.4: The process example with different placements of the TPM with reconfiguration of inventory loops. Inventory control is in direction of flow downstream TPM and in direction opposite to flow upstream TPM. The time constants for each unit is displayed together with the effective dead time (θ_{eff}) for each location for the throughput manipulator.

- ***B:*** *the TPM is located inside the process,*
- ***C:*** *the recommended approach in this paper where the TPM is located at the bottleneck.*

Each unit is represented by a second order model where the time constants (τ_1, τ_2) are stated in Figure 3.4. In addition unit 1 has a delay $\theta_1 = 1$. The disturbances d_1 to d_5 enter between the units. This gives the following disturbance transfer functions (G_{d_i}) from the disturbances (d_1, d_2, d_3, d_4, d_5) to the bottleneck flow (y):

$$G_{d_j} = k_d \cdot \prod_{i=1}^{j} \frac{e^{-\theta_1 s}}{(\tau_{i1}s+1)(\tau_{i2}s+1)}$$

The disturbance gain is given by k_d and is here selected to $k_d = 1$. The process transfer functions G_A, G_B and G_C from the input (TPM at location A, B, or C) are the same as for the disturbances, except that the process gain is given by k and here selected to $k = 2$.

The TPM (u) is adjusted using a PI feedback controller ($y = Ku$, $K = K_c(1+\frac{1}{\tau_I s})$) that controls the bottleneck flow (y) and tuned using the SIMC tuning rules with $\tau_c = 3\theta_{eff}$. The resulting sensitivity function $S = (I+GK)^{-1}$ for the three alternatives is showed in Figure 3.5. Note that the response is much faster with the TPM located close to the bottleneck (location C).

The minimum back off b_{min} for each disturbance $|Sg_d|$ is displayed as a function of frequency for the TPM located at feed (A), in the middle (B) and at the bottleneck (C) in Figure 3.6(a), 3.6(b) and 3.6(c), respectively. Note that a linear scale on back off b is used since the cost is linear in back off (Equation (3.8)).

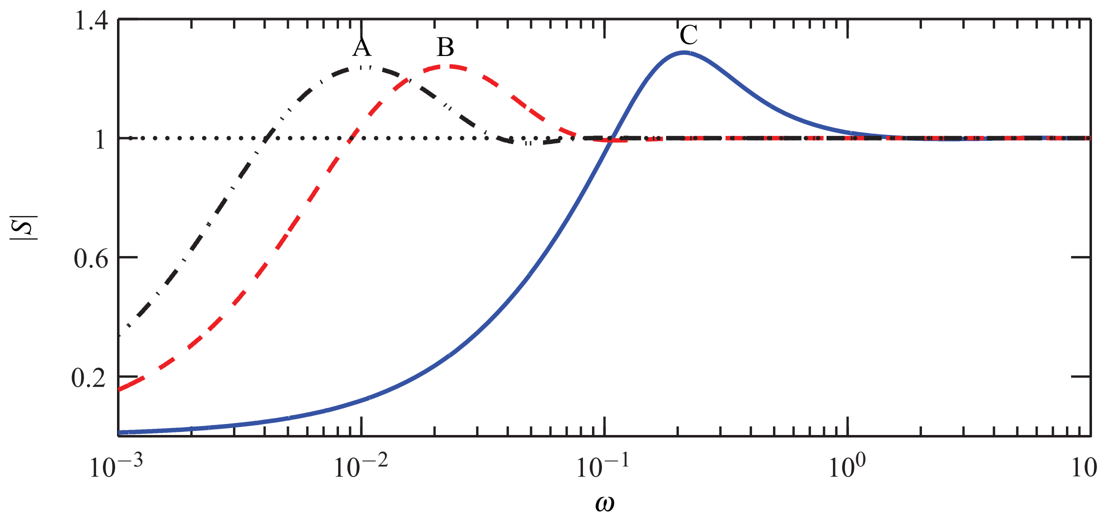


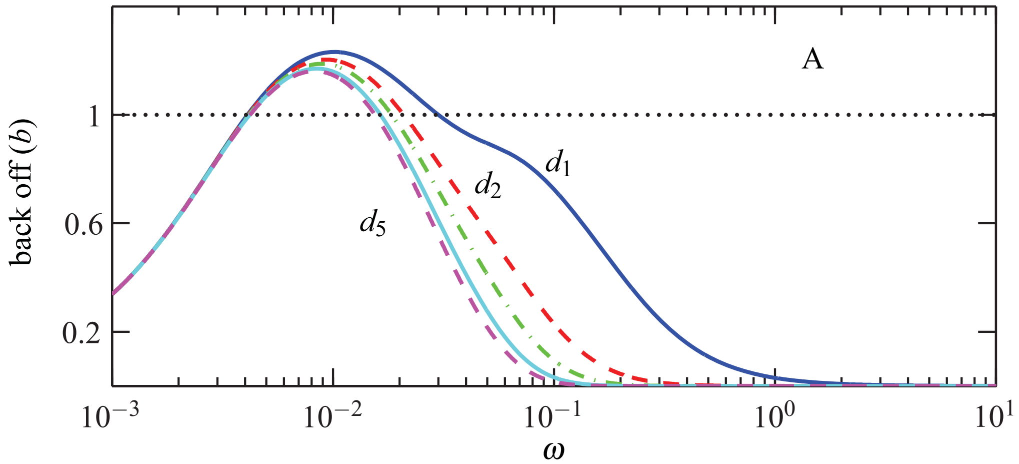
Figure 3.5: Sensitivity $|S|$ as a function of frequency for different placement of the TPM (location A,B and C) in Example 3.5. $S = (I + GK)^{-1}$ and K is a PI-controller.

With the TPM located at the bottleneck (Figure 3.6(c)), the peak of $|Sg_d|$ is reduced significantly, and especially disturbances d_2 to d_5 (upstream the TPM) have a very small effect on the bottleneck flow. With the TPM placed at the feed (Figure 3.6(a)), all the disturbances have almost the same effect on the bottleneck. At the worst-case frequency, the peak of $|Sg_d|$ is about 1.25 which is higher than the value of 1 (because the peak of $|S|$ is $M_s = 1.25$). Of course, we need to apply control to avoid steady-state drift, but this indicates that further detuning of the controller should be considered (the larger τ_c will reduce M_S), but this will lead to poorer set point tracking. For the TPM located inside the process string (Figure 3.6(b)), the peak of $|Sg_d|$ for d_1 (the most important disturbance) has almost the same magnitude as for TPM located at the feed, but the effect of the disturbances d_2 to d_5 is reduced.

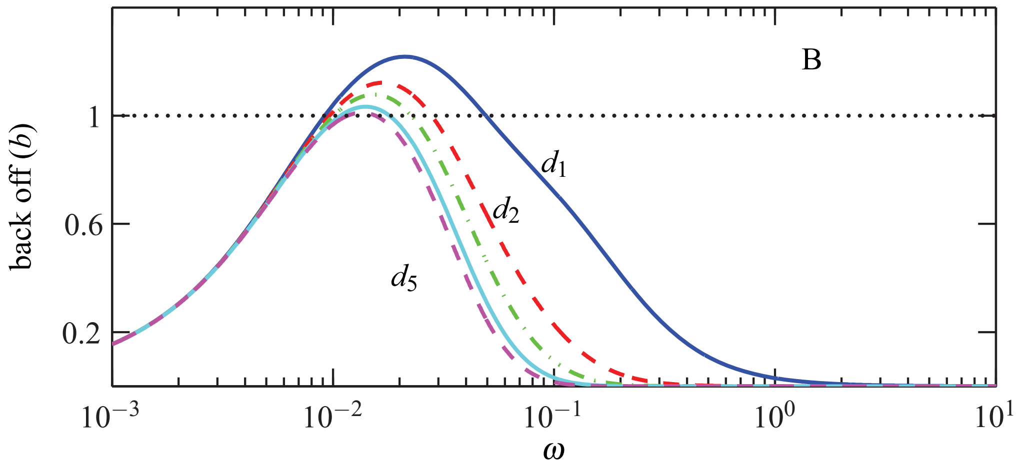
The peak of $|Sg_d|$ with TPM located at the bottleneck is reduced from 0.7 to 0.3 by using a PID-controller instead of a PI. For the two other locations there is only a very small difference in the peak of $|Sg_d|$ between PI- and PID-controllers. In practice, PI-controllers are more common to use than PID since the latter is sensitive to noise and therefore a PI-controller is used here.

From the more detailed derivations of estimating minimum back off (Appendix 3.A.1) we have:

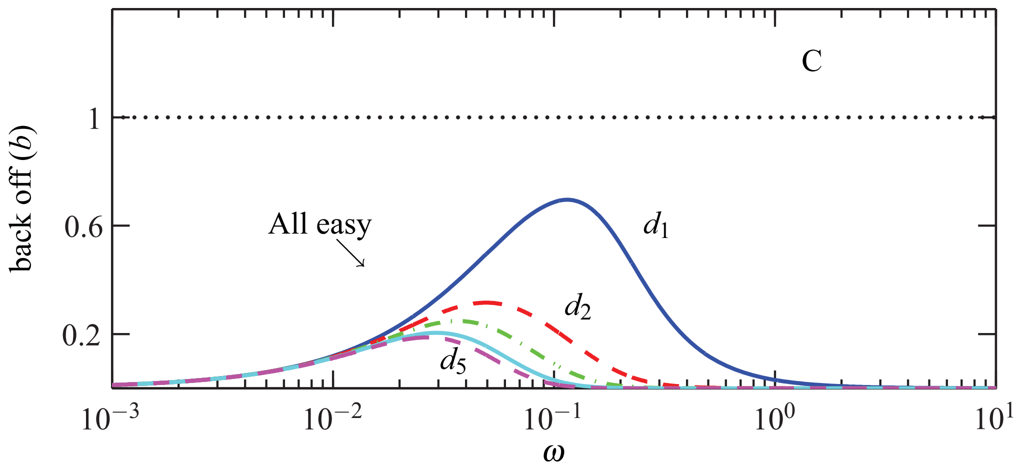
- An "easy" (slow) disturbance has a time constant $\tau_d > 4\theta_{\text{eff}}$. In this case tight bottleneck control (tight control of y) is helpful for rejecting the disturbance. The worst-case frequency is $\omega_{wc} \approx \frac{1}{\tau_d}$ and the resulting minimum back off assuming PI-control with "tight" control is given by $b_{\min} \approx \frac{2\theta_{\text{eff}}}{\tau_d} \cdot k_d|d_0| \leq$



(a) TPM at feed (location A) where $\tau_c = 3\theta_{\text{eff}}$, $\theta_{\text{eff}} = 87$



(b) TPM in middle (location B) where $\tau_c = 3\theta_{\text{eff}}$, $\theta_{\text{eff}} = 39$



(c) TPM at bottleneck (location C) where $\tau_c = 3\theta_{\text{eff}}$, $\theta_{\text{eff}} = 3$

Figure 3.6: Minimum back off ($|Sg_d|$) as a function of frequency for the disturbances d_1 to d_5 on the bottleneck flow, for the three different locations of TPM (A, B, C) in Figure 3.4.

$k_d|d_0|$ (assuming a SIMC PI-controller with $\tau_c = \theta$). This shows that the back off can be significantly reduced if θ_{eff} is small compared to τ_d.

- A "difficult" (fast) disturbance has a time constant $\tau_d < 2\theta_{\text{eff}}$. In this case, control actually gives a larger back off than no control. However, control is necessary for set point tracking. The worst-case frequency is $\omega_{wc} \approx \omega_{peakS}$ where ω_{peakS} is the peak frequency of $|S|$ defined as $|S(j\omega_{peakS})| = \max_\omega |S(j\omega)| = M_S$. To reduce the peak M_S, it is recommended to used "smooth" control (with $\tau_c \geq 2\theta$), that is , for following slow changes in the bottleneck constraints. The minimum back off is given by $b_{\min} \approx M_S \cdot k_d|d_0|$.

In summary, the requirement $\theta_{\text{eff}} < \frac{\tau_d}{4}$ to have benefit of control implies that the TPM must be located very close to the bottleneck to have any benefit of improved control and reducing back off. This also explains in most cases why the loss with manual control, where the operator adjusts the TPM, is usually small.

A more detail mathematical model-based approach for estimating the minimum back off is discussed by Narraway *et al.* (1991); Heath *et al.* (1996) and Loeblein and Perkins (1999) (see Appendix 3.A.2 for more details). The approach requires a nonlinear dynamic model of the process and optimizes simultaneously the control structure and controller parameters in order to find the minimum back off required accommodating the effects of disturbances. However, this approach is too rigorous to be useful as a practical engineering tool.

3.6.2 On-line identification

On-line identification or simply manual adjustment based on experience is the most common approach to determine the back off. In practice, instead of identifying the disturbances itself, it is easier to identify from plant data the output variance. The back off must be set larger than the observed variations to ensure feasible operation even with worst-case disturbances. The back off may be successively reduced from the initial value with increasing disturbance experience. On-line identification is the simplest method, but may be time consuming and requires extensive monitoring of the plant.

3.7 Reducing the back off

Reducing the back off may possibly increase the throughput and give large improvements in profit. To reduce the back off, the first step is to reduce the dynamic variation (squeeze) in the variables with the most costly back off. In the following, suggestions to obtain less dynamic variation are listed.

Improvement 1: Retune the control loops, especially those associated with the bottleneck unit in order to reduce dynamic variations, primarily in the active "hard" constraints variables.

Improvement 2: Move, add or make use of additional degrees of freedom, that influence the flow through the bottleneck (e.g. throughput manipulator, crossovers, splits, extra feeds, inventories) to obtain tighter dynamic control of the bottleneck unit.

Improvement 3: Introduce feedforward control from measured disturbances to obtain tighter control.

Improvement 4: Introduce feedforward control from expected changes in the active constraint variable ($y_{\text{constraint}}$) to the set point (y_s) to keep the back off b unchanged.

Improvement 5: Adjust the back off b depending on expected disturbance level. Importantly, the back off b can be reduced (move y_s closer to $y_{\text{constraint}}$) when the expected disturbance level is low ("calm periods").

Improvement 6: Exploit the hold-up volume in buffer volumes as a dynamic degree of freedom to obtain tighter bottleneck control.

Improvement 7: Add buffer tank to dampen disturbances that affect the active constraints.

A more detailed discussion of each Improvement is given below.

Improvement 1: Retune control loops

As shown in Section 3.6, the possibility to reduce the back off by achieving tight control of the bottleneck unit itself is limited in most cases, unless the TPM is located close to the bottleneck. However, this does not mean that retuning is not important, because retuning the control loop may avoid *unnecessary* variations in variables that may propagate dynamic variations to the bottleneck unit. An example is a poorly tuned temperature controller in a distillation column upstream the bottleneck unit. The temperature controller performance can be acceptable for composition control in the distillation column itself, but it may lead to unnecessary flow variations that disturb the downstream (bottleneck) unit(s).

Improvement 2: Move, add or use additional degrees of freedom

As mentioned in Section 3.5.3, the TPM should be moved close to the bottleneck unit in order to reduce the effective time delay from the TPM to the bottleneck.

However, other alternatives should be considered because moving the TPM requires reconfiguration of the inventory loops to obtain a self-consistent inventory control system (see Section 3.4). Note that it is possible to move the TPMs without reconfiguration, but then the inventory control system will only be consistent and may consist of “long loops”. Such a “long loop” requires larger hold-up volume because of longer physical distance and hence longer effective time delay. Other ways to shorten the possible “long loop” from the TPM to the bottleneck unit is to use other variables that affect the throughput, like crossovers between parallel units and feed splits (see Rule 3.2). The key point for using additional degrees of freedom is to reduce the effective time delay from the manipulated variable to the active constraint in the bottleneck unit.

Improvement 3: Feedforward control from measured disturbances

Feedforward control from (important) disturbances can reduce the dynamic variation in the controlled variable (bottleneck constraint) y. This leads to tighter control and the back off can be reduced.

Improvement 4: Follow changes in $y_{\text{constraint}}$ (feedforward action)

From (3.7), the back off is $b = |y_{\text{constraint}} - y_s|$, so the actual set point y_s is set by $y_{\text{constraint}}$ and the back off b. The “hard” constraint $y_{\text{constraint}}$ may change due to disturbances and we want y_s to follow these variations (at least to some extent) to avoid an unnecessary change in back off (b). For example, consider a distillation column operating at maximum throughput. The maximum feed rate to the column depends on the feed composition, and a change in the feed composition may increase the maximum feed rate, hence an increase in $y_{\text{constraint}}$ occurs. By increasing y_s correspondingly to $y_{\text{constraint}}$, the back off b will remain constant. With available disturbance measurements, feedforward can be applied to adjust y_s.

Improvement 5: Adjust back off depending on disturbance level (feedforward action)

Compared to Improvement 4, where y_s is adjusted to keep a *constant* back off, we want here to *adjust the back off b* itself depending on the expected disturbance level. The idea is that the back off can be reduced in (expected) “calm periods”. For example, consider a plant that receives feed gas at high pressure through a long pipeline, where the feed composition is monitored at the pipe inlet. The feed composition is an important disturbance, and by monitoring the feed composition in the pipeline, one will know in advance when the changes will occur. In periods with no feed composition changes, the back off b can be reduced. It is important that the monitoring of disturbance level is reliable, so that the back off can be increased again during periods with larger disturbances.

Improvement 6: Buffer volume as dynamic degree of freedom

The hold-up volume in a process can be exploited as *dynamic degree of freedom* to obtain faster (short-term) corrections of the flow to the downstream unit. When using inventories, the hold-up volume must be refilled from upstream source to avoid emptying, so this requires acceptable speed of the inventory control systems. The hold-up volume should be large enough to change the throughput in the downstream unit for the period it takes to refill it. Implementing hold-up volumes can be done by by using ratio control (single-loop) or a multivariable dynamic controller (e.g. MPC) that manipulate on the buffer volume (level). These issues are discussed in more detail in Chapter 4.

Improvement 7: Add buffer volume

The buffer volume can dampen the variations (or the disturbances) by exploiting its hold-up volume. This requires smooth tuning of the buffer volume, otherwise inflow $\approx$ outflow and no smoothing will be obtained. Buffer volumes that is added to smooth out disturbances that affect the bottleneck must be placed upstream the bottleneck. Buffer volumes downstream the bottleneck has no effect on the bottleneck (the active constraint) and no reduction in back off will be obtained. However, note that hold-up volumes placed between the throughput manipulator and the bottleneck increases the effective time delay for flow rate changes, and tight control of the bottleneck unit becomes more difficult if the buffer volume is not exploited.

Example 3.6. ***Using buffer volumes as dynamic degrees of freedom to obtain tighter bottleneck control***. *This example illustrates tighter bottleneck control by using hold-up volumes as dynamic degrees of freedom. Consider three units, each followed by a buffer (hold-up) volume, as displayed in Figure 3.7. Maximum capacity for each unit changes due to disturbances and the bottleneck moves. The objective is maximum throughput and the throughput manipulator is located at the feed but the hold-up volumes are exploited for tighter control of the bottleneck.*

Three different control structures are studied:

1. *Manual control where the TPM is set at a rate that ensures feasibility in spite of the predefined disturbances.*

2. *An MPC controller that uses only the TPM as manipulator to maximize throughput and consider the constraints in each unit.*

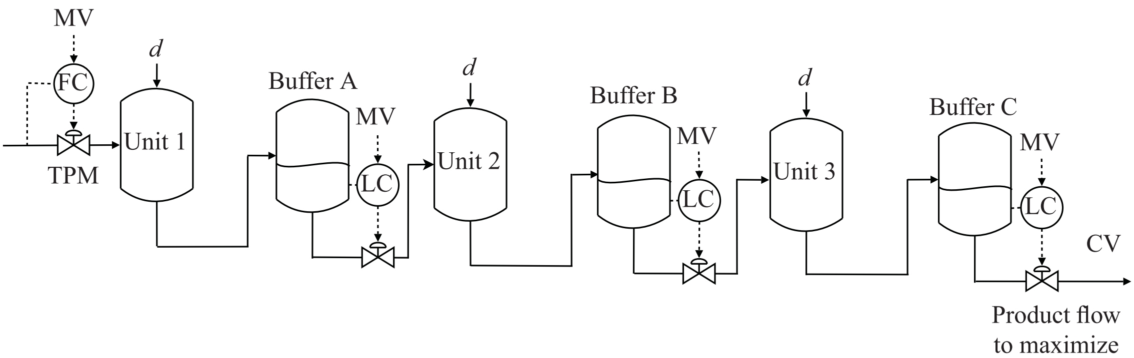


Figure 3.7: Diagram of the simulated process in Example 3.6 with MPC that uses the feed rate and the buffer volumes to maximize throughput (control structure 3).

3. *An MPC controller that uses the TPM in addition to the set point to the level controller in the three buffer volumes as manipulated variables to maximize throughput and consider the constraints in each unit.*

The predefined step-disturbances are regarded as unmeasured and not included as disturbance variables (DVs) in the MPC controller. The necessary back off from the hard capacity constraints in the units are found by trial-and error. With the predefined step-disturbances present, accumulated production for the three control structures is given in Figure 3.8. Using the hold-up volumes (structure 3) tighter bottleneck control is obtained and the accumulated production is increased. Using only the feed rate is only marginally better than manual control. This is due to the long effective delay ("long loop") from the feed valve to the constraint and hence a large back off is needed.

3.8 Discussion

3.8.1 Network theory

The maximum throughput case in production systems is closely related to the maximum flow problem in networks considered in operations research. Such a network consists of sources (feeds), arcs, nodes and sinks (products) (e.g. Phillips *et al.*, 1976). An arc is like a pipeline or unit with a given (maximum) capacity and the nodes may be used to add or split streams. We assume that the network is linear, which requires that the splits are either free variables (”actual” splits or crossovers in process networks) or constant (typically, internal splits in the units in process networks, for example, a distillation column that splits into two products). We then have a linear programming problem, and the trivial but important conclusion is that the maximum flow is dictated by the network bottleneck. To see this, one introduces ”cuts” through the network, and the capacity of a cut is the sum of the

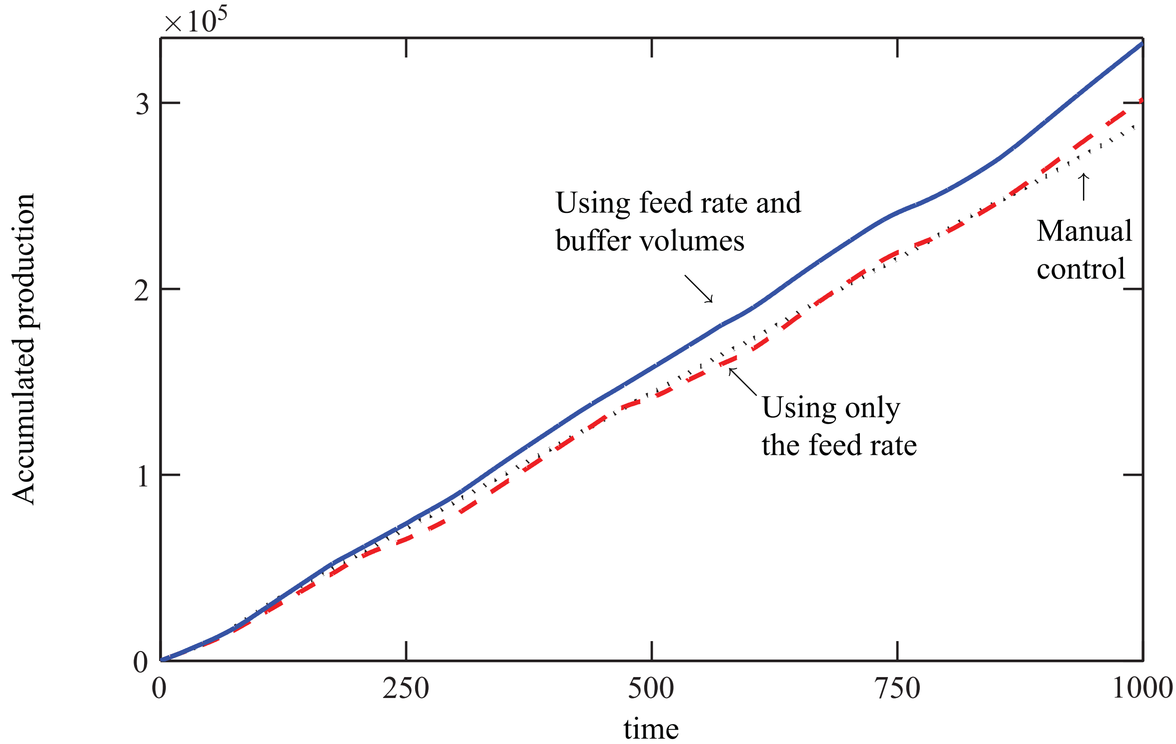


Figure 3.8: Accumulated product rate manual control (TPM constant, dotted line), TPM in closed-loop (dashed) and both using TPM and hold-up volumes (solid).

capacity of the forward arcs (arcs that is leaving the node) that it cuts through. The *max-flow min-cut theorem* (Ford and Fulkerson, 1962) says that the maximum flow through the network is equal to the minimum capacity of all cuts (the minimal cut). We then reach the important insight that maximum network flow (maximum throughput) requires that all arcs in some cut have maximum flow, that is, they must all be bottlenecks (with no available capacity left). Figure 3.9 illustrates parts of a chemical plant with sources ($s1 - s3$), arcs, nodes (units $u1 - u11$ and junctions $m1 - m3$ in our terms) and sinks ($n1 - n12$) and a possible location of the minimal cut. The location of the minimum cut shows that the units $u1$ and $u11$ are bottlenecks units. Note that a cut separating the source and the sink is a partition of the nodes into two subset S and $\overline{S}$ where the source nodes are in S and the sink nodes are in $\overline{S}$ (e.g. Phillips *et al.*, 1976). The arc denoted c (crossover) is not included in the summation of the capacity in the minimal cut since it is directed from a node in $\overline{S}$ to a node in S. A network like the one displayed in Figure 3.9 with multiple sources and sinks can be converted to a single-source single-sink by creating an imaginary super source and an imaginary super sink (Phillips *et al.*, 1976), but this is not included here. Therefore it does not seem like all the sink nodes are located in the subset $\overline{S}$ in Figure 3.9.

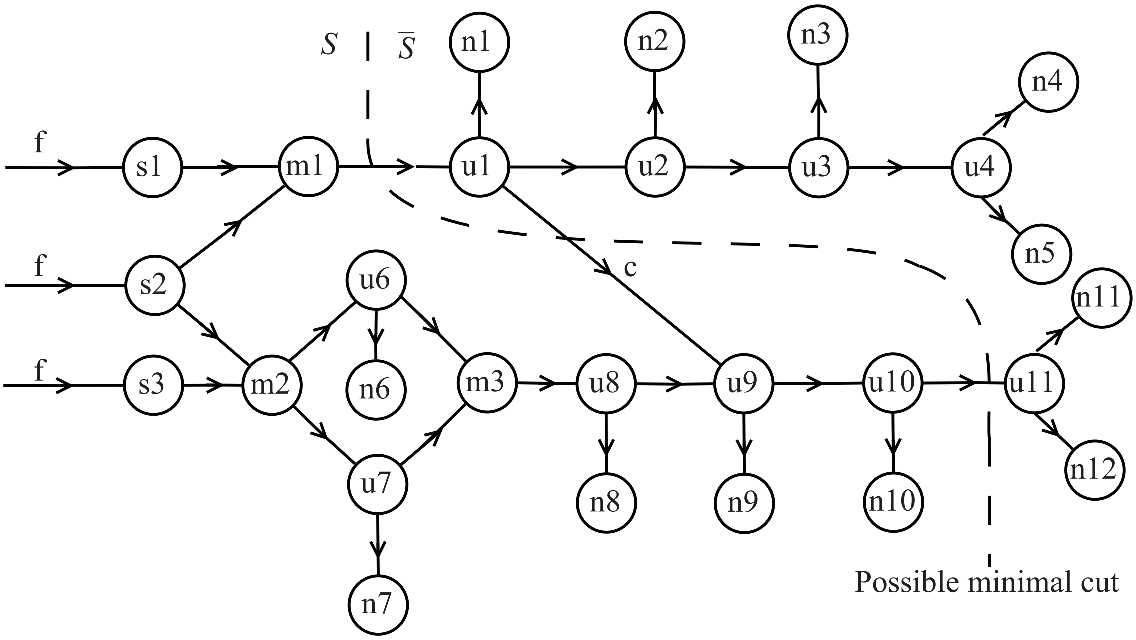


Figure 3.9: A part of a gas plant illustrated as a flow network, with a possible minimal cut. The corresponding flow sheet of the gas processing plant is displayed in Figure 5.3.

To apply network theory to production systems, we first need to obtain the capacity (maximum flow) of each unit (arc). This is quite straightforward, and involves solving a (nonlinear) feasibility problem for each unit (see Definition 3.3). The capacity may also be computed on-line, for example, by using local MPC implementations as proposed by Aske *et al.* (2008).

The main assumption for applying network theory is that the mass flow through the network is represented by linear flow connections. Note that the nonlinearity of the equations within a unit is not a problem, but rather the possible nonlinearity in terms of flows between units. The main problem of applying linear network theory to production systems is therefore that the flow split in a unit, e.g. a distillation column, is not constant, but depends on the state of its feed, and, in particular, of its feed composition. The main process unit to change composition is a reactor, so decisions in the reactor may strongly influence the flow in downstream units and recycles. Another important decision that affects composition, and thus flows, is the amount of recycle. One solution to avoid these sources of nonlinearity is to treat certain combinations of units, like a reactor-recycle system, as a single combined unit as seen from maximum throughput (bottleneck) point of view.

Although the linearity assumptions will not hold exactly in most of ”our” systems, the bottleneck result is nevertheless likely to be optimal in most cases. The reason is that the location of active constraints (bottleneck) is a structural issue.

3.8.2 Issues on estimation of back off

Estimating the dynamic variation in a controlled variable y by using controllability analysis has some limitations. The back off estimation is only valid for single-loop control where the controller is tuned by using the SIMC-tuning rules. The tuning rules are not really a limitation, since the speed of the closed-loop response is a degree of freedom. However, the simplified analytic estimation needs a model of the disturbance and assumes that the shape of G_d is flat up to the break frequency where the disturbance rolls off. The asymptotic consideration of the disturbance will be wrong, especially for higher order. For a higher order disturbance, the assumption that G_d is "flat" up to ω_{bd} will not be correct, since the disturbance starts to roll of at a lower frequency.

With our experience from industry today, on-line identification is by far the most used. A model is not required in this case, only plant data. For a new plant, estimating necessary back off has minor importance; because during a plant start up, optimal production is not the issue, but rather to obtain stabilized production. After reaching nominal production, reducing back off and optimal production becomes an operating issue, but at that time plant data is available. Operating margins is typically reduced gradually. With close follow-up from personnel, the time spent to move the plant from nominal to optimal production can be reduced.

Back off is based on experience and therefore the importance of the manual control should not be underestimated. However, a new regime of closed-loop control of the throughput can be fulfilled, but now with the back off as the available manipulator for the operators instead of the throughput. This makes the back off (and also the loss) more visual instead of being "baked into" the throughput set point.

3.9 Conclusion

In this paper, we have shown that "maximum throughput" is an optimal economic operation policy in many cases. To implement maximum throughput, the key is to achieve maximum flow through the bottleneck unit(s). However, to achieve feasible operation (no constraint violation), is usually necessary to "back off" from the optimally active constraints. Back off leads to a lower flow through the bottleneck and an unrecoverable economic loss. This leads to the obvious but important conclusion that "throughput maximization requires tight bottleneck control". However, achieving tight bottleneck control in practice is not so simple because the throughput manipulator is often located too far away from the bottleneck unit (with a large effective delay θ_{eff}) to be effective for reducing the effect of disturbances on the key bottleneck variables. For example, to significantly reduce the effect of a first-order disturbance (and be able to reduce the back off), we must require $\theta_{\text{eff}} < \tau_d/4$ where τ_d is the first-order response times for the disturbance. In practice, the requirement $\theta_{\text{eff}} < \tau_d/4$ is unlikely to be satisfied unless the TPM is located at the bottleneck unit. Thus, "tight bottleneck control" (and reducing the back off) in practice requires that the TPM is located close to the bottleneck unit. This can either be achieved by moving TPM (which requires reconfiguration of the inventory control system) or for some plants, to utilize "extra" TPMs such as crossovers and splits (Chapter 5). Another alternative is to make use of dynamic degrees of freedom (variations in the inventories) as is further discussed in Chapter 4. Increased throughput can also be achieved by strategies where the back off is reduced in "calm" periods where there are less disturbances. Possible improvements to reduce back off are listed in Section 3.7.

3.A Estimation of minimum back off

We here use a controllability analysis for identifying the dynamic control variations. This requires a model of the process together with assumption of the expected frequency and amplitude of the disturbances. Controllability is a property that is independent of the detailed controller tuning, but here we assume that IMC-tuning are used. The issue here is to estimate the minimum required back off from a model without designing a controller.

3.A.1 Simplified analytic estimation for single-loop control

Let y denote the controlled active constraint in the bottleneck unit, for which we want to estimate the expected dynamic variation which is equal to the minimum back off. Let u denote the manipulated variable (e.g. TPM or a dynamic variable that affects y) and d the disturbance. For the linearized system $y = Gu + G_d d$, the closed-loop transfer function from a disturbance d to y is (e.g. Skogestad and Postlethwaite, 2005)

$$y = (I + GK)^{-1} \cdot G_d d = SG_d d \tag{3.10}$$

where G is the process model, K is the feedback controller, $S = (I + GK)^{-1}$ is the sensitivity function and G_d is the disturbance model. Assume that the disturbances are sinusoidal, $d(t) = d_0 \sin(\omega t)$, and that $|d_0|$ is bounded. We consider only scalar disturbances (i.e. one disturbance at a time). The worst-case amplification (peak output variation as a function of disturbance frequency) from d to y then gives the optimal (minimum) back off, thus

$$b \geq b_{\min} = \max_{\omega,d} |y| = \max_{d} ||Sg_d||_\infty \cdot |d_0| \tag{3.11}$$

where $\max_{\omega,d} |y|$ represents the effect of the worst-case disturbance over all frequencies and directions and therefore represents the minimum back off. Note that

$$||Sg_d||_\infty \stackrel{\triangle}{=} \max_{\omega} |Sg_d(j\omega)| = |Sg_d(j\omega_{wc})| \tag{3.12}$$

where ω_{wc} is the worst-case frequency where $|Sg_d|$ has its peak.

Worst-case frequency

The minimum back off for a given disturbance is given by $||Sg_d||_\infty = Sg_d(j\omega_{wc})$, but what is the worst-case frequency (peak frequency) ω_{wc}? It is difficult to know ω_{wc} beforehand, but typically the peak frequency for $|Sg_d|$ is located around the closed-loop bandwidth frequency. Thus, two interesting frequencies are the peak

$\frac{\tau_c}{\theta}$	0	1	2	3	4	5
$\omega_S \cdot \theta$	0.741	0.511	0.414	0.357	0.319	0.291
$\omega_{peakS} \cdot \theta$	1.38	1.14	1.02	0.947	0.891	0.849
M_s	3.13	1.59	1.35	1.25	1.19	1.16

Table 3.1: Frequencies for sensitivity function (S) and robustness margins for different τ_c using SIMC-settings ($K_c = \frac{1}{k}\frac{\tau_1}{\theta+\tau_c}$, $\tau_I = \tau_1$, $\tau_D = \tau_2$) in the PID-controller.

frequency for $|S|$ defined as $|S(j\omega_{peakS})| = \max_\omega |S(j\omega)| = M_S$, and the frequency ω_S defined as $|S(j\omega_S)| = 1$. Using these two specific frequencies we have

$$b_{\min} \geq |Sg_d(j\omega_{wc})| \cdot |d_0| \geq \begin{cases} |Sg_d(j\omega_s)| \cdot |d_0| = |g_d(j\omega_s)| \cdot |d_0| \\ |Sg_d(j\omega_{peakS})| \cdot |d_0| = M_S \cdot |g_d(j\omega_{peakS})| \cdot |d_0| \end{cases} \tag{3.13}$$

These two lower bounds on the minimum back off are fairly tight for a first-order model of g_d. For a disturbance model g_d of higher order, general rules for estimating the minimum back off $b_{\min} = \max_\omega ||SG_d||_\infty$ is difficult to state. For example, a g_d of high order will roll off quickly at higher frequencies and ω_S and ω_{peakS} may not represent the worst-case frequencies.

Nevertheless, the two frequencies will always provide a lower bound, so it is interesting to estimate ω_S and ω_{peakS}. Table 3.1 gives the peak of $|S|$ (= M_s) and the frequencies ω_s and ω_{peakS} for a first-order process with time delay, $G_1 = ke^{-\theta s}/(\tau_1 s+1)$, controlled with a PI-controller using the SIMC-tunings rules ($K_c = \frac{1}{k}\frac{\tau_1}{\theta+\tau_c}$, $\tau_I = \tau_1$) as a function of the tuning parameter τ_c (the closed-loop time constant). The same values apply to a second order with time delay process delay ($G_2 = e^{-\theta s}/((\tau_1 s+1)(\tau_2 s+1))$ controlled with a PID-controller if we select the derivative time $\tau_D = \tau_2$. In both cases the closed-loop transfer function becomes $L = GC = \frac{e^{-\theta s}}{(\tau_c+\theta)s}$.

Selection of the tuning variable τ_c

The sensitivity function depends on the controller tuning K, that is, the closed-loop time constant τ_c. Here we want to state some recommendations for selection of τ_c in our further development of an assumption of minimum back off.

1. We want to minimize $\|SG_d\|_\infty$ to minimize the back off. This leads to selecting a small τ_c to reject "easy" disturbances upstream the input u (tight control) and a large τ_c to reject "difficult" disturbances after the input u (leads to M_s small).

2. For robustness we want $\|S\|_\infty = M_s \leq 1.6$, which implies $\tau_c \geq \theta$ approximately, see Table 3.1.

3. We want to minimize τ_c to have fast set point tracking.

To make some more specific recommendations of what τ_c should be, consider the disturbance break frequency ω_{bd} defined as

$$\omega_{bd} = \frac{1}{\tau_d} \tag{3.14}$$

where τ_d is the largest disturbance time constant in g_d. In other words, ω_{bd} is the frequency where the disturbance gain starts dropping. Consider two cases:

Case 1: ''Difficult" ("fast") disturbances with $\boldsymbol{\omega_{bd} > \omega_S}$. Here, $|g_d|$ is "flat" at the frequency ω_S (and approximately "flat" at ω_{peakS}), so the use of feedback will give worse response than with no control at some frequencies because $|S|$ has an unavoidable peak at the resonance frequency ω_{peakS}. This leads to the worst-case frequency $\omega_{wc} \approx \omega_{peakS}$, and we have $\|Sg_d\|_\infty \approx M_S|g_d(j\omega_{peakS})| \cdot |d_0| \approx M_S \cdot k_d|d_0|$. To reduce M_S we want τ_c large (but on the other hand we want τ_c small for set point tracking (y_s)). In summary, a steady-state analysis is sufficient for back off estimation and we have $b_{\min} \approx M_S \cdot k_d|d_0|$ where $k_d = g_d(0)$ is the steady-state disturbance gain. To minimize M_S we want τ_c large.

Case 2: "Easy" ("slow") disturbance with $\boldsymbol{\omega_{bd} < \omega_S}$. In this case ω_{bd} is approximately the worst-case frequency because $|S| \approx \frac{\omega}{\omega_S}$ increases linearly with ω in a log-log plot in the frequency region up to ω_S (Skogestad and Postlethwaite, 2005) and $|g_d| \approx k_d$ up to ω_{bd}. In summary, $b_{\min} \approx |Sg_d(j\omega_{bd})| \approx k_d \frac{\omega_{bd}}{\omega_S}$ and we want ω_S as large as possible for disturbance rejection, which corresponds to τ_c small.

In the above case definitions, ω_S is used to determine the disturbance case and hence decide the tuning parameter τ_c. However, ω_S depends on the selection of τ_c. From Table 3.1 a relation between ω_S, θ and τ_c are given, and we can state ω_S approximately

$$\omega_S \approx \frac{1}{\tau_c + \theta} \tag{3.15}$$

From the arguments above, we can suggest a "rule of thumb" for selection of τ_c:

$$\tau_c = \begin{cases} 3\theta, & \text{for } \omega_{bd} > \frac{1}{2\theta} \text{ or } \tau_d < 2\theta \\ \theta, & \text{for } \omega_{bd} < \frac{1}{4\theta} \text{ or } \tau_d > 4\theta \end{cases} \tag{3.16}$$

The choice of $\tau_c = 3\theta$ is a trade-off between disturbance rejection and set point trajectory: we want to minimize τ_c to track set points, but at the same time we want to maximize τ_c to reduce M_S. The choice $\tau_c = 3\theta$ gives $M_S \approx 1.25$ (see Table 3.1), so the use of feedback gives 25% extra back off.

The recommendations (3.16) do not state a selection of τ_c in the intermediate range $2\theta < \tau_d < 4\theta$. The disturbances with $\tau_d > 4\theta$ are "slow" disturbances and the control system are able to reject them fairly good. For $\tau_d < 2\theta$ the disturbances are fast and here the control is poorer for disturbance rejection than no control because of the peak of $|S|$. In the intermediate range τ_c should be increased from θ up to 3θ.

Summary of simplified analytic estimation of back off

The minimum back off $b_{\min}$ is given by (3.11). The frequencies ω_S and ω_{peakS} are expressed by θ and τ_c in Table 3.1, and the recommendations for τ_c are given in (3.16). In the idealized case we assume that $\frac{1}{\tau_d} = \omega_{bd}$ and that g_d is approximately "flat" at frequencies below ω_{bd}. In addition, we assume that $|S| \approx \frac{\omega}{\omega_S}$ between ω_S and ω_{peakS}, in other words, the slope of $|S|$ is approximately $+1$ in the given range. Then the location of the peak frequency and the magnitude of the necessary back off can be summarized as:

$$\text{For "difficult" disturbance with } \boldsymbol{\tau_d < 2\theta}: \quad \begin{aligned} \omega_{wc} &\approx \omega_{peakS} \\ b_{\min} &\approx M_S \cdot k_d |d_0| \end{aligned} \tag{3.17}$$

$$\text{For "easy" disturbance with } \boldsymbol{\tau_d > 4\theta}: \quad \begin{aligned} \omega_{wc} &\approx \frac{1}{\tau_d} \\ b_{\min} &\approx \frac{2\theta}{\tau_d} \cdot k_d |d_0| \leq k_d |d_0| \end{aligned} \tag{3.18}$$

To conclude the estimation of back off, we see from (3.17) and (3.18) that control is helpful for $\tau_d > 4\theta_{\text{eff}}$. Otherwise the back off is given by steady-state disturbance effect.

To illustrate the estimation of back off, consider the introductory example.

Example 3.5 (continued). ***Minimum back off for different TPM locations.*** *The necessary back off for the "difficult" disturbance d_1 (difficult because it is located close to the bottleneck) is calculated using Table 3.1 and Equations* (3.17)-(3.18). *The tuning variable is selected to $\tau_c = 3\theta$ for all three TPM locations. The disturbance time constant for d_1 is $\tau_d = 8$ or equivalent $\omega_{bd} = 0.125$. The calculated frequencies and minimum back off are compared with the observed ones in Table 3.2. Note that location C with $\theta_{eff} = 3$ is in the intermediate range $2\theta < \tau_d < 4\theta$ and it is not clear if* (3.17) *or* (3.18) *should be used. Here,* (3.18) *is selected since the disturbances have started to roll off and a stationary analysis will be less correct.*

Location	A ($\theta_{\text{eff}} = 87$)	ω_S	ω_{peakS}	ω_{wc}	$b_{\min}$
Estimated	Tab. 3.1, Eq. (3.17)	0.004	0.011	0.011	1.25
Observed	Fig. 3.6(a)	0.004	0.010	0.010	1.23
Location	B ($\theta_{\text{eff}} = 39$)	ω_S	ω_{peakS}	ω_{wc}	$b_{\min}$
Estimated	Tab. 3.1, Eq. (3.17)	0.009	0.024	0.024	1.25
Observed	Figure 3.6(b)	0.009	0.022	0.021	1.22
Location	C ($\theta_{\text{eff}} = 3$)	ω_S	ω_{peakS}	ω_{wc}	$b_{\min}$
Estimated	Tab. 3.1, Eq. (3.18)	0.12	0.32	0.13	0.75
Observed	Figure 3.6(c)	0.11	0.22	0.12	0.70

Table 3.2: Estimated and observed frequencies (ω_s, ω_{peakS} and ω_{wc}) and minimum back off ($b_{\min}$) to account for disturbance d_1 (with $\tau_d = 8$) from Example 3.5. The frequencies and back off are estimated by using Table 3.1, Equation (3.17) and (3.18). The observations are from Figure 3.6.

We see that ω_{peakS} provides a good estimate of the worst-case frequency for processes with long effective time delay θ (location A and B) whereas ω_{bd} provides a good estimate for the worst-case frequency for processes with a short effective time delay θ (location C). For the back off calculation, $M_s \cdot k_d|d_0|$ gives a good estimate for long effective time delay. For a short effective time delay θ the back off estimate is also good. However, by using the estimated frequency of ω_S instead of the approximation of $\omega_S \approx \frac{1}{2\theta}$, the estimated minimum back off becomes larger than the observed minimum back off, since the disturbance has started to roll off (it is not really a "fast" disturbance but an "intermediate"). Note that the location of the peak to $|Sg_d|$ moves from ω_{peakS} towards ω_{bd} with smaller effective time delay between TPM and bottleneck. To move the TPM from location A to location B has very little effect in terms of reducing minimum back off. The disturbances are still fast compared to the closed-loop response and control is not helpful for rejecting the major disturbance.

Assume that it is possible (and preferable in terms of costs) to increase the hold-up between the inlet of the plant and the middle of the plant (refer to location A and B in Example 3.5). To evaluate the effect of larger holdups between location A and B in terms of minimum back off, consider a new example.

Example 3.7. ***Minimum back off in a process with large hold-up volumes.*** *Consider the same process string as in Example 3.5, but now with significantly larger hold-up volumes in unit* 1 *and* 2*. The bottleneck flow (y) is considered fixed at the outlet of the last unit. The time constants for each unit are displayed in Table 3.3.*

The minimum back off b_{min} for each disturbance $|Sg_d|$ is displayed as a function of frequency for the TPM located at feed (A), in the middle (B) and at the bottleneck (C) in Figure 3.10., With the TPM located at the bottleneck (Figure 3.10(c)), the

Unit	τ_1	τ_2
1	200	100
2	50	50
3	16	8
4	20	10
5	8	1

Table 3.3: Time constants τ_1 and τ_2 for the units in Example 3.7.

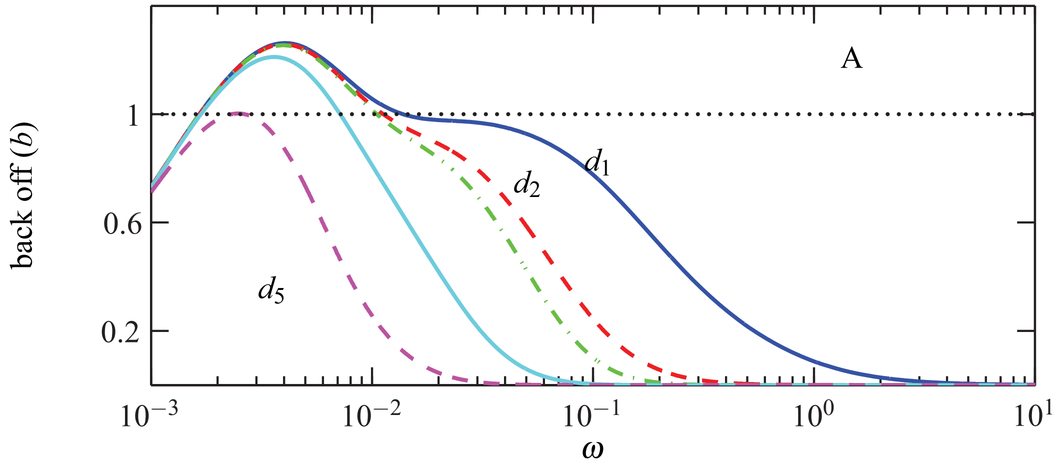
peak of $|Sg_d|$ is reduced significantly compare to when the TPM is located in A and B. For TPM located in A an B there is almost no difference for the worst disturbance d_1, but the effect of the disturbances d_2 to d_5 is reduced when TPM is moved from location A to B.

By using Table 3.1 together with (3.17) *and* (3.18)*, the frequencies ω_S, ω_{peakS} and ω_{wc} are estimated together with minimum back off. The observed and the estimated frequencies and back off are compared in Table 3.4. Here location A and B is in the area for steady-state analysis ($\tau_d < 2\theta$). For location C the worst disturbance d_1 is fast compared to the closed-loop response ($\tau_d > 4\theta$).*

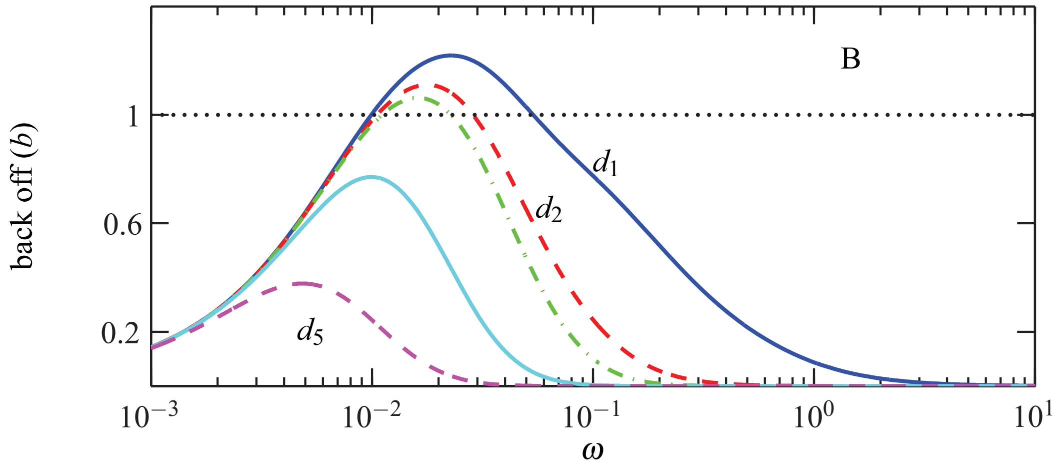
Location	A ($\theta_{\text{eff}} = 214$)	ω_S	ω_{peakS}	ω_{wc}	$b_{\min}$
Estimated	Tab. 3.1, Eq. (3.17)	0.0017	0.0044	0.0044	1.25
Observed	Fig. 3.6(a)	0.0017	0.0040	0.0040	1.26
Location	B ($\theta_{\text{eff}} = 36$)	ω_S	ω_{peakS}	ω_{wc}	$b_{\min}$
Estimated	Tab. 3.1, Eq. (3.17)	0.010	0.026	0.026	1.25
Observed	Figure 3.6(b)	0.010	0.024	0.023	1.22
Location	C ($\theta_{\text{eff}} = 1.5$)	ω_S	ω_{peakS}	ω_{wc}	$b_{\min}$
Estimated	Tab. 3.1, Eq. (3.18)	0.24	0.62	0.13	0.38
Observed	Figure 3.6(c)	0.22	0.48	0.18	0.49

Table 3.4: Estimated and observed frequencies (ω_s, ω_{peakS} and ω_{wc}) and the minimum back off ($b_{\min}$) to account for disturbance d_1 (with $\tau_d = 8$) from Example 3.7. The frequencies and back off are estimated by using Table 3.1, Equation (3.17) and (3.18). The observations are from Figure 3.10.

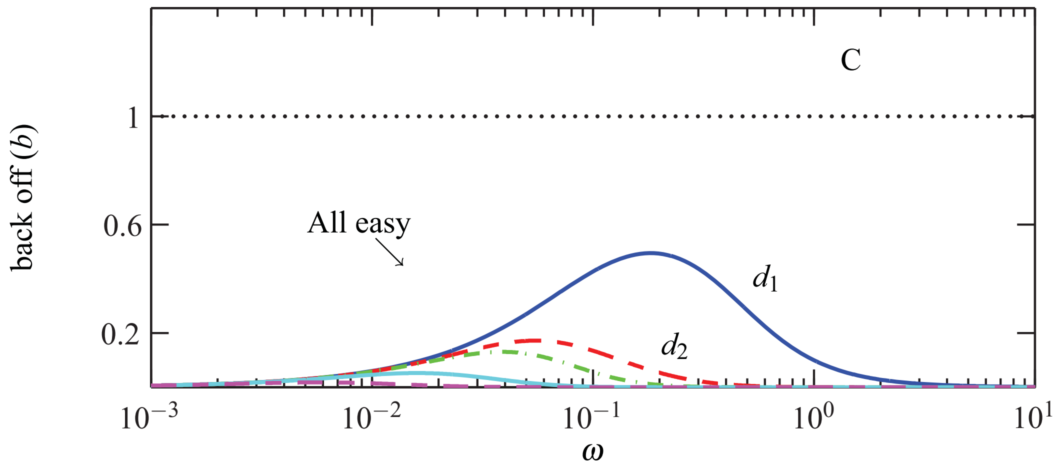
We see that ω_{peakS} provides a good estimate of the worst-case frequency for processes with long effective time delay θ (location A and B) whereas ω_{bd} provides a good estimate for the worst-case frequency for processes with a short effective time delay θ (location C). For the back off calculation, $M_s \cdot k_d|d_0|$ gives a good estimate for long effective time delay. For location C the worst-case disturbance is categorized as "easy" and here the estimate is lower than the observed minimum back off. However, by using the estimated frequency of ω_S instead of the approxi-



(a) TPM at feed (location A) where $\tau_c = 3\theta_{\text{eff}}$, $\theta_{\text{eff}} = 214$



(b) TPM in middle (location B) where $\tau_c = 3\theta_{\text{eff}}$, $\theta_{\text{eff}} = 36$



(c) TPM at bottleneck (location C) where $\tau_c = 3\theta_{\text{eff}}$, $\theta_{\text{eff}} = 1.5$

Figure 3.10: $|Sg_d|$ as a function of frequency; effect of the disturbances d_1 to d_5 on the bottleneck flow, for the three different locations of TPM given in Example 3.7.

mation of $\omega_S \approx \frac{1}{2\theta}$*, the estimated minimum back off becomes slightly larger than the observed back off. Note that even though the difference in effective time delay between location A and B is now much larger than in Example 3.5, the minimum back off is almost the same. The effective time delay with TPM at location B is still large compared to the most important disturbance time constants, so a stationary analysis is still valid.*

3.A.2 Comments on mathematical approach

A mathematical approach to estimate the necessary back off is treated by e.g. Perkins and coauthors (Narraway *et al.*, 1991; Narraway and Perkins, 1993, 1994; Heath *et al.*, 1996; Loeblein and Perkins, 1998, 1999) and Romagnoli and coauthors (Bandoni *et al.*, 1994; Bahri *et al.*, 1996; Figueroa *et al.*, 1996).

Narraway *et al.* (1991) present a method to assess the impact of disturbances on plant economics. Their approach is to perform an economic evaluation of the necessary back off (dynamic economics) to select the control structure (pairing) that minimize the economic impact of disturbances on the process economics. They consider so-called stationary disturbances that are fast disturbances which do not change the steady-state optimum but requires back off since they affect the size of the dynamic operating region. The analysis is performed to a linearized plant dynamic model with assumption of perfect control to the chosen control objectives.

Narraway and Perkins (1993) presents a modification of the method proposed in Narraway *et al.* (1991) for the *a priori* assessment of the effect of disturbances on the economics, in addition to a branch and bound algorithm for the choice of control structure based on the economic criteria. Further, Heath *et al.* (1996) modifies the method by using multiloop PI structures tuned by Ziegler Nichols gains/resets instead of the assumption of perfect control in the control structure selection algorithm.

Loeblein and Perkins (1999) integrate dynamic economics and average deviation from optimum in order to obtain a unified measure for the economic performance by adding the back off from the dynamic economics and from average deviation from optimum. Regulatory back off is evaluated using the unconstrained MPC law with QP algorithm for a stochastic description of disturbances. This leads to a quadratic program which can be solved analytically since the inequality constraints on the input variables are neglected during the back off calculation. The statistical variation of the variables to which constraint are to be applied is described by a density function of a Gaussian distribution with zero mean and known covariance. The regulatory back off is described with a probability that is specified a priori.

To find the necessary back off by using a detailed model-based approach is unrealistic to solve exact for real systems. It requires a dynamic model of the plant together with disturbance characteristics, where the information is limited, especially prior to plant operation. In addition, the variations in the controlled variables are dependent on the regulatory control structure and its parameters and the use of advanced process control (e.g. MPC).

Chapter 4

Dynamic degrees of freedom for tighter bottleneck control

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In many cases, optimal plant operation is the same as maximum throughput. To realize maximum throughput, tight control of the bottleneck unit(s) is necessary. Dynamic degrees of freedom can be used to obtain tighter bottleneck control. Here, "dynamic" means that the degree of freedom has no steady-state effect on plant operation. For example, most inventories (levels) have no steady-state effect. Nevertheless, temporary changes of inventories can allow for dynamic changes in the flow through the bottleneck that keeps the process closer to its bottleneck constraint and increase the throughput. A simple structure is to use a single-loop bottleneck controller that adjusts the feed flow, combined with a simple ratio control scheme that adjusts the dynamic degrees of freedom. The idea is to change all the flows upstream of the bottleneck simultaneously, instead of waiting for inventory loops to move the feed rate change through the units. The required buffer volume for plant design is analyzed for upstream disturbances and bottleneck set point changes.

4.1 Introduction

In many cases, prices and market conditions are such that optimal operation is the same as maximizing plant throughput. In this case, the optimum lies at constraints, and in order to maximize throughput, the flow through the bottleneck(s) should be at its maximum at all times (Chapter 3). If the actual flow through the bottleneck is not at its maximum at any given time, then this gives a loss in production that can never be recovered (sometimes referred to as a ”lost opportunity”). Tight bottleneck control is therefore important for maximizing throughput and avoiding losses.

In existing plants, the most common approach for controlling the throughput is to set the feed flow at the inlet of the plant and use inventory control in the direction of flow (Price *et al.*, 1994). One reason for this is that most of the control structure decisions are done at the design stage (before the plant is built), where one usually assumes a fixed feed rate. However, tight bottleneck control requires that the throughput manipulator (TPM) is located close to the bottleneck (Skogestad, 2004). The term “close to the bottleneck” means that there is a short effective delay from the input (TPM) to the output (bottleneck flow).

Ideally the TPM should be located at the bottleneck, but this may not be desirable (or even possible) for other reasons. First, if the TPM is moved, the inventory loops must be reconfigured to ensure self-consistency (Chapter 2). Second, there may be dynamical reasons for avoiding a so-called on-demand control structure with inventory control opposite the direction of flow, which is required upstream of the TPM to ensure self-consistency. Luyben (1999) points out several inherent dynamic disadvantages with the on-demand structure, including propagation of disturbances, dynamic lags, process time constants and interactions. Third, if a bottleneck(s) moves in the plant due to disturbances, then single-loop control requires relocation of TPM and reconfiguration of inventory loops. Thus, in practice one is often left with a fixed throughput manipulator, usually the feed rate. This usually leads to a large effective delay (“long loop”) because the bottleneck is usually located inside the plant. This leads to an economic loss because of a large required back off from the bottleneck constraints.

Instead, with the TPM fixed, for example at the feed, one may introduce additional degrees of freedom to reduce the back off:

1. For plants with parallel trains one may use crossover and splits (Aske *et al.*, 2008). This are “extra” degrees of freedom that usually cannot be used by a single unit.

2. More generally, one may use “dynamic” degrees of freedom. This is the topic of the present paper. By “dynamic” degrees of freedom we mean manipulated variables with no steady-state effect. The most common examples are liquid inventories (levels) and buffer tank inventories.

The idea is to change the inventory to make temporary flow rate changes in the units between the TPM (feed) ant the bottleneck. This may give tighter bottleneck control, but the cost is that the inventory itself will be less tightly controlled. However, in many cases, inventories need only to be kept within a given range and tight set point control is not needed.

Faanes and Skogestad (2003) defined a buffer tank (surge tank) as a unit where the holdup (volume) is exploited to provide improved operation. They applied control theory to the design of buffer tanks, including deciding on the number of tanks and tank volumes required to dampen the fast (i.e., high-frequency) disturbances, which cannot be handled by the feedback control system. In this paper, the issue is to use the buffer volume to introduce dynamic flow rate changes.

There are also related issues in business systems. Supply chains are sometimes modelled as continuous processes and Schwartz *et al.* (2006) used simulation to study decision policies for inventory management. To improve the financial benefits, they use the inventory set points for intermediate storage subject to maintain acceptable performance in the presence of significant supply and demand variability and forecast error as well as constraints on production, inventory levels, and shipping capacity.

The organization is as follows. Section 4.2 explains how to include dynamic degrees of freedom using either single-loop with ratio control or using a multivariable controller. The use of dynamic degrees of freedom for tighter bottleneck control is demonstrated by an example in Section 4.3. Transfer functions are developed for the single-loop with ratio control structure in Section 4.4 and these functions are further analyzed to estimate the required inventory for disturbances (Section 4.5). A discussion follows in Section 4.6. A summary of the implications for design of inventory tanks is given in Section 4.7 before the paper is concluded in Section 4.8.

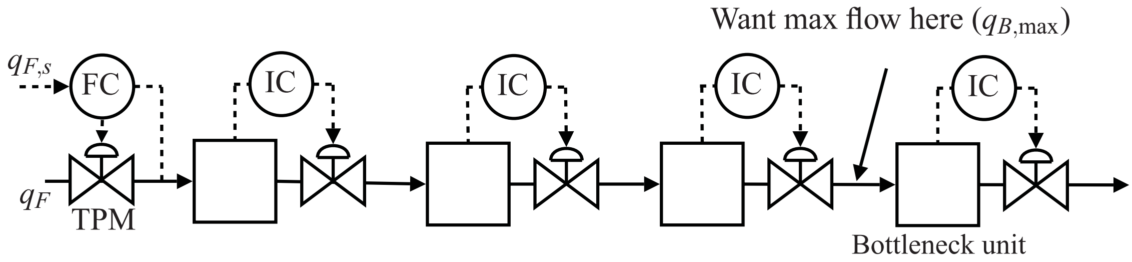
4.2 Alternative strategies for bottleneck control

Assume that the objective is to maximize the flow through the bottleneck and that the feed rate is available as a degree for freedom (throughput manipulator, TPM). Figure 4.1 shows four ways of achieving this using simple single-loop control structures.

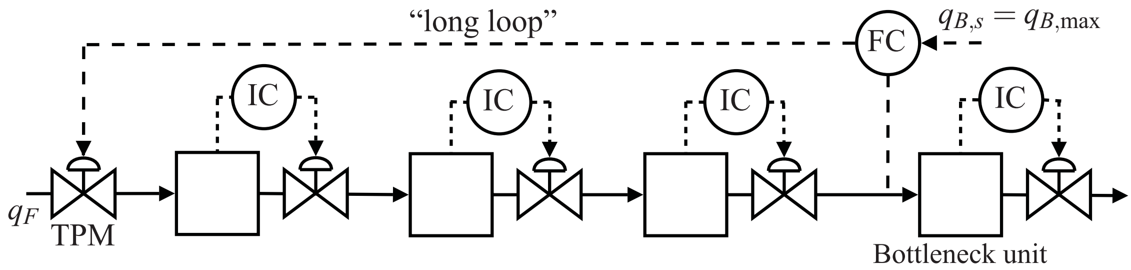
In the traditional configuration in Figure 4.1(a), the feed rate is the degree of freedom for manipulating throughput (TPM), and inventory control is in the direction of flow. To maximize the flow through the bottleneck, the operators change the feed valve manually based on information about the plant operation and experience. However, careful attention by the operators is required in order to keep the bottleneck flow close to its maximum at all times, so we want to use automatic control.

Alternative 1: Single-loop control of bottleneck flow using the feed rate. (Figure 4.1(b))

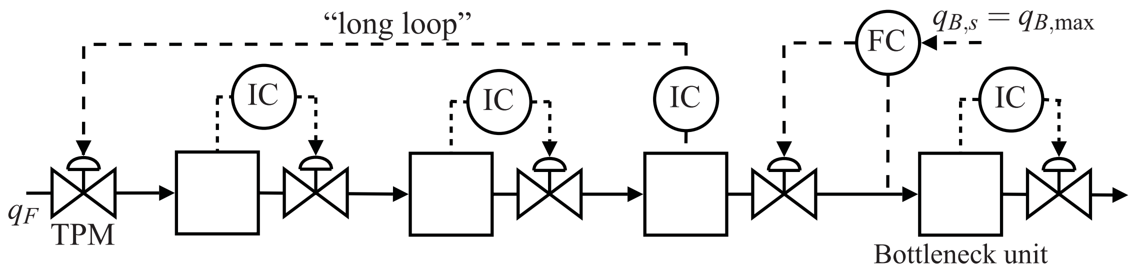
The simplest is to use single-loop feedback control where the feed rate (TPM) is



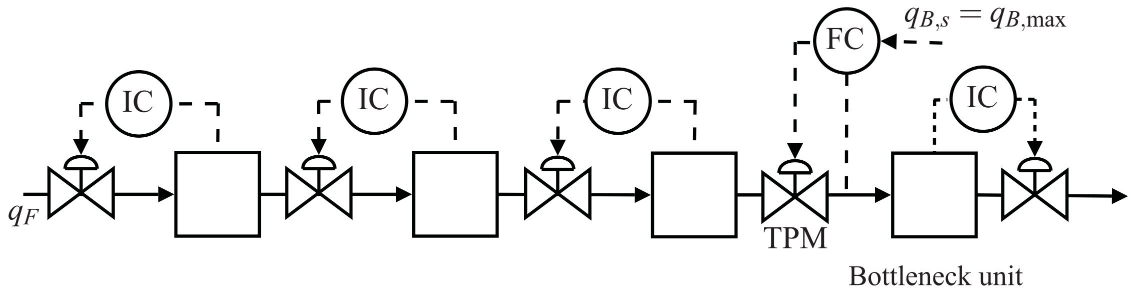
(a) Traditional configuration (manual control of feed rate)



(b) Alternative 1: Single-loop control where the feed rate controls the bottleneck flow (Problem: "long loop" with large effective delay).



(c) Alternative 2: Throughput manipulator moved to bottleneck without reconfiguration of the inventory loops in the other units. Feed rate controls the "lost task", in this case the upstream inventory (Problem: "long loop" with large effective delay).



(d) Alternative 3: Throughput manipulator moved to bottleneck with reconfiguration of inventory loops upstream of bottleneck (Problem: reconfiguration).

Figure 4.1: Simple single-loop control structures for maximizing bottleneck flow in serial process. IC stands for inventory controller (e.g. level controller).

manipulated to keep the bottleneck flow close to its maximum. However, there is often a large effective delay from the feed flow (input) to the bottleneck flow (output), so tight control of the bottleneck flow is not possible because of disturbances.

Alternative 2: Move TPM from feed to bottleneck and let feed control "lost task". (Figure 4.1(c))
The bottleneck flow is set directly at its maximum, which corresponds to moving the throughput manipulator to the bottleneck. The inventory loops are not reconfigured, so the feed rate now needs to take over the "lost task" which in this case is control of the inventory upstream of the bottleneck. In this case, tight bottleneck control is achieved, but inventory control may be poor, leading to possibly emptying or overflowing the upstream tank because of a large effective delay from the feed flow (input) to the tank (output).

Alternative 3: Reconfigure inventory control. (Figure 4.1(d))
The TPM is moved to the bottleneck and all the upstream inventory loops are reconfigured to be in the opposite direction of flow upstream the bottleneck. In this case, both tight bottleneck control and good inventory control may be achieved. However, the reconfiguration of inventory loops is usually very undesirable from a practical point of view.

In summary, none of these alternatives are desirable. To improve control and keep the flow through the bottleneck closer to its maximum at all times, we would like to have additional degrees of freedom, and the only ones that are normally available are the inventories (holdups) in the buffer tanks, which can be used to make dynamic flow changes. The word "dynamic" is used because most inventories have no steady-state effect on plant operation.

The main idea is as follows: To change the flow through the bottleneck, for example, to increase it, we temporarily reduce the inventory in the upstream holdup volume. However, this inventory needs to be kept within bounds, so if we want to increase the bottleneck flow permanently, we need to increase the flow into this part of the process and so on, all the way back to the feed (throughput manipulator). The simplest (but not generally optimal) approach is to use a "ratio" control system where all flows upstream the bottleneck are increased simultaneously by the same relative amount. The idea is illustrated in Figure 4.2.

Alternative 1D: Single-loop plus ratio control. (Figure 4.2(a))
The idea is to control the bottleneck flow by simultaneously changing all the flows upstream of the bottleneck by the same relative amount. The advantage is that the effective delay from the feed to the bottleneck may be significantly reduced and even eliminated in some cases. However, the dynamic flow changes are counteracted by the inventory controllers. In particular, note that the feed flow is the only

degree of freedom that has a steady-state effect on the bottleneck flow. The strategy may also be viewed as a "ratio feedforward controller" from the feed flow to the downstream flows.

Alternative 2D: Move TPM to bottleneck and add ratio control to "lost task". (Figure 4.2(b))
The TPM is moved to the bottleneck and the "lost task" (inventory upstream the bottleneck) is controlled by the feed rate. The use of ratio control is the same as for Alternative 1D. The effective delay from the feed rate to the lost task is reduced by using ratio control.

Alternative 4: Multivariable controller. (Figure 4.2(c))
A multivariable controller (e.g. MPC) uses the feed rate and the inventories as manipulated variables (MVs). The controlled variables (CVs) are the bottleneck flow and inventory constraints.

In this paper we focus on Alternative 1D. One reason is that the analytic treatment is quite simple. To understand how the "ratio control" works, consider first inventory control of an individual buffer tank. The "normal" feedback inventory controller (IC) can be written

$$q = K(s)(I - I_s) + q_0 \tag{4.1}$$

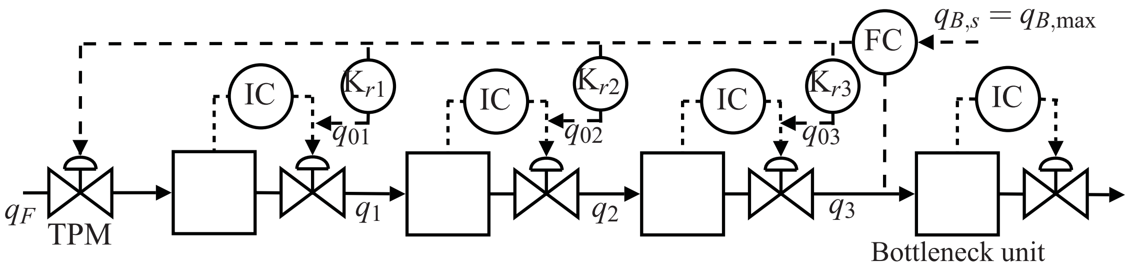
where I is the inventory (e.g. level), I_s is its set point, q is the flow in our out of the tank (output from controller) and q_0 is the flow bias term of the controller. The feedback controller $K(s)$ has a negative sign if q is an inflow and a positive sign if q is an outflow. Now, to introduce the inventory as a degree of freedom one can either adjust the inventory set point (I_s) or adjust the bias (q_0). The most obvious is to adjust the inventory set point I_s, but it is more direct in terms of flow changes to adjust the bias. Actually, the two approaches are not very different, because a change in q_0 can equivalently be implemented as a set point change by choosing $I_s = -q_0/K(s)$. In this paper, we choose to use the bias q_0 as the dynamic degree of freedom for ratio control.

Let now q_F be the feed flow computed by the flow controller (FC) in Figure 4.2(a). Then, the bias adjustment in all the inventory controllers (IC) in the figure is

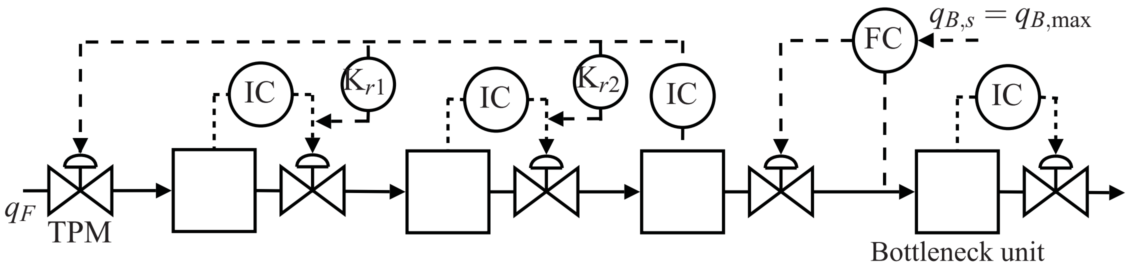
$$\Delta q_0 = K_r \Delta q_F \tag{4.2}$$

where K_r is the steady-state gain for the effect of q_F on q_0. The overall IC then becomes

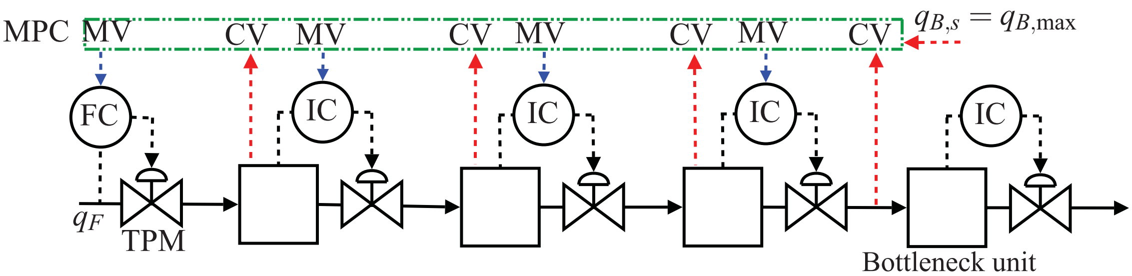
$$\Delta q = K(s)(I - I_s) + \underbrace{K_r \Delta q_F}_{\Delta q_0} \tag{4.3}$$



(a) Alternative 1D: The feed rate (TPM) controls the bottleneck flow with use of inventories as additional dynamic degrees of freedom (here shown using a "bias" adjustment of the flow from each unit).



(b) Alternative 2D: The TPM is moved without reconfiguration of inventory loops. The feed rate controls the lost task, in this case the inventory upstream the bottleneck (large effective delay) and inventories are used as dynamic degrees of freedom.



(c) Alternative 4: Multivariable control structure (e.g. MPC) where the feed rate and the inventory controller set points are MVs.

Figure 4.2: Structures for controlling bottleneck flows that use inventories as dynamic degrees of freedom (with no reconfiguration of the inventory loops). Alternative 1D is studied in this paper. IC stands for inventory controller (e.g. level controller) and K_i is a constant gain (ratio controller).

	De-ethanizer	De-propanizer	De-butanizer	Butane splitter
Number of trays	32	48	40	92
Feed tray	20	20	19	45
Nominal feed [*kmol/min*]	75.6	29.4	11.6	8.1
Holdup sump (M_B) [*Kmol*]	121	38	18	66
Holdup condenser (M_D) [*Kmol*]	226	227	62	88
Nominal boil up [*Kmol/min*]	88.6	32.3	9.9	34.5
Nominal reflux [*Kmol/min*]	71.5	27.1	8.6	25.9

Table 4.1: Sizes and nominal flows for the distillation columns in Example 4.1.

The important point to note is that there are no dynamics in K_r. This means that all the flows q are changed simultaneously when q_F changes. This is not generally optimal, but it is the simplest and is used in this paper.

4.3 Introductory example

The example given below illustrates how tight bottleneck control can be obtained by use of dynamic degrees of freedom.

Example 4.1. ***Four distillation columns in series.*** *Consider four distillation columns in series, as shown in Figure 4.3. The four columns represent the liquid upgrading part of a gas processing plant and consist of a deethanizer, a depropanizer, a debutanizer and a butane splitter. Assume that the butane splitter (the last unit) has the lowest processing capacity and is therefore the bottleneck unit. The throughput is manipulated at the feed to the first column. The idea is to use the column inventories (sump or condenser drum holdup) as dynamic degrees of freedom to obtain tighter bottleneck control.*

The distillation column models are implemented in Matlab/Simulink. Each of the four columns is modelled as multicomponent distillation with one feed and two products, constant relative volatilities, no vapor hold-up, constant molar flows, total condenser and liquid flow dynamics represented by the Francis weir formula. All columns use the "LV-configuration" where distillate (D) and bottoms flow (B) are used for inventory control (M_D and M_B). To stabilize the column composition profile, all columns have temperature control in the bottom section by manipulating the boilup. Some relevant sizes and flows for the columns are given in Table 4.1. Note that there is a crossover flow from the bottoms of the deethanizer where $q_{cross} = 15.8$ kmol/min, as displayed in Figure 4.3.

Four different control structures for maximizing throughput are tested:

1. ***Manual****. Traditional (manual) control of the throughput.*

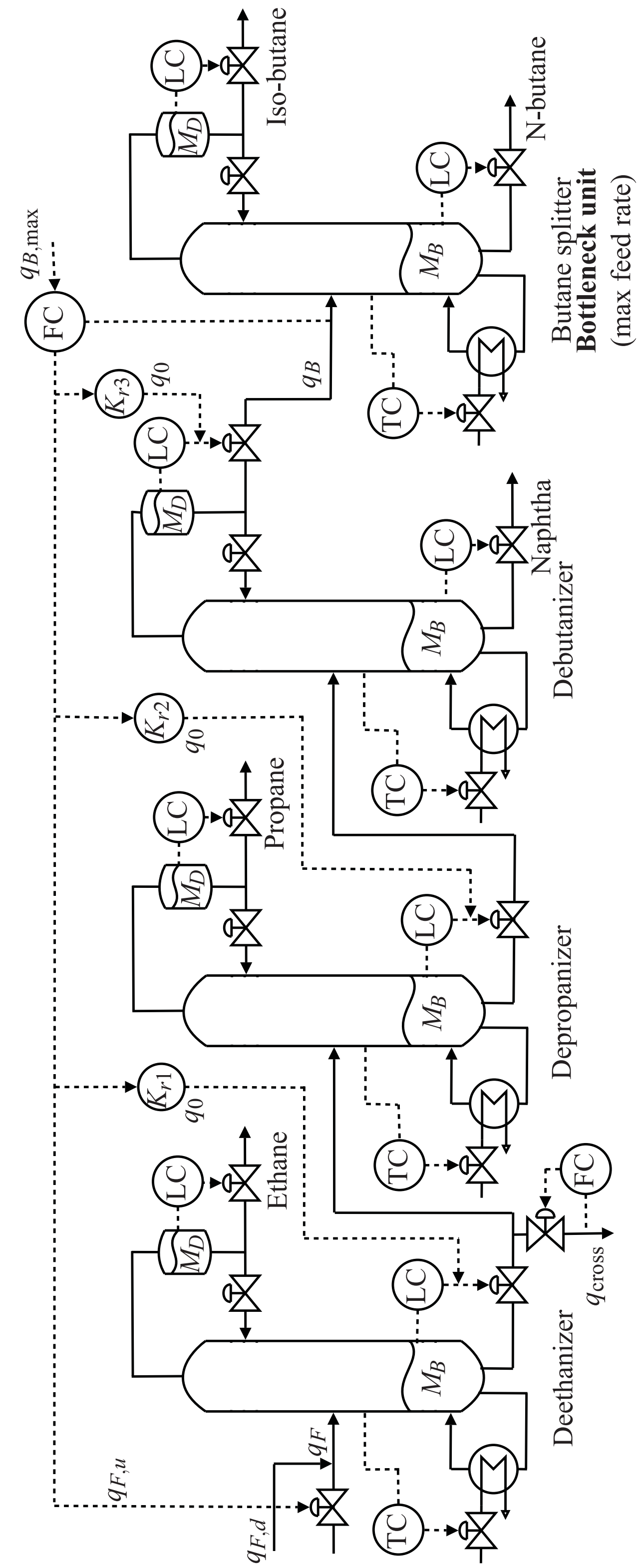


Figure 4.3: Distillation process: Four columns in series, here shown with throughput controlled by using single-loop with ratio control (Alternative 1D).

2. ***Single-loop***. *Single-loop control where the bottleneck flow is controlled using the feed rate (Alternative 1 in Section 4.2).*

3. ***Single-loop with ratio***. *Use of the inventories as dynamic degrees of freedom by adding a bias (q_0) to the inventory controller outputs as in Figure 4.3. (Alternative 1D in Section 4.2).*

4. ***Multivariable***. *MPC with the feed rate and the inventory set points as MVs and the bottleneck flow and level constraints as CVs (Alternative 4 in Section 4.2).*

The column inventories M_D and M_B are controlled with P-controllers with gain $K_c = 1/\tau_V$. Here we use "smooth" level control where we set $\tau_V = V_{tank}/q_{out}$ (Skogestad, 2006) where q_{out} is the flow out of the volume (D or B). With a nominal half-full tank we can then handle a 50% change in the product flow (D and B) without emptying or overfilling. Actually, the flow into the inventory is considerably larger, but disturbances in boilup (or reflux) are counteracted by the temperature controller (Skogestad, 2007). The temperature controllers (TC) are tuned with SIMC PI-tuning (Skogestad, 2003) with $\tau_c = 0.5$ min. The TCs and ICs tunings are identical in all four columns and in the four tested control structures.

Two disturbances are considered. First, at $t = 10$ min, we make a set point change in the bottleneck flow, for example, caused by a disturbance in the bottleneck unit (the butane splitter). Second, at $t = 210$ min, there is an unknown change in the feed rate.

1. *For manual control, we assume that a skilled operator can immediately change the feed rate to the value corresponding to the new bottleneck flow set point. However, we assume that the operator does not notice the unmeasured feed flow disturbance, so no adjustment is therefore done for the feed rate disturbance.*

2. *For the single-loop control structure we want smooth tuning to avoid overshoot and "aggressive" use of the feed valve. Therefore, the bottleneck flow controller (FC) is tuned with SIMC tunings with $\tau_c = 3\theta$ for smooth tuning (Skogestad, 2006). This gives a PI-controller with $K_c = 3.0$ and $\tau_I = 14$ min.*

3. *For the single-loop control with ratio (bias) adjustment (Alternative 1D), there is no effective delay and the bottleneck flow controller (FC) is tightly tuned with a short integral time ($K_c = 0.5$ and $\tau_I = 0.3$ min), which are typical FC tuning parameters.*

4. *In the multivariable structure the FC at the feed is omitted and the MPC manipulates directly the feed valve. The built-in MPC toolbox in Matlab is*

used and tuned with a low penalty on the use of inventories (MV moves) and a high penalty on the deviation from the bottleneck flow set point (CV set point).

The four control structures are evaluated in terms of how tightly the bottleneck flow (q_B) is controlled in spite of disturbances. As mentioned, two disturbances are considered:

- *At $t = 10$ min: 5% increase in bottleneck flow set point ($q_{B,s}$).*
- *At $t = 210$ min: 8% decrease in the feed rate to the deethanizer (q_F). The net feed flow is $q_F = q_{F,u} + q_{F,d}$, where $q_{F,u}$ is the flow contribution from the controller (initially $q_{F,d} = 0$ and $q_F = q_{F,u} = 100$, but then $q_{F,d} = -8$ at $t = 210$).*

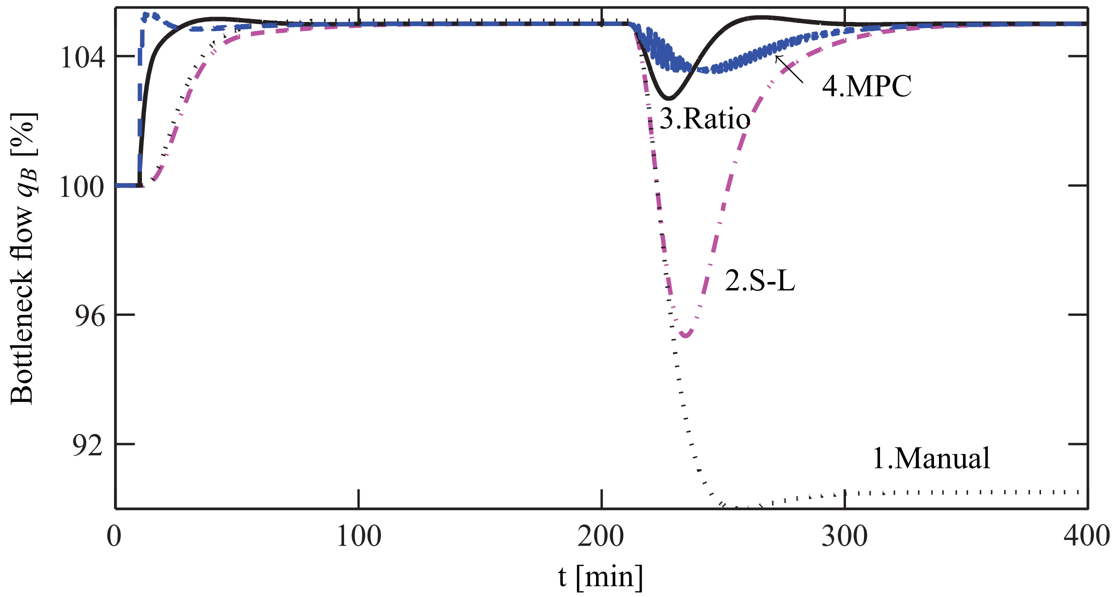
The resulting bottleneck flow (q_B), the net feed flow (q_F) and the inventories used as dynamic degrees of freedom (deethanizer M_B, depropanizer M_B and debutanizer M_D) for the four different control structures are displayed in Figure 4.4. The first observation is that we have significantly tighter bottleneck control with ratio control and MPC (Alternative 3 and 4) where inventories are used as dynamic degrees of freedom (Figure 4.4(a)). The inventories (levels) are quite tightly controlled with surprisingly small variations as shown in Figure 4.4. There is some steady-state offset because we use P-control (no integral action).

In summary, we can operate closer to the capacity constraint of the butane splitter (reduce the back off) and hence increase the throughput when dynamic degrees of freedom (inventories) are used.

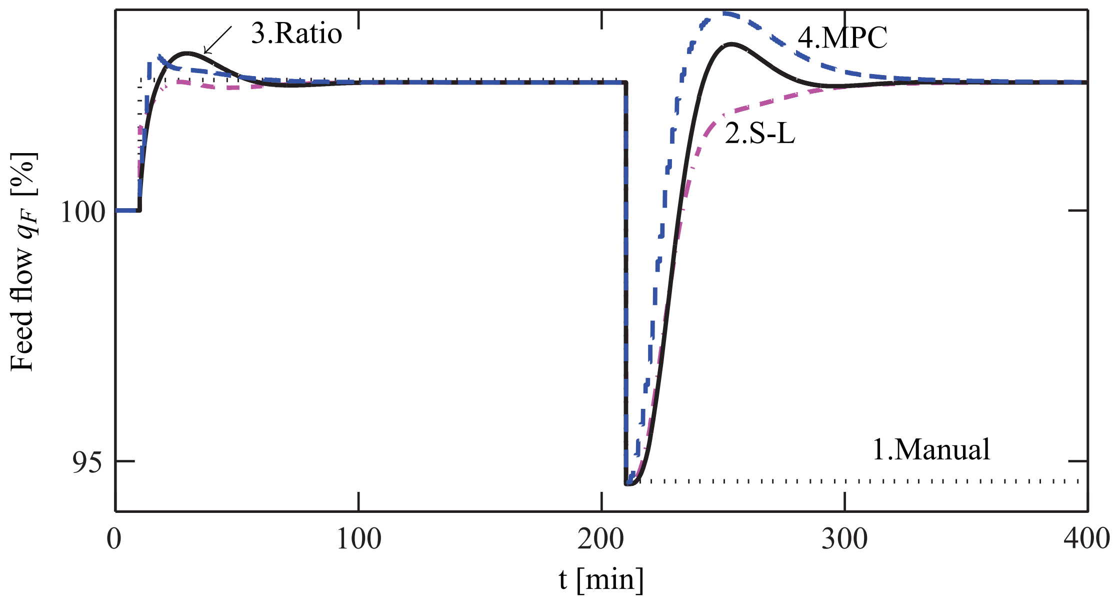
4.4 Analysis of use of dynamic degrees of freedom

In this section, the single-loop with ratio control scheme (Alt. 1D in Section 4.2) is analyzed in more detail. The main reason is to later use the results to estimate the required buffer volume for dynamic degrees of freedom (Section 4.5). The dynamic degrees of freedom are either the inventory set point (V_s) or the bias adjustment (q_0), but here we only consider q_0.

To make the control structure in Figure 4.2(a) clearer, consider a similar structure, which consists of only one unit, or more precisely, a process unit (G) followed by an inventory (G_V), as displayed in Figure 4.4. The outflow q_B from the inventory is assumed to be the bottleneck flow that should be tightly controlled. However, q_B cannot be set freely because it is already used for level control. Thus, to improve the dynamic response, we add a bias term q_0 which is set in proportion to the net feed flow q_F, computed by the bottleneck controller. This single-loop

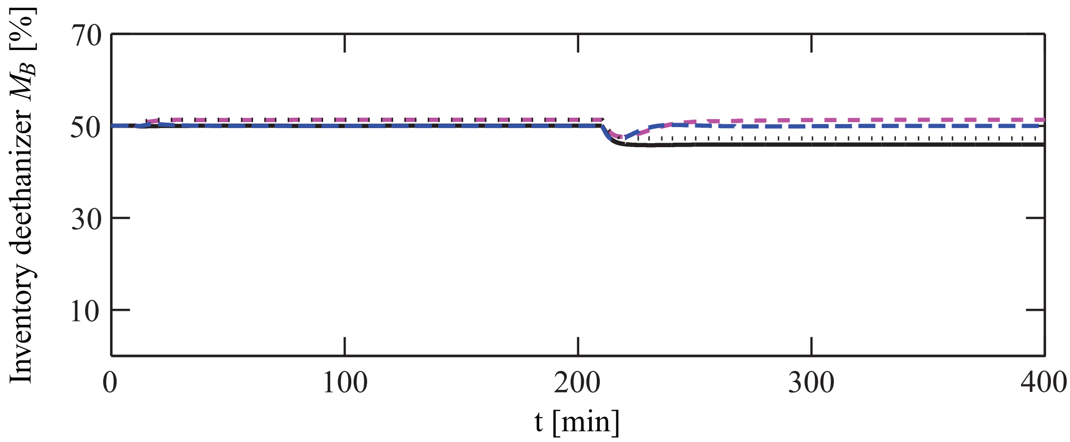


(a) Response in Bottleneck flow q_B (CV)

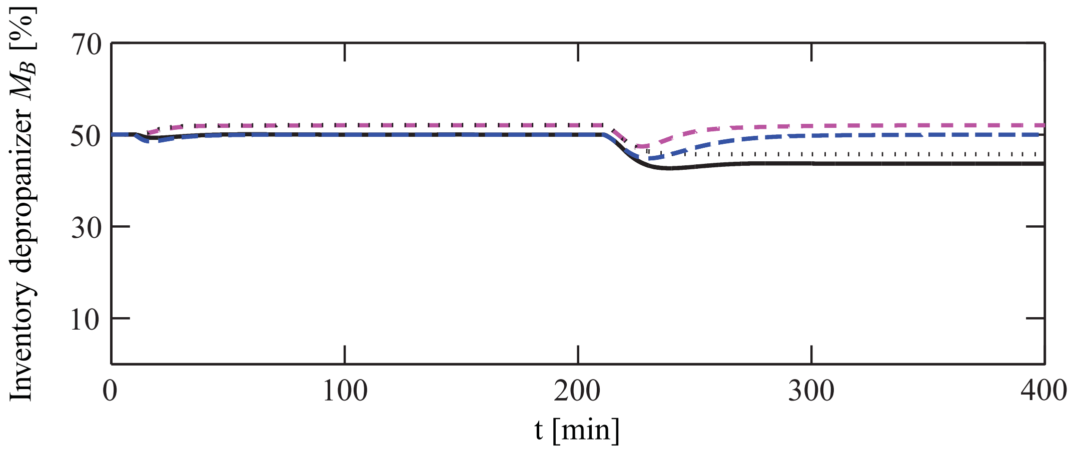


(b) Responses in Feed flow q_F (MV)

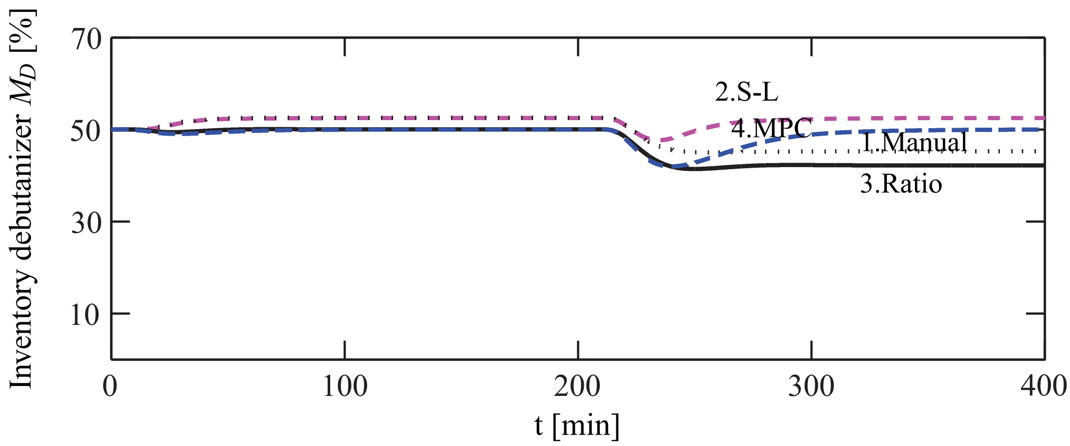
Figure 4.4: Continued on next page.



(c) Responses in deetanizer bottoms level M_B (dynamic degree of freedom no. 1)



(d) Responses in depropanizer bottoms level M_B (dynamic degree of freedom no. 2)



(e) Responses in debutanizer distillate level M_D (dynamic degree of freedom no. 3)

Figure 4.4: Bottleneck control of the distillation process for four different control structures. 1) Manual control (dotted), 2) Single-loop control (dash-dotted), 3) Single-loop with ratio (bias adjustment on inventory flows, solid), 4) MPC using both feed rate and inventories as MVs (dashed). Disturbances: 5% increase in bottleneck flow set point $q_{B,s}$ at $t = 10$ and 8% unknown decrease in feed rate q_F at $t = 210$.

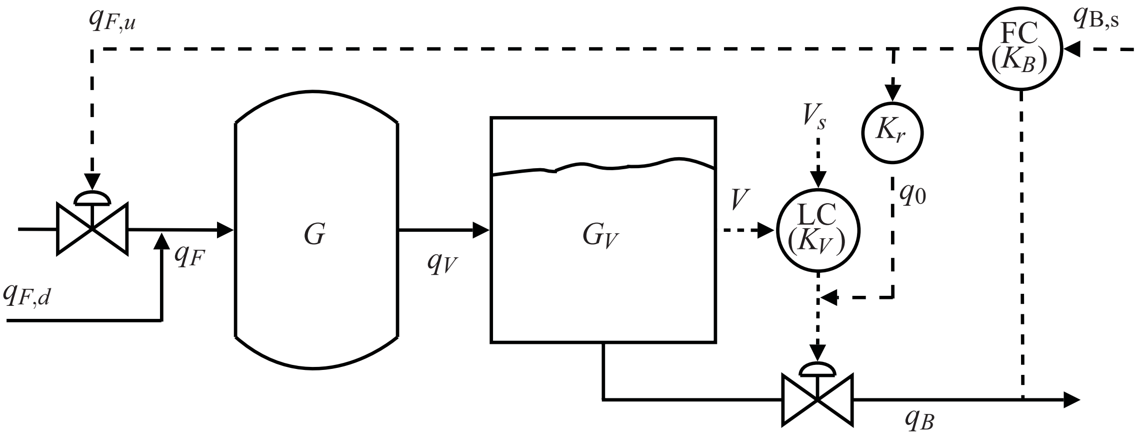


Figure 4.4: Example of single-loop control with a linear bias adjustment added to the level controller output.

with static ratio control structure can be viewed as feedforward control combined with feedback, where the flows in downstream units are increased proportionally to the feed rate q_F. This idea is also used sometimes by skilled operators, e.g. during start-up of a plant. We will now analyze this system in more detail.

The mass balance for the holdup volume V, assuming constant density, is

$$\frac{dV}{dt} = q_V - q_B \tag{4.4}$$

where q_V is the inflow and q_B is the outflow (see Figure 4.4). Upon taking the Laplace transform and introducing deviation variables, we get

$$V(s) = \frac{1}{s}(q_V - q_B) \tag{4.5}$$

Thus, the transfer function for the inventory is $G_V(s) = \frac{1}{s}$. Next, assume that the inlet flow to the buffer volume q_V is given by

$$q_V = G(s) \cdot q_F \tag{4.6}$$

where G is the process transfer function for the upstream process between the feed and the buffer volume. The net feed flow q_F is defined as

$$q_F = q_{F,u} + q_{F,d} \tag{4.7}$$

where $q_{F,u}$ is the flow contribution from the bottleneck (flow) controller and $q_{F,d}$ is an unmeasured disturbance in the flow. The bottleneck flow q_B is given by the level controller with transfer function $K_V(s)$ plus the ratio (bias) contribution q_0,

$$q_B = K_V(s)(V - V_s) + q_0 \tag{4.8}$$

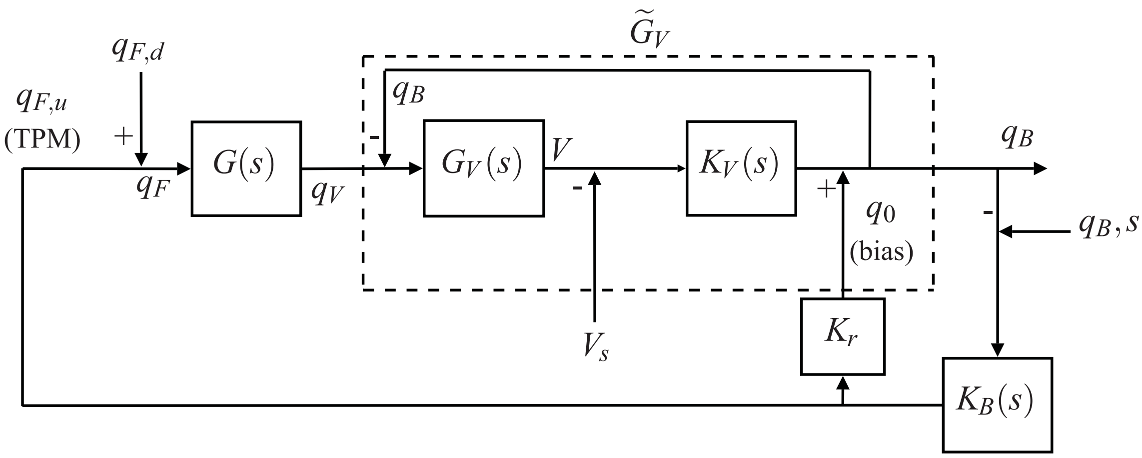


Figure 4.5: Corresponding block diagram of Figure 4.4 in the Laplace domain. q_0 (bias) and V_s (inventory set point) are the dynamic degrees of freedom for control of the bottleneck flow q_B.

where V_s is the set point for the inventory volume. Note that we want the level controller to be a "slow" (averaging) level controller, because otherwise no exploitation of the holdup volume can be obtained. In most cases, we use a proportional-only controller, where $K_V(s) = 1/\tau_V$ (a constant). Typically, to be able to exploit all the volume, τ_V is chosen equal to the nominal residence time (V/q) of a half-full tank (Skogestad, 2006).

The corresponding block diagram of the control structure in Figure 4.4 is given in Figure 4.5. The block K_B is the bottleneck flow controller (FC in Figure 4.4), K_V is the level controller (LC in Figure 4.4) and K_r is the ratio (bias) controller. The block $\widetilde{G}_V$ gives the closed-loop transfer function from the flow into the inventory q_V to the bottleneck flow q_B and consists of the buffer volume plus the level controller. This block also has the two dynamic degrees for bottleneck control as inputs, namely V_s and q_0.

Without active bottleneck control

With only the inventory controller (i.e., without the bottleneck control active, $K_B = 0$) we get from the block diagram (in deviation variables)

$$q_B = \frac{K_V G_V G}{1+K_V G_V} \cdot q_F + \frac{1}{1+K_V G_V} \cdot q_0 - \frac{K_V}{1+K_V G_V} \cdot V_s \tag{4.9}$$

$$V = \frac{G_V G}{1+K_V G_V} \cdot q_F - \frac{G_V}{1+K_V G_V} \cdot q_0 + \frac{K_V G_V}{1+K_V G_V} \cdot V_s \tag{4.10}$$

Introducing $G_V(s) = 1/s$ gives

$$q_B = \frac{K_V G}{s+K_V} \cdot q_F + \frac{s}{s+K_V} \cdot q_0 - \frac{K_V s}{s+K_V} \cdot V_s \tag{4.11}$$

$$V = \frac{G}{s+K_V} \cdot q_F - \frac{1}{s+K_V} \cdot q_0 + \frac{K_V}{s+K_V} \cdot V_s \tag{4.12}$$

The steady-state effect is obtained by setting $s = 0$. Thus, we note, as expected, that only q_F has a steady-state effect on the bottleneck flow q_B.

For the further analysis, we assume that the process $G(s)$ is first-order with gain K_r and time constant τ_G

$$G = \frac{K_r}{\tau_G s + 1} \tag{4.13}$$

We assume that the level controller is a proportional controller

$$K_V \triangleq \frac{1}{\tau_V} \tag{4.14}$$

Now, Equations (4.11) and (4.12) become:

$$q_B = \frac{K_r}{(\tau_G s+1)(\tau_V s+1)} \cdot q_F + \frac{\tau_V s}{\tau_V s+1} \cdot q_0 - \frac{s}{\tau_V s+1} \cdot V_s \tag{4.15}$$

$$V = \frac{K_r \tau_V}{(\tau_G s+1)(\tau_V s+1)} \cdot q_F - \frac{\tau_V}{\tau_V s+1} \cdot q_0 + \frac{1}{\tau_V s+1} \cdot V_s \tag{4.16}$$

The effective delay from q_F to q_B in this simple case with PI control is, using the half rule (Skogestad, 2003), $\theta_{\text{eff}} = \min(\frac{\tau_V}{2}, \frac{\tau_G}{2})$. From Equation (4.15) and (4.16), the block $\widetilde{G}_V$ in Figure 4.5 is summarized in Table 4.2. The transfer functions given in Table 4.2 are of interest also for MPC.

q_V $\downarrow$	q_0 $\downarrow$	V_s $\downarrow$	
$\frac{1}{\tau_V s+1}$	$\frac{\tau_V s}{\tau_V s+1}$	$\frac{-s}{\tau_V s+1}$	$\rightarrow q_B$
$\frac{\tau_V}{\tau_V s+1}$	$\frac{-\tau_V}{\tau_V s+1}$	$\frac{1}{\tau_V s+1}$	$\rightarrow V$

Table 4.2: Block $\widetilde{G}_V$ in Figure 4.5 with $G_V(s) = 1/s$ and $K_V(s) = 1/\tau_V$ (P-control).

4.5 Analysis of single-loop with ratio control

In this section, the objective is to find the required buffer tank volume V_{min}. In principle, this can be done by either dynamic simulation or analytically. Here we choose to use the single-loop with ratio control result from the previous section to derive an analytical expression for the required inventory to find an estimate. The most common control structure for dynamic degrees of freedom would probably be MPC, but as shown in the introductory example (Figure 4.4), the inventory volume variations in these two control structure were similar (see also Tables 4.4 and 4.5), although they will depend on the MPC tuning. Note that V denotes the volume of the liquid in the tank and V_{tank} is the actual tank volume.

4.5.1 Developing transfer functions for single-loop with ratio control

Consider Figure 4.4, with one unit followed by a volume where its inventory is exploited dynamically by single-loop with ratio control structure.

Response with ”perfect” bias adjustment (ratio controller)

We assume “perfect” static bias adjustment where a feed change is accomplished by a corresponding relative change in downstream flows. This corresponds to the static bias adjustment

$$q_0 = K_r^* q_{F,u} \tag{4.17}$$

where K_r^* is the nominal steady-state ratio $\Delta q_B / \Delta q_{F,u}$. If there are no flow splits or junctions between the feed and the bottleneck unit, then $K_r = 1$. We now want to study the effect of adding the bias ratio adjustment. We assume that the inventory set point is constant ($V_s = 0$). Then, from Equation (4.15), the effect of $q_{F,u}$ and $q_{F,d}$ on the bottleneck flow q_B is

$$\begin{aligned} q_B &= \frac{1 + \tau_V s(\tau_G s + 1)}{(\tau_V s + 1)(\tau_G s + 1)} \cdot K_r \cdot q_{F,u} + \frac{1}{(\tau_V s + 1)(\tau_G s + 1)} \cdot K_r \cdot q_{F,d} \\ &= h_{q_B q_{F,u}}(s) \cdot q_{F,u} + h_{q_B q_{F,d}}(s) \cdot q_{F,d} \end{aligned} \tag{4.18}$$

Note that there is a “direct effect” from $q_{F,u}$ to q_B, because of the bias from the static ratio controller. Thus, the effective delay from $q_{F,u}$ to q_B is zero and “perfect” control of q_B is in theory possible. However, one must take into account the variations in $q_{F,u}$ and the volume (level) constraints.

Similarly, from Equation (4.16), the effect of $q_{F,u}$ and $q_{F,d}$ on the volume (level) V is

$$V = \frac{-\tau_G \tau_V s}{(\tau_V s + 1)(\tau_G s + 1)} \cdot K_r \cdot q_{F,u} + \frac{\tau_V}{(\tau_V s + 1)(\tau_G s + 1)} \cdot K_r \cdot q_{F,d} \tag{4.19}$$

Response with "perfect" bottleneck flow controller

To study the expected variations in volume (level), assume a "perfect" bottleneck flow controller K_B that gives $q_B = q_{B,s}$ at all times. This assumption requires the fastest variations in the manipulated input may be expected to lead to the worst-case variation in inventory (V) with the given inventory controller tuning.

Setting $q_B = q_{B,s}$ (perfect bottleneck control), the resulting change in the feed rate from (4.18) is:

$$\begin{aligned} q_{F,u} &= \frac{1}{h_{q_B q_{F,u}}} \cdot q_{B,s} - \frac{h_{q_B q_{F,d}}}{h_{q_B q_{F,u}}} \cdot q_{F,d} \\ &= \frac{(\tau_V s+1)(\tau_G s+1)}{1+\tau_V s(\tau_G s+1)} \frac{1}{K_r} \cdot q_{B,s} - \frac{1}{1+\tau_V s(\tau_G s+1)} \cdot q_{F,d} \end{aligned} \tag{4.20}$$

and from Equations (4.19) and (4.20), the resulting change in the inventory with perfect bottleneck control is:

$$\begin{aligned} V &= \frac{-\tau_G \tau_V s}{1+\tau_V s(\tau_G s+1)} \cdot q_{B,s} + \frac{\tau_V}{1+\tau_V s(\tau_G s+1)} \cdot K_r \cdot q_{F,d} \\ &= h_{V q_{B,s}} \cdot q_{B,s} + h_{V q_{F,d}} \cdot q_{F,d} \end{aligned} \tag{4.21}$$

We note that a feed disturbance $q_{F,d}$ has a steady-state effect on the volume (level) because we use a P-only level controller. However, these should be within the allowed bounds when we use an averaging (smooth) level controller when gain $K_V = 1/\tau_V = |\Delta q_0|/|\Delta V_{max}|$ (Skogestad, 2006, Eq.25). A bottleneck flow change $q_{B,s}$ has no steady-state effect of V, but there will be dynamic variations, as studied in more detail below.

4.5.2 Required inventory volume for single unit

The following results are for a single unit (Figure 4.4).

Requirements for bottleneck flow $q_{B,s}$

From (4.21), the transfer function from bottleneck flow changes ($q_{B,s}$) to volume changes (V) with "perfect" bottleneck control is

$$h_{V q_{B,s}} = \frac{-\tau^2 s}{\tau^2 s^2 + 2\tau\zeta s + 1} \text{ where } \tau = \sqrt{\tau_G \tau_V}\ ;\ \zeta = \frac{1}{2}\sqrt{\frac{\tau_V}{\tau_G}} \tag{4.22}$$

The peak magnitude for $h_{Vq_{B,s}}$ occurs at frequency $\omega_{peak} = \frac{1}{\tau} = 1/\sqrt{\tau_G \tau_V}$ (see Appendix 4.A for details) and we get

$$|\Delta V_{peak,B}| = \frac{-\tau^2 \omega_{peak}}{\sqrt{(1-\omega_{peak}^2 \tau^2)^2 + (2\omega_{peak}\tau\zeta)^2}} \cdot |\Delta q_{B,s}| = \frac{\tau}{2\zeta} \cdot |\Delta q_{B,s}| = \tau_G \cdot |\Delta q_{B,s}| \tag{4.23}$$

This means that the peak of $|V|$ is equal to $\tau_G \cdot |\Delta q_{B,s}|$ and is independent of the level tuning τ_V. This somewhat surprising result follows because of the assumption of perfect bottleneck control, which means that the bottleneck flow controller will counteract the level controller actions.

Requirements for upstream disturbances $q_{F,d}$

Consider next unmeasured disturbances in the feed rate. From (4.21), assuming no overshoot (i.e. $\zeta \geq 1$ or $\tau_V \geq 4\tau_G$), the largest volume change is found at steady-state and is given by

$$|\Delta V_{peak,d}| = \left. \frac{K_r \tau_V}{1 + \tau_V s(\tau_G s + 1)} \right|_{s=0} \cdot |\Delta q_{F,d}| = K_r \tau_V \cdot |\Delta q_{F,d}| \tag{4.24}$$

Note here that the volume variation depends directly on the level tuning τ_V, so we may use (4.24) to derive the slowest allowed level tuning.

Acceptable variations in feed rate $q_{F,u}$

We want to avoid too large variations in the feed rate caused by bottleneck set point changes. The transfer function from $q_{B,s}$ to $q_{F,u}$ is given by $1/h_{q_B q_{F,u}}(s)$ (Equation (4.20)). Let us assume $q_{B,s}$ can vary sinusoidally and that we do not want more than 50% overshoot in the manipulated feed rate, that is, $|q_{F,u}/q_{F,ss}| \leq M = 1.5$ at all frequencies, where the steady-state change is $q_{F,ss} = q_{B,s}/K_r$. To achieve this we must require

$$\tau_V \geq \frac{\tau_G}{M-1} = 2\tau_G \tag{4.25}$$

as derived in Appendix 4.B.

4.5.3 Required inventory volume for units in series

We here consider units in series, for example, as shown for the distillation columns in Figure 4.3. In this case, the above expressions do not strictly hold, even for the case when we can approximate the flow dynamics in each part of the process by a first-order response with time constant τ_G. Consider three units in series, where

1 is the first unit, 2 is the intermediate unit, and 3 is the last unit upstream of the bottleneck.

The above expressions do not hold because the counteracting effect of the level control in upstream units is neglected. Nevertheless, let as assume that with perfect bottleneck control the resulting feed rate change is given by (4.20), except that we must use the dynamics for the last unit (unit 3). We then have for the effect of $q_{B,s}$ on $q_{F,u}$:

$$\frac{q_{F,u}}{q_{B,s}} = \frac{(\tau_{V,3}s+1)(\tau_{G,3}s+1)}{1+\tau_{V,3}s(\tau_{G,3}s+1)} \frac{1}{K_{r,3}} \tag{4.26}$$

This is the flow rate change into the first unit (unit 1). Note that if we assume that $\tau_{V,3} >> \tau_{G,3}$ then this transfer function approaches $(1/K_{r,3})$, which means that $q_{F,u}$ changes to its steady-state value (which is $q_{B,s}/K_{r,3}$) and stays there (with no overshoot). We assume in the following that this holds, that is

$$q_{F1,u} = q_{F,u} = (1/K_{r,3})q_{B,s} \tag{4.27}$$

For the other units we similarly get if we neglect the counteracting effect of the upstream level controller.

$$q_{F2,u} = q_{0,1} = (K_{r,1}/K_{r,3})q_{B,s} \tag{4.28}$$

$$q_{F3,u} = q_{0,2} = (K_{r,2}/K_{r,3})q_{B,s} \tag{4.29}$$

$$q_{F4,u} = q_{0,3} = q_{B,s} \tag{4.30}$$

Requirements for bottleneck flow $q_{B,s}$

In (4.23), $\Delta q_{B,s}$ is the flow into the next bottleneck unit (unit 4 in our case). With our assumptions of immediate flow changes, the same expression applies also to the other units and we have that the expected maximum change in inventory volume is

$$|\Delta V_{peak,B,i}| = \tau_{G,i} \cdot \Delta q_{B,i} \tag{4.31}$$

where $\Delta q_{B,i} = (K_{r,i}/K_{r,3}) \cdot \Delta q_{B,s}$ is the steady-state flow change in tank i resulting from a change in the bottleneck flow. We note from the derivation that this formula is only approximate, but nevertheless we find by comparing with simulations that it holds quite well (see below).

Requirements for upstream disturbances

The maximum volume change for disturbances occurs at steady state, which means that (4.24) will hold well also for units in series. The general expression for tank i becomes

$$|\Delta V_{peak,d,i}| = \tau_{V,i} \cdot \Delta q_{d,i} \tag{4.32}$$

where $\Delta q_{d,i}$ is the effect of a disturbance on the flow in tank i. For a feed flow disturbance we have $\Delta q_{d,i} = K_{r,i} \cdot \Delta q_{F,d}$.

Acceptable variation in feed rate $q_{F,u}$

The feed rate change is primarily determined by the dynamics in the last unit, see (4.26). Equation (4.25) therefore applies to the last unit only, that is, for the last unit (here denoted 3) we must require to have an overshoot in the feed rate of less than a factor M for sinusoidal variations in $q_{B,s}$:

$$\tau_{V,3} \geq \frac{1}{M-1} \tau_{G,3} \tag{4.33}$$

which is equal to $2\tau_{G,3}$ when $M = 1.5$ (50% overshoot).

4.5.4 Example of required inventory size using single-loop with ratio control

To check the required inventory, we compare for the introductory example the observed volume variations with the estimated volume variation derived in (4.31) and (4.32).

Example 4.1 (continued). ***Required buffer volume for four distillation columns in series.*** *The relevant flow dynamics for each column is approximated by a first-order transfer function* $K_r/(\tau_G s+1)$ *where* τ_G *is found from simulations. The time constant* τ_G *was found as the time for the flow rate into the inventory to reach 63% of its steady-state change following a step change in column feed rate (outflow of previous inventory). The time constants and gains are summarized in Table 4.3. For example, following a step change in the deethanizer column feed rate, it takes* $\tau_G = 0.85$ *min before the liquid flow into the column reboiler has changed 63%.*

Inventory	τ_G [min]	τ_V [min]	K_r
1. Deethanizer sump (M_B)	0.85	2.7	0.602
2. Depropanizer sump (M_B)	3.9	3.3	0.254
3. Debutanizer condenser (M_D)	1.2	7.7	0.209

Table 4.3: Time constant flow change (τ_G, approximated), inventory (τ_V) and the static ratio gain (K_r) for the distillation columns in Example 4.1.

The observed variations in the volumes (deethanizer M_B*, depropanizer* M_B *and debutanizer* M_D*) are normalized to find* $\Delta V/\Delta q_{F,d}$ *and* $\Delta V/\Delta q_{B,s}$ *and are compared with the estimated volume variations given by* (4.31) *and* (4.32). *For example, for the deethanizer the estimate from* (4.31) *is* $|\Delta V|/|\Delta q_{B,s}| = \tau_{G,1} \cdot K_{r,1}/K_{r,3} =$

0.85 *min* $\cdot\, 0.602/0.209 = 2.4$ *min, and from* (4.32) *the estimate is* $|\Delta V|/|\Delta q_{F,d}| = \tau_{V,1}K_{r,1} = 2.7$ *min* $\cdot\, 0.602 = 1.6$ *min. From Table 4.4 we see that the estimated volume variations compare well with the observed variations. There is some difference for the bottleneck set point change, but this is expected since the time constant* τ_G *is only an approximation. For the feed rate disturbance the steady-state volume is the same as estimated, but there are slight overshoots in the volume. This is caused by the overshoot in the manipulated feed rate* $q_{F,u}$ *(see Figure 4.4(b)).*

		1. Deethanizer M_B [min]	2. Depropanizer M_B [min]	3. Debutanizer M_D [min]
$\frac{\lvert\Delta V\rvert}{\lvert\Delta q_{B,s}\rvert}$	Observed at $t=\infty$	0	0	0.026
	Observed max	0.69	1.4	1.8
	Estimated max (4.31)	2.4	4.7	1.2
$\frac{\lvert\Delta V\rvert}{\lvert\Delta q_{F,d}\rvert}$	Observed at $t=\infty$	1.6	0.83	1.6
	Observed max	1.7	0.97	1.8
	Estimated max (4.32)	1.6	0.84	1.6

Table 4.4: Calculated and observed volumes variations in Example 4.1 for single-loop with static bias adjustment (Alternative 1D in Section 4.2).

The corresponding volume variations with MPC are given in Table 4.5. The inventory usage is about the same initially for the two control alternatives, but the MPC has integral action so the inventories return to their set points. However, note that the variations depend on the specific set points weights and penalty on MV moves used in MPC.

		1. Deethanizer M_B [min]	2. Depropanizer M_B[min]	3. Debutanizer M_D[min]
$\frac{\lvert\Delta V\rvert}{\lvert\Delta q_{B,s}\rvert}$	Observed at $t=\infty$	0	0	0
	Observed max	2.2	3.7	3.0
$\frac{\lvert\Delta V\rvert}{\lvert\Delta q_{F,d}\rvert}$	Observed at $t=\infty$	0	0	0
	Observed max	1.2	0.86	1.6

Table 4.5: Observed volumes variations in Example 4.1 with MPC (Alternative 4 in Section 4.2).

The advantages of including dynamic degrees of freedom in throughput maximization are clear. Including buffer volumes leads to tighter control at the bottleneck unit and less back off is required under presence of disturbances, leading to improvement of the plant throughput. The simple formulas developed here can be used to determine the buffer tank volume in plant design. For upstream disturbances the required buffer volume is given by (4.32), and for bottleneck set point changes the required buffer volume is given by (4.31); see also the discussion.

4.6 Discussion

Effect of level control tuning

In the above simulations, the level controllers were actually quite tightly tuned (Figure 4.4). A tight inventory controller counteracts the bias added to the inventory output (q_0) and this leads to poorer bottleneck control. It may also lead to some overshoot in q_F, because the flow controller must generate a larger signal to q_0. On the other hand, with a smoother tuning there is a risk for overfilling or emptying the tank. Thus, tuning of the level controller is a trade-off. This is illustrated by simulation in Figure 4.6 where smoother level tunings are used (τ_c about 7 times larger). The results are summarized in Table 4.6. We see, as expected, that the volume variations are significantly larger, but the control of the bottleneck is better. There is now no overshoot in q_F for the ratio structure. Again, the observed and estimated volume variations are close (Table 4.6).

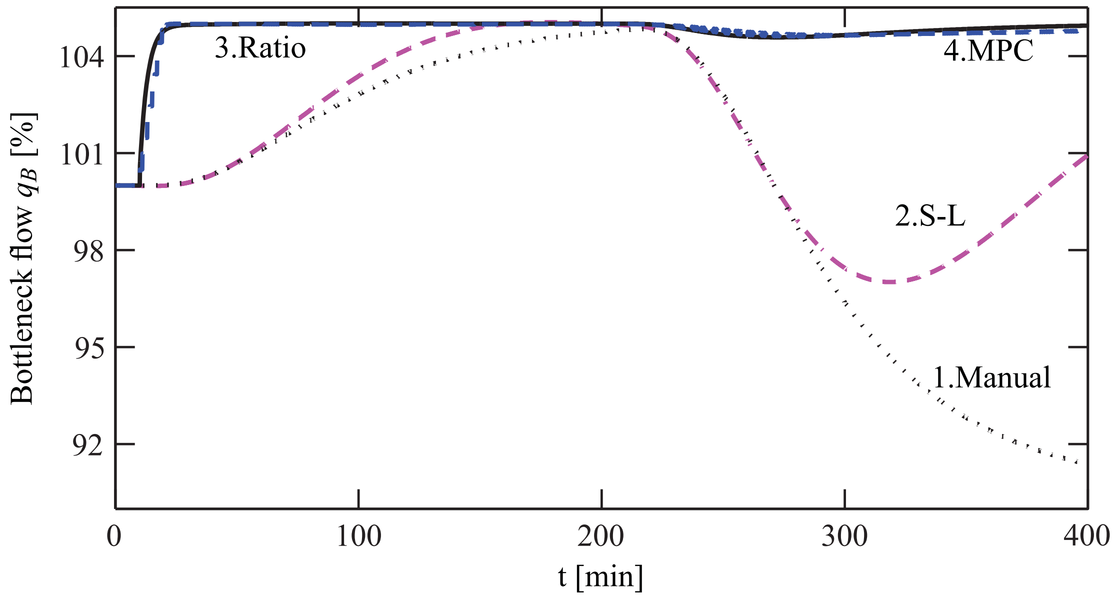
		1. Deethanizer M_B [min]	2. Depropanizer M_B [min]	3. Debutanizer M_D [min]
$\frac{\lvert\Delta V\rvert}{\lvert\Delta q_{B,s}\rvert}$	Observed at $t=\infty$	0	0.02	0.05
	Observed max	1.42	3.2	2.5
	Estimated max (4.31)	2.4	4.7	1.2
$\frac{\lvert\Delta V\rvert}{\lvert\Delta q_{F,d}\rvert}$	Observed at $t=\infty$	12	5.1	10
	Observed max	12	5.1	10
	Estimated max (4.32)	12	5.1	10

Table 4.6: Calculated and observed volumes variations for the introductory example with smooth inventory tunings. The control structure is single-loop with static bias adjustment.

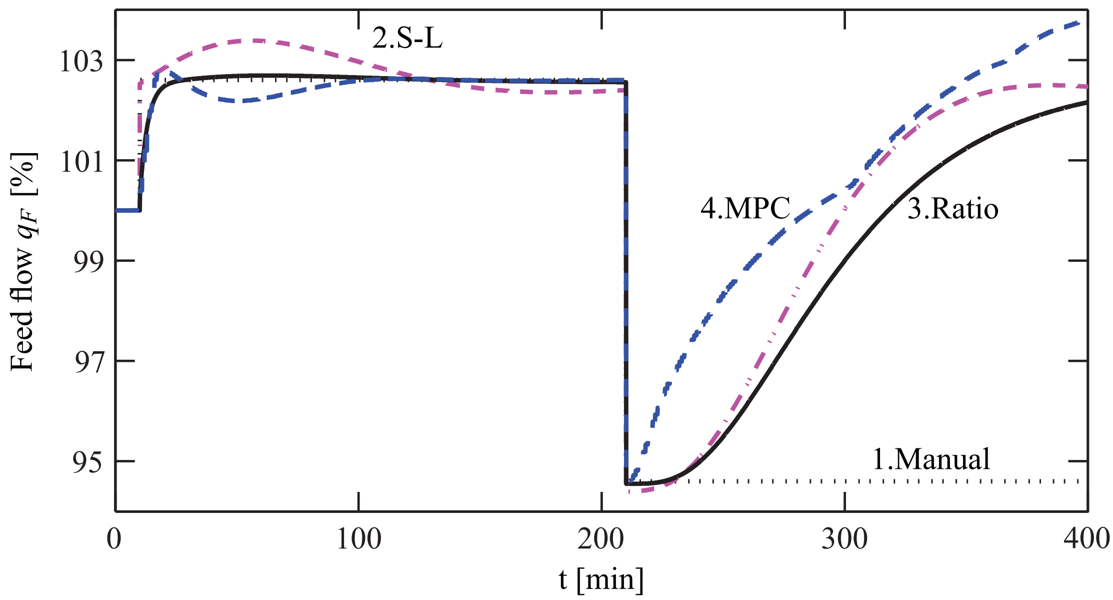
Finally, note that with smoother level tunings, manual or single-loop bottleneck control is poorer, because it then takes longer time for the flow rate change to move through the system. An important conclusion is that for manual or single-loop bottleneck control we should have tight the level control on the path from the feed (TPM). However, the conclusion is opposite of we make use of the levels as dynamic degrees of freedom. In practice, this may imply that we may need to detune the level loops if we want to use the levels as dynamic degrees of freedom.

Bias or set point adjustment?

Use of the inventories as dynamic degrees of freedom can be realized with either bias adjustment (used here for the ratio scheme) or with set point changes (used here in MPC). Use of bias adjustment does not affect the control system directly, and the inventory set point is still available to operators. However, it may not be

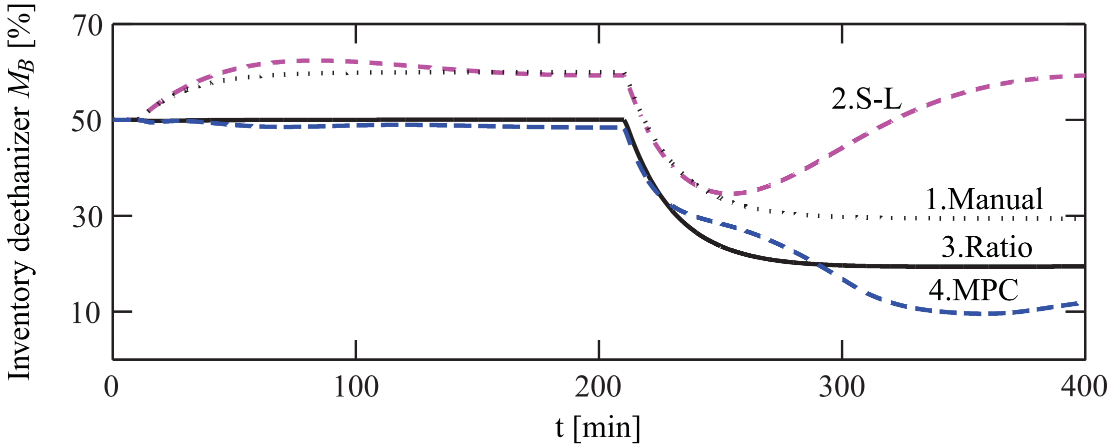


(a) Response in Bottleneck flow q_B (CV)

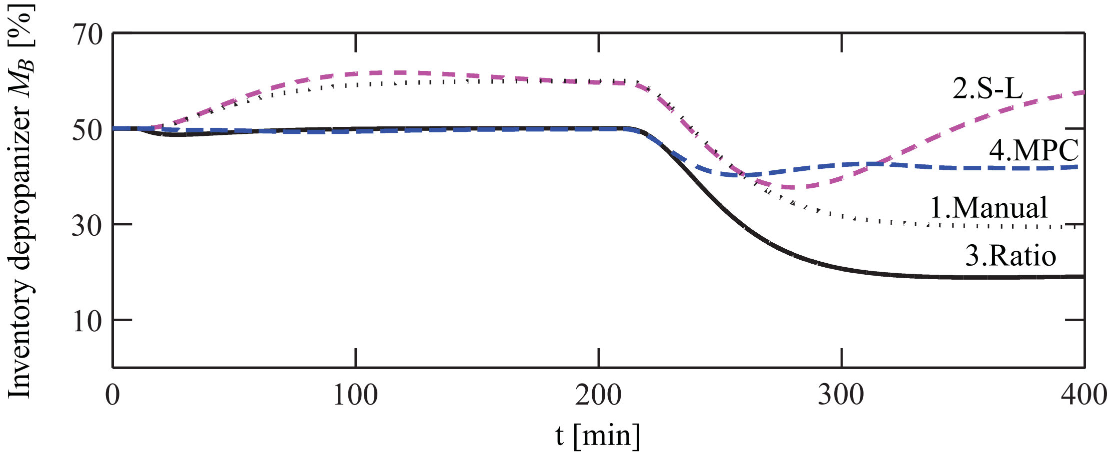


(b) Responses in Feed flow q_F (MV)

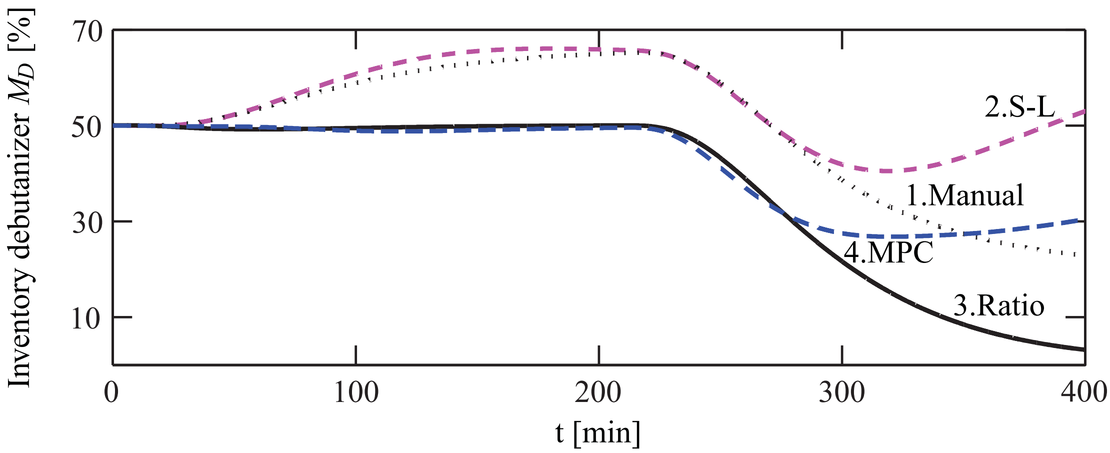
Figure 4.6: Continued on next page.



(c) Responses in deetanizer bottoms level M_B (dynamic degree of freedom no. 1)



(d) Responses in depropanizer bottoms level M_B (dynamic degree of freedom no. 2)



(e) Responses in debutanizer distillate level M_D (dynamic degree of freedom no. 3)

Figure 4.6: Same as in Figure 4.4, but with slower (smoother) inventory control.

possible in practice to include bias adjustment because it is not available in the digital control system (DCS). On the other hand, with use of set point adjustment, the use of inventories is very dependent on the inventory tuning.

Placement of the buffer volume

When the feed is the throughput manipulator, the inventory must be placed (and exploited) upstream the bottleneck on the path from the throughput manipulator. Alternatively, they may be placed at the path from important disturbances. If the bottleneck is fixed, then all inventories should be upstream the bottleneck. If the bottleneck is moving, then inventories should be distributed in the plant.

Variations in static ratio gain

The single-loop with ratio control scheme is sensitive to errors in the static ratio gains. This follows because the static ratio gain gives a feedforward control action and feedforward is in general sensitive to modelling errors. In particular, with a too small value of K_r, one will get an overshoot in the feed rate (q_F).

4.7 Summary: Implications for design of inventory tanks

We have derived two formulas, (4.31) and (4.32), for the expected volume variations when inventories are used as dynamic degrees of freedom to achieve bottleneck control. The validity of (4.32) and to some extent (4.31) have been confirmed by simulations. We here summarize the practical use of these formulas for design of inventory tanks.

Tank size

A desired change in tank throughput Δq_B results in a volume variation ΔV and from (4.31) we have

$$|\Delta V| = \tau_G \cdot |\Delta q_B| \tag{4.34}$$

where τ_G is the time constant for "refilling" the tank. In practice, τ_G is the time for the flow rate into V to reach 63% of its steady-state change following a step in flow rate out of the (closest) upstream inventory. This is for the normal case when the TPM is upstream the bottleneck; the same formula applies also when it is downstream. For design purposes, the flow change $|\Delta q_B|$ is the (steady-state) flow change through tank resulting from the largest expected throughput (bottleneck flow) change. (Here, "largest change" should be evaluated over a time period shorter than τ_G, approximately, because slower changes do not pose a problem in terms of dynamic changes in tank volume).

Equation (4.34) is useful for sizing the tank (inventory volume). In words, (4.34) says that the expected volume variation for an inventory used for bottleneck control (V $[m^3]$) is approximately the expected variation in flow through the unit (Δq_B $[m^3/min]$) multiplied by the time constant (τ_G $[min]$) for the flow dynamics for "refilling" V from the upstream inventory. As expected, a large tank is required if τ_G is large.

For our distillation columns process, we get from (4.34) the following minimum inventories if we assume a 5% desired change in the throughput (bottleneck flow). Note that we here give the inventory in *kmol* (M) rather than in m^3 (V).

$$\text{Deethanizer: } M_B = 0.85\ min \cdot 29.4\ kmol/min \cdot 0.05 = 1.2\ kmol$$
$$\text{Depropanizer: } M_B = 3.9\ min \cdot 11.6\ kmol/min \cdot 0.05 = 2.3\ kmol$$
$$\text{Debutanizer: } M_D = 1.2\ min \cdot 8.1\ kmol/min \cdot 0.05 = 0.49\ kmol$$

For comparison, the actual holdups are 121 *kmol*, 38 *kmol* and 62 *kmol*, respectively, which is from about 40 to 200 times larger than the minimum. This explains why the variations in the inventories for the first 200 *min* in the simulations (Figure 4.4 and Figure 4.6) are so small for the cases 3 and 4 where the inventories are used as degrees of freedom for bottleneck control.

Level control tuning

Next consider (4.32), which involves the closed-loop time constant (τ_V) for the level control loop in the inventory tank. We get

$$|\Delta V_{peak}| = \tau_V \cdot |\Delta q_d| \tag{4.35}$$

where Δq_d is the flow rate change through the tank in question. Equation (4.35) can be used to tune the level controller, and then gives the well-known formula for smooth (averaging) level control. To see this, note that for a nominally half-full tank we must require $|\Delta V_{peak}| < 0.5\ V_{\text{tank}}$ to avoid overfilling or emptying. If we furthermore assume that the maximum expected change in flow through the tank is 50% of the nominal flow, then $q_d = 0.5\ q$. Inserting into (4.35) then gives

$$\tau_V < \frac{V_{\text{tank}}}{q} \tag{4.36}$$

where τ_V is the closed-loop time constant for the level control loop. Thus, selecting $\tau_V = V_{\text{tank}}/q$ (the well-known value for smooth level control, (e.g. Skogestad, 2006), gives the slowest possible controller tuning subject to not overfilling or emptying the tank for 50% flow rate changes.

Applying the formula $\tau_V = V_{\text{tank}}/q$ to our distillation column example gives (the factor 2 is because we assume that the tank is nominally half full).

$$\begin{aligned}
\text{Deethanizer: } \tau_V &= 2 \cdot 121\ kmol/(29.4 + 15.8)\ kmol/min = 5.4\ min \\
\text{Depropanizer: } \tau_V &= 2 \cdot 38\ kmol/11.6\ kmol/min = 6.6\ min \\
\text{Debutanizer: } \tau_V &= 2 \cdot 62\ kmol/8.1\ kmol/min = 15.3\ min
\end{aligned}$$

The actual values used in the simulations were 2.7 *min*, 3.3 *min* and 7.7 *min*, respectively, which is half of the values given above and results in smaller variations in the volumes. In addition, the flow rate disturbance was only 8%, and this is why the variations in the inventories for the last 200 *min* in the simulations were so small. In the later simulations (Figure 4.6), τ_V was increased by about a factor 7 in all three level loops. As expected, this resulted in much larger variations in the inventories (about 7 times larger for the last 200 *min* of simulations), but it also resulted in better bottleneck control (for ratio control and MPC where the inventories are used as dynamic degrees of freedom).

We have also derived a formula (4.33) which applies for the level tuning in the last tank upstream of the bottleneck. It says that we should have τ_V for the last tank significantly larger than τ_G. In our case we have $\tau_G = 1.2\ min$ for the last unit upstream of the bottleneck (debutanizer), whereas $\tau_V = 7.7\ min$ for the last tank, so this is satisfied.

By comparing Figure 4.4(a) and 4.6(a) we note that bottleneck control is only weakly dependent on the inventory control tuning (value of τ_c) for cases 3 and 4 where the inventories are used as degrees of freedom for bottleneck control (bottleneck control is slightly better in Figure 4.6(a) with smoother inventory control). This is good, because it means that the inventory controllers (value of τ_c) can be tuned independently of the plantwide issue of throughput control.

On the other hand, for cases 1 and 2 where we only use the feed rate as a degree of freedom, bottleneck control is much better with tight inventory control (Figure 4.4(a)) because the effective deadtime from the feed flow to the bottleneck is then reduced. On the other hand, tight inventory control results in little damping of flow disturbances. Thus, there will be a trade-off between wanting tight inventory control (for good bottleneck control) and slow inventory control (to dampen flow disturbances).

4.8 Conclusion

Tight bottleneck control is important for maximizing throughput and avoiding economic losses. However, achieving tight bottleneck control in practice is not so simple because the throughput manipulator is often located away from the bottleneck unit (with a large effective delay θ_{eff}). In this paper we propose to reduce the effective delay by using dynamic degrees of freedom. The main idea is as follows: To change the flow through the bottleneck, for example, to increase it, we temporarily reduce the inventory in the upstream holdup volume. However, this inventory needs to be kept within bounds, so if we want to increase the bottleneck flow permanently, we need to increase the flow into this part of the process and so on, all the way back to the feed (throughput manipulator). The simplest approach is to make a control system where all flows upstream the bottleneck are increased simultaneously by the same relative amount, like a single-loop bottleneck controller that adjusts the feed flow, combined with ratio controllers that adjust the dynamic degrees of freedom. In this paper a static bias adjustment is studied. Two formulas (4.31) and (4.32) are derived for the expected volume variations when inventories are used as dynamic degrees of freedom to achieve bottleneck control. These two formulas can be used for inventory design purposes.

4.9 Acknowledgments

The introductory example (four columns in series) was modelled in Matlab and Simulink by MS student Théogéne Uwarwema (Uwarwema, 2008).

4.A Derivation of the peak frequency for second order transfer function

4.A.1 Peak frequency for a second order system

The transfer function $h_{Vq_{B,s}}$ is of second order. To analyze the transfer function, consider first a general second order system

$$G(s) = \frac{K}{\tau^2 s^2 + 2\tau\zeta s + 1} \tag{4.37}$$

where K is gain of the second order model, τ is the system time constant and ζ is the damping factor. The magnitude $|G|$ as a function of frequency ω is given by (e.g., Seborg *et al.* (1989, eq. 14-35a))

$$|G| = \frac{K}{\sqrt{(1-\omega^2\tau^2)^2 + (2\omega\tau\zeta)^2}} \tag{4.38}$$

The transfer function $h_{Vq_{B,s}} = (-\tau_G\tau_V s)/(1+\tau_V s(\tau_G s+1))$ has a differentiation (s) in the numerator and a second order system in the denominator. The differentiation has a slope of $+1$ in the whole frequency range. The peak frequencies of $h_{vB,s}$ is where the derivative with respect to frequency are zero, thus the denominator should have slope -1 in this point, since the integrator in the numerator always has the slope $+1$.

The phase to a second order system is always $-90°$ at $\omega = \frac{1}{\tau}$, see Seborg *et al.* (1989, Figure 14.3). For stable minimum-phase systems the slope is approximately -1 at $\phi = -90°$ (Skogestad and Postlethwaite, 2005, Eq. 2.12), and this is a commonly used approximation. Thus, the peak frequency of $h_{Vq_{B,s}}$ is located at the break frequency, $\omega_{peak} = \frac{1}{\tau}$. The peak frequency can also be found analytically by differentiating (4.22) with respect to ω and let the derivative be zero, as shown in Appendix 4.A.2. Note that in this case the peak frequency is independent of the damping factor ζ.

4.A.2 Analytic derivation of peak frequency

Here the peak frequency for Equation (4.22) is derived analytically and we confirm the arguments in Section 4.A.1. To evaluate the magnitude of $h_{Vq_{B,s}}$, replace s with $j\omega$ in (4.22)

$$h_{Vq_{B,s}} = \frac{-\tau^2 j\omega}{\tau^2(j\omega)^2 + 2\zeta\tau\omega + 1} \tag{4.39}$$

The magnitude is given by

$$|h_{Vq_{B,s}}| = \frac{\tau^2\omega}{\sqrt{(1-\tau^2\omega^2)^2 + (2\zeta\tau\omega)^2}} \tag{4.40}$$

Differentiation with respect to ω

$$
\begin{aligned}
\frac{d|h_{Vq_{B,s}}|}{d\omega} &= \left(\frac{u}{v}\right)' = \frac{u' \cdot v - v' \cdot u}{v^2} \quad \text{where} \\
u &= \tau^2\omega \\
v &= [(1-\tau^2\omega^2)^2 + (2\tau\zeta\omega)^2]^{\frac{1}{2}} = n^{\frac{1}{2}} \\
\frac{du}{d\omega} &= \tau^2 \\
\frac{dv}{d\omega} &= \frac{1}{2}n^{-\frac{1}{2}}\frac{dn}{d\omega} \\
\frac{dn}{d\omega} &= [(1-\tau^2\omega^2)^2 + (2\omega\tau\zeta)^2]' \\
&= -4\tau^2\omega + 4\tau^4\omega^3 + 8\tau^2\zeta^2\omega \\
\frac{dv}{d\omega} &= \frac{1}{2}n^{-\frac{1}{2}} \cdot (-4\tau^2\omega + 4\tau^4\omega^3 + 8\tau^2\zeta^2\omega) \\
&= (-2\tau^2\omega + 2\tau^4\omega^3 + 4\tau^2\zeta^2\omega) \cdot n^{-\frac{1}{2}}
\end{aligned}
\tag{4.41}
$$

Inserting for u and v in (4.41) gives

$$
\frac{d|h_{Vq_{B,s}}|}{d\omega} = \frac{\tau^2 n^{\frac{1}{2}} - (-2\tau^2\omega + 2\tau^4\omega^3 + 4\tau^2\zeta^2\omega) \cdot n^{-\frac{1}{2}} \cdot \tau^2\omega}{n} \tag{4.42}
$$

Multiply numerator and denominator with $n^{\frac{1}{2}}$ gives

$$
\frac{d|h_{Vq_{B,s}}|}{d\omega} = \frac{\tau^2 n - (-2\tau^2\omega + 2\tau^4\omega^3 + 4\tau^2\zeta^2\omega) \cdot \tau^2\omega}{n^{\frac{3}{2}}} \tag{4.43}
$$

We want to find the peak frequency, which corresponds to setting the derivative to zero. Here it is sufficient to evaluate the numerator in Equation (4.43). This yield

$$
\begin{aligned}
\tau^2 n - (-2\tau^2\omega + 2\tau^4\omega^3 + 4\tau^2\zeta^2\omega) \cdot \tau^2\omega &= 0 \\
1 - 2\tau^2\omega^2 + \tau^4\omega^4 + 4\tau^2\zeta^2\omega^2 + 2\tau^2\omega^2 - 2\tau^4\omega^4 - 4\tau^2\zeta^2\omega^2 &= 0 \\
1 - \tau^4\omega^4 &= 0 \\
\omega^4 &= \frac{1}{\tau^4} \\
\omega &= \frac{1}{\tau}
\end{aligned}
$$

Hence, the peak frequency for $h_{Vq_{B,s}}$ is derived analytically to be $\omega = \frac{1}{\tau} = \frac{1}{\sqrt{\tau_G \tau_V}}$.

4.B Analytic derivation of acceptable variations in feed rate

The variations in feed rate caused by bottleneck set point changes is given by (4.20) and we have $|q_{F,u}/q_{F,ss}| = |q_{F,u}K_r/q_{B,s}|$ where

$$\frac{q_{F,u}K_r}{q_{B,s}} = \frac{\tau_V\tau_G s^2 + (\tau_V + \tau_G)s + 1}{\tau_V\tau_G s^2 + \tau_V s + 1} \tag{4.44}$$

which can be written as a second order system

$$\begin{aligned} \frac{q_{F,u}K_r}{q_{B,s}} &= \frac{\tau s^2 + 2\tau\zeta_n s + 1}{\tau s^2 + 2\tau\zeta_d s + 1} \quad \text{with} \\ \tau &= \sqrt{\tau_G\tau_V} \\ \zeta_n &= \frac{\tau_V + \tau_G}{2\sqrt{\tau_G\tau_V}} = \frac{1}{2}\sqrt{\frac{\tau_V}{\tau_G}}\frac{\tau_V + \tau_G}{\tau_G} \geq \zeta_d \\ \zeta_d &= \frac{1}{2}\sqrt{\frac{\tau_V}{\tau_G}} \end{aligned} \tag{4.45}$$

The magnitude of a second-order system is given in Equation (4.38).

$$\left|\frac{q_{F,u}\cdot K_r}{q_{B,s}}\right| = \frac{\sqrt{(1-\omega^2\tau^2)^2 + (2\omega\tau\zeta_n)^2}}{\sqrt{(1-\omega^2\tau^2)^2 + (2\omega\tau\zeta_d)^2}} \tag{4.46}$$

From Section 4.A.1, a stable minimum-phase, second-order system has its magnitude peak at frequency $\omega = 1/\tau = 1/\sqrt{\tau_G\tau_V}$ and inserting this gives:

$$\left|\frac{q_{F,u}\cdot K_r}{q_{B,s}}\right|_{\max} = \frac{\frac{\tau_V+\tau_G}{\sqrt{\tau_G\tau_V}}}{\sqrt{\frac{\tau_V}{\tau_G}}} = 1 + \frac{\tau_G}{\tau_V} \tag{4.47}$$

Let M denote the allowed overshoot (e.g. $M = 1.5$ if us allow 50% overshoot). Then we must require

$$\left|\frac{q_{F,u}\cdot K_r}{q_{B,s}}\right| \leq M \tag{4.48}$$

and from (4.47) we get

$$\begin{aligned} 1 + \frac{\tau_G}{\tau_V} &\leq M \\ \tau_V &\geq \frac{\tau_G}{M-1} \end{aligned} \tag{4.49}$$

For example, with $M = 1.5$ we get $\tau_V \geq 2\tau_G$.

Chapter 5

Coordinator MPC for maximizing plant throughput

Comput. Chem. Eng. **32**(1-2), 195-204 (2008)

In many cases economic optimal operation is the same as maximum plant throughput, which is the same as maximum flow through the bottleneck(s). This insight may greatly simplify implementation. In this paper, we consider the case where the bottlenecks may move, with parallel flows that give rise to multiple bottlenecks and with crossover flows as extra degrees of freedom. With the assumption that the flow through the network is represented by a set of units with linear flow connections, the maximum throughput problem is then a linear programming (LP) problem. We propose to implement maximum throughput by using a coordinator model predictive controller (MPC). Use of MPC to solve the LP has the benefit of allowing for a coordinated dynamic implementation. The constraints for the coordinator MPC are the maximum flows through the individual units. These may change with time and a key idea is that they can be obtained with almost no extra effort using the models in the existing local MPCs. The coordinator MPC has been tested on a dynamic simulator for parts of the Kårstø gas plant and performs well for the simulated challenges.

5.1 Introduction

Real-time optimization (RTO) offers a direct method of maximizing an economic objective function. Most RTO systems are based on detailed nonlinear steady-state models of the entire plant, combined with data reconciliation to update key parameters, such as feed compositions and efficiency factors in units, see for example Marlin and Hrymak (1997). Typically, the RTO application reoptimizes and updates on an hourly basis the set points for the lower-layer control system, which may consists of set points of local MPCs based on simple linear dynamic models. A steady-state RTO is not sufficient if there are frequent changes in active constraints of large economic importance. For example, this could be the case if the throughput bottleneck in a plant moves frequently, which is the case for the application studied in this paper. At least in theory, it is then more suitable to use dynamic optimization with a nonlinear model, which may be realized using dynamic RTO (DRTO) or nonlinear MPC with an economic objective (Tosukhowong *et al.*, 2004; Kadam *et al.*, 2003; Strand, 1991). However, a centralized dynamic optimization of the entire plant is undesirable (Lu, 2003). An alternative is to use local unit-based MPCs, but the resulting steady-state target calculation may be far from optimal (Havlena and Lu, 2005). Coordination of multiple local MPCs has been studied by several authors. Cheng *et al.* (2004, 2006, 2007) have suggested to approach this "coordination" problem by identifying appropriate interactions for linking constraints to find the steady-state targets for the local MPCs. Rawlings and Stewart (2007) discuss a cooperative distributed MPC framework, where the local MPC objective functions are modified to achieve systemwide control objectives. Ying and Joseph (1999) propose a two-stage MPC complement that track changes in the optimum caused by disturbances. The approach permits dynamic tracking of the optimum which is not achievable with a steady-state RTO used in conjunction with a single-stage MPC.

In this paper, we present a different and simpler solution that achieves economic optimal operation without any of these complexities. This solution applies to the common case where prices and market conditions are such that economic optimal operation of the plant is the same as maximizing plant throughput. The main objective is then to maximize the feed to the plant, subject to achieving feasible operation (satisfying operational constraints in all units). This insight may be used to implement optimal operation, without the need for dynamic optimization based on a detailed model of the entire plant.

The *max-flow min-cut* theorem (Ford and Fulkerson, 1962) from linear network theory states that the maximum throughput in a linear network is limited by the "bottleneck(s)" of the network (Aske *et al.*, 2007). In order to maximize the throughput, the flow at the bottlenecks should always be at their maximum. In particular, if the actual flow at the bottleneck is not at its maximum at any given time, then this gives a loss in production that can never be recovered (sometimes referred to as a "lost opportunity").

The throughput manipulators (TPMs) are the degrees of freedom available for implementing maximum throughput. They affect the flow through the entire plant (or at least in more than one unit), and therefore cannot be used to control an individual unit or objective. Ideally, in terms of maximizing plant production and

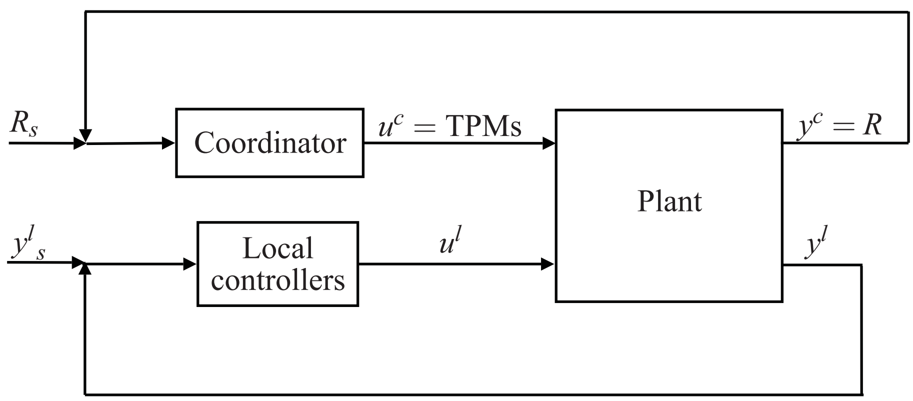


Figure 5.1: The coordinator uses the throughput manipulators (u^c = TPMs) to control the remaining capacity ($y^c = R$) in the units.

minimizing the back off, the TPM should be located at the bottleneck (Aske *et al.*, 2007). However, the bottleneck may move depending on plant operating conditions (e.g. feed composition), and it is generally very difficult to change the TPM, once a decision on its location has been made. The reason is that the location of the TPM affects the degrees of freedom available for local control, and thus strongly affects the structure of the local control systems and in particular the structure of the inventory control system (Buckley, 1964; Price and Georgakis, 1993). The TPM will therefore generally be located away from the bottleneck, for example at the feed. For dynamic reasons it will then not be possible to achieve maximum flow through the bottleneck at all times, and a loss in production is inevitable.

The use of a coordinator controller that uses the throughput manipulators (u^c = TPMs) to control the remaining local capacity ($y^c = R = F^l_{max} - F^l$) in the units as illustrated in Figure 5.1. In the simplest case with a fixed bottleneck and feed rate as the TPM, the coordinator may be a single-loop PI-controller with the feed rate as the manipulated variable (u^c) and the bottleneck flow as the controlled variable (y^c) (Skogestad, 2004). However, more generally the coordinator must be a multivariable controller. Note from Figure 5.1 that the “coordinator” and the “local” controllers for the individual units are actually on the same level in the control hierarchy, like in decentralized control. Nevertheless, the term coordinator is used because the TPMs strongly affect all the units and because in general the coordinator controller must be designed based on a flow network model of the entire plant. An alternative to the decentralized structure is to combine all the local MPCs into a large combined MPC application that include the throughput manipulators as degrees of freedom.

Optimal operation corresponds to $R = 0$ in the bottleneck, but if the maximum flow through the bottleneck is a hard constraint, then to avoid infeasibility ($R < 0$)

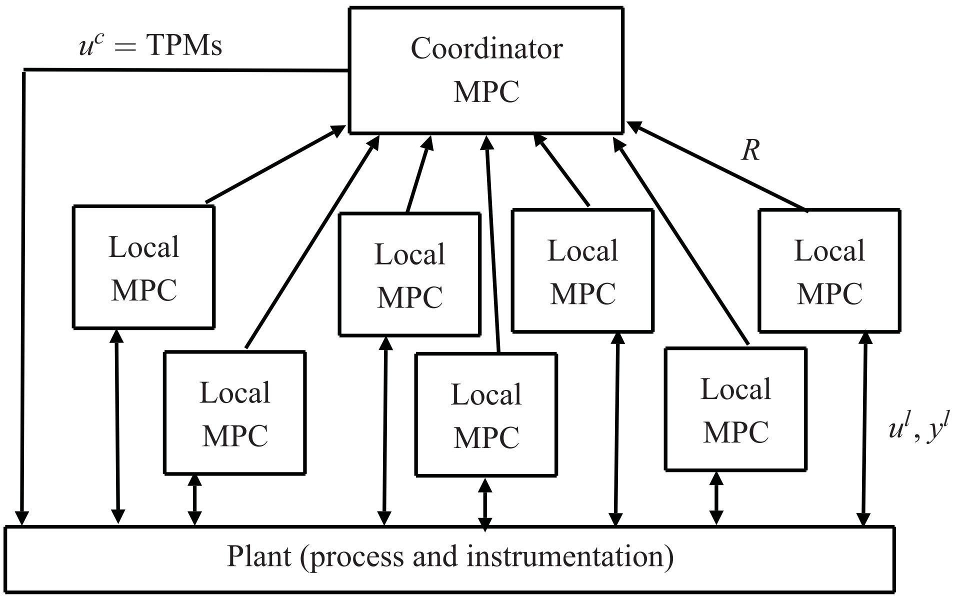


Figure 5.2: Proposed control structure where the coordinator MPC receives information from the local MPC about the remaining capacity (R) in the units.

dynamically, we need to "back off" from the optimal point

$$\text{Back off}\,(b) = R_s = F^l_{max} - F^l_s \tag{5.1}$$

More generally, the back off is the distance to the active constraint needed to avoid dynamic infeasibility in the presence of disturbances, model errors, delay and other sources for imperfect control (Narraway and Perkins, 1993, Govatsmark and Skogestad, 2005). The back off is a "safety factor" and should be obtained based on information about the disturbances and the expected control performance.

In this paper, we consider cases where the bottlenecks may move and with parallel trains that give rise to multiple bottlenecks and multiple throughput manipulators. This requires multivariable control and the proposed coordinator MPC both identifies the bottlenecks and implements the optimal policy. The constraints for the coordinator MPC are non-negative remaining capacities ($R \geq b \geq 0$) in all units. The values of R may change with time and a key idea is that they can be obtained with almost no extra effort using the existing local MPCs, as illustrated in Figure 5.2.

The paper is organized as follows. Economic optimal operation and the special case of maximum throughput is discussed in Section 5.2. Section 5.3 describes the coordinator MPC in addition to the capacity calculations in the local MPCs. Section 5.4 describes a dynamic simulation case study for a gas plant. A discussion follows in Section 5.5 before the paper is concluded in Section 5.6.

5.2 Maximum throughput as a special case of optimal operation

Mathematically, the optimum is found by minimizing the cost J (i.e., maximize the profit (-J)), subject to satisfying given specifications and model equations ($f = 0$) and operational constraints ($g \leq 0$). At steady-state:

$$\begin{aligned} &\min_u J(x,u,d) \\ &\text{s. t. } f(x,u,d) = 0 \\ &\qquad g(x,u,d) \leq 0 \end{aligned} \tag{5.2}$$

Here u are the degrees of freedom (or manipulated variables, MVs), d the disturbances and x the (dependent) state variables. The degrees of freedom are split into those used for local control (u^l) and the TPMs used for throughput coordinator (u^c),

$$u = \begin{bmatrix} u^l \\ u^c \end{bmatrix} \tag{5.3}$$

A typical profit function is

$$(\text{-}J) = \sum_j p_{P_j} \cdot P_j - \sum_i p_{F_i} \cdot F_i - \sum_k p_{Q_k} \cdot Q_k \tag{5.4}$$

where P_j are the product flows, F_i the feed flows, Q_k the utility duties (heating, cooling, power), and p denote the prices.

In many cases, and especially when the product prices are high, optimal operation of the plant (maximize -J) is the same as maximizing throughput. To understand this, let F denote the overall throughput in the plant, and assume that all feed flows are set in proportion to F,

$$F_i = k_{F,i}F \tag{5.5}$$

Then, under the assumption of constant efficiency in the units (independent of throughput) and assuming that all intensive (property) variables are constant, all extensive variables (flows and heat duties) in the plant will scale with the throughput F e.g, Skogestad (1991). In particular, we have that

$$P_j = k_{P,j}F;\ Q_k = k_{Q,k}F \tag{5.6}$$

where the gains $k_{P,j}$ and $k_{Q,k}$ and are constants. Note from (5.6) that the gains may be obtained from nominal (denoted 0) mass balance data:

$$k_{P,j} = P_{j0}/F_0;\ k_{F,i} = F_{i0}/F_0;\ k_{Q,k} = Q_{k0}/F_0 \tag{5.7}$$

Substituting (5.5) and (5.6) into (5.4) gives

$$(\text{-}J) = \left(\sum_j p_{P_j} \cdot k_{P,j} - \sum_i p_{F_i} \cdot k_{F,i} - \sum_k p_{Q_k} \cdot k_{Q,k} \right) F = pF \tag{5.8}$$

where p is the operational profit per unit of feed F processed. From the above derivation, p is a constant for the case with constant efficiencies. We assume $p > 0$ such that we have a meaningful case where the products are worth more than the feedstocks and utilities. Then, from (5.8) it is clear that maximizing the profit $(\text{-}J)$ is equivalent to maximizing the throughput F. However, F cannot go to infinity, because the operational constraints ($g \leq 0$) related to achieving feasible operation (indirectly) impose a maximum value for F.

In practice, the gains $k_{P,j}$ and $k_{Q,k}$ and are not constant, because the efficiency of the plant changes. Usually, operation becomes less efficient and p decreases when F increases. Nevertheless, as long as p remains positive, $d(\text{-}J)/dF = p > 0$ is nonzero, and we have a constrained optimum with respect to the throughput F. From (5.8) we see that p will remain positive and optimal operation is the same as maximum throughput if the feed is available and product prices $p_{P,j}$ are sufficiently high compared to the prices of feeds and utilities.

5.3 Coordinator MPC for maximizing throughput

The overall feed rate (or more generally the throughput) affects all units in the plant. For this reason, the throughput is usually not used as a degree of freedom for control of any individual unit, but is instead left as an "unused" degree of freedom to be set at the plant-wide level. Most commonly, the throughput manipulators (u^c) are set manually by the operator, but the objective here is to coordinate them to achieve economic optimal operation.

It is assumed that the local controllers (e.g. local MPCs) are implemented on the individual units. These adjust the local degrees of freedom u^l such that the operation is feasible. However, local feasibility requires that the feed rate to the unit F_k^l is below its maximum capacity, $F_{k,max}^l$, and one of the tasks of the plant-wide coordinator is to make sure that this is satisfied. $F_{k,max}^l$ may change depending on disturbances (e.g. feed composition) and needs to be updated continuously. One method is to use the already existing models in the local MPCs, as discussed in Section 5.3.2.

5.3.1 The coordinator MPC

The steady-state optimization problem (5.2) can be simplified when the optimal solution corresponds to maximizing plant throughput. Consider the steady-state optimization problem

$$\max_{u^c}(\text{-}J) \quad \text{s. t.} \tag{5.9}$$

$$F^l = Gu^c \tag{5.10}$$

$$R = F^l_{max} - F^l \geq b \geq 0 \tag{5.11}$$

$$u^c_{min} \leq u^c \leq u^c_{max} \tag{5.12}$$

Here F^l is a vector of local feeds to the units and R is a vector of remaining capacities in the units. If the objective is to maximize throughput with a single feed, then $(\text{-}J) = F$. More generally, with different values of the feedstocks and products, the profit function in (5.4) is used. G is a linear steady-state network model from the throughput manipulators u^c (independent feed and crossover flows) to all the local flows F^l. In order to achieve feasible flow through the network, it is necessary that $R \geq 0$ in all units. However, to guarantee dynamic feasibility, an additional back off from the capacity constraint may be required, which is represented by the vector b in (5.11). The main difference from the original optimization problem (5.2) is that only u^c (TPMs) are considered as degrees of freedom for the optimization in (5.9)-(5.12) and that the original constraints for the units ($f = 0, g \leq 0$) are replaced by a linear flow network and flow constraints ($R \geq b$).

It is assumed that the local controllers generate close-to optimal values for the remaining degrees of freedom u^l, while satisfying the original equality ($f = 0$) and inequality constraints ($g \leq 0$). This implies that no coordination of the local controllers is required, or more specifically that constant set points for the local controllers give close to optimal operation. In other words, it is assumed that we for the local units can identify ”self-optimizing” controlled variables Skogestad (2000*b*). If this is not possible then centralized optimization (RTO or maybe even DRTO) is required.

With the linear profit function $(\text{-}J)$ in (5.4), the optimization problem in (5.9)-(5.12) is an LP problem. The optimal solution to an LP problem is always at constraints. This means that the number of active constraints in (5.11) and (5.12) must be equal to the number of throughput manipulators, u^c. Note that an active constraint in (5.11) corresponds to having $R_k = F^l_{max,k} - F^l_k = b_k$, that is, unit k is a bottleneck. This agrees with the max-flow min-cut theorem of linear network theory. However, to solve the LP problem, we will not make use of the max-flow min-cut theorem.

The steady-state optimization problem in (5.9)-(5.12) can be extended to the dynamic optimization problem:

$$\min_{u^c} (J - J_s)^2 + \Delta u^{cT} Q_u \Delta u^c \quad \text{s. t.} \tag{5.13}$$

$$F^l = G_{dyn} u^c \tag{5.14}$$

$$R = F^l_{max} - F^l \geq b \geq 0 \tag{5.15}$$

$$u^c_{min} \leq u^c \leq u^c_{max} \tag{5.16}$$

$$\Delta u^c_{min} \leq \Delta u^c \leq \Delta u^c_{max} \tag{5.17}$$

Maximum throughput under the presence of disturbances is dynamic in nature, and here, G_{dyn} is a linear dynamic model from u^c (manipulated variables, MVs) to the remaining capacity in each unit, R_k. Obtaining the dynamic models may be time consuming. However, it is possible to use simple mass balances to calculate the steady-state gains of G_{dyn}, see (5.7).

The dynamic cost function (5.13) includes penalty on the MV moves to ensure robustness and acceptable dynamic performance. The constraints are: back off on capacity to each unit (5.15), MV high and low limits (5.16) and MV rate of change limits (5.17). MV rate of change is mainly a safeguard for errors and is normally not used for tuning.

The term $\Delta u^{cT} Q_u \Delta u^c$ makes the objective function quadratic, whereas the objective function in the original problem (5.9) is linear. To obtain a quadratic objective function that fits directly into the MPC software used here, we have used a common trick of introducing a quadratic term $(J - J_s)^2$. The profit set point J_s is high and unreachable with a lower priority than the capacity constraints. An alternative approach would be to include a linear term in J in (5.13).

Standard MPC implementations perform at each time step two calculations (Qin and Badgwell, 2003). First, the steady-state optimization problem with all the constraints is solved to obtain a feasible steady-state solution. Second, the dynamic problem is solved using the feasible targets obtained from the steady-state calculation. In our case, the steady-state part gives a feasible set point for the profit (or total flow) that replaces J_s in the subsequent solution of the dynamic problem. The dynamic terms involving Δu^c do not matter in the steady-state part, so the steady-state solution is identical to the LP problem in (5.9)-(5.12).

It is assumed that the local controllers (including local MPCs) are closed before obtaining the dynamic flow model G_{dyn}. To ensure good performance, it is then advisable that the coordinator operates with a longer time horizon than the local MPCs.

5.3.2 Capacity calculations using local MPCs

An important parameter for the coordinator is the maximum flow for the individual (local) units, F^l_{max}. A key idea in the present work is to obtain updated values using on-line information (feedback) from the plant. Note that it is not critical that the estimate of the maximum capacity is correct, except when the unit is actually approaching its maximum capacity and the corresponding capacity constraint $R = F^l_{max} - F^l \geq b$ becomes active. The use of on-line information from the actual plant will ensure that this is satisfied.

In simple cases, one may update the maximum capacity using the distance (Δconstraint ≥ 0) to a critical constraint in the unit,

$$F^l_{max} = F^l + c \cdot \Delta\text{constraint}$$

where c is a constant and F^l is the present flow through the unit. For example, for a distillation column $\Delta\text{constraint} = \Delta p_{max} - \Delta p$ could be difference between the pressure drop corresponding to flooding and the actual pressure drop.

In more complex cases, there may be more than one constraint that limits the operation of the unit and thus its maximum capacity. MPC is often implemented on the local units to improve dynamic performance and avoid complex logic. The maximum feed for each unit k can then be easily estimated using the already existing models and constraints in the local MPC applications. The only exception may be that the model must be updated to include the feed to the unit, F^l_k, as an independent variable. The maximum feed to the unit k is then obtained by solving the additional steady-state problem:

$$F^l_{k,max} = \max_{u^l_k, F^l_k} F^l_k \tag{5.18}$$

subject to the linear model equations and constraints of the local MPC, which is a LP problem. Here u^l_k is the vector of manipulated variables in the local MPC, and the optimization is subject to satisfying the linear constraints for the unit. To include past MV moves and disturbances, the end predictions of the variables should be used instead of the present values.

5.4 Kårstø gas processing case study

The Kårstø plant treats gas and condensate from central parts of the Norwegian continental shelf. The products are dry gas, which is exported through pipelines, and natural gas liquids (NGL) and condensate, which are exported by ships. The Kårstø plant plays a key role in the pipeline structure in the Norwegian Sea and

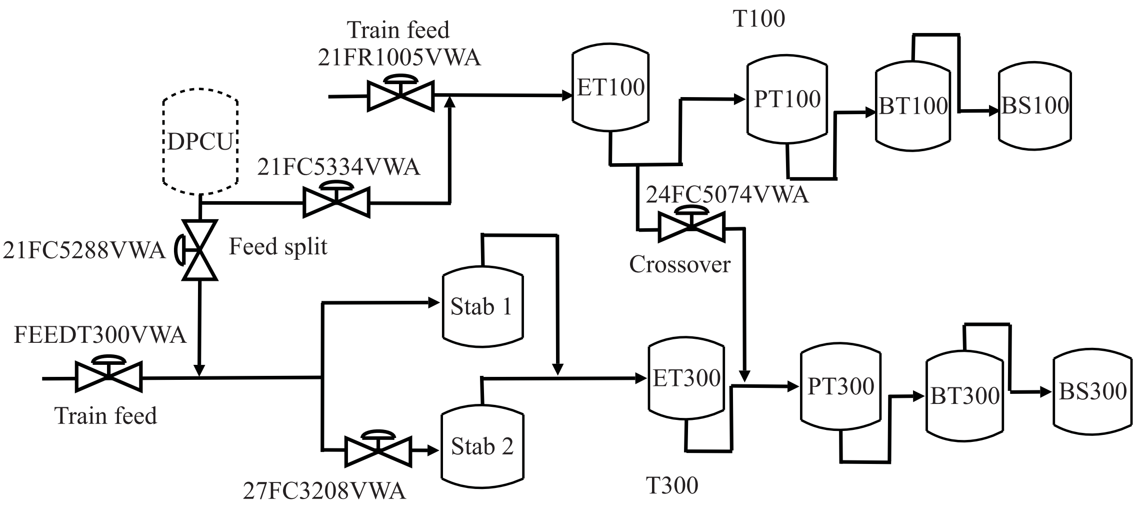


Figure 5.3: The simulated parts of the Kårstø plant

therefore is maximum throughput usually the main objective. Also, from an isolated Kårstø point of view, the plant has relative low feed and energy costs and high product prices that favor high throughputs. There are no recycles in the plant. Usually, feed is available and can be manipulated within given limits.

The feed enters the plant from three different pipelines and the feed composition may change frequently in all three lines. Changes in feed compositions can move the main bottleneck from one unit to another and affect the plant throughput. The coordinator MPC approach has been tested with good results using the Kårstø Whole Plant simulator. This is a dynamic simulator built in the software D-SPICE®.

5.4.1 The case

To demonstrate the applicability of the coordinator MPC, we use a detailed simulator model of parts of the Kårstø plant. To avoid the need for large computer resources to run the process simulator, only parts of the whole plant are used in the case study, see Figure 5.3. The selected parts include two fractionation trains, T100 and T300. Both trains have a deethanizer, depropanizer, debutanizer and a butane splitter. In addition T300 has two stabilizers in parallel. There are six throughput manipulators (u^c) as indicated by valves in Figure 5.3: two main train feeds, two liquid streams to the trains from the dew point control unit (DPCU), a crossover from train T100 to T300, and a flow split for the parallel stabilizers in train T300.

The local MPCs and the coordinator are implemented in Statoils SEPTIC*

*Statoil Estimation and Prediction Tool for Identification and Control

MPC software (Strand and Sagli, 2003). Data exchange between the simulator and the MPC applications is done by the built-in D-SPICE® OPC server. The detailed dynamic simulator was used to obtain "experimental" step response models (G_{dyn}) in the coordinator MPC. This approach has been found to work well in practice (Strand and Sagli, 2003).

5.4.2 Implementation of the local MPCs

The main control objective for each column is to control the quality in the top and bottom streams, by manipulating boil-up (V) and reflux flow (L). In addition the column must be kept under surveillance to avoid overloading, which is an important issue when maximizing throughput. Column differential pressure (Δp) is used as an indicator of flooding (Kister, 1990). The remaining feed capacity for each column (R_k) is calculated in the local MPC.

The LV-configuration with a temperature loop is used for regulatory control of the columns (Skogestad, 2007), and the local MPCs are configured as follows:

- CV (set point + constraint): Impurity of heavy key component
- CV (set point + constraint): Impurity of light key component
- CV (constraint): Column differential pressure
- MV: Reflux flow rate set point
- MV: Tray temperature set point in lower section
- DV: Column feed flow

These MVs correspond to u^l (local degrees of freedom), and CVs are the same as y^l. The feed rate is a disturbance variable (DV) for the local MPC, and is used as a degree of freedom when solving the extra LP problem to obtain the remaining capacity (R) to be used by the coordinator. Some of the columns have additional limitations that are included as CVs in the local MPC. The product qualities are described as impurity of the key component and a logarithmic transformation is used to linearize over the operating region (Skogestad, 1997). The high limits on the product qualities are given by the maximum levels of impurity in the sales specifications and the differential pressure high limit is placed just below the flooding point.

The control specification priorities for solving the steady-state feasibility problem for the local MPC are as follows:

1. High limit differential pressure

2. Impurity limits

3. Impurity set points

where 1 has the highest priority. The priority list is used in the steady state part in the MPC solver and leads to relaxation of the impurity set points (and in worst case limits) to avoid exceeding the differential pressure high limit (Strand and Sagli, 2003). By quality relaxation the column can handle the given feed rate without flooding the column. The low-priority quality set points are not used when solving the extra steady-state LP problem to obtain the remaining capacity R, because set point deviations are acceptable if the alternative is feed reduction. In the dynamic optimization part the constraints violations are handled by adding penalty terms to the objective function.

The local MPC applications are built with experimental step response models as described in Aske *et al.* (2005). The prediction horizon is 3 to 6 hours, which is significantly longer than the closed-loop response time. The sample time in the local MPC is set to 1 minute. From experience this is sufficiently fast for the distillation column applications and is the actual sample time used in the plant today.

5.4.3 The design and implementation of the coordinator MPC

The objective function for the coordinator is to maximize the total plant feed, $-J = F = \sum F_i$, which is the sum of the train feeds and the flows from the DPCU (FEEDT300VWA + 21FC5288VWA + 21FC5334VWA + 21FR1005VWA). The CVs and MVs for the coordinator MPC are:

- CV (high set point): Total feed flow F to the plant (PLANT FEED).
- CVs (constraints): Remaining feed capacity R_k in columns, 10 in total (R-ET100, R-PT100, R-BT100, R-BS100, R-STAB1, R-STAB2, R-ET300, R-PT300, R-BT300, R-BS300)
- CV (constraint): T100 deethanizer sump level controller output (LC OUTLET)
- MV: Feed train 100 (21FR1005VWA)
- MV: Feed train 300 (FEEDT300VWA)
- MV: Feed from DPCU to train 100 (21FC5334VWA)
- MV: Feed from DPCU to train 300 (21FC5288VWA)

- MV: Crossover flow from T100 to T300 (24FC5074VWA)
- MV: Stabilizers feed split (27FC3208VWA)

These MVs correspond to u^c (coordinator degrees of freedom). The deethanizer sump level controller output CV (gives the feed to PT100) is used to avoid emptying or overfilling up the sump level in ET100 when manipulating the crossover. The total plant feed has a high unreachable set point with low priority. The remaining feed capacity low limits, and high and low limits of the level controller output have high priority.

Note that each train has two feeds; one train feed and one from the dew point control unit (DPCU). The two feeds have different compositions, and this makes it possible for the coordinator to adjust the feed composition, and thus adjust the load to specific units. The two stabilizers are identical in the simulator, so the stabilizer split (27FC3208VWA) will ensure equal load to the stabilizers. The coordinator uses experimental step response models, obtained in the same way as for the local MPCs. The models were obtained at 80-95% of the maximum throughput, which is typical for the current plant operation. The coordinator execution rate is slower than in the local MPCs to ensure robustness and is here chosen to be 3 minutes. The prediction horizon is set to 20 hours.

The coordinator attempts to maximize the total feed rate while satisfying the capacity constraints for the units. Since the capacity constraints are “hard”, it is necessary to introduce at steady-state a back off b to ensure $R \geq 0$ also dynamically. Tuning of the coordinator MPC is a trade-off between robustness and MV (feed) variation on the one side and keeping the flows through the bottlenecks close their maximum on the other side. The required back off b needs to be obtained after observing over some time the performance of coordinator MPC. In the case study, the value of b is about 1-2% of the feed to the unit.

5.4.4 Results from the simulator case study

The coordinator MPC performance is illustrated with three different cases:

1. Take the plant from unconstrained operation (with given feed rate) to maximum throughput (at $t = 0$ min)
2. Change in feed composition (at $t = 360$ min)
3. Change in a CV limit in a local MPC (at $t = 600$ min)

All three cases are common events at the Kårstø plant. Feed composition changes are the most frequent ones. The coordinator should also be able to handle operator changes in the local MPCs as illustrated by changing a local CV limit.

The most important CVs in the coordinator MPC are displayed in Figure 5.4 and the corresponding coordinator MVs are shown in Figure 5.5. CVs far from their constraints are omitted. The vertical lines in the Figures indicate the time where disturbances are introduced (Cases 2 and 3). The back off from the capacity constraints is indicated by dashed horizontal lines in Figure 5.4. Figure 5.6 shows the response of a local MPC application (BS100).

Case 1: Take the plant to maximum throughput

Initially, the plant is not operating at maximum throughput, and Figure 5.5 shows that all four feed rates are ramped up over the first hour. The crossover (named 24FC5074VWA in Figure 5.5) is reduced to unload train 300 where BS300 is close to its capacity limit even initially (the plant is not steady state at $t = 0$ min). From Figure 5.4, ET100 and the T300 stabilizers (Stab1 and Stab2) impose a bottleneck upstream of the crossover, whereas BS300 is a bottleneck downstream the crossover, at least for some period. The remaining capacity in BS300 violates its lower limit of $b = 1.6$ t/h, and is actually just below zero for some time. Hence the back off b is not sufficiently large to keep the remaining capacity just above zero in this case. From Figure 5.6, we see that the local MPC application for BS100 relaxes the quality set points because the column reaches the differential pressure high limit.

Case 2: Change in feed composition

A feed composition step change is introduced to the train 100 feed (which is sum of 21FR1005VWA and 21FC5335VWA). The composition change is given in Table 5.1 and occurs at time $t = 360$ minutes, at the first vertical line in Figures 5.4, 5.5 and 5.6. The reduction in ethane content leads to an increase in the remaining feed capacity in ET100, which is a bottleneck at that time, and the coordinator can increase the train feed. However, the increase in iso-butane content reduces the remaining feed capacity in the further downstream butane splitter (BS100), which becomes a new bottleneck. The coordinator increases the crossover to make use of some remaining capacity in train 300.

Case 3: Change in a CV limit in a local MPC

The bottom quality high limit in BS100 is reduced at a time where BS100 is already operating at its capacity limit, as can be seen at $t = 600$ minutes in Figure 5.6. This leads to a reduction in the remaining feed capacity in BS100 of about 2 t/h. The coordinator MPC responds by increasing the crossover flow from T100 to T300 in addition to T100 feed reduction. The two butane splitters (BS100 and BS300) are

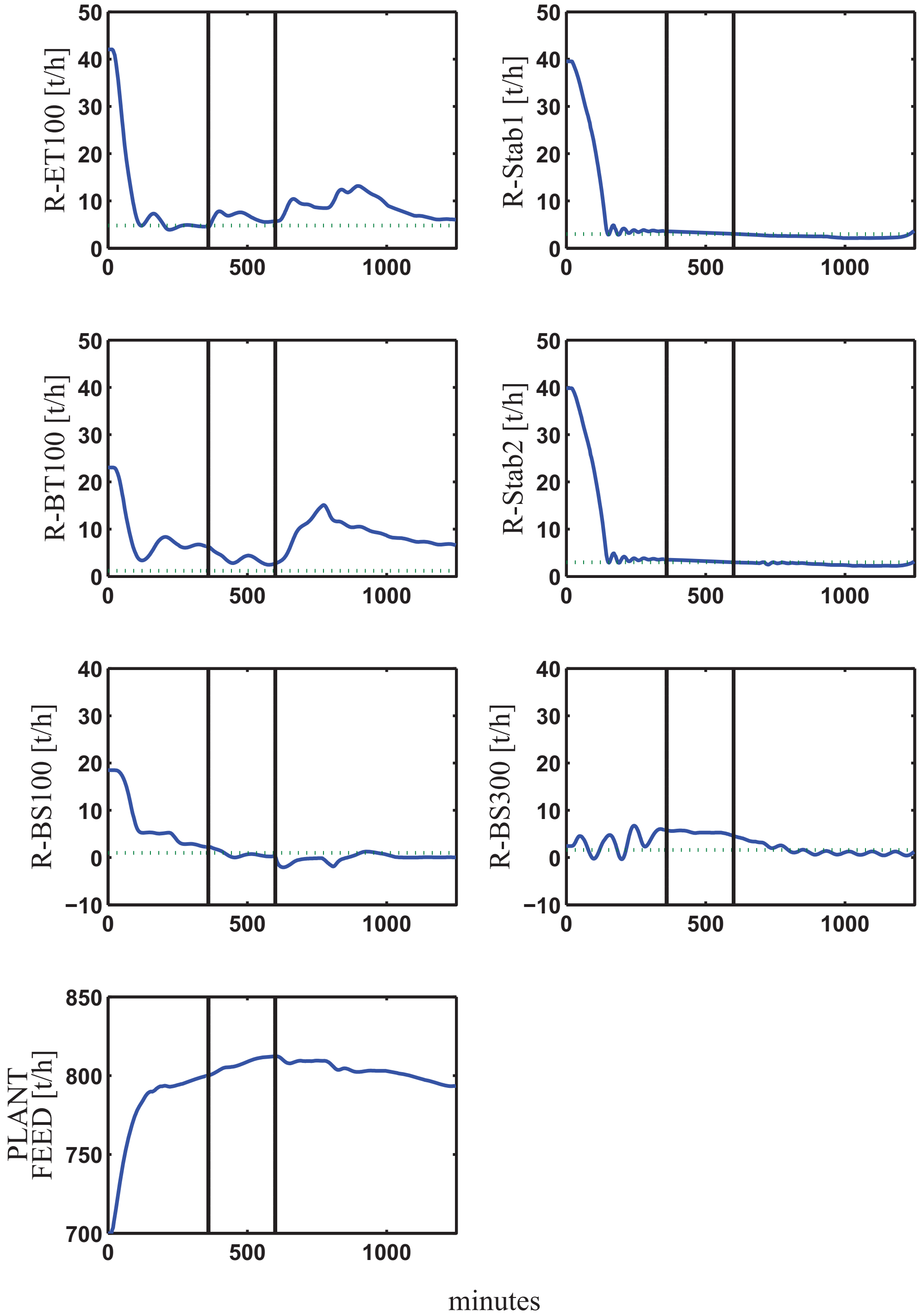


Figure 5.4: The most important CVs in the coordinator MPC (solid) with CV limits (dotted)

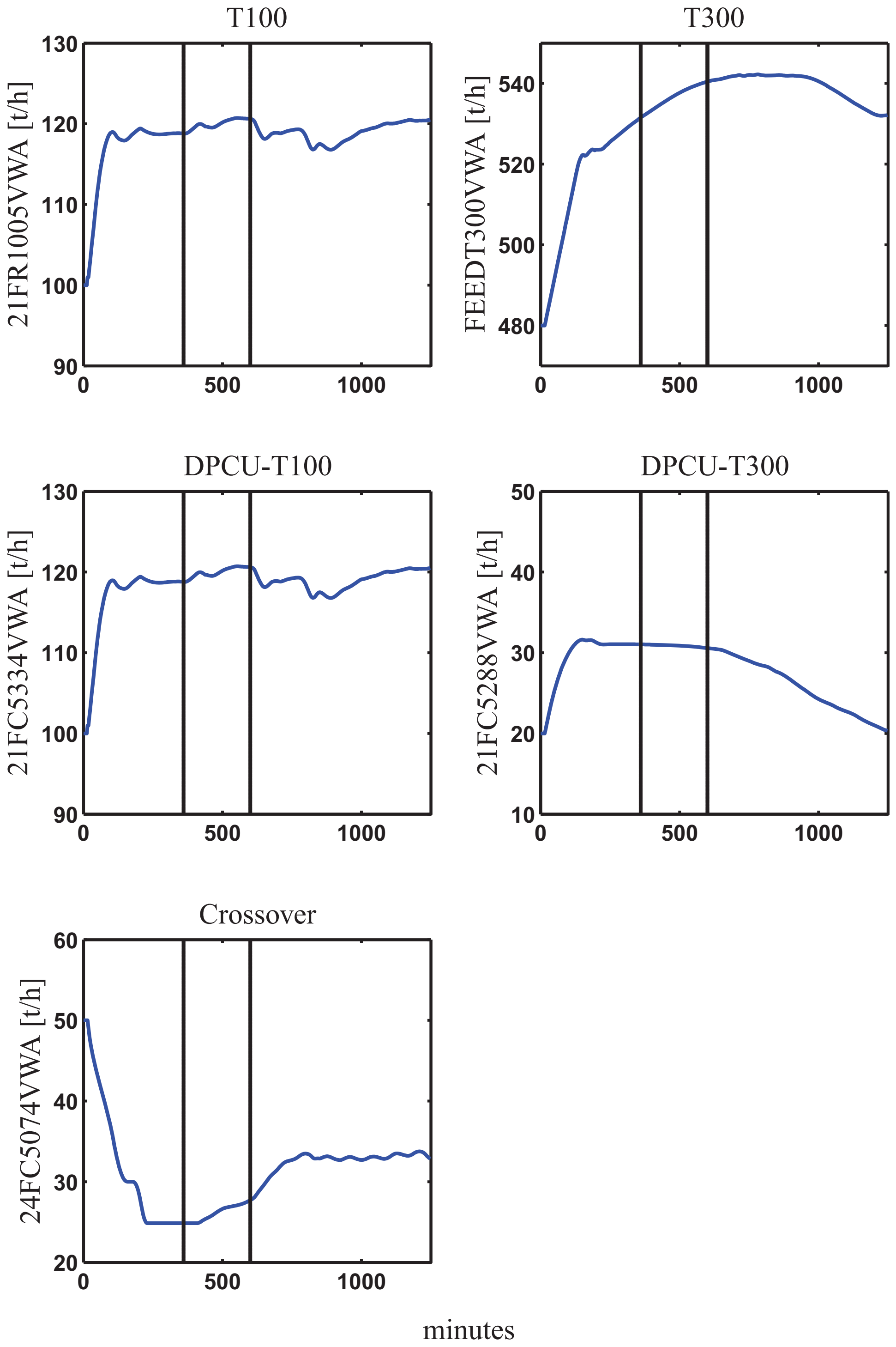


Figure 5.5: MVs in the coordinator MPC. Vertical lines indicate new case.

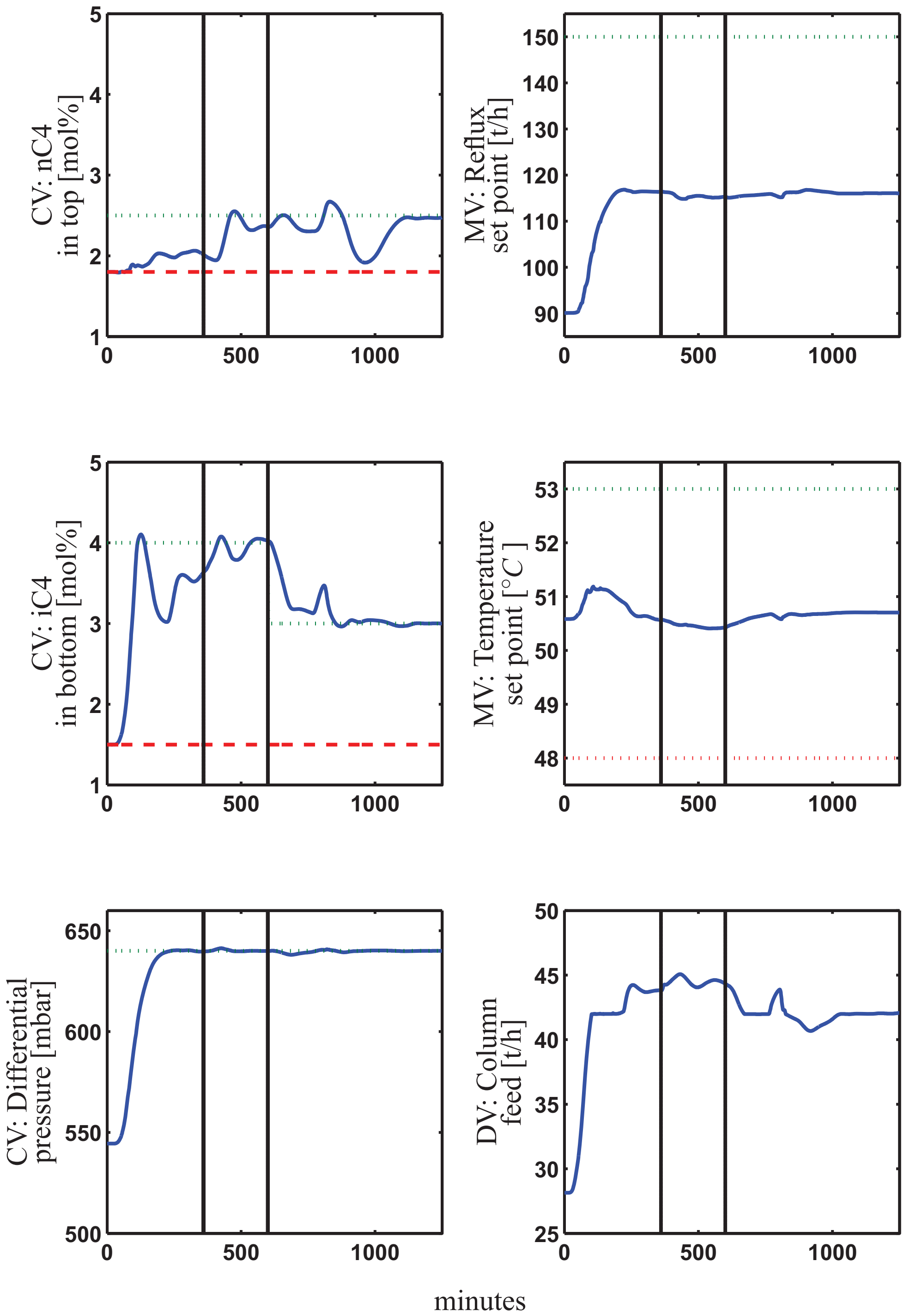


Figure 5.6: CVs, MVs and DV in BS100 MPC. Horizontal lines are set points (dashed) and limits (dotted).

Component	Nominal [mol%]	Points change [%]
Ethane	37.3	-1.1
Propane	35.4	0.71
Iso-butane	5.64	5.6
N-butane	11.3	-0.34
Iso-pentane	1.79	0.09
N-pentane	1.79	0.10

Table 5.1: The feed composition change in the T100 feed at $t = 360$ minutes

now the bottlenecks together with the stabilizers. As expected, the overall effect of the stricter quality limit is reduction in the total plant feed. The reduction takes a long time, however, because the bottleneck in the butane splitters is quite far from the plant feeds.

5.5 Discussion

The main assumption behind the proposed coordinator MPC (see (5.13)-(5.17)), is that optimal operation corresponds to maximum throughput. This will always be the case if the flow network (G_{dyn}) is linear because we then have a LP problem. However, as discussed in Section 5.2, even a nonlinear network will have maximum throughput as the optimal solution provided the product prices are sufficiently high. Thus, the use of a linear flow network model (G_{dyn}) in the coordinator MPC is not a critical assumption. The coordinator identifies the maximum throughput solution based on feedback about the remaining capacity in the individual units, and the main assumption for the network model is that the gains (from feed rates to remaining capacities) have the right sign. Nevertheless, a good network model, both static and dynamic, is desired because it improves the dynamic performance of the coordinator MPC.

In this application, the remaining capacity is obtained for individual units. However, in some cases, for example, reactor-recycle systems, it may be better to consider *system bottleneck* caused by the combination of several units (Aske *et al.*, 2007).

By using a decoupled strategy based on the remaining feed capacity in each unit, the coordinator MPC exploits the already existing models in the local MPCs. This leads to a much smaller modelling effort compared to alternative approaches, like RTO based on a detailed nonlinear model of the entire plant. The computation time in the coordinator MPC is small, and facilitates fast corrections of disturbances, model errors and transient dynamics. The coordinator MPC effectively solves the DRTO problem with acceptable accuracy and execution frequency.

An alternative coordinator MPC strategy would be to combine all the local MPCs into one large combined MPC application including the throughput manipulators. However, for a complete plant the application will be over-complex leading to challenging modelling and maintenance. The improvement by using a combined approach compared to our simple coordinator MPC is expected to be minor since the set points to the MPC are not coordinated. Set point coordination would require a nonlinear model for the entire plant, for example, RTO.

A back off from the maximum throughput in the units is necessary due to unmeasured disturbances and long process response times. The back off should be selected according to the control performance and acceptable constraint violations. In general, the back off can be reduced by improving the dynamic network model and including more plant information to allow for feed-forward control. For example, feed composition changes could be included in the coordinator MPC to improve performance. Due to the lack of fast and explicit feed composition measurements in the plant, feed composition changes are treated as unmeasured disturbances in the simulations in the current concept. However, the concept can be extended by using intermediate flow measurements as indicator for feed composition changes. Therefore, the use of alternative model structures that will simplify and propagate model corrections from intermediate flow measurements should be evaluated.

The most effective way of reducing the back off is to introduce throughput manipulators that are located closer to the bottlenecks. This reduces the dynamic response time and gives tighter control of the flow through the bottleneck. In the case study, the crossover flow introduces a throughput manipulator in the middle of the plant, which improves the throughput control of the units downstream the crossover. It is also possible to include additional dynamic throughput manipulators that make use of the dynamic buffer capacity in the various units and intermediate tanks in the network.

The coordinator requires that the local MPC are well tuned and work well. If the local MPC is not well tuned, a larger back off is needed to avoid constraint violation in the coordinator MPC. In the case study, the BS300 MPC should be retuned to give less oscillation at high throughputs.

The term ”coordinator” is used by authors (Venkat *et al.* and Cheng *et al.*) to describe coordination of multiple MPCs where the coordinator is at the level above and generates set points to the local MPCs. In this work the term ”coordinator” is used in the meaning of coordinating the flow through the plant, and the coordinator at the same level in the control hierarchy as the local MPCs (see Figure 5.1). However, the tuning is assumed to be done sequentially, with the local MPCs being closed before obtaining the flow network model and tuning the coordinator MPC.

5.6 Conclusion

In many cases, optimal operation is the same as maximum throughput. In terms of realizing maximum throughput there are two issues, first identifying bottleneck(s) and second, implementing maximum flow at the bottleneck(s). The first issue is solved by using the models and constraints from the local unit MPC applications to obtain an estimate of the remaining feed capacity of each unit. The second issue is solved using a standard MPC framework with a simple linear flow network model. The overall solution is a coordinator MPC that manipulates on plant feeds and crossovers to maximize throughput. The coordinator MPC has been tested on a dynamic simulator for parts of the Kårstø gas plant, and it performs well for the simulated challenges.

Chapter 6

Industrial implementation of a coordinator MPC for maximizing throughput at a large-scale gas plant

Based on a paper submitted to International Symposium on Advanced Control of Chemical Processes, July 2009, Istanbul, Turkey.

A coordinator MPC has been earlier proposed as a way to implement maximum throughput (Aske *et al.*, 2008). The coordinator manipulates feed rates, crossovers and flow splits that affect the flows through the units, but which are not used as degrees of freedom by the local MPCs on the individual units. In this paper, an industrial implementation of a coordinator MPC at the large-scale Kårstø gas plant is described, including design, modelling and tuning. The local MPC applications estimate the remaining capacity of each unit. Although not fully implemented, the coordinator MPC is found to be a promising tool for implementing maximum throughput.

6.1 Introduction

In this paper, we describe an actual industrial implementation of the method for maximum throughput proposed earlier by Aske *et al.* (2008). The application is the Kårstø gas processing plant, which plays a key role in the transport and treatment of gas and condensate from the Norwegian continental shelf. The products from the plant are dry gas, which is exported in pipelines, and natural gas liquids (NGL) and condensate, which are exported by ships. The plant receives rich gas and unstabilized condensate through pipelines from more than 30 producing fields. This set high demands, not only to the plant efficiency and its regularity, but also to the plant throughput. Limited gas plant processing capacity means that one or more fields must reduce production or even shut down. Therefore, it is important that the Kårstø plant does not become a "bottleneck" in the Norwegian gas transport system. The Kårstø plant has no recycles or reactors, but it has several independent feeds and parallel flows that make it possible to have multiple bottlenecks at the same time. In addition, the bottlenecks may move due to disturbances. The coordination problem of maximizing the throughput is thus a challenging multivariable problem.

The overall feed rate (or more generally the throughput) affects all units in the plant. For this reason, the throughput is usually not used as a degree of freedom for control of any individual unit, but is instead left as an "unused" degree of freedom (u^c) to be set at the plant-wide level.

The throughput at the Kårstø plant is presently set by the operators who manipulate the feed valves to satisfy orders from the gas transport system (operated by another company). The orders may be given as pipeline pressures, feed rates and export gas rates, which may change on an hourly basis. The objective of this work is to coordinate the throughput manipulators (u^c) to achieve economic optimal operation.

In general, to optimize the economic operation of a plant, one may use real-time optimization (RTO), normally based on (rigorous) steady-state models. Standard RTO methods require the plant to be close to steady state before performing a reoptimization based on data reconciliation or parameter estimation (Marlin and Hrymak, 1997). However, many plants are rarely at steady state or important economic disturbances occur more frequent than the controlled plant response times. At least in theory, it is then more suitable to use dynamic optimization with a non-linear model, which may be realized using dynamic RTO (DRTO) or non-linear model predictive controller (MPC) with an economic objective, e.g. Engell (2007); Kadam *et al.* (2003); Backx *et al.* (2000); Strand (1991).

In this study, a different approach is used. We assume that optimal economic operation is the same as maximizing plant throughput, subject to achieving feasible operation (satisfying operational constraints in all units) with the available feeds. This corresponds to a constrained operation mode with maximum flow through the bottleneck(s). At maximum throughput, all throughput manipulators (u^c) are used to satisfy active constraints (bottleneck). Thus a nonlinear model of the entire plant is not needed, and instead linear MPC may be used (Aske *et al.*, 2008). One option is to combine all the MPCs in the plant into a single application. However, here we choose to decompose the problem by keeping the local MPC applications and introducing a coordinator MPC (Aske *et al.*, 2008) to maximize throughput.

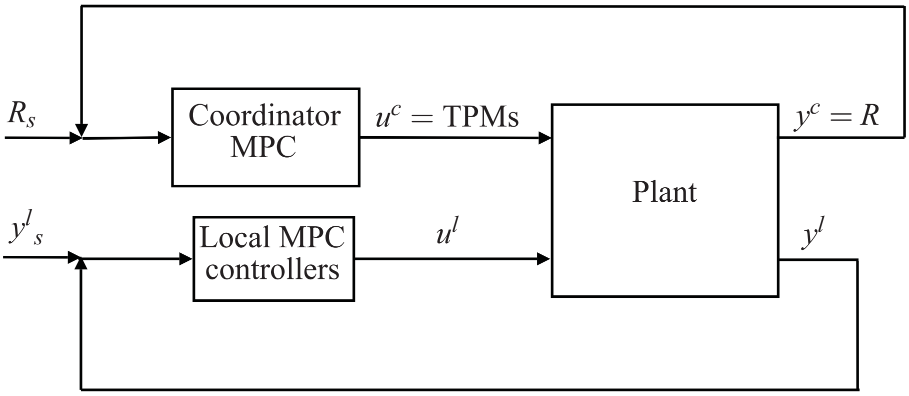


Figure 6.1: Plant decomposition by coordinator MPC. The local MPC applications uses u^l to control the local targets y^l_s, whereas the coordinator uses the throughput manipulators (u^c = TPMs) to control the remaining capacity ($y^c = R$) in the units.

The coordinator uses the remaining degrees of freedom (u^c) to maximize the flow through the network subject to satisfying given constraints. The remaining degrees of freedom (u^c) include feed rates, feed splits and crossovers. The constraints are the feasible remaining capacities of the individual units ($R_k > 0$). The feasible remaining capacity R_k is how much more feed unit k can receive while operating within its constraints. For most units, R_k is not a quantity that can be measured, because it depends on the operation of the unit. For example, the capacity may be increased by producing less pure products. A key idea in the approach of Aske *et al.* (2008) is to use the local MPC to estimate R_k. By estimating R_k for each unit, the plant-wide control problem is decomposed. The main advantage of decomposition is that the application becomes smaller in size and hence easier to understand and maintain. The plant decomposition is illustrated in Figure 6.1.

All MPC applications at the Kårstø plant use the in-house SEPTIC* technology (Strand and Sagli, 2003). SEPTIC minimizes a quadratic objective function using linear models and constraints and handles relaxation of the constraints. Even though SEPTIC is capable of using non-linear models, linear SISO step response models are used in all applications described here.

This paper considers about half of the Kårstø gas processing plant. The application presently includes 12 distillation columns, 2 compressor stages, 4 feed valves and 2 crossovers (splits). The main reason for not including the entire plant is that local MPC applications are yet not implemented on all units.

This paper is organized as follows. The local MPC controllers for the individual units are discussed briefly in Section 6.2. The local MPCs adjust the local degrees of freedom (u^l) such that the operation is locally feasible. However, local

***S**tatoil **E**stimation and **P**rediction **T**ool for **I**dentification and **C**ontrol feasibility requires that the feed rate to the unit F_k^l is below its maximum capacity, $F_{k,max}^l$, and one of the tasks of the plant-wide coordinator is to make sure that this is satisfied ($R_k = F_{k,max}^l - F_k^l > 0$). The maximum capacity for a unit ($F_{k,max}^l$) may change depending on disturbances (e.g. feed composition) and needs to be updated continuously. A key idea of this work is to use the already existing models in the local MPCs to estimate $F_{k,max}^l$ and is discussed in Section 6.3. Section 6.4 discusses the coordinator MPC, including control design choices, model development, tuning issues and test runs. Experience from the implementation at the Kårstø site is summarized in Section 6.5. All the time series displayed in this paper are from closed-loop operation of the coordinator MPC at the Kårstø plant. The experience with the coordinator MPC is so far limited, but it seems to be a promising tool for implementing maximum throughput (Section 6.6).

6.2 Local MPC applications

Presently, all the local MPC applications for the coordinator are on two-product distillation columns. A short description of these applications is given below.

The main control objective for each distillation column is to control the quality of the distillate- (D) and bottoms (B) products. In addition, the column must be kept under surveillance to avoid overloading, which is an important issue for maximizing throughput. Column differential pressure (Δp) is used as an indicator of flooding (Kister, 1990), but so far the differential pressure is controlled for only a few of the columns. The LV-configuration is used for the distillation columns, which means that reflux L and boilup V remain as degrees of freedom after closing the level loops using D and B. In addition, column temperature is controlled using boilup V in the regulatory control layer.

The local MPCs are configured with the following controlled variables (CVs), manipulated variables (MVs) and disturbance variables (DVs):

CV (set point + max constraint): Impurity of heavy key component in D.

CV (set point + max constraint): Impurity of light key component in B.

CV (max constraint): Column differential pressure (Δp).

MV: Reflux flow rate set point (L).

MV: Tray temperature set point (T_s).

DV: Column feed flow.

These MVs correspond to the local degrees of freedom (u^l) and the CVs correspond to the local outputs (y^l), see Figure 6.1. Some of the columns have additional

CV constraints, like valve opening, temperatures and levels. One column has an additional MV and some columns have additional DVs, but in principle, all the columns have the same control configuration.

The product qualities are given by the mole fraction of the key component ratios and a logarithmic transformation is used to linearize over the operating region (Skogestad, 1997). The high limits on the product impurities follow from the sales specifications and the differential pressure high limit is set to avoid flooding.

The local MPC problems are solved at each sample time using a standard two-step approach, where first a steady-state problem is solved with constraint relaxation until the predicted final steady state is feasible, and then the "standard" dynamic MPC problem is solved with the possibly recalculated (reachable) set points and constraints. The priority order for solving the steady-state feasibility problem in the local MPC (Strand and Sagli, 2003) is:

1. High limit differential pressure
2. Impurity limits
3. Impurity set points

This priority hierarchy may lead to a relaxation of the impurity set points (and in worst case the limits) to avoid exceeding the differential pressure high limit. By using relaxation, the column can handle the given feed rate without flooding the column, but note that the exceeding the limits may result in an unsellable product. In the dynamic optimization part, constraints are handled by adding penalty terms to the objective function.

The local MPC applications are based on experimental step response models as described in Appendix A. The prediction horizon is 3 to 6 hours, which is longer than the closed-loop response time. The sample time is 1 minute, which is sufficiently fast for the distillation column applications.

6.3 Estimate of remaining capacity

In this section, the procedure used by the local MPCs for estimating the remaining capacity in each unit (R_k) is explained.

The remaining capacity for unit k is the difference between the current feed F_k^l and the feasible maximum feed $F_{k,max}^l$

$$R_k = F_{k,max}^l - F_k^l \tag{6.1}$$

The feed to the local unit F_k^l is assumed to be a DV in the local MPC application. The maximum feed to the unit k is then easily obtained by solving an additional steady-state LP-problem:

$$F^l_{k,max} = \max_{u^l_k, F^l_k} F^l_k \tag{6.2}$$

subject to the present initial state, linear model equations and constraints used in the local MPC. Here u^l_k is the vector of manipulated variables in the local MPCs, and at the optimal solution $F^l_{k,max}$, all these degrees of freedom (u_k) are used to satisfy constraints (feasibility limit). Note that $F^l_{k,max}$ is calculated using the end predictions (steady-state model) for the variables. This is to include both past MV moves and disturbances and future MV moves for the local MPC. This indirectly assumes that the closed-loop response time for the local MPC is faster than for the coordinator. The algorithm included in the MPC software uses a Simplex method to solve the LP problem.

Note that $F^l_{k,max}$ can change due to updated measurements, disturbances (e.g. feed compositions changes), changes in the constraints and model changes (that is, the steady-state gain in the models) in the local MPCs. The current feed to the unit (F^l_k) is measured, either by a flow transmitter or by a level controller output (valve opening) if a flow transmitter is not available.

The accuracy of the estimated remaining capacity depends on:

- The validity of the models used in the local application. The algorithm uses the end prediction; hence, the steady-state gain in the models is important.
- The appropriate use of gain scheduling for CV-MV pairs with larger nonlinearities, in particular, for distillation column flooding indicators (differential pressure). Here "gain scheduling" means that the model gain is updated (scaled) based on the current operation point. Gain scheduling on differential pressure is included for some columns.
- The CV constraints must reflect the true operational limits and the MV constraints must be reasonable.

Let us explain the first two points in more detail. An incorrect steady-state gain leads to a poor estimate of the remaining capacity (controlled variable) and because the coordinator MPC has slow dynamics, it will take a long time before the feedback can correct for the error. A too high remaining capacity estimate (too small steady-state gain) lead to a oscillating behavior because of the long delays in the flow network. In such cases, detuning may be necessary (high move penalty on MVs) to avoid amplifying the oscillations. Another issue is that the operators will not trust the remaining capacity estimates if the estimates are far away compared to their own experience.

Ideally, the calculation of remaining capacity uses directly the model and constraints used in the existing local MPC. However, in some cases "artificial" (non-physical) constrains are added for tuning reasons in the local MPCs and these should not be included. For example, in the demethanizer MPC application there is a temperature constraint in the column mid-section (high limit) with the same priority as the CO_2 content in distillate (high limit). Here, only the CO_2 content should be a limiting factor on the feed rate. The temperature high limit is included to obtain better boiler distribution in the column and should not limit the throughput. In this case we choose to replace the "artificial" constraints with a wider constraint, since our version of the LP algorithm does not handle relaxation of constraints and may risk infeasibility. Another option would be to omit the constrained variable from the remaining capacity calculation, but for the temperature variable mentioned above, it has a low limit that must be considered in the capacity calculation and the variable must therefore be included.

For distillation columns that frequently operate close to their capacity limit, the estimated capacity is generally good. For these units we have more experience in the actual operation range, and the models in the local MPC applications are typically obtained in this range. For some columns, the differential pressure is included in the remaining capacity calculation, and for these columns, the estimate of remaining capacity is better. Another issue is that the estimate uses the CV constraints and not the CV set points. For a distillation column, the distillate and bottoms quality constraints are used instead of the CV set points because set point deviations are acceptable if the alternative is feed reduction. This leads to an estimated capacity that is larger than expected by the operators.

For units with several feeds, the LP optimization will maximize the feed with the smallest steady-state gain (smallest predicted effect on capacity), whereas the other feeds will go to zero. However, some feeds cannot be set to zero, because they are outlet from an upstream unit with no possibility for routing it elsewhere. In this case, the LP optimization is set to maximize the feed from the flow line the unit must process and the other feeds are held constant in the optimization.

The estimation of remaining capacity described above is given for distillation columns. However, compressors are also included in the application, but at present, there are no MPC applications implemented on these. To estimate the remaining capacity of the compressors one option could be to consider the percent load (given by the speed). However, it may not always be possible to reach 100% load due to other constraints, for instance the turbine exhaust gas temperature. To consider several constraints, we therefore use "dummy" MPC applications, with only CVs and DVs and models between them to estimate the remaining capacity for the compressors.

The use of the local MPCs to estimate the remaining capacity decomposes the control problem to a large extent, and the coordinator MPC has a "reasonable" size, even though if it is a plantwide controller. At present, the estimate is based on experimental models. However, rigorous models for local units can also be used to predict the remaining capacity. This is attractive for units where experimental modelling is difficult, for example, due to nonlinearities. This illustrates the flexibility with this decomposition where the best available model can be used to predict the remaining capacity.

6.4 Coordinator MPC

In this section the objective, variables, modelling and tuning of the coordinator MPC is described. Note that the coordinator MPC coordinates the network flows and not the local MPC applications. Its main objective is to maximize plant throughput subject to achieving feasible operation.

6.4.1 Objective, variables and constraints

The Kårstø plant is shown in Figure 6.2 where most of the CVs, MVs and DVs for the coordinator MPC are indicated. The coordinator MPC maximizes sum of the total plant feed which is the sum of the feeds to train 100 (T100), train 200 (T200), train 300 (T300), train 400 (T400) and the dew point control unit (DPCU). The application consists of:

- 6 MVs: 4 feed rates, 1 crossover, 1 feed split.
- 22 CVs: Remaining capacity of 12 distillation columns and 2 compressors steps, 7 other constraints plus the main objective: total plant feed with a high, unreachable set point with lower priority.
- 7 DVs: 3 feed rates, 2 feed compositions, 1 crossover, 1 feed split.

The MVs (throughput manipulators) are the feed rates, a crossover between parallel trains (from T100 to T300) and a feed split to T300. Other throughput manipulators that affect the CVs in the sub-application are included as DVs. Later, if the coordinator MPC is extended to the whole plant, most of these DVs will become MVs. The feed compositions (DVs) reflects the gas/liquid split, and determine the split between gas flow to the compressors and liquid flow to the fractionation and are estimated from flow- and temperature measurements.

The CVs are the remaining capacities of the units, in total 2 compressor stages and 12 distillation columns. Even though there are three compressors at each stage, the remaining capacity of each stage is used as a CV, because local control handles the distribution between parallel compressors (equal distance to the compressor control line). The “other” 7 CV constraints are related to the use of MVs, that is, levels constraints to avoid filling or emptying of buffer tanks and sump volumes, pressure constraints in the pipelines and pressure controller outputs.

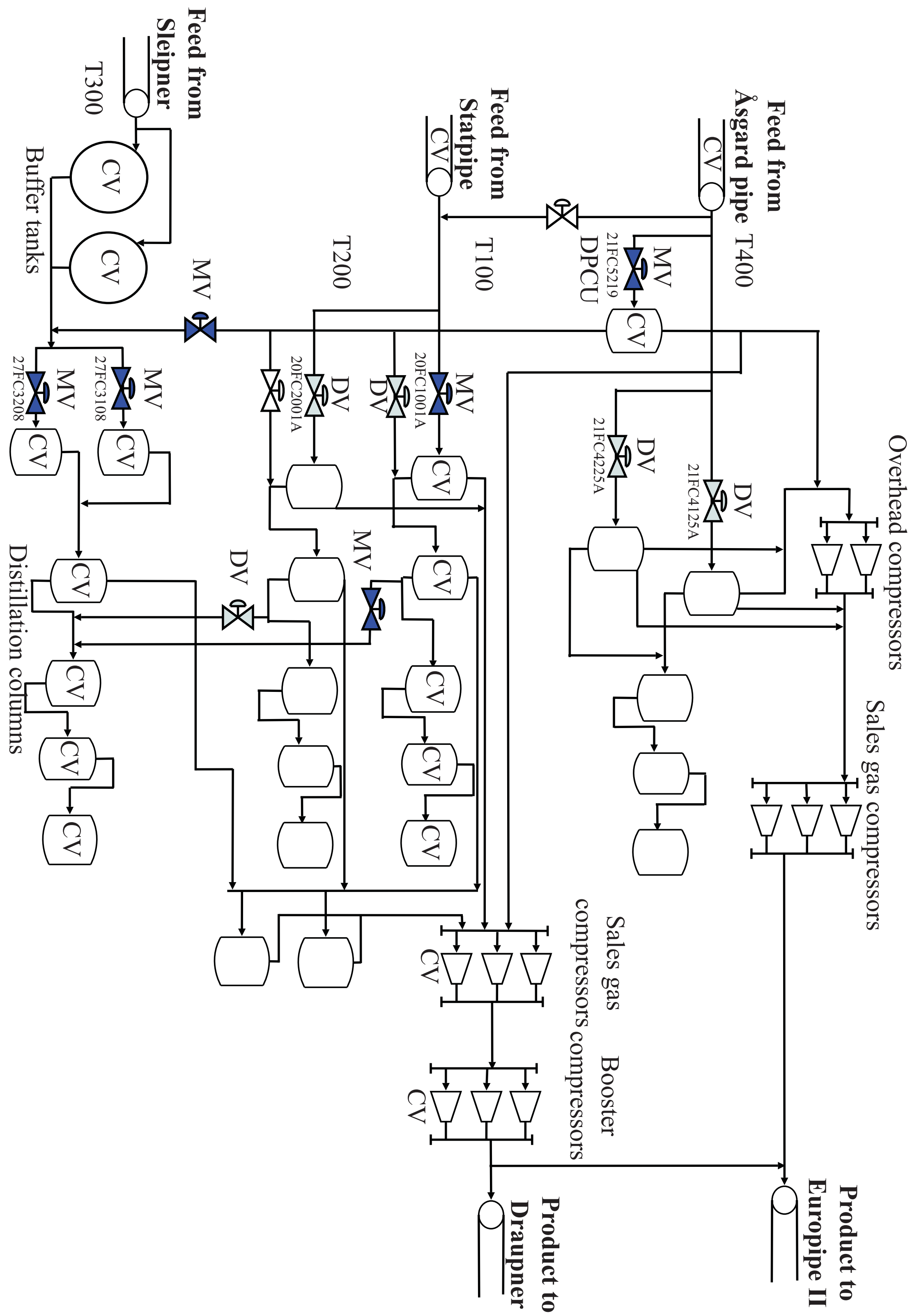


Figure 6.2: Overview of the Kårstø plant, including the coordinator MPC variables.

Each variable (CV, MV and DV) belongs to one or more sub-groups that will be deactivated if one critical variable in the sub-group is deactivated. For instance, if a local MPC application is turned off, the corresponding remaining capacity CV is deactivated, and this critical variable suspends the whole sub-group. By using this condition-based logic, the coordinator MPC can operate even if parts of the plant are not running or not available for throughput maximization. For the coordinator MPC, each MV defines a sub-group with corresponding CVs as members.

The CV "total plant feed" is the sum of the plant feeds and is given by

$$\begin{aligned}\mathrm{TOTALFEED} = {} & \mathrm{20FC1001A} + \mathrm{20FC2001A} + \mathrm{27FC3108} \\ & + \mathrm{27FC3208} + \mathrm{21FC4125A} + \mathrm{21FC4225A} + \mathrm{21FC5219}\end{aligned} \tag{6.3}$$

where the variables are marked in Figure 6.2. In general, the feeds could have different weighting, but at present, their weights are equal. Of the 22 CVs, only the total plant feed is set point controlled; the other CVs are constraints. The objective function in the SEPTIC MPC algorithm is quadratic, while the objective function for the the maximum throughput problem is linear

$$J = -\mathrm{TOTALFEED} \tag{6.4}$$

To obtain a quadratic objective function that fits directly into our quadratic MPC algorithm, we have used the common trick of introducing a quadratic set point deviation term with a high and unreachable set point $\mathrm{TOTALFEED}_s$ with a lower priority than the capacity constraints, $J = (\mathrm{TOTALFEED}_s - \mathrm{TOTALFEED})^2$. (Of course, the actual case function used by the coordinator MPC has additional terms and weights). The first step of the coordinator MPC solution will then result in a recalculated (reachable) set point for the total feed.

The MV feeds have ideal values (IV) for dynamic reasons that are discussed in Section 6.4.3. The crossover has an ideal value to keep its flow in the middle of the operation range when constraints do not determine the crossover flow. The detailed control structure including priorities (CV limits, CV set points and MV ideal values) and groupings is summarized in Table 6.1.

The decomposition requires that the coordinator receives three variables from each of the 12 local MPC applications:

- Estimated remaining capacity (value)
- Quality of the remaining capacity value (good/bad)
- Status of the local MPC (on/off)

If the estimated remaining capacity has a bad value, that is, the LP formulation is not feasible, then the status of the remaining capacity CV is set to ERROR and

MVs								
Name	Description	Priority	Sub-group					
20FC1001A	Feed T100	3	1*					
24FC5074	Crossover T100 to T300	4		2*				
27FC3108	Feed Stabilizer 1 T300	3			3*			
27FC3208	Feed Stabilizer 2 T300	3				4*		
21FC5219	Feed DPCU (T500)	3					5*	
21FC5288	From DPCU to T300	3						6*
CVs								
Name	Description	Priority	Sub-group					
RemCapMT100	*R* demethanizer T100	1	1*	2				
RemCapET100	*R* deethanizer T100	1	1*	2			5*	6*
RemCapPT100	*R* depropanizer T100	1	1*	2*			5*	6*
RemCapBT100	*R* debutanizer T100	1	1*	2*			5*	6*
RemCapBS100	*R* butane splitter T100	1	1*	2*			5*	6*
RemCapSTB1	*R* stabilizer 1 T300	1			3*		5	6
RemCapSTB2	*R* stabilizer 2 T300	1				4*	5	6
RemCapET300	*R* deethanizer T300	1			3*	4*	5	6*
RemCapPT300	*R* depropanizer T300	1		2*	3*	4*	5	6*
RemCapBT300	*R* debutanizer T300	1		2*	3*	4*	5	6*
RemCapBS300	*R* butane splitter T300	1		2*	3*	4*	5	6*
RemCapDPCU	*R* DPCU	1					5	
RemCapSTPSGC	*R* Statpipe sales gas compressors	1	1*	2				
RemCapSTPCC	*R* Statpipe booster compressors	1	1*	2				
15PI0039	Pressure Statpipe	1	1*					
15PC0002VYA	Pressure control output Statpipe	1	1*					
24LC1001VYA	Sump level output deethanizer T100	1	1	2*			5	
36LI3054	Level buffer volume 1	1			3*	4*		6*
36LI3914	Level buffer volume 2	1			3*	4*		6*
15PI2025	Pressure Åsgard pipe	1					5*	
15PI2028VYA	Pressure control output Åsgard pipe	1					5*	
TOTALFEED	Total plant feed	2	1		3	4	5	
DVs								
Name	Description		Sub-group					
FEEDCOMPT100	Feed composition T100		1	2			5	
FEEDCOMPT200	Feed composition T200		1	2			5	
20FC2001A	Feed T200		1	2			5	
24FC5071	Crossover T200 to T300			2	3	4	5	
21FC5334DEV	From DPCU to T100		1	2			5	6
21FC4125A	Feed T410						5	
21FC4225A	Feed T420						5	

Table 6.1: MVs, CVs and DVs in coordinator MPC with its 6 subgroups. *: Critical variable for the sub-group.

the corresponding MVs, given by the sub-grouping in the coordinator, are then suspended. If a local MPC application is deactivated, then the unit remaining capacity CV is set to OFF in the coordinator and the sub-group in the coordinator is suspended. The coordinator still runs, but the MVs in the sub-group are deactivated. This is done because we require that the local MPC application is active before the coordinator can operate.

6.4.2 Dynamic modelling for the coordinator MPC

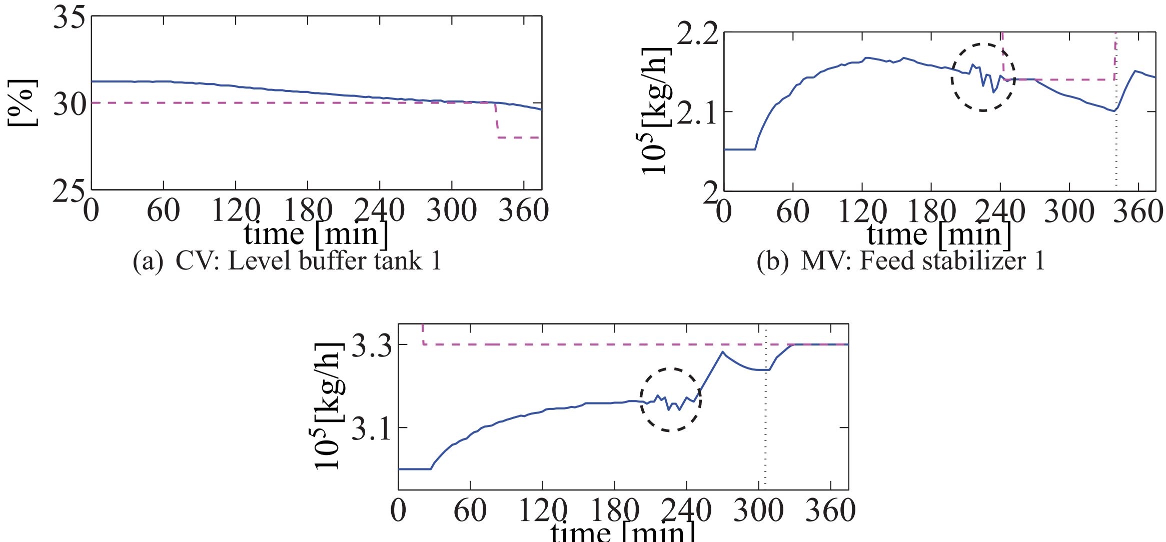
The model for the coordinator MPC is a linear dynamic model for the flows through the plant network with the local MPC applications in service. The current implementation of the coordinator uses individual (SISO) step response models, or more precisely a single-input multiple-output representation of a multi-input multi-output system. The advantage with SISO models is that it is easy to adjust the models independently for input-output pairs. However, SISO models imply that the structure of the model is lost and, for instance, disturbances may not propagate as they would in a state-space model. The loss of structure leads to some additional work around the DPCU. The feed to the DPCU is an MV, and from the DPCU there are three liquid streams, where two are DVs and one is a MV in the coordinator. The two DVs need to be corrected for the changes caused by the two MVs, to avoid modelling the same effect twice. This is done by let the two DVs be the difference between measured and modelled response instead of the measurement directly. In other words the changes in the DVs caused by the two MVs are “subtracted”.

The models are obtained from step tests and historical plant data. The steady-state gains found from step-tests are verified by calculating the gains using typical feed compositions.

The sampling time for the coordinator MPC is 3 minutes. The prediction and control horizon are set to 6 hours, whereas the longest response models reach steady state at approximately 4.5 hours.

6.4.3 Tuning the coordinator MPC

The tuning of the coordinator MPC is a trade-off between robustness and MV (e.g. feed) variations on one side and keeping the flows through the bottlenecks close to their maximum on the other side. The coordinator MPC was gradually operating in closed-loop and tuned in several tests in February 2008.



(a) CV: Level buffer tank 1

(b) MV: Feed stabilizer 1

(c) MV: Feed T100

Figure 6.3: From test run 05 Feb. 2008: why IV is needed on MV feed rates. MV and CV values (solid), high and low limits (dashed) and ideal values (dotted).

MV tuning

From the early tests, it became clear that the trick of using a CV of total plant feed with a high, unreachable set point to maximize throughput, requires ideal values on the MV plant feeds to obtain satisfactory dynamic performance. This is illustrated from a plant test using two MVs and a CV in Figure 6.3. The buffer tank level CV (Figure 6.3(a)) is predicted to reach its low limit (prediction not shown here), and the recalculated (reachable) set point for the CV total plant feed is then reduced. To reach the new recalculated set point for CV total plant feed, *all* MVs that constitute the CV total plant feed (see Equation (6.3)) are reduced dynamically (two of them shown in Figures 6.3(c) and 6.3(b)), even though only the latter affects the buffer tank level. This leads to the “jagged” use of the MVs at $t = 215$ min (marked with a circle). In this case, only the MVs that effects the CV that meets its constraint should be used to reach the recalculated set point for CV total plant feed. This is solved by introducing ideal values on the MV plant feeds. The dotted vertical line in the MV plots indicates the time where ideal values are turned on and then the MV Feed T100 are increased up to its high limit. The reduction in MV Feed T100 at around $t = 270$ min is due to another constraint not shown here. The ideal values that are added to the MV plant feeds are high and unreachable with a lower priority than the total plant feed set point and have a low penalty on the deviation from the ideal value.

When ideal values (IV) for the MVs are introduced, the rate of change towards the ideal value is specified to obtain ramping rate independent of the penalty on the deviation from ideal value (Strand and Sagli, 2003). The ideal ramping rate is set to typical 500-750 kg/h. Maximum increase and decrease of the MV at each sample is chosen based on typically rate changes operators choose to implement.

CV tuning

The most important tuning variables for the CVs are the penalties on constraint violation (see Appendix A) used in the dynamic step of the MPC algorithm. The constraint violation is "balanced" by using penalties on MV moves to obtain a satisfactory dynamic behavior when CV constraints are violated. Even though a CV constraint is violated, the use of MVs should not be too aggressive to avoid unnecessary throughput variations. Importantly, the CV constraints are not absolute because back off is included to handle disturbances and imperfect control. Specifically, the lower value of the remaining capacities is not set to zero, but rather to a positive back off value, $R_k^l > \text{back off}_k > 0$. The value of the back off is a tuning parameter decided by disturbance handling and model accuracy.

The coordinator MPC has four integrating CVs; two buffer volumes (levels) and two pipelines pressures. For an integrator, the horizon length is a tuning parameter. To see this, consider an increase in feed rate that draws more out of the controlled volume, hence the derivative to the integrating CV is negative. The maximum allowed change in level (CV) or slope (the derivative) is given by the current distance to the level constraint divided by the horizon length. A shorter horizon length will give a larger slope and allow for larger feed rate changes. The integrating variables have a prediction horizon of 3 hours, which is half the prediction length to the other variables. The prediction horizon is shortened because it is likely that disturbances occur within the 6-hour period that counteracts the level change.

6.5 Experience from implementation

Some experiences from the implementation at the Kårstø site are summarized in this Section.

6.5.1 Estimate of remaining capacity

To estimate the remaining capacity in each unit, the corresponding local MPC application requires, in general, acceptable product quality control within some operational constraints. One observation is that when a large disturbance occurs, the predicted steady-state values may violate their limits and, if this violation is sufficiently large, the LP optimization does not find a feasible solution and the estimate of maximum capacity ($F^l_{k,max}$) fails. The end prediction values are in such cases often not reasonable because the MPC application assumes that the disturbances will maintain constant (possible reduced with a low-pass filter) throughout the prediction horizon, which is rarely the case.

We have observed oscillations in the estimated capacity with periods of 1-2 hours. These variations are challenging because this corresponds to the closed-loop time constant of the coordinator; hence, these variations cannot be reduced by signal filtering. The variations in the estimated capacities usually arise due to model errors from the feed to the unit (DV). A systematic evaluation of the inferential models (estimators of product quality) and models in the local MPC applications is necessary to obtain satisfactory performance of the coordinator MPC. Since some of the local MPC applications were commissioned several years ago, a validation of the models was found necessary.

To improve the estimation of remaining capacity, several approaches are used:

- With a known, measured, short-time disturbance: The maximum capacity ($F^l_{k,max}$) is held constant during the period of the disturbance. For example, this is used for the disturbances that occur at each dryer exchange.

- For each unit, a minimum value of the maximum capacity ($F^l_{k,max}$) is included.

- CV constraints included in the local MPCs that should not limit the throughput were replaced with wider constraints. This applies to "non-physical" constraint that may have been added in the MPC for tuning reasons.

- Gain scheduling is included for some differential pressure models.

During implementation and test-runs of the coordinator MPC, the local MPC applications were followed up closely and some changes were made. The changes include updating inferential models, updating response models and adding new models in the local applications (mostly for differential pressures).

The main structural weakness in the estimation of remaining capacity is that the LP solver may "give up" to find a solution because there is no possibility for relaxation of constraints. When the LP solver does not find a solution, it returns a "bad quality" value to the coordinator and its variable subgroup is turned off. It would be preferable that the coordinator finds the best possible solution instead of "giving up". This can be realized with a LP solver that includes relaxation of the constraints. This improvement of the LP algorithm is planned to be included in the future.

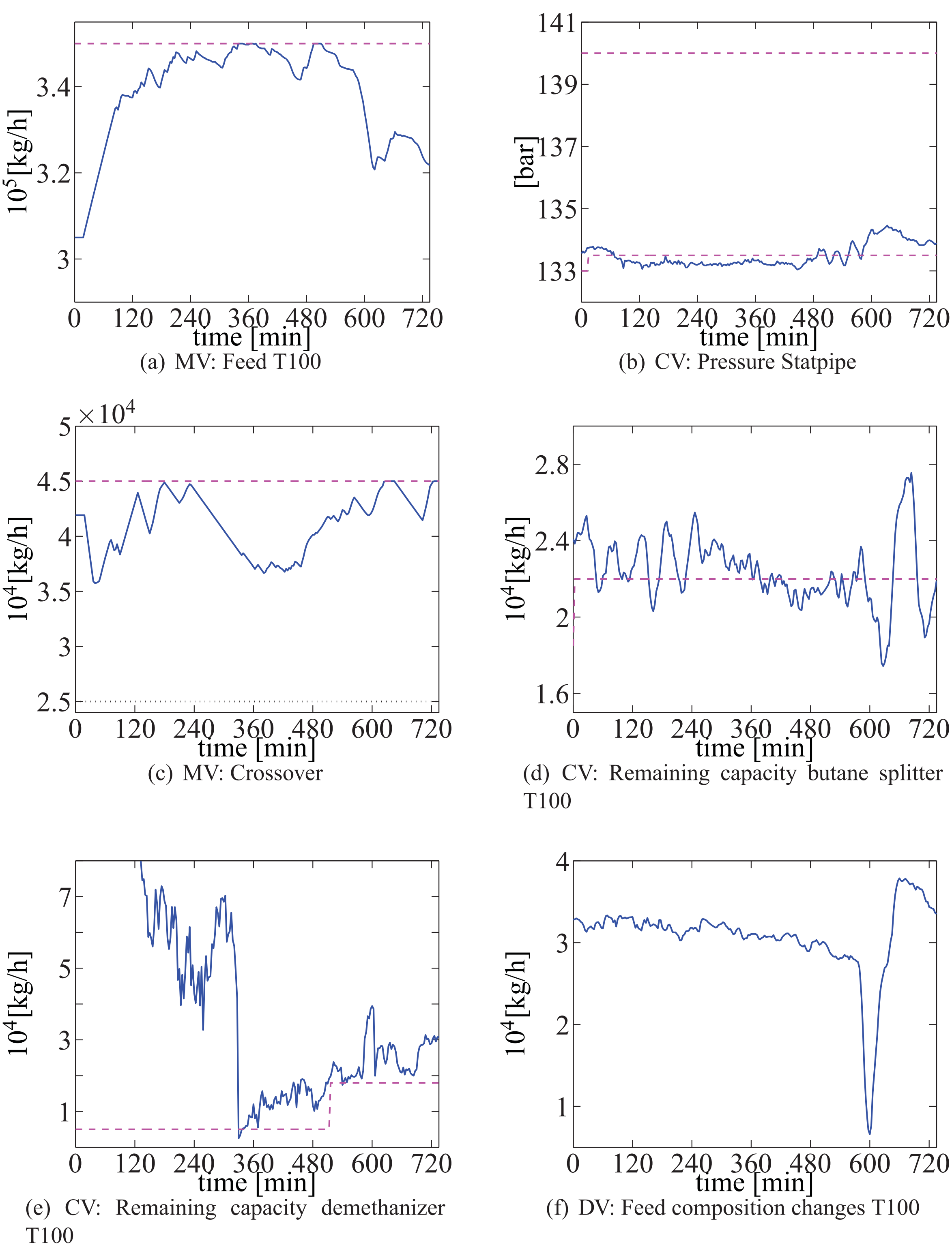
6.5.2 Experience with the coordinator MPC

A test run of the coordinator MPC from 07 Feb. 2008 is displayed in Figure 6.4. The coordinator is turned on at $t = 18$ min and the coordinator starts to increase the feed to T100 (Figure 6.4(a)) until the pipeline pressure in Statpipe reaches its low constraint (Figure 6.4(b)). During this start-up period, the crossover flow ramps towards its ideal value (Figure 6.4(c)). The remaining capacity in the butane splitter T100 reaches its low constraint (Figure 6.4(d)) and the crossover increases again to avoid reduction in the throughput. However, the use of the crossover is "aggressive" and actually generates oscillations in the downstream remaining capacities because of the delays in the flow network if the model gain was too low. The adjustment of the model gain was based on comparing the model prediction and actual value (not included) that showed that the model gain was too low. To avoid the oscillations, the model gain was almost doubled around $t = 250$ minutes and the crossover is now able to control the remaining capacity towards its low constraint.

The accuracy of the estimate of remaining capacity for demethanizer T100 (Figure 6.4(e)) was poor. This column has operation problems like gas flooding (that occurs at different differential pressures), and large duty changes in side boilers because of large shift in the column temperature profile. In this test, the model gain from column feed to differential pressure was increased at $t = 320$ minutes, and the new value seems to give a more correct estimate of the remaining capacity for the column. Again, this adjustment of the model gain was based on comparing the model prediction an the actual value that showed that the model gain was too low. Note that the remaining capacity of the demethanizer T100 became close to zero at about $t = 330$ min. To avoid this, the lower constraint value (back off) was increased at $t = 500$ min.

A key idea with the coordinator MPC is that the coordinator should maintain maximum throughput in spite of feed composition changes. Feed composition changes are important disturbances and affect the remaining capacity to the units. The feed composition in the Statpipe (T100) (Figure 6.4(f)) is rather stable until $t = 580$ min when the feed becomes significantly heavier and thereafter (at $t =$ 610 min) significantly lighter. In this case, the coordinator uses the crossover (Figure 6.4(c)) and the T100 feed rate (Figure 6.4(a)) to control the remaining capacity for the butane splitter T100 (Figure 6.4(d)) at its constraint.

In another test run of the coordinator MPC (08 Feb. 2008), one of the three booster compressors was not running due to maintenance, so the capacity of the booster compressor was a bottleneck. During the test period, the feed composition became slightly lighter (increased gas content) and this change was large enough to affect the capacity of the booster compressors. The back off in the booster compressors was reduced to be able to maintain the production with higher gas



(a) MV: Feed T100

(b) CV: Pressure Statpipe

(c) MV: Crossover

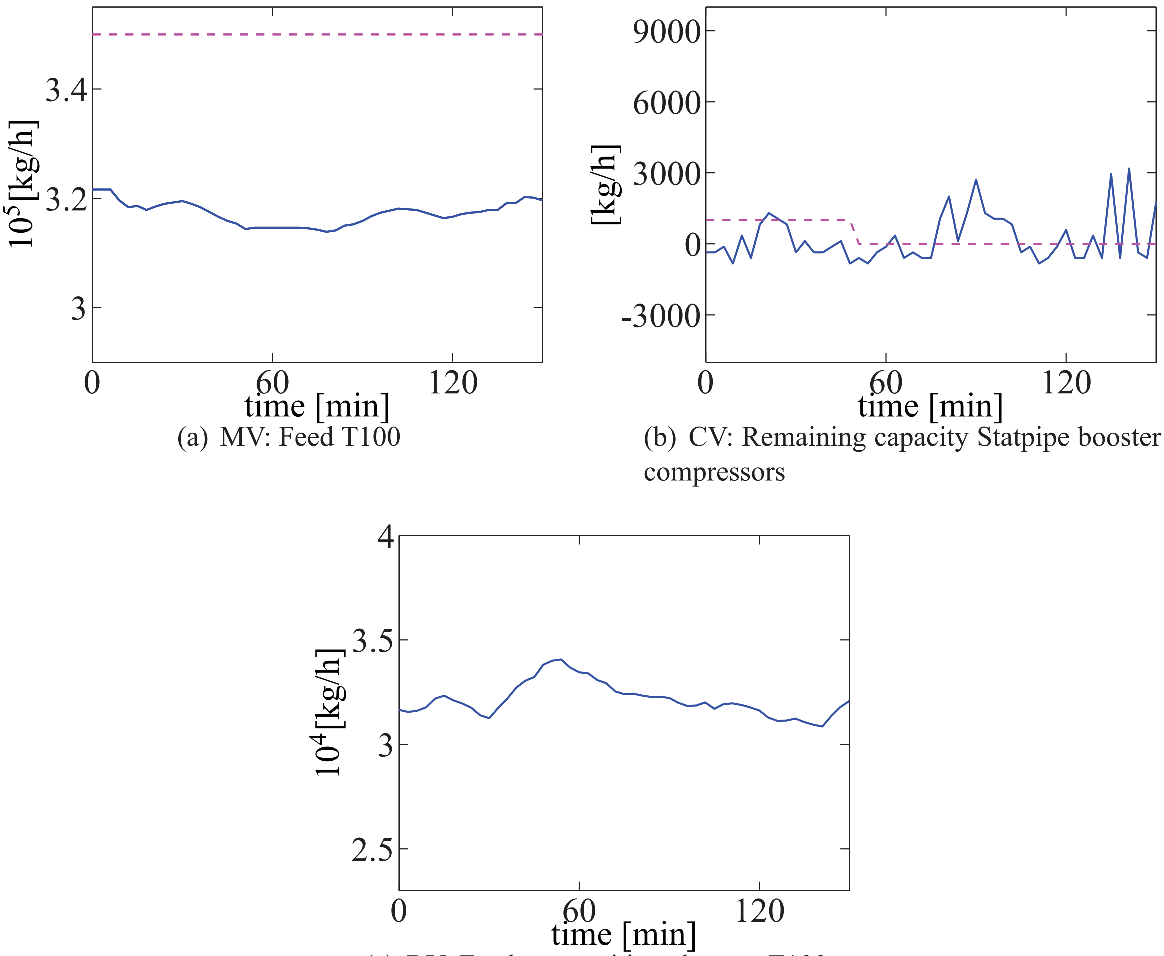
(d) CV: Remaining capacity butane splitter T100

(e) CV: Remaining capacity demethanizer T100

(f) DV: Feed composition changes T100

Figure 6.4: From test run 07 Feb. 2008: $t = 18$ min: turn on, $t = 250$ min: change in model gain for crossover, $t = 320$ min: change in model grain for feed to differential pressure in the demethanizer, $t = 580$ and $t = 610$ min: feed composition change. MV and CV values (solid), high and low limits (dashed) and ideal values (dotted).

content in the feed. Running the compressors at this high load is possible, but is not recommended over longer periods.



(a) MV: Feed T100

(b) CV: Remaining capacity Statpipe booster compressors

(c) DV: Feed composition changes T100

Figure 6.5: From test run 08 Feb. 2008 where Statpipe booster compressors are bottleneck. MV, CV and DV values (solid), high and low limits (dashed).

The guidelines from the gas pipeline network manager are typically given as “reduce the feed 40 t/h to keep the pipeline pressure above 134 bar”. However, while testing the coordinator it became clear that these two values do not coincide. For the gas pipeline network operation, it is the pressure profile in the pipeline which is most important, but for the gas plant operators it easier to relate to the feed flows. With the coordinator MPC, it is possible to specify a low limit on the pipeline pressure, and let the feed to the plant be given by the pipeline pressure (if the plant itself is not limiting the feed).

When in closed loop, the coordinator MPC manipulates directly on the plant production. This directly involves the shift manager at Kårstø and also close cooperation with the manager at the gas pipeline (which is operated by another company) is necessary. The plant is operated by three control panels, so a close dialog between the operator personnel and the shift manager is crucial.

The operators are familiar with the MPC interface from several years of experience with local MPC applications. This is a big advantage because the coordinator MPC has the same interface and operates in the same manner, so it is easier to get operator acceptance. However, the coordinator MPC introduces a "new way of thinking" for both operators and shift managers. The coordinator introduces the back off constraint as a new handle, in addition to pressure pipeline constraints, instead of the feed valves.

Using an in-house MPC tool has the advantage of allowing for quick and appropriate software changes, which has been very useful, for example, in changing the algorithm for estimate remaining capacity. In addition, the use of own personnel, from the research center and the plant site, keeps the knowledge within the company. A successful implementation also requires that the project gets priority by the managers, especially since this application is plant-wide and involves most of the control room operators.

6.6 Conclusion

A coordinator MPC to maximize production is currently under implementation on a large-scale gas plant. The Kårstø gas plant is an important part of the Norwegian gas transport system and the plant should process as much as possible to avoid being a bottleneck in the gas transport network. There are frequent changes in feed composition, pipeline pressures and other disturbances which require a dynamic model for optimization, and a coordinator MPC was earlier proposed as a way to implement maximum throughput (Aske *et al.*, 2008).

A key factor in the implementation is the estimate of the remaining capacity R_k for each unit, which tells how much more feed unit k can receive while operating within its constraints. The remaining capacity for each unit is estimated by the local MPC applications and is treated as CVs in the coordinator MPC. This decomposition leads to a plantwide application with "reasonable" size. The first part of the implementation includes about half of the plant and has 22 CVs, 6 MVs and 7 DVs. A future coordinator that includes the whole plant will have about twice as many CVs and MVs. The coordinator MPC is built with SISO step response models, similar to the local MPC applications.

There are some pitfalls in estimating the remaining capacity. The estimate relies on the accuracy of the steady-state models in the local MPC application, correct and reasonable CV and MV constraints and the use of gain scheduling to cope with larger nonlinearities. We have found that it is crucial to inspect the models and tuning of the local applications in a systematic manner. The estimate of the remaining capacity was found to be reasonable for the distillation columns where the differential pressure is included as a CV and the flooding point is approximately known.

Although the experience with the actual implementation of the coordinator MPC is limited, it is nevertheless clear that this is a promising tool for implementing maximum throughput at the Kårstø gas plant. The implementation leads to more focus among operating personnel on the capacity of each unit and it became clearer that several units were operating far from their constraints. In addition, the coordinator provides a plant-wide perspective which is required to properly adjust the plant- and crossover flows.

6.7 Acknowledgment

The implementation at the Kårstø gas plant was performed together with Kjetil Meyer, Roar Sørensen, shift managers and operating personnel at the Statpipe and Sleipner panels. All are gratefully acknowledged. The plant operator Gassco and technical services provider StatoilHydro is acknowledged for plant data.

Chapter 7

Conclusions and directions for further work

7.1 Conclusions

This thesis has discussed plantwide control configuration with focus on the maximum throughput case. In the general case, an important task for the plantwide control system, if not the most important, is to maintain the plant mass balances. The proposed *self-consistency rule* in **Chapter 2** fills this lack of a general rule that applies to all cases. It may be regarded as an obvious rule, but is often forgotten in a plantwide perspective.We believe the self-consistency rule states the mass balances in a clear manner and will be very useful for students and newcomers in the field.

In **Chapter 3** we have shown that "maximum throughput" is an optimal economic operation policy in many cases. This occurs when product prices are sufficiently high and feed is available and the throughput F is a degree of freedom. Optimal economic operation then corresponds to maximizing the throughput F subject to achieving feasible operation.

From a literature search and based on our own industrial experience, it seems like the feed valve (or more general the throughput manipulator) is very rarely used in practice for closed-loop control, in spite of its great importance on the plant economics in cases where maximum throughput is optimal. The reason is probably the large effect of feed rate on the operation of the entire plant, but the result may be a loss in economic performance.

This thesis discussed several methods for implementing maximum throughput in the control layer. The nature of maximum throughput simplifies the implementation because the optimum is constrained and corresponds to maximum throughput in the bottlenecks(s). Maximum throughput can then be implemented in the control layer and the approaches discussed in this thesis are:

Chapter 3: To obtain tight bottleneck control, move the throughput manipulator to the bottleneck unit and control the bottleneck flow with single-loop control. The approach requires the bottleneck to be fixed in one unit. The disadvantage is that the inventory loops upstream the bottleneck must be reconfigured when moving the throughput manipulator to ensure self-consistency.

Chapter 4: In cases where it is not desired to move the throughput manipulator, dynamic degrees of freedom can be included to shorten the effective time delay from the throughput manipulator to the bottleneck. With dynamic degrees of freedom, we mean manipulated variables with no steady-state effect. The most common examples are liquid levels and buffer tank levels. To include dynamic degrees of freedom in single-loop control, the structure *single-loop with ratio control* is proposed. This control structure uses the original location of the throughput manipulator (usually the feed rate) and use inventories dynamically by adding bias to the inventory controller outputs. The structure can be used for cases with fixed bottleneck. The single-loop with ratio control structure has no need for reconfiguration of the inventory loops, even the control parameter tunings can remain unchanged (except if the inventories are poorly tuned). An multivariable controller (e.g. MPC) can also be used to include dynamic degrees of freedom with throughput manipulator (feed rate) and inventories (inventory controller set point or directly manipulating the valve) as manipulated variables.

Chapters 5 and 6: In larger plants, there are often independent feeds and parallel trains with crossovers and splits between them that give rise to multiple bottlenecks and multiple throughput manipulators. This requires multivariable control and the proposed coordinator MPC both identifies the bottlenecks and implements the optimal policy. The coordinator uses the remaining degrees of freedom (u^c) to maximize the flow through the network subject to given constraints. The remaining degrees of freedom (u^c) include feed rates, splits and crossovers and the local MPCs provide estimates of the available capacity constraints ($R_k > 0$) in each node for the network. The constraints for the coordinator MPC are non-negative remaining capacities (R_k) for each unit k, that is, how much more the unit is able to receive within feasible operation. The values of R_k may change with time and a key idea is that they can be obtained with almost no extra effort using the existing local MPCs.

In the latter approach, coordinator MPC for maximizing throughput, the plant-wide control problem is decomposed by estimating the remaining capacity of each unit in the local MPC applications. The remaining capacity (R_k) is estimated from the present initial state, linear model equations and constraints used in the local MPC. To calculate the current maximum feed for each unit, the end predictions (steady-state gain) for the variables are used. In this thesis, the estimate is based on experimental models, most of them linear (some are gain scheduled). However, rigorous models for local units can also be used to predict the remaining capacity and makes decomposition flexible where the best available model can be used to predict the remaining capacity. The major advantage of decomposition is that the overall plant application becomes smaller in size and hence easier to understand and maintain. The coordinator MPC can also easily be built in steps with successive local MPC applications included in the coordinator.

The coordinator MPC is an effective tool for plantwide dynamic optimization. It uses simple models and by estimating remaining capacity of each unit, the plant is decomposed in an effective way. Dynamic optimization with simple models and decomposition of the plantwide control problem is satisfactorily in many cases compared to traditional (steady-state) RTO. This thesis discusses an objective function equal to maximum throughput and dynamic optimization using linear models. However, the coordinator MPC is not imitated to this. The objective function can be economic, for example with a price weighting between the feeds. The coordinator can also use non-linear, rigorous models when it is necessary.

To implement maximum throughput, the key is to achieve maximum flow through the bottleneck unit(s). However, to achieve feasible operation it is usually necessary to “back off” from the optimally active constraints. Back off leads to a lower flow through the bottleneck and an unrecoverable economic loss. This leads to the obvious conclusion that “throughput maximization requires tight bottleneck control”. It is important to know (or estimate) the expected back off in order to quantify the possible benefits of moving the throughput manipulator (changing the inventory control system), adding dynamic degrees of freedom, changing or retuning the supervisory control system etc. The magnitude of the back off should be obtained based on information about the disturbances and the expected control performance. In practice, determining the expected dynamic variation is difficult. In this thesis, we obtain a rough estimate of the necessary back off based on controllability analysis. In summary, the requirement that that the effective time delay in the bottleneck controller loop should be less than 1/4 of the disturbance time constant to have benefit of control. This implies that the throughput manipulator must be located very close to the bottleneck to have any benefit of improved control and reducing back off.

7.2 Directions for further work

Within the scope of this thesis, some issues for further work are listed below.

Uncertainty in the static ratio gain

In the single-loop with ratio control, the bias adjustment is considered constant (static). However, this gain may change, for example due to feed composition changes. The performance of the control structure is not considered if the static ratio changes significantly. An alternative implementation can be a nonlinear bias adjustment to account for significant gain changes, but this structure is not studied in detail.

Information loss in plantwide control decomposition

In the estimate of the remaining capacity of a unit, only a single unit is considered in the local MPC application. Thus, some information between the units is therefore lost in the decomposition. For example, the capacity of one unit may depended on how an another unit is operated. Are there any effective ways to add cross-information between the units but still be able to decompose the plant and not include all variables? How large is this loss in cross-information in terms of economics? How much more effort must be added to avoid this loss?

Further implementation of the coordinator MPC

The coordinator MPC is implemented at the Kårstø gas plant, covering about half of the processing units. This should be extended to cover the whole plant and include export gas quality to achieve the real maximum plant throughput. In the estimation of remaining capacity, an LP solver that includes relaxation of the constraints should be implemented. It is preferable that the estimate returns the best possible solution instead of "giving up" and this improves the robustness of the coordinator MPC.

Throughput maximization in recycle systems

The maximum throughput case in production systems is closely related to the maximum flow problem in networks considered in operations research. The main assumption for applying network theory is that the mass flow through the network is represented by linear flow connections. The main process unit that creates nonlinearity in terms of flows between the units is a reactor. Another important decision that affects composition, and thus flows, is the amount of recycle. In this thesis, these sources of nonlinearity are viewed as a single combined unit as seen from maximum throughput (bottleneck) point of view. Combined units are not treated in detail and should be understand better in terms of maximum throughput. However, such systems with reactors will often be in Mode 2b, optimized throughput, with an unconstrained optimum with no bottlenecks, but there might be cases when such plants are in Mode 2a, maximum throughput.

Obtain an back-off estimate on more realistic example

In Chapter 3, controllability analysis is used to obtain necessary back off to ensure feasibility in spite of disturbances. The controllability analysis should be performed on more realistic example.

Bibliography

Ahuja, R. K., T. L. Magnanti and J. B. Orlin (1993). *Network Flows: Theory, Algorithms, and Applications*. Prentice Hall, Englewood Cliffs, N.J.

Araújo, A. and S. Skogestad (2008). Control structure design for the ammonia synthesis process. *Comput. Chem. Eng.* **32**, 2920–2932.

Arkun, Y. and G. Stephanopoulos (1980). Studies in the synthesis of control structures for chemical processes. Part IV: Design of steady-state optimizing control structures for chemical process units. *AIChE J.* **26**, 975–991.

Aske, E.M.B., S. Skogestad and S. Strand (2007). Throughput maximization by improved bottleneck control. In: *8th International Symposium on Dynamics and Control of Process Systems (DYCOPS)*. Vol. 1. Cancun, Mexico. pp. 63–68.

Aske, E.M.B., S. Strand and S. Skogestad (2005). Implementation of MPC on a deethanizer at Kårstø gas plant. In: *16th IFAC World Congress, paper We-M06-TO/2*. Prague, Czech Republic. pp. CD–rom published by International Federation of Automatic Control.

Aske, E.M.B., S. Strand and S. Skogestad (2008). Coordinator MPC for maximizing plant throughput. *Comput. Chem. Eng.* **32**(1-2), 195–204.

Backx, T., O. Bosgra and W. Marquardt (2000). Integration of model predictive control and optimization of processes. *International Symposium on Advanced Control of Chemical Processes (ADCHEM), Pisa, Italy, June 14-16, 2000* pp. 249–260.

Bahri, P.A., J.A. Bandoni and J.A. Romagnoli (1996). Effect of disturbances in optimizing control: Steady-state open-loop backoff problem. *AIChE J.* **42**(4), 983–994.

Bandoni, J.A., J.A. Romagnoli and G.W. Barton (1994). On optimising control and the effect of disturbances: Calculation of the open-loop backoffs. *Comput. Chem. Eng.* **18**, S505–S509.

Bauer, M. and I.K. Craig (2008). Economic assessment of advanced process control - A survey and framework. *J. Proc. Control* **18**, 2–18.

BenAmor, S., F. J. Doyle III and R. McFarlane (2004). Polymer grade transition control using advanced real-time optimization software. *J. Process Contr.* **14**, 349–364.

Buckley, P. S. (1964). *Techniques of Process Control*. John Wiley & Sons, Inc., NY, USA.

Cheng, R., J.F. Forbes and W.S. Yip (2004). Dantzig-Wolfe decomposition and large-scale constrained MPC problems. In: *International Symposium on Dynamics and Control of Process Systems (DYCOPS)*. Boston, USA. pp. paper 117, in CD rom.

Cheng, R., J.F. Forbes and W.S. Yip (2006). Coordinated decentralized MPC for plant-wide control of a pulp mill benchmark problem. In: *International Symposium on Advanced Control of Chemical Processes (ADCHEM)*. Vol. 2. Gramado, Brazil. pp. 971–976.

Cheng, R., J.F. Forbes and W.S. Yip (2007). Price-driven coordination method for solving plant-wide MPC problems. *J. Proc. Control* **17**, 429–438.

Cheng, Y.-C., K.-L. Wu and C.-C. Yu (2002). Arrangement of throughput manipulator and inventory control in plantwide control. *J. Chin. Inst. Chem. Engrs.* **33**(3), 283–295.

Diehl, M., H.G. Bock, J.P. Sclöder, R. Findeisen, Z. Nagy and F. Allgöwer (2002). Real-time optimization and nonlinear model predictive control of processes governed by differential-algebraic equations. *J. Proc. Control* **12**, 577–585.

Downs, J. J. (1992). Distillation control in a plantwide control environment. In: *Practical Distillation Control* (William L. Luyben, Ed.). pp. 413–439. Von Nostrand Reinhold, New York, USA.

Edgar, T.F., D.M. Himmelblau and L. S. Lasdon (2001). *Optimization of Chemical Processes*. 2nd ed.. McGraw Hill, NY, USA.

Engell, S. (2007). Feedback control for optimal process operation. *J. Proc. Control* **17**, 203–219.

Faanes, A. and S. Skogestad (2003). Buffer tank design for acceptable control performance. *Ind. Eng. Chem. Res.* **42**, 2189–2208.

Figueroa, J.L., P.A. Bahri, J.A. Bandoni and J.A. Romagnoli (1996). Economic impact of disturbances and uncertain parameters in chemical processes - a dynamic back-off analysis. *Comput. Chem. Eng.* **20**(4), 453–461.

Finco, M.V., W.L. Luyben and R.E. Polleck (1989). Control of distillation columns with low relative volatility. *Ind. Eng. Chem. Res.* **28**, 75–83.

Findeisen, W., F.N. Nailey, M. Brdys, K. Malinowski, P. Tatjewski and A. Wozniak (1980). *Control and Coordination in Hierarchical Systems*. John Wiley & Sons.

Forbes, J. F., T.E. Marlin and W.S. Yip (2006). Real-time optimization: Status, issues and opportunities. In: *Encyclopedia of Chemical Processing* (Sunggyu Lee, Ed.). Vol. 1. pp. 2585–2598. Taylor & Francis.

Ford, L.R. and D.R. Fulkerson (1962). *Flows in Networks*. Princeton University Press.

Garcia, C.E., D.M. Prett and M. Morari (1989). Model predictive control: Theory and practice - A survey. *Automatica* **25**(3), 335–348.

Govatsmark, M.S. and S. Skogestad (2005). Selection of controlled variables and robust setpoints. *Ind. Eng. Chem. Res.* **44**, 2207–2217.

Havlena, V. and J. Lu (2005). A distributed automation framework for plant-wide control, optimisation, scheduling and planning. In: *16th Triennial World Congress of the International Federation of Automatic Control* (P. Horacek, M. Simandl and P. Zitek, Eds.). Prague, Czech Republic. pp. 80–94.

Heath, J., J. Perkins and S. Walsh (1996). Control structure selection based on linear dynamic economics - Multiloop PI structures for multiple disturbances. In: *Preprints IFAC '96, 13th World Congress of IFAC*. Vol. M. San Francisco, CA. pp. 85–90.

Kadam, J.V., M. Schlegel, B. Srinivasan, D. Bonvin and W. Marquardt (2007). Dynamic optimization in the presence of uncertainty: From off-line nominal solution to measurement-based implementation. *J. Proc. Control* **17**(5), 389–398.

Kadam, J.V., W. Marquardt, M. Schlegel, T. Backx, O.H. Bosgra, P.-J. Brouwer, G. Dünnebier, D. van Hessem, A. Tiagounov and S. de Wolf (2003). Towards integrated dynamic real-time optimization and control of industrial processes. In: *Proceedings Foundations of Computer-Aided Process Operations (FOCAPO2003)*. Coral Springs, Florida. pp. 593–596.

Kida, F. (2004). Plant wide control system (1) which the process engineer designs. On the consistency of the plant wide control loop composition. Simple judgement and composition standard tactics rule of the erratum of the control loop composition.. *Chemical Engineering (Tokyo)* **49**(2), 144–151. In Japanese.

Kida, F. (2008). Private communication. Kida has published 6 papers about plantwide control configuration in Chemical Engineering (Japan:Tokyo) in February, March, April, June, July and September; 2004, all in Japanese.

Kister, H. Z. (1990). *Distillation Operation*. McGraw Hill, NY, USA.

Larsson, T. and S. Skogestad (2000). Plantwide control - A review and a new design procedure. *Model. Ident. Control* **21**(4), 209–240.

Larsson, T., M. Govatsmark, S. Skogestad and C.C. Yu (2003). Control structure selection for reactor, separator, and recycle processes. *Ind. Eng. Chem. Res.* **42**, 1225–1234.

Litzen, D.B. and J.L. Bravo (1999). Uncover low-cost debottlenecking opportunities. *Chem. Eng. Prog.* **95**, 25–32.

Loeblein, C. and J.D. Perkins (1998). Economic analysis of different structures of on-line process optimization systems. *Comput. Chem. Eng.* **22**(9), 1257–1269.

Loeblein, C. and J.D. Perkins (1999). Structural design for on-line process optimization: I. Dynamic economics of MPC. *AIChE J.* **45**(5), 1018–1029.

Lu, J.Z. (2003). Challenging control problems and emerging technologies in enterprise optimization. *Control Engineering Practice* **11**, 847–858.

Luyben, M.L., B.D. Tyreus and W.L. Luyben (1997). Plantwide control design procedure. *AIChe J.* **43**, 3161–3174.

Luyben, W. L. (1993*a*). Dynamics and control of recycle systems 1. Simple open-loop and closed-loop systems. *Ind. Eng. Chem. Res.* **32**(3), 466–475.

Luyben, W. L. (1993*b*). Dynamics and control of recycle systems 2. Comparison of alternative process design. *Ind. Eng. Chem. Res.* **32**(3), 476–486.

Luyben, W. L. (1993*c*). Dynamics and control of recycle systems 3. Alternative process design in a ternary system. *Ind. Eng. Chem. Res.* **32**(6), 1142–1153.

Luyben, W. L., B. D. Tyerus and M. L. Luyben (1998). *Plantwide process control*. McGraw-Hill.

Luyben, W.L. (1994). Snowball effects in reactor/separator processes with recycle. *Ind. Eng. Chem. Res.* **33**, 299–305.

Luyben, W.L. (1999). Inherent dynamic problems with on-demand control structures. *Ind. Eng. Chem. Res.* **38**(6), 2315–2329.

Maarleveld, A. and J.E. Rijnsdorp (1970). Constraint control on distillation columns. *Automatica* **6**(1), 51–58.

Marlin, T. E. and A. N. Hrymak (1997). Real-time operations optimization of continuous processes. In: *Fifth International Conference on Chemical Process Control (CPC-5)* (J.C. Kantor, C.E. Garcia and B. Carnahan, Eds.). Lake Tahoe, Nevada. pp. 156–164.

Moore, C.F. and E.S. Percell (1995). Analysis of the operation and control of a simple plant-wide module. In: *Proc. American Control Conference*. Seattle, Washington. pp. 230–234.

Morari, M., Y. Arkun and G. Stephanopoulos (1980). Studies in the synthesis of control structures for chemical processes. part I: Formulation of the problem. process decomposition and the classification of the control task. analysis of the optimizing control structures. *AIChE J.* pp. 220–232.

Narraway, L. and J. Perkins (1994). Selection of process control structure based on economics. *Comput. Chem. Eng.* **18**, S511–5S15.

Narraway, L.T. and J.D. Perkins (1993). Selection of process control structure based on linear dynamic economics. *Ind. Eng. Chem. Res.* **32**(11), 2681–2692.

Narraway, L.T., J.D. Perkins and G.W. Barton (1991). Interaction between process design and process control: Economic analysis of process dynamics. *J. Proc. Control.* **1**, 243–250.

Perry, R. H. and C. H. Chilton (1973). *Chemical Engineers' Handbook*. 5th ed.. McGraw-Hill.

Phillips, D. T., A. Ravindran and J. J. Solberg (1976). *Operations Research Principles and Practice*. John Wiley.

Price, R. M. and C. Georgakis (1993). Plantwide regulatory control design procedure using a tiered framework. *Ind. Eng. Chem. Res.* **32**, 2693–2705.

Price, R. M., P. R. Lyman and C. Georgakis (1994). Throughput manipulation in plantwide control structures. *Ind. Eng. Chem. Res.* **33**, 1197–1207.

Qin, S.J. and T.A. Badgwell (2003). A survey of industrial model predictive control technology. *Control Engineering Practice* **11**, 733–764.

Rawlings, J.B. and B.T. Stewart (2007). Coordinating multiple optimization-based controllers: New opportunities and challenges. In: *8th International Symposium on Dynamics and Control of Process Systems (DYCOPS)*. Vol. 1. Cancun, Mexico. pp. 19–28.

Richalet, J. (2007). Origin and industrial applications of predictive control. Presentation at Nordic Process Control Workshop, Espoo, Finland. Available at the homepage of S. Skogestad, *www.nt.ntnu.no/users/skoge/npc/richalet.pdf*.

Richalet, J., A. Rault, J.L. Testud and J. Papon (1978). Model predictive heuristic control: Applications to industrial processes. *Automatica* **14**, 413–428.

Schwartz, J.D., W. Wang and D.E. Rivera (2006). Simulation-based optimization of process control policies for inventory management in supply chains. *Automatica* **42**, 1311–1320.

Seborg, D. E., T. F. Edgar and D. A. Mellichamp (1989). *Process Dynamics and Control*. Wiley International Edition.

Shinskey, F.G. (1984). *Distillation Control - For Productivity and Energy Conservation*. McGraw Hill, NY, USA.

Skogestad, S. (1991). Consistency of steady-state models using insights about extensive variables. *Ind. Eng. Chem. Res.* **30**, 654–661.

Skogestad, S. (1997). Dynamics and control of distillation columns - A tutorial introduction. *Trans. IChemE* **75**(Part A), 539–562.

Skogestad, S. (2000*a*). Plantwide control: the search for the self-optimizing control structure. *J. Proc. Control* **10**, 487–507.

Skogestad, S. (2000*b*). Self-optimizing control: the missing link between steady-state optimization and control. *Comput. Chem. Eng.* **24**, 569–575.

Skogestad, S. (2003). Simple analytic rules for model reduction and PID controller tuning. *J. Proc. Control* **13**, 291–309.

Skogestad, S. (2004). Control structure design for complete chemical plants. *Comput. Chem. Eng.* **28**, 219–234.

Skogestad, S. (2006). Tuning for smooth pid control with acceptable disturbance rejection. *Ind. Eng. Chem. Res.* **45**, 7817–7822.

Skogestad, S. (2007). The dos and don'ts of distillation column control. *Trans. IChemE, Part A* **85**(A1), 13–23.

Skogestad, S. and I. Postlethwaite (2005). *Multivariable Feedback Control: Analysis and Design*. 2nd ed.. John Wiley & Sons.

Skogestad, S., E.W. Jacobsen and M. Morari (1990). Inadequacy of steady-state analysis for feedback control: Distillate-bottom control of distillation columns. *Ind. Eng. Chem. Res.* **29**(12), 2339–2346.

Strand, S. (1991). Dynamic Optimization in State-Space Predictive Control Schemes. PhD thesis. Norwegian Institute of Technology (NTH). Trondheim.

Strand, S. and J.R. Sagli (2003). MPC in Statoil - Advantages with in-house technology. *International Symposium on Advanced Control of Chemical Processes (ADCHEM), Hong Kong, 2004* pp. 97–103.

Tosukhowong, T., J.M. Lee, J.H Lee and J. Lu (2004). An introduction to a dynamic plant-wide optimization strategy for an integrated plant. *Comput. Chem. Eng.* **29**, 199–208.

Tyreus, B. D. and W. L. Luyben (1993). Dynamics and control of recycle systems 4. ternary systems with one or two recycle streams. *Ind. Eng. Chem. Res.* **32**(6), 1154–1162.

Uwarwema, Théogéne (2008). Use of dynamic degrees of freedom for tighter bottleneck control. Master's thesis. Norwegian University of Science and Technology. Trondheim, Norway. Available at the homepage of S. Skogestad, *www.nt.ntnu.no/users/skoge/diplom/diplom08/uwarwema_theo/*.

Venkat, A.N., J.B. Rawlings and S.J. Wright (2006). Stability and optimality of distributed, linear model predictive control. Part I: State feedback. Technical report. 2006-03, TWMCC, Department of Chemical Engineering, University of Wisconsin-Madison.

Ying, C-M. and B. Joseph (1999). Performance and stability analysis of LP-MPC and QP-MPC cascade control systems. *AlChE J.* **45**(7), 1521–1534.

Zhu, Y. (1998). Multivariable process identification for MPC: the asymptotic method and its applications. *J. Proc. Control* **8**(2), 101–115.

Appendix A

Implementation of MPC on a deethanizer at Kårstø gas plant

Presented at
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Model predictive control (MPC) is implemented on several distillation columns at the Kårstø gas processing plant, Norway. The paper describes the procedure in the implementation of MPC at a deethanizer using the SEPTIC* MPC tool, including design, estimator development, model development and tuning. For the deethanizer, the variance in the product quality has been reduced with about 50%. The number of flaring episodes has also been reduced. An increase in impurities has not been challenged yet, so the average reflux flow and steam consumption to feed ratios are almost unaltered.

*SEPTIC: Statoil Estimation and Prediction Tool for Identification and Control

A.1 Introduction

A.1.1 Plant description

The Kårstø gas processing plant plays a key role in the transport and treatment of gas and condensate from central parts of the Norwegian continental shelf. This plant receives rich gas and unstabilized condensate through pipelines and separates the feed into its various components. The products from the plant are sales gas, which is exported in pipelines, and ethane, propane, iso-, normal butane, naphtha and condensate, which are exported by ships. The rich gas processing design capacity at Kårstø is today at 74 MSm3/d. The facility had 575 ship calls in 2002 to load the liquid products, and is one of the largest producers of liquefied petroleum gases (LPG) in the world.

A.1.2 Model predictive control

MPC is sometimes defined as the family of controllers where there is a direct use of an explicit and separately identifiable model, where the model provides predictions of the process response to future changes in the manipulative variables and to predicted process disturbances (Garcia *et al.*, 1989). In practice, MPC is characterized by its ability to handle constraints in both manipulated and controlled variables. MPC techniques provide the only *methodology* to handle constraints in a systematic way during the design and implementation of the controller. Moreover, in its most general form MPC is not restricted in terms of the model, objective function and/or constraint functionality. These are the primary reasons for the success of these techniques in numerous applications in the chemical process industries (Garcia *et al.*, 1989; Qin and Badgwell, 2003).

The most important issues for the Kårstø processing plant are regularity and capacity, to avoid being a bottleneck in the large gas transportation system in the Norwegian Sea. While several extension projects gradually increase the plant size and complexity, the resulting regularity challenges are met with MPC implementation. Moreover, large value creations take place, and pushing the capacity limits requires a control tool like MPC to handle the varying set of active constraints.

A.2 SEPTIC MPC

SEPTIC is an in-house software system for MPC, real-time optimization (RTO), dynamic process simulation for simpler case studies, and off- and on-line parameter estimation in first principle based process models. At Kårstø, SEPTIC was selected as a tool for MPC. The MPC issues of SEPTIC are described by Strand and Sagli (2003).

Currently, most SEPTIC MPC applications in Statoil use experimental SISO step response models. SEPTIC is also capable of running generally non-linear models implemented in a compact model object. However, the SISO models represent to a large extent the process dynamics sufficiently accurate to achieve good controller performance.

The SEPTIC MPC is configured with

- controlled variables (CV), specified with setpoint (SP), high limit and low limit,

- manipulated variables (MV), specified with rate of change, high and low limit and ideal value (IV),
- disturbance variables (DV).

The control specifications are explicitly prioritized by:

1. MV rate of change limits
2. MV high and low limits
3. CV hard constraints, hardly ever used
4. CV setpoints, CV high and low limits and MV ideal values with priority level *1*
5. CV setpoints, CV high and low limits and MV ideal values with priority level *n*
6. CV setpoints, CV high and low limits and MV ideal values with priority level *99*

MV rate of change and MV high and low limits are always activated and respected unless there is a dynamic conflict between those two specifications. Then a sequence of steady-state quadratic programs is solved to respect the remaining specifications 3) - 6), giving the achievable steady-state targets. The control specifications are adjusted accordingly for the dynamic optimization problem.

A.3 Deethanizer MPC

The implementation of MPC for the Sleipner train deethanizer is described in the following chapter.

A.3.1 Column description

The deethanizer has 34 trays, a partial condenser with propane coolant, a reflux drum, and a reboiler with LP steam as heating medium. The gas from the reflux drum goes to the steam boilers as fuel gas, and the liquid splits to reflux and distillate. The column feed is the top product from two stabilizers that consists of butane and lighter components. The feed passes through the gas dryers to remove water before it enters the column.

The deethanizer basic control structure can be summarized as follows:

- Reflux drum level control with distillate

- Reflux flow control
- Column bottom level control
- Tray 1 temperature control with condensate
- LP steam pressure control
- Column pressure control by reflux drum gas valve

The column including the basic control structure is displayed in figure A.1. The performance to the PID controllers around the column is verified and tuned if necessary before any MPC modelling take place.

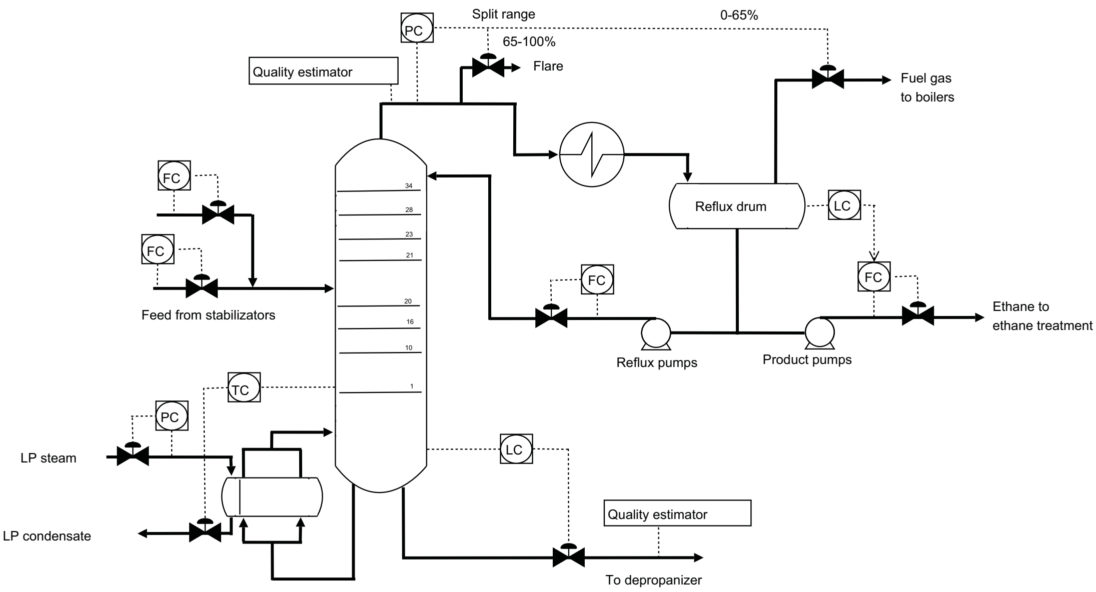

Figure A.1: The deethanizer including the basic control

There are three main disturbances to consider in operation. First, the feed rate may be reduced to the half of its nominal value in less than 15 minutes. This occurs when one of the two stabilizers are taken out of production. Second, the feed flow composition may change. There are analyzers on both feed streams, but the sampling time is about 15 minutes, so the column responds to the variations before the analyzers. The third disturbance is feed temperature variations due to the 1-2 days gas drier regeneration cycle.

A.3.2 MPC design

The MPC design starts with MV, CV and DV selection. The system components are the column, condenser, reflux drum and reboiler, while the input and output

	MV:Reflux	MV: Temperature	DV:Column Feed
CV: C3 in C2	-	+	+
CV: C2 in C3	+	-	+
CV: PC output	+	-	0

Table A.1: The selected variables in the MPC including steady state gain

streams are feed and products. The main control objective is to control the quality of the top and bottom streams, by manipulating boil-up and reflux flow.

The temperature controller set point is selected as an MV. An option is to manipulate the steam flow, which is a direct manipulation on the energy input. However, the original configuration is kept and leaves the basic control scheme unchanged for the operators. Manipulating the temperature controller set point requires that the temperature controller dynamics must be included in the MPC models.

Also, the column must be kept under surveillance to avoid overloading. The differential pressure is a good indicator for flooding (Kister, 1990), but is not measured for the actual column. In addition, limitations in the basic level control and in the process equipment must be considered. The pressure controller output is included as a CV to avoid the flare valve opening when the controller exceeds 65%.

Only the feed flow is included as a DV in the MPC. The unmeasured feed composition changes are suppressed by the MPC feedback action. The feed temperature is measured and may be used as a DV if some special gas drier considerations are made.

Manipulating the column pressure is a trade-off between energy savings and flooding limit. The pressure is not included as an MV, but could have lead to a more optimal operation of the column.

The steady state gain between the reflux flow and the bottom quality is positive. The temperature controller is in closed loop and to some extent compensates for the reflux flow. However, if the temperature controller was located higher in the column, the steady state gain may have been negative. The other steady-state gains are as expected. The deethanizer MPC design including the steady state gains is summarized in table A.1.

The top and bottom product qualities must be measured in some way. The top quality is expressed in propane mol% in ethane (C3 IN C2), whereas the bottom quality is expressed in ethane mol% in propane (C2 IN C3). There are on-line gas chromatographs (GC) at the deethanizer distillate and at the depropanizer distillate. The GC sample rate is 10 minutes, which from a control point of view is too infrequent. In addition, the GC is occasionally inoperative due to maintenance. The product qualities are therefore estimated by the temperature profile in the column. A more detailed description of the quality estimators is found in section A.3.3.

The CV prioritizing for the deethanizer application is as follows:

1. High and low limit pressure controller output, high limit top and bottom quality

2. Set point top and bottom quality

where 1 is the highest priority. The priority list leads to relaxation of the quality set points when the application predicts on one of the limits to the pressure controller output.

Application subgrouping must be considered in the design. In this MPC, the top quality and the reflux flow are in one subgroup and both are critical variables. The bottom quality and the temperature are in another subgroup and both are critical members of the group. The pressure controller output and the column feed are members of both subgroups but are stated as non-critical members. This means that top quality is still allowed to be controlled with reflux but not with temperature if bottom quality is deactivated and vice versa.

A.3.3 Obtaining estimators

The deethanizer data history had sufficient variance in the product qualities, so no test period was needed to enrich the data. The calibration data represented a two month period with 20 minute averages.

The deethanizer and depropanizer GC values are time shifted 10 and 25 minutes respectively, to account for sampling delay and process dynamics.

Distillation columns are known to be strongly nonlinear due to the vapor-liquid equilibrium (VLE). Logarithmic compositions reduce the nonlinearity and the behavior becomes much less dependent on the operation point (Skogestad, 1997). Different quality transformations were tried for the estimator calibration, and the square root gave the best fit.

The least squares regression gave that to describe the top product quality only the tray 28 and top temperatures are needed, whereas the tray 10 and bottom temperatures are needed for the bottom product quality.

A.3.4 Dynamic modelling

The deethanizer modelling took two days with step testing, with the MV steps and DV (feed rate) variations shown in figure A.2. The resulting CV's are displayed in figure A.3. The GC is compared with the estimator and shows a satisfactory match, illustrated by the top quality in figure A.4.

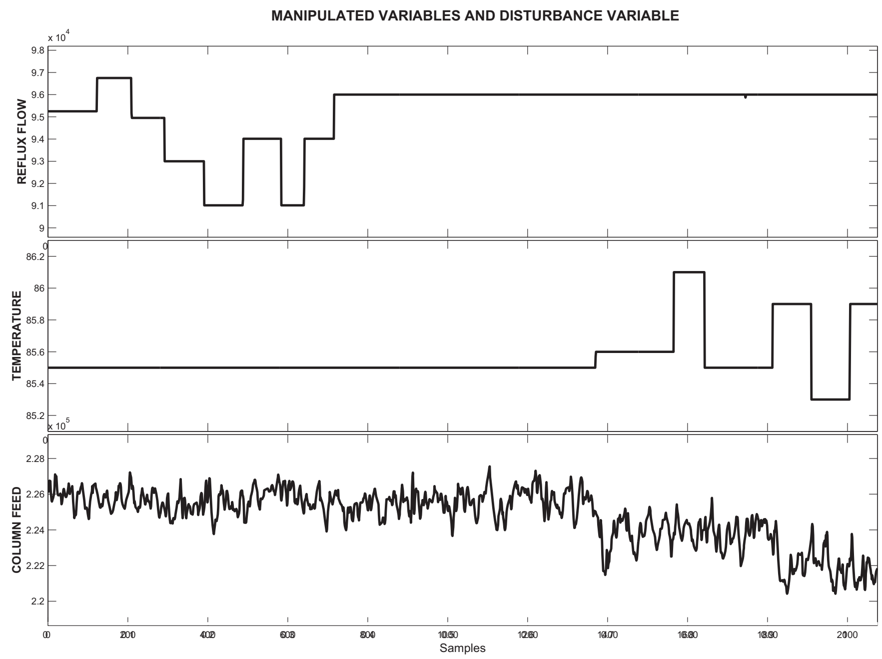

Figure A.2: Step test period for MVs and DV

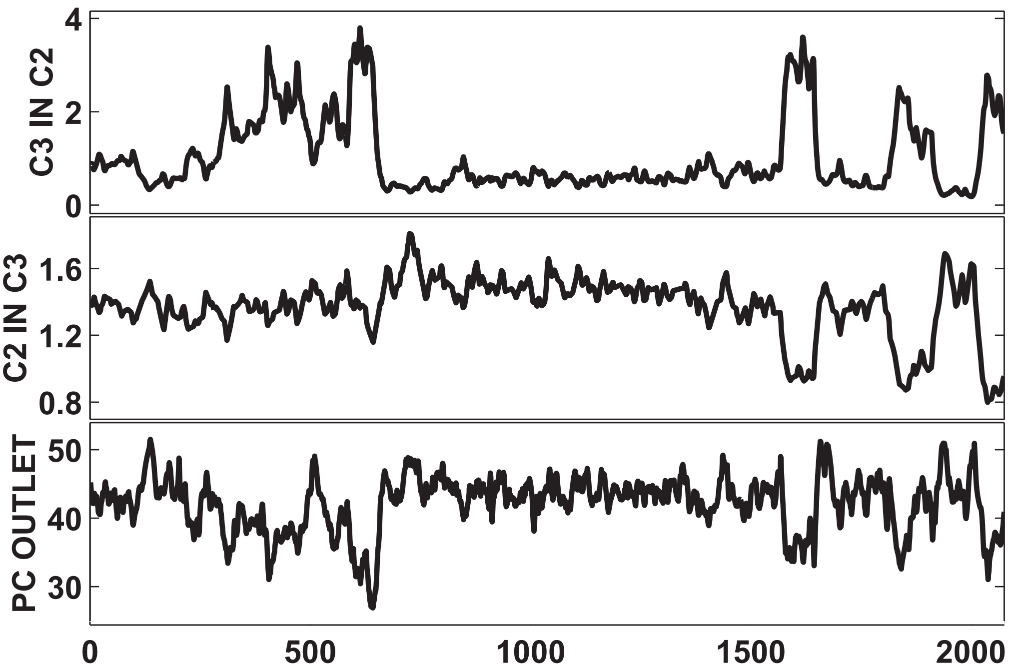

Figure A.3: Resulting CV's from the step test period

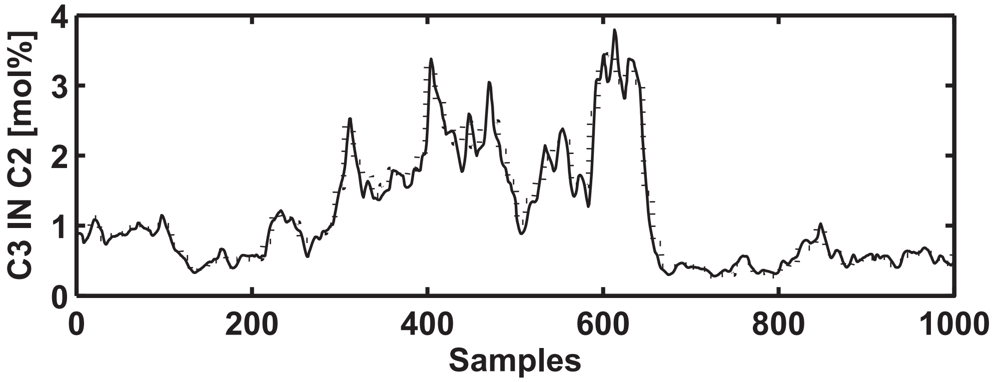

Figure A.4: Top quality, GC (dotted) versus estimator (solid)

The dynamic models are identified by Tai-Ji ID (Zhu, 1998). The Tai-Ji ID identification is based on the asymptotic method (ASYM), which calculates time domain parametric models using frequency domain criterion. The step response models from the Tai-Ji ID tool is displayed in figure A.5. The grading A to D is determined from the upper error bounds in a frequency plot. The steady state gains in the models are as expected, except the column feed influence on the top quality that turns out to be negative. A positive steady state gain effect for this model is found from data with more variations in the feed. The model fit is displayed in figure A.6.

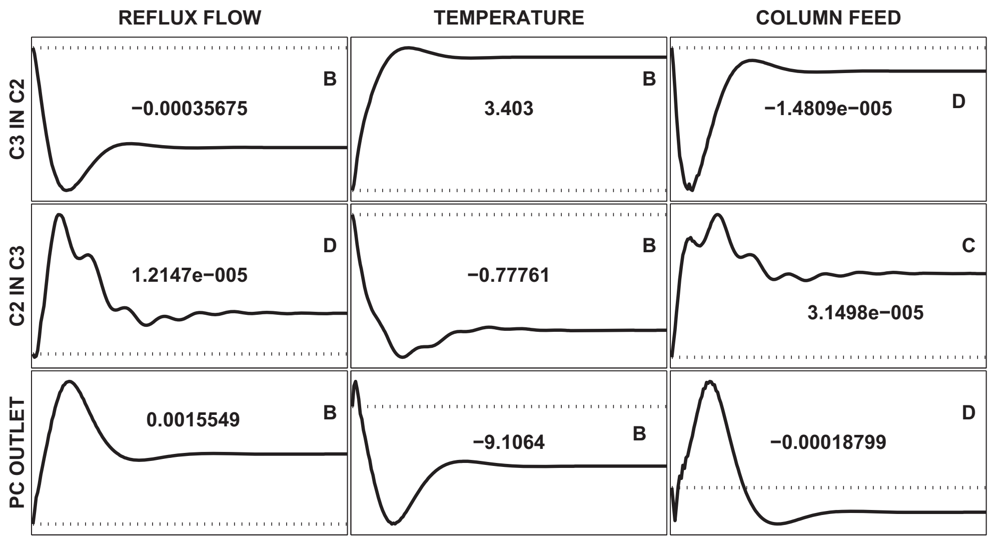

Figure A.5: Step response models for the deethanizer application

Experience from other MPC applications have shown that using the logarith-

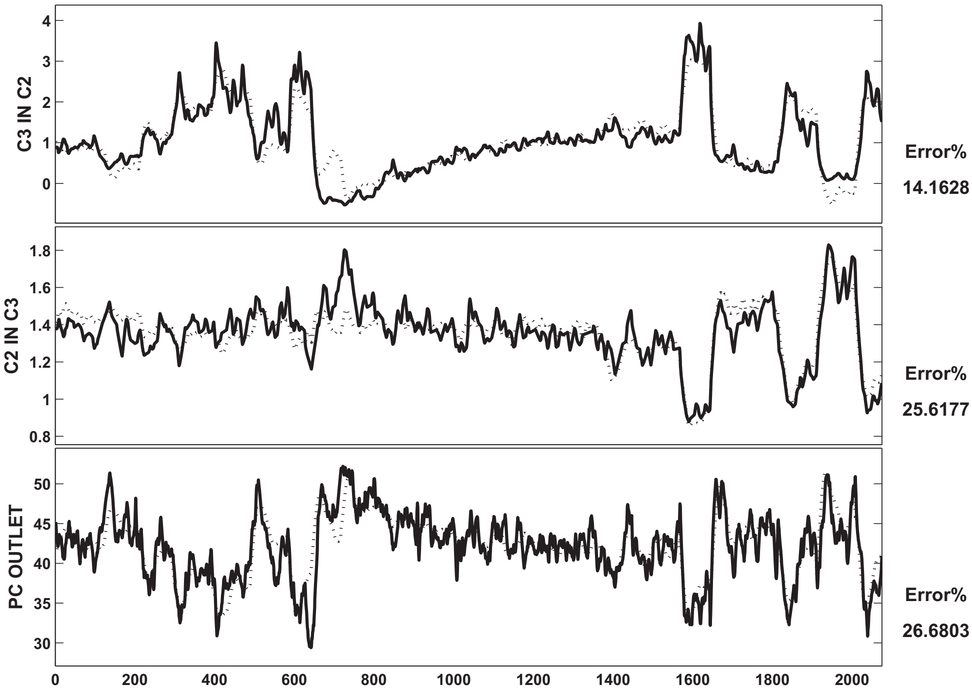

Figure A.6: The model fit. Measured CV's (solid) and simulated CV's (dashed)

mic qualities gives better adaption to step response models. The logarithmic composition is defined as the logarithm between the ratio of the key components (Skogestad, 1997) and is written as

$$X = \log \frac{0.01 \cdot y}{1 - 0.01 \cdot y} \tag{A.1}$$

where y is the impurity component in mol% . The step response models and the model fit of the transformed CV's are displayed in figure A.7 and A.8 respectively.

The improvement by using logarithmic quality is not that clear in this application. There is reduced error in the models between the logarithmic qualities versus the column feed, leads to an improvement from C to B model for the top and D to C model for the bottom, indicating a better initial response. Changes in reflux have a small effect on the bottom quality, and the identification found only a D model in both cases. The frequency plot of the error bounds show a acceptable initial response, which is caused by the temperature controller do not compensate for the reflux change immediately, so the D model is kept in the application.

The models between the CV's and the column feed are verified through a new data set with more variation in the feed. In the new models from column feed, the

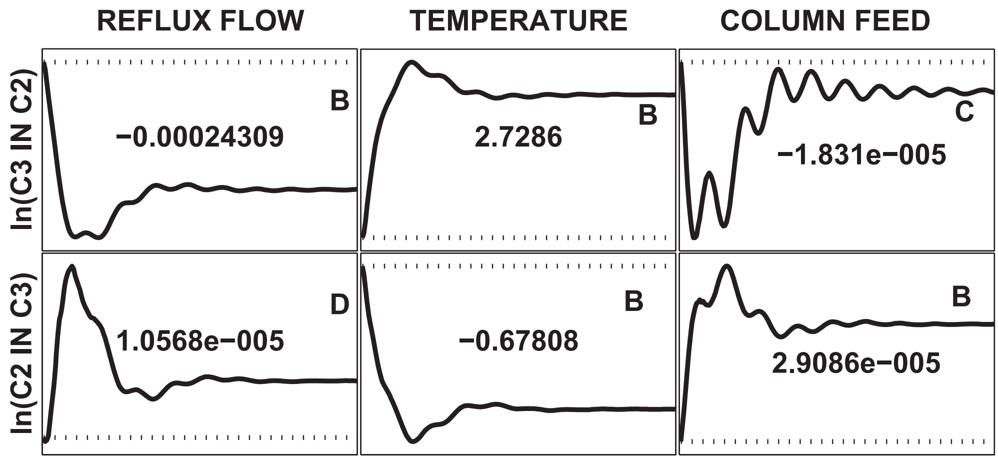

Figure A.7: Step response models with logarithmic transformation of the qualities

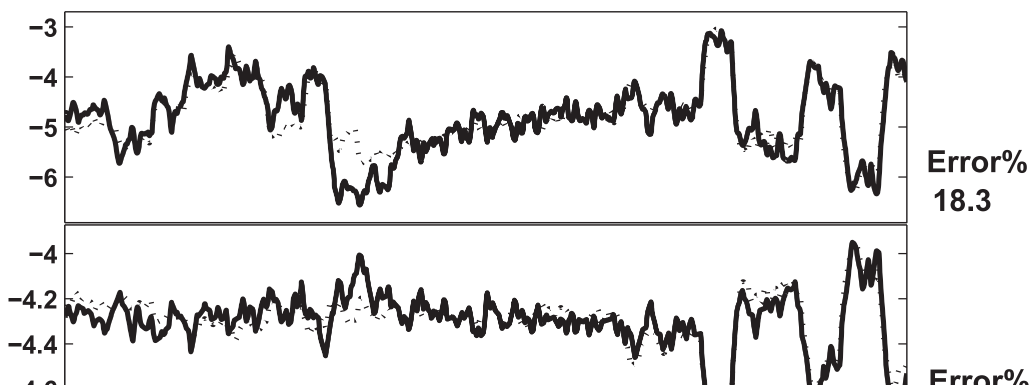

Figure A.8: The logaritmic model fit. Measured CV's (solid) and simulated CV's (dashed) steady state gain for the top quality and the pressure controller outlet changed sign. The column feed have a small influence on the pressure controller outlet in general so the model is omitted from the application.

A.3.5 MPC tuning

Several tuning parameters must be decided to obtain a rational use of the MV's to reach the control targets. The available set of SEPTIC MPC tuning parameters are:

CV and MV span internal scaling reflecting the ”acceptable” standard deviation of each variable

CV Fulf set point deviation penalty

MV Fulf ideal value deviation penalty

CV HighPnlty/ LowPnlty high and low limit violation penalty

CV SetpTref time constant for first order low pass filtering of set point changes

CV ConsTfilt time constant for first order low pass filtering of high and low limit changes

MV MovePnlty change penalty

MV MaxUp/ MaxDown rate of change limits

MV IvROC desired rate of change for IV fulfillment

All penalties are quadratic, including the ones for deviation, violation and move penalty.

A summary of the MPC tuning parameters are given in table A.2. The *HighPnlty* and *LowPnlty* for the pressure controller output are lower than for the qualities to avoid too aggressive use of the MV's when pressure controller outlet operates close to its limits. The scaling have already proportionate the variables, so the *MovePnlty* parameter is set to 1. *SetpTref* and *ConsTfilt* are not used in the application. Also typical operation values are listed in table A.2. The qualities are specified with a set point value and a high limit value, while the pressure controller output is specified with a high limit and a low limit. The bottom quality high limit is lower than the product specification because of too high ethane content in propane leads to condensation problems in the depropanizer condenser. The pressure controller output high limit is the limitations in the fuel gas system whereas the low limit is introduced to provide a minimum fuel gas stream.

Parameter	CV: C3 in C2 [mol%]	CV: C2 in C3 [mol%]	CV: PC output [%]	MV: Reflux flow [kg/h]	MV: Temperature [$°C$]
SP/IV	1.2 (2)	1.2 (2)			
High Limit	4 (1)	2.5 (1)	60 (1)	110000	86.3
Low Limit			15 (1)	55000	84.5
Span	0.3	0.3	1	2000	0.2
Fulf	0.5	0.5			
HighPnlty	5	5	2.5		
LowPnlty			2.5		
MovePnlty				1	1
MaxUp				2000	0.15
MaxDown				-500	-0.15

Table A.2: Typical operation values and MPC tuning parameters for the deethanizer, CV priority level in parenthesis

At last, the parameters that specify the model updating are determined. The bottom quality has some noise and the deviation between the model and the CV is filtered through a 2 minutes low pass filter. Both the top quality and the pressure controller outputs have non-modeled disturbances that influence on the variables. Letting the MV's react fast suppresses these disturbances, so both variables have a first order prediction of the disturbances with 5 minutes time constant. The cost is a more aggressive use of the reflux flow.

A.4 Results from implementation

A.4.1 Column operation without MPC

As opposed to other distillation columns at Kårstø, the deethanizer did not operate with particularly high purity in both ends. However, the deethanizer is one of the most sensitive columns with respect to disturbances and changes in reflux flow and boil-up. The basic control scheme gave large variations in product quality due to feed disturbances.

Finding the right combination of temperature set point and reflux flow rate was not easy. This combination changes with feed flow and feed composition, so the operator must be awake and adjust the temperature and the reflux flow several times during a shift.

A.4.2 Column operation with MPC

A 20 days period with 20 minutes interval have been sampled, to compare operation before and after MPC implementation. The most distinctive improvement is the variance in the product qualities. The standard deviation for the top product is reduced with 46% for the collected data series, whereas the standard deviation for the bottom product is reduced with 56%. The top and bottom quality without and with MPC operation is displayed in figure A.9.

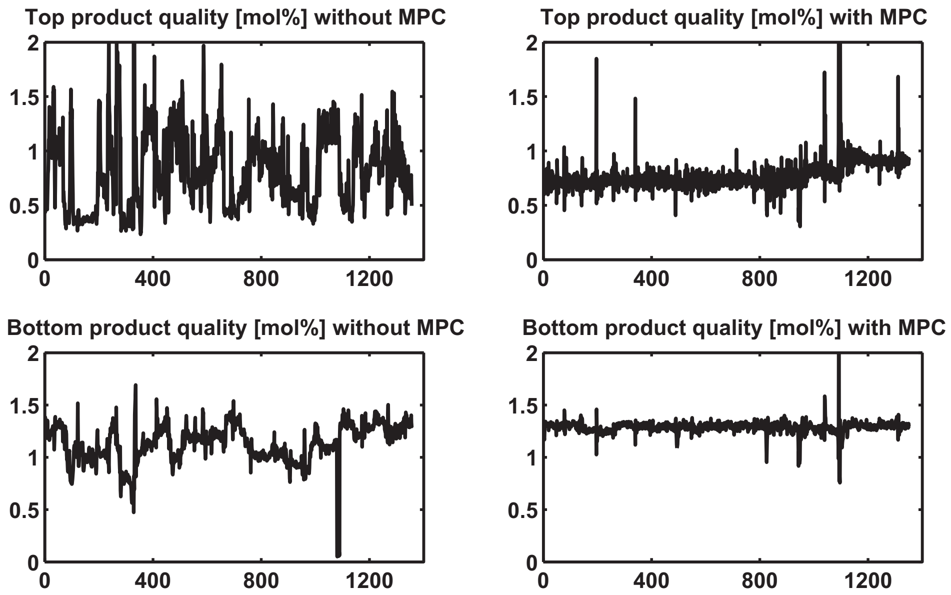

Figure A.9: Product quality from the column without (left) and with(right) MPC

The product qualities have not been changed significantly. The impurities can be increased 1-1.5 mol%, but the limits have not been challenged yet. Therefore the average changes in reflux flow and steam consumption are small. From the data period, the reflux flow per unit feed is unaltered. The steam consumption per unit feed has decreased with 2%. The average bottom impurity is slightly higher, which can explain the steam consumption reduction.

With too much methane in the feed, flaring is unavoidable since the fuel gas system has limited capacity. However, data from a two months period indicates a 20-40% flaring frequency reduction and the flaring episodes have most often a shorter duration.

A.5 Conclusions

A successful MPC implementation at the Kårstø gas processing plant has been described in detail. Reduced variance in the product qualities and less flaring have been obtained. Also the opportunity to specify the product qualities directly is an advantage gained with MPC. The product qualities have not been changed significantly after implementation of MPC and therefore the average reflux flow and steam consumption to feed ratios are almost unaltered.

A.6 Acknowledgment

The authors will thank the personnel working in the ”Optimization project” in Statoil. We would also thank Gassco for financial support.