Antonis Kokossis, Michael C. Georgiadis, Efstratios N. Pistikopoulos (Eds.)
PROCEEDINGS OF THE 33rd European Symposium on Computer Aided Process Engineering (ESCAPE33), June 18-21, 2023, Athens, Greece

A comparative study of distributed feedback-optimizing control strategies

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Abstract

In industry, the processes often consist of several subsystems with a common constraint, for example, a shared resource. This paper considers the problem of steady-state real-time optimization (RTO) for a subsea gas-lifted oil production network with multiple wells and constrained access to shared gas-lift supply. Such problems can be solved by a centralized numerical optimization, which can be computationally expensive. To avoid the use of numerical optimization, one can utilize either online primal or dual decomposition instead, where the problem is converted into a feedback-based problem. The main benefit of primal method is that it distributes local setpoints which complies with primal feasibility, however, the dual method is more general. Both the primal and dual methods allow for a distributed implementations. The dual method is more general in terms for allowing for many constraints, but as shown for the simulations with uncertainty and measurement noise, the primal method may give better dynamic constraint satisfaction.

Keywords: Distributed optimization, Feedback control, Production optimization.

1. Introduction

Industrial process often consists of several subsystems with a common constraint, for example, a shared resource. Problems like this can be decomposed and solved using distributed optimization. However, this can be computationally expensive as it solves several rounds of numerical optimization problems online at each sample time. This can be addressed by indirectly moving the optimization problem into the control layer (Morari et al. (1980)). Such problems are known as feedback-optimizing control, which can be implemented using simple tools such as Proportional-Integral-Derivative (PID) controllers.

In our previous work (Dirza et al., 2022(b)), we have experimentally validated a recently developed method of feedback-optimizing control called distributed feedback-based RTO. This method is developed based on dual decomposition and optimally handles steady-state changes in active constraints. However, the constraints are controlled in a slower time scale by updating the dual variables. This leads to the need for significant “back-off” strategy, which could lead to profit loss in the long run. To eliminate or reduce the “back-off,” Dirza et al. (2022(a)) introduces an alternative distributed feedback-optimizing control based on online primal decomposition using feedback and constraint controller(s) which distribute local setpoints without violating the common constraint to avoid or minimize use of a “back-off” strategy.

In this paper, we provide a comparative analysis of the two distributed feedback-optimizing control approaches. The model we consider represents our lab-scale experimental rig that consider uncertainty and measurement noise. The rig emulates a subsea oil production network with gas-lift rate as the manipulated variables. The experimental results will be provided as the future work.

2. Feedback-optimizing control

Consider the optimization problem for the entire system built by a network of N subsystems. We assume each subsystem is optimized locally and that we always have active constraint. Thus, the steady-state optimization problem can be expressed as follows,

$$\min_{u_i, \forall i \in N} \quad J_N = \sum_{i \in N} J_{N,i} \tag{1a}$$

$$s.t. \qquad \boldsymbol{f}_i(\boldsymbol{x_i}, \boldsymbol{u_i}, \boldsymbol{d_i}) = 0 \quad i \in [1, N] \tag{1b}$$

$$\sum_{i \in N} g_i(\boldsymbol{x_i}, \boldsymbol{u_i}, \boldsymbol{d_i}) - g^{max} = 0 \tag{1c}$$

where $\boldsymbol{x}_i, \boldsymbol{u}_i$ *and* $\boldsymbol{d}_i$ denote the vector of states, inputs and disturbances respectively. Constraint (1b) is related to the entire system model, and (1c) is an equality constraint.

2.1. Distributed Feedback-Optimizing Control using Online Primal Decomposition

To solve problem (1) using simple feedback controller, it is possible to construct a method based on online primal decomposition (Dirza et. al (2022(a))). The main motivation for this method is that we want to achieve optimal steady-state operation in a distributed manner, with minimal dynamic constraint violation and without demand of solving numerical optimization problems online. By introducing an initial value of local constraint g_i^{sp}, (1c) can be written as

$$g_i(\boldsymbol{x_i}, \boldsymbol{u_i}, \boldsymbol{d_i}) - g_i^{sp} = 0 \quad i \in [1, N] \tag{1d}$$

$$\sum_{i \in N} g_i^{sp} = g^{max} \tag{1e}$$

instead. As long as (1e) is satisfied, primal feasibility is guaranteed. Each subproblem estimates local Lagrange multipliers, which is used in central constraint controllers. These controllers update the setpoints iteratively, where the goal is to provide setpoints that satisfy the primal feasibility. In this paper we assume that the constraint is always active, therefore we have two types of approaches of updating the local setpoints. To ensure that (1e) is satisfied, one of the local setpoints is updated as follows,

$$g_N^{sp,k+1} = g^{max} - \left(g_1^{sp,k+1} + \cdots + g_{N-1}^{sp,k+1}\right) \tag{2}$$

This is called the compensator subsystem, and the objective is to ensure primal feasibility. For the remaining subsystems, $i = \{1, \dots, N-1\}$, the local setpoints is updated by

$$g_i^{sp,k+1} = g_i^{sp,k} + K_{I,i}(-\lambda_1^k + \lambda_N^k) \tag{3}$$

We may use integrating controllers with integral gain $K_{I,i} = \frac{1}{K_i(\tau_{c,i})}$, where K_i is the step response gain and $\tau_{c,i}$ is the closed-loop time constant. However, a proportional integral (PI) controller can also be used. The local Lagrange multipliers, λ_i, can be estimated

$$\lambda_i = -\nabla_{u_i}\hat{J}_{N,i}\left(\nabla_{u_i}\hat{g}_i(x_i, u_i, d_i)\right)^{-1} \tag{4}$$

Where $\nabla_{u_i}\hat{J}_{N,i}$ and $\nabla_{u_i}\hat{g}_i$ are the estimated gradient of local cost and local setpoints respectively. In figure 2.1 the online primal decomposition framework is illustrated. The central constraint controllers, which contains both normal and compensator subsystems, provide new set points for the local constraints. As the constraints always are active, these set points are considered as inputs to the subsystems. When there is a presence of disturbance, we can use the plants current information to estimate the current state and parameters by implementing a local dynamic estimator, for instance Extended Kalman Filter (EKF). By applying the inputs, estimated states and parameters, we can estimate cost gradient as well as constraint gradient to compute the local Lagrange multipliers. The multipliers are then implemented in the central constraint controller to calculate the new set points.

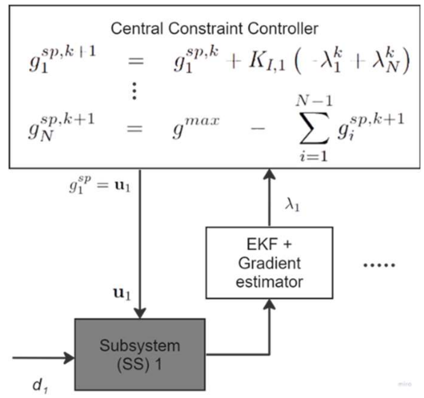

Figure 2.1 The online primal decomposition control structure.

3. Simulation descriptions

3.1. Simulation model setup

To emulate the subsea gas-lifted oil production system, we use a model created in MATLAB R2021a based on a lab-scale experimental rig from a paper by Matias et al. (2022) and has been tested and used in earlier papers related to the lab-rig. It is implemented with noise similar to what is expected from the lab-rig. The reservoir section of the model is implemented by valve openings $\boldsymbol{p} = [\boldsymbol{p_1}, \boldsymbol{p_2}, \boldsymbol{p_3}]$ which determine the liquid flow. These valves represent the disturbances from the reservoir. In the MATLAB model the gas-lift flow controllers are given $\mathbf{u}_i$ as setpoints. They are implemented as a 5 second delay plus input noise to simulate controller action instead of coding the actual controllers. The setpoint controllers in the central constraint controller are integral controllers tuned using SIMC tuning rules (Skogestad (2003)).

3.2. Optimization problem setup

The objective of the optimization problem in this model setup is to maximize the liquid flow rate, which equals the sum of the liquid production of the three wells, with a limited amount of gas-lift injection, which is input shared constraint. Considering problem (1). the economic objective can be expressed as below,

$$\boldsymbol{J}(\boldsymbol{u},\boldsymbol{p}) = \sum_{i=1}^{3} f_i(u_i, p_i) = -20Q_{l,1}(u_1,p_1) - 25Q_{l,2}(u_2,p_2) - 30Q_{l,3}(u_3,p_3) \quad (5)$$

where $Q_{l,1}$, $Q_{l,2}$ and $Q_{l,3}$ are the produced liquid flow rates of well 1, 2 and 3 respectively. We assume different values of the hydrocarbon flows as shown in eq. 6, to illustrate how different values affect the behavior of the subsystems. The input vector is defined as $\boldsymbol{u} = [Q_{gl,1}^{\mathbf{sp}} \quad Q_{gl,2}^{\mathbf{sp}} \quad Q_{gl,3}^{sp}]^{\boldsymbol{T}}$ where $Q_{gl,1}^{sp}$, $Q_{gl,2}^{sp}$ and $Q_{gl,3}^{sp}$ are the gas-lift set points from the central controller of well 1, 2 and 3 respectively. In addition, the reservoir valve $\mathbf{p}$ is time varying.

3.3. Comparative method

We consider our previous work, distributed feedback-based optimization with dual decomposition, to compare with primal decomposition. The implemented dual decomposition method is based on Dirza et al. (2022(b)) and Krishnamoorthy et al. (2021). In figure 3.1 the implemented dual decomposition framework is shown.

4. Results and Discussion

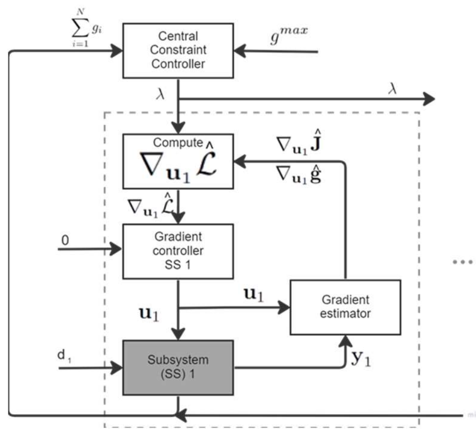

Figure 3.1 Block diagram of dual decomposition control structure for one well. The area within the dashed gray box is duplicated N times. For a more detailed version the reader is referred to Dirza et al. (2022b).

Figure 4.1 shows the disturbance in this simulation, which corresponds to the reservoir valve openings **p** in the lab-rig model. The first disturbance occurs when p_1 gradually decreases from t = 6 to t =12.5 min. During this interval, we expect the gas-lift injection in well 1 to decrease, and redistribution of the gas to the other wells. The second disturbance occurs when also p_3 gradually decreases from t = 14 to t = 18 min. As before, we expect that the gas-lift injection rate in well 3 will go down. At the same time, we expect the other wells will gain a higher gas-lift injection. The third and fourth disturbance occur from t = 21 to t = 24.5 and t = 28 to t = 34.5 respectively. During this time p_3 and p_1 are gradually increased back up to the initial values. This is because we want to see how the controllers behave for both decrease and increase in the disturbance.

In this paper subsystem 3 has been used as the compensator for the primal decomposition. This subsystem was chosen because it has the highest gain magnitude, which results in most profit for cases with active shared constraint.

In figures 4.2 – 4.7 we compare the simulation results from primal decomposition and dual decomposition. The calculated input setpoints are shown in figure 4.2, the actual gas-lift flow rate will deviate slightly from the setpoints due to the implemented measurement noise in the model. Another cause is how the gas flow rate controllers are implemented in the model, which is described in section 3.1.

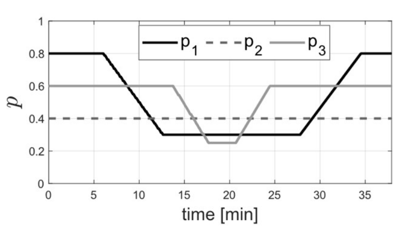

Figure 4.1 Disturbance profile during the simulation.

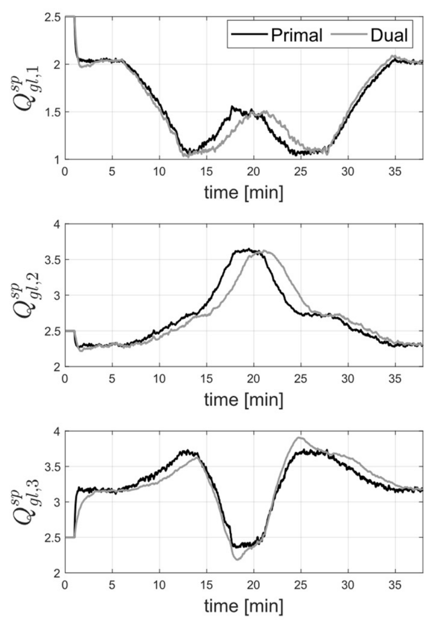

Figure 4.2 The gas-lift flow rate setpoint $\boldsymbol{u}^{sp} = \boldsymbol{Q}^{sp}_{gl}$ of every well due to disturbance from the simulation.

In figure 4.2 and 4.3 we see that the dual control is responding slower to the disturbances than the primal control for correcting the gas-lift setpoints. This is a consequence of the time-scale separation between the central controller and the gradient controllers needed in the dual structure (Dirza et al (2022b)).

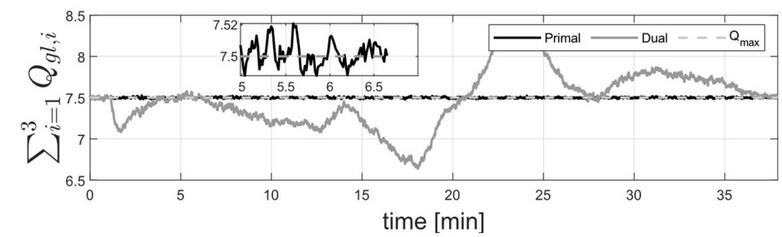

Figure 4.3 The constraint satisfaction of both primal and dual. There is a magnifying plot in time window 5 to 6.5 min showing only constraint satisfaction for primal.

Figure 4.3 shows the constraint satisfaction. As expected, the Primal decomposition performs much better than the dual decomposition here. This is because the compensator system, subsystem 3, "absorbs" the deviations from the constraint. The absorbing ability comes from how the compensator set point is calculated, see eq. 2, and is the reason for the primal decomposition's capability to maintain the active constraint. In terms of what this means for the operation, with the use of primal decomposition we can run the system without any significant back-off and remain feasible at all times. On the other and, for the dual decomposition we must implement back-off, especially from t = 21 when the disturbances start to increase again.

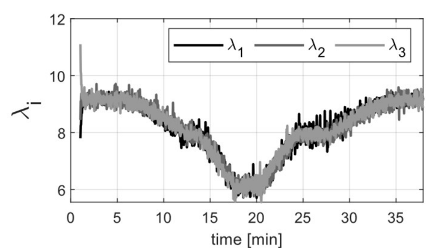

Figure 4.4 Local Lagrange multipliers for primal decomposition.

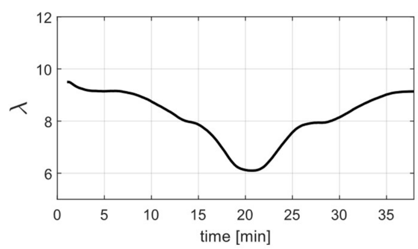

Figure 4.5 Lagrange multiplier for dual decomposition

Figure 4.4 depicts that the local Lagrange multipliers for Primal Decomposition converge to the same value. Figure 4.5 show the Lagrange multiplier for the dual decomposition. We can observe that the multiplier converges slower and is smoother for this method. This is because the central controller for dual decomposition operates on a slower timescale. When the Lagrange multiplier converges around t = 3.5 we can see that the active constraint is controlled almost as good as for the Primal case. It is also worth to notice that the Lagrange multipliers converges to the same value for both Primal and Dual.

To analyze the optimization performance of primal and dual decomposition, we compare the profit obtained by the two methods with a naive approach, where it is considered fixed inputs, $\boldsymbol{u}_i = Q_{gl}^{max}/3$. The naive approach represents the case where no information is available, therefor, the best approach is to divide the available gas equally among the

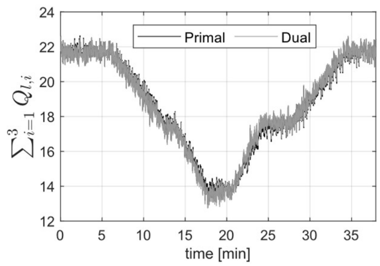

Figure 4.6 Total liquid flow of the system

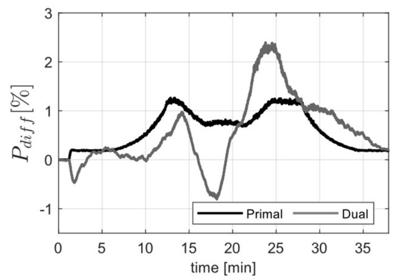

Figure 4.7 Instantaneous profit compared to naive approach.

wells. This additional approach is used as benchmark to show how primal and dual decomposition compares to not do any optimization at all. To display the performance, we plot the difference in percentage between the instantaneous profit of primal/dual and the naive approach. The difference is calculated as $J_{diff} = \frac{J - J_{naive}}{J_{naive}} \cdot 100$, where J is the profit of the method of interest and $\boldsymbol{J_{naive}}$ is the profit of the naive approach. Figure 4.7 shows that primal is more profitable than dual until t = 21 min, after this dual appears to be favorable. However, in figure 4.3 we see that after t = 21 dual does not achieve primal feasibility and therefore the profit here is not viable.

5. Conclusion

In this work, we have done simulation with uncertainty and measurement noise to compare feedback-based real-time optimization with online primal and dual decomposition. Based on the results we can conclude that primal decomposition is able to effectively ensure primal feasibility for separable systems with an active shared input constraint. It performs better than dual decomposition in this aspect, mainly because of the timescale difference of the central constraint controllers. This significantly reduces the need for any significant back off, which results in more profitable operations. While primal ensures constant feasibility, it is less general than dual. As the continuation of this work, we consider obtaining the experimental result, implementing dual with override and develop a primal structure able to switch between active constraints.

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