

# Comparison of Simple Feedback Control Structures for Constrained Optimal Operation<sup>\*</sup>

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**Abstract:** Optimal operation of systems subject to constraints can be challenging, especially when the set of active constraints changes during operation due to disturbances. The use of traditional model-based real-time optimization strategies has limitations related to model-plant mismatch and computational effort, and therefore the use of simple feedback strategies in lower layers is a good alternative to avoid these issues on fast timescales. This work aims to evaluate the viability of region-based control structures using selectors and the use of a primal-dual feedback optimizing control framework on this kind of problem, through the study of two process systems with changing active constraints. While region-based strategies focus on the effective control in specific active constraint regions, the primal-dual approach allows for control in all possible regions, with the introduction of intermediate variables that estimate the Lagrange multiplier values. In the first case study, the traditional region-based strategy could not handle all constraint regions, and an additional logical switching was necessary to account for the remaining region. The implementation of primal-dual feedback optimizing control was flexible enough to control the system in all regions without the need for additional logic. The second case study presents more constraints than the first and increased nonlinearities, which makes finding controlled variables for the unconstrained degrees of freedom challenging. The primal-dual control framework was able to drive the system to the optimum in all considered regions. Therefore, this framework is deemed as a promising control structure for optimal operation in the presence of changing active constraints.

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## 1. INTRODUCTION

Optimal operation is one of the main objectives in process operation, as it is always desired that losses are minimized when possible. Optimal operation requires optimization of economic objectives, i.e. maximization of profit or minimization of costs, subject to constraints related to intrinsic or external conditions, such as operational capacity, product specification, or emission limit values. This is often formulated as a steady-state optimization problem, which can be solved through a plethora of methods (Nocedal and Wright, 2006), given that a full model for the system is known. While all constraints are satisfied in the solution of such problems, some constraints influence the location of the solution, but others do not. The former type of constraints is typically referred to as active constraints.

The main challenge related to implementation of real-time optimization (RTO) strategies lies on the lack of knowledge about the system. This can be detected through the available system measurements, and how far these mea-

surements are from the model predictions gives a metric of how inaccurate the model is. If the model is parametrized by disturbances which cannot be known or measured, online parameter estimation can be used to fit the model to the measurements and thus allowing for better predictions (Roberts and Williams, 1981). This translates into a two-step approach for RTO implementation, consisting of parameter estimation and reoptimization, which is very simple, but depends heavily on the structural similarity between model and plant. If this condition is not met, operation might converge to a suboptimal or even infeasible point (Marchetti et al., 2009).

Therefore, the use of model-based RTO approaches has fundamental limitations, as model-plant mismatch is always present in some degree, and may not be completely removed even in the presence of measurements. In this context, an interesting area of research is attempting to satisfy optimality conditions without solving the model-based optimization problem, and at the same time requiring the least amount of knowledge of the system. One particularly useful concept is self-optimizing control (Skogestad, 2000), which is based on the translation of the

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optimization problem into a feedback control problem, and the focus becomes the selection of variables that, when kept controlled under a fixed setpoint, allow for optimal operation. The resulting control structure minimizes the effect of disturbances by design, based on the available model, and any model-plant mismatch is dealt by the upper control layers through setpoint changes. With this, the magnitude of setpoint changes that the upper layers must perform is minimized, guaranteeing that near-optimal operation is attained even in the faster timescales, when the necessary update is not yet available.

This class of strategies, however, has limitations regarding the treatment of constraints, see Gros et al. (2009); François et al. (2005). If the set of active constraints is fixed through operation, a single set of variables can be controlled for optimal operation. In particular, if there are no active constraints, the ideal self-optimizing variables are the gradient of the cost with relation to the inputs (Jäschke and Skogestad, 2011), and in presence of active constraints, the ideal self-optimizing variables become the active constraints themselves and the reduced cost gradient projected in the unconstrained directions (Krishnamoorthy and Skogestad, 2019). However, if the set of active constraints change during operation, these control objectives no longer apply, and restructuring of the control system is required.

Another setback in the implementation of RTO strategies is related to the computational effort necessary for implementation. As the solution of optimization problems is computationally expensive, RTO is often performed in a slow timescale, and operation must always be performed with the aid of fast controllers that stabilize the process and control key variables for operation. This means that rapid changes in active constraints may not be counteracted efficiently if changes in active constraints are only dealt by the RTO layer, even if these RTO strategies have the capability of completely eliminating the offset in steady state (Marchetti et al., 2020). There is therefore great interest in the implementation of fast and feedback-based approaches to optimal operation of processes with changes in active constraints.

For this end, a classic approach is analyzing the possible active constraint regions, and designing a control structure that is able to switch between the controlled variables (CVs) (Krishnamoorthy and Skogestad, 2019; Reyes-Lúa et al., 2018). A limitation of this approach is the necessity of pairing, which becomes problematic when constraints are independent and may activate at the same time. In such cases, the pairing needs to be adaptive, but proposing adaptive structures may be cumbersome or even infeasible. In this work, such types of case study are explored, and we aim to evaluate the viability of region-based control structures and the use of more general feedback control structures in the optimal operation of these systems. Specifically, we propose the use of a primal-dual feedback optimizing control structure, based on the work presented in Krishnamoorthy (2021), which can be applied to solve most steady-state optimal operation problems of interest. In this structure, the Lagrange multipliers are introduced as extra degrees of freedom that can be used for constraint control, and therefore the resulting approach presents both primal and dual decision variables as manipulated vari-

ables, which enables for tracking of all necessary conditions of optimality.

## 2. CONTROL STRUCTURES FOR OPTIMAL OPERATION

In this section we present the control structures considered in the present work, which aim to solve a steady-state optimization problem through feedback. This generic optimization problem can be defined as:

$$\begin{aligned} \min_u \quad & J(u, d) \\ \text{s.t.} \quad & g(u, d) \leq 0 \end{aligned} \quad (1)$$

In this definition,  $u \in \mathbb{R}^{n_u}$  represents the manipulated variables (MVs),  $d \in \mathbb{R}^{n_d}$  represents process disturbances,  $J: \mathbb{R}^{n_u} \times \mathbb{R}^{n_d} \rightarrow \mathbb{R}$  represents the objective function, and  $g: \mathbb{R}^{n_u} \times \mathbb{R}^{n_d} \rightarrow \mathbb{R}^{n_g}$  represents all process inequality constraints. For this problem, by introducing  $\lambda \in \mathbb{R}^{n_g}$  as the Lagrange multipliers associated to the inequality constraints, the Lagrangian function is written as:

$$\mathcal{L}(u, d, \lambda) = J(u, d) + g(u, d)^T \lambda \quad (2)$$

The necessary Karush-Kuhn-Tucker (KKT) conditions for the optimization problem state that the optimal pair  $(u^*, \lambda^*)$  satisfies:

$$\nabla_u \mathcal{L}(u^*, d, \lambda^*) = \nabla_u J(u^*, d) + \nabla_u g(u^*, d)^T \lambda^* = 0 \quad (3a)$$

$$g(u^*, d) \leq 0 \quad (3b)$$

$$\lambda^* \geq 0 \quad (3c)$$

$$g(u^*, d)^T \lambda^* = 0 \quad (3d)$$

The main challenge in solving this type of problem is related to the lack of knowledge about the set of active constraints  $g_A$ , which is here written as the vector composed of the elements of  $g$  such that  $g_A(u^*, d) = 0$ . If this set is known beforehand, the problem is simplified to an equality-constrained optimization problem, which is written as:

$$\nabla_u \mathcal{L}(u^*, d, \lambda^*) = \nabla_u J(u^*, d) + \nabla_u g_A(u^*, d)^T \lambda_A^* = 0 \quad (4a)$$

$$g_A(u^*, d) = 0 \quad (4b)$$

### 2.1 Active constraint region-based control using selectors

This strategy can be regarded as a generalization of the classic approach to designing an advanced supervisory control layer for optimal operation. In this strategy, each possible set of active constraints define a set of control objectives for optimal operation (Krishnamoorthy and Skogestad, 2019). For a given region defined by the active set  $A$ , controlling  $g_A(u, d) = 0$  is the first straightforward choice of CVs, which fills  $n_a$  degrees of freedom. The remaining  $n_u - n_a$  unconstrained degrees of freedom are filled with a projection of the cost gradient  $N^T \nabla_u J(u, d)$  such that  $N$  is the nullspace of  $\nabla_u g_A(u, d)$ . This leads to optimal operation because it automatically satisfies the KKT condition given in Equation (4a), since  $N^T \nabla_u J(u, d) =$

$-N^T \nabla_u g_A(u, d)^T \lambda_A = 0$  at the stationary point. Therefore, the control objectives in this strategy are calculated from the plant gradients, and fully independent of the optimal Lagrange multipliers.

A scheme of the general control strategy is presented in Figure 1, where the gradient  $\nabla_u J$  is considered to be obtained with the aid of the model and measurements. The control structure is designed taking into account the constraints  $g$  and all possible gradient projections  $N_i \nabla_u J$ , each of them being paired to specific plant inputs, and logic must be applied to select between the corresponding control actions  $u_g$  and  $u_0$ . This logic serves therefore as a detection mechanism of the active constraints, which can rapidly change during operation. In this work, we attempt to implement the switching logic through the use of selectors, as it is usually done in practice. However, this strategy alone is not effective when it is necessary to switch pairings for different regions. This shall be discussed within the first case study considered in this work.

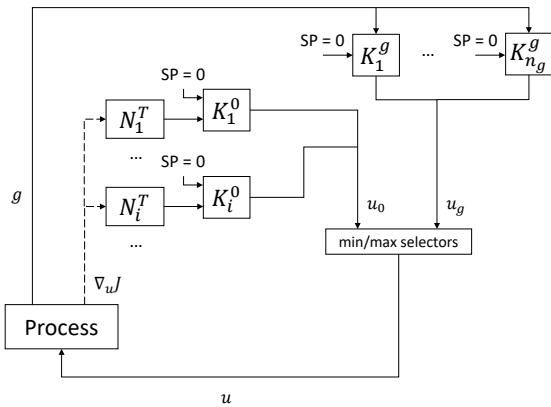


Fig. 1. Region-based control strategy using selectors

### 2.2 Primal-dual feedback optimizing control

In this strategy, a single control structure is used for all regions, with controllers arranged in a cascade layout, according to Figure 2. The components of the Lagrangian gradient  $\nabla_u \mathcal{L}$  are paired to the respective process inputs  $u$  with simple controllers, and each constraint  $g$  is controlled in an outer loop by manipulating the estimate of the respective Lagrange multipliers  $\lambda$ , entailing the use of  $n_u + n_g$  controllers, labelled as  $K^u$  and  $K^\lambda$  in Figure 2. The constraint controllers must become inactive when the constraints are not violated ( $g(u, d) < 0$ ), and a switching logic to enforce  $\lambda \geq 0$  is thus introduced, guaranteeing steady-state primal feasibility (3b), dual feasibility (3c), and complementarity conditions (3d). This control structure is based on the works of Krishnamoorthy (2021); Dirza et al. (2021), which were written under a distributed optimization perspective, but it can also be applied to a generic constrained optimal operation problem (Krishnamoorthy and Skogestad, 2022).

This structure gives up on tight control of constraints in fast timescales, as their control is placed in an internal loop mediated by the Lagrange multiplier estimates. However, given that the gradient calculations are accurate, and that integral action is present in all controllers, optimality

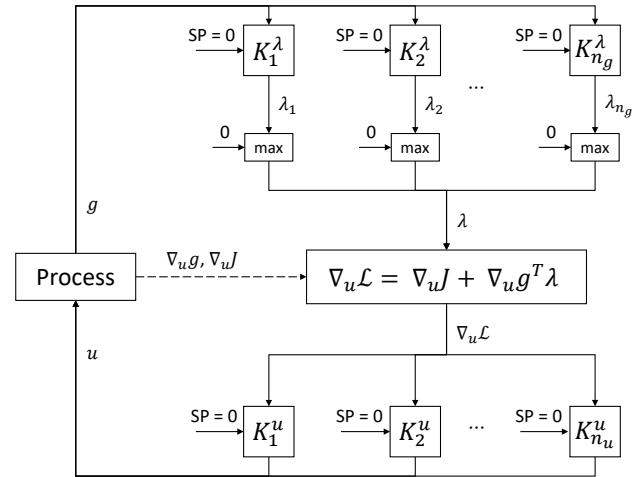


Fig. 2. Primal-dual feedback optimizing control framework, based on the DFRTO framework (Krishnamoorthy, 2021).

is attained at steady state. The switching logic between active constraints is mediated by the max blocks, as the calculated  $\lambda$  shifts the relevant directions for control in the actuator layer when the constraints are violated, and zero is selected when the calculated  $\lambda$  value becomes negative, which happens when the constraint is no longer being violated. One advantage of this strategy is that pairing between constraints and MVs is not required, since this association is done through the Lagrangian gradient calculation.

### 3. CASE STUDY 1: HEAT EXCHANGER NETWORK

The first system considered in this work, based on the work of Jäschke and Skogestad (2014), consists of three heat exchangers in parallel. The network is fed with a cold stream, which is split to be heated into each line by different hot streams. The goal of the process is therefore to maximize the final temperature of the heated stream,  $T$ , but subject to constraints of maximum allowed temperature in each branch,  $T_i \leq T_{max}$ . For this system, the available manipulated variables are the splits for each line,  $u = \alpha$ , and the possible disturbances are  $d = [T_0, w_0, T_{h,i}, w_{h,i}, UA_i]$ , namely the cold stream inlet temperature and flow, hot streams inlet temperatures and flows, and heat transfer coefficients for the exchangers. A representation of the system is shown in Figure 3, and the respective optimization problem is written as:

$$\begin{aligned} \min_u \quad & J = -T \\ \text{s.t.} \quad & g_i = T_i - T_{max} \leq 0, \quad i = 1, 2, 3 \end{aligned} \quad (5)$$

#### 3.1 Active constraint region-based control

The case study has a total of  $2^3 = 8$  possible regions, but only 7 are feasible regions, as it is not possible to satisfy all 3 constraints with 2 manipulated variables. In the case that all constraints are violated, it is acceptable that one constraint is given up for the operation, but this is beyond the scope of this analysis. For each region, the control objectives that allow for optimal operation are

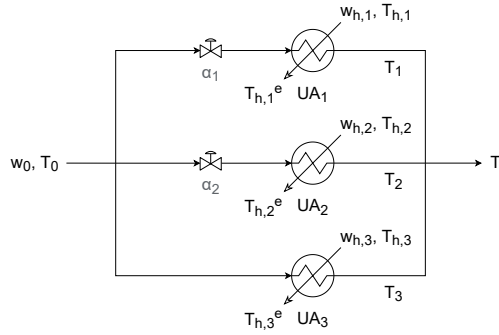


Fig. 3. Heat exchanger network scheme.

given in Table 1. In this case study, due to the nature of the constraints, the derivation of linear combinations of the gradient,  $N^T$ , for the different regions results in constant coefficients inside the region.

Table 1. Control objectives per region for case study 1

Active constraints	Control objectives
-	$\nabla_u J$
$g_1$	$g_1, [0 \ 1] \nabla_u J$
$g_2$	$g_2, [1 \ 0] \nabla_u J$
$g_3$	$g_3, [-1 \ 1] \nabla_u J$
$g_1, g_2$	$g_1, g_2$
$g_2, g_3$	$g_2, g_3$
$g_1, g_3$	$g_1, g_3$

Upon inspection of the control objectives, it is clear that a single pairing strategy cannot account for all regions, especially due to the regions with 2 active constraints. When  $g_1$  is paired to  $u_1$  and  $g_2$  is paired to  $u_2$ , which is a natural pairing choice,  $g_3$  cannot be attributed to a single MV if control over all regions is desired. If  $g_3$  is paired to a single MV in this case, a selector strategy can account for 6 regions at most. One pairing example is given at Table 2. In this case, following a classic approach of pairing and implementing a switching logic, the region where  $g_1$  and  $g_3$  are simultaneously active cannot be optimally controlled.

Table 2. Example of classic pairing for region-based control of case study 1

$u_1$	$u_2$
$g_1$	$g_2$
$[1 \ 0] \nabla_u J$	$[0 \ 1] \nabla_u J$
$g_3$	$[-1 \ 1] \nabla_u J$

Figure 4 shows the performance of this control structure over a disturbance sequence that activates all possible operation modes. In spite of it being able to handle most regions correctly, there is steady-state constraint violation for  $g_3$  in the region that cannot be handled, from  $t = 200$  to  $t = 300$ .

From these results, it becomes clear that a control structure that handles all regions must be more flexible. Specifically,  $g_3$  must also be controlled using  $u_2$  in some cases. This introduces external conditions on the controllers activation, since this possibility should only be accessed when  $g_1$  is being controlled by  $u_1$ . Implementing this logical statement should guarantee optimal operation over all possible regions. This adaptive pairing is presented in Table 3.

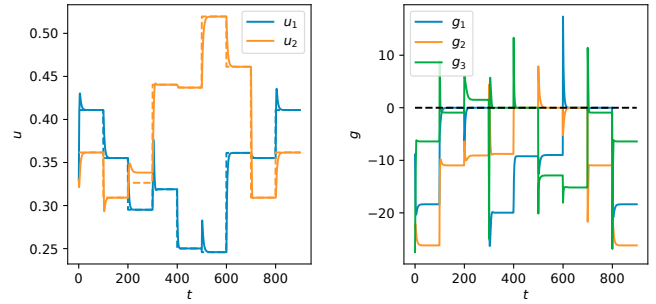


Fig. 4. Operation of case study 1 with region-based control using classic pairing, along with optimal inputs (dashed).

Table 3. Adaptive pairing for region-based control of case study 1

$u_1$	$u_2$ ( $g_1$ inactive)	$u_2$ ( $g_1$ active)
$g_1$	$g_2$	$g_2$
$[1 \ 0] \nabla_u J$	$[0 \ 1] \nabla_u J$	$[0 \ 1] \nabla_u J$
$g_3$	$[-1 \ 1] \nabla_u J$	$g_3$

The performance of the adaptive region-based control structure over the same disturbance realization is presented in Figure 5. In this case, no steady-state constraint violation is obtained, and the expected peaks that happen when disturbances change are quickly corrected. At  $t = 200$ , quick oscillations in  $u_2$  can be noticed, due to the changes in the active control structure.

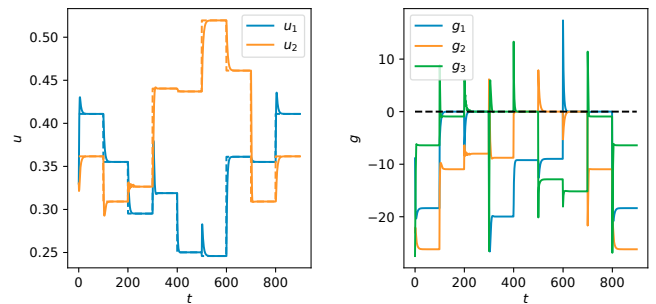


Fig. 5. Operation of case study 1 with region-based control using adaptive pairing, along with optimal inputs (dashed).

### 3.2 Primal-dual feedback optimizing control

The performance of the primal-dual feedback optimizing control structure is presented in Figure 6. The steady-state performance of this structure is the same when compared to what is attained by selectors, and constraint violation is efficiently corrected even though it is regulated by an extra control layer.

In addition, Figure 7 shows the dual variables of the primal-dual control structure. The combination of layers leads to slower responses in some disturbance changes, especially when there are big changes in the values of Lagrange multipliers. Nevertheless, the estimated Lagrange multipliers smoothly converge to the optimal values in each region, and spikes due to instantaneous constraint violation are corrected at steady state.

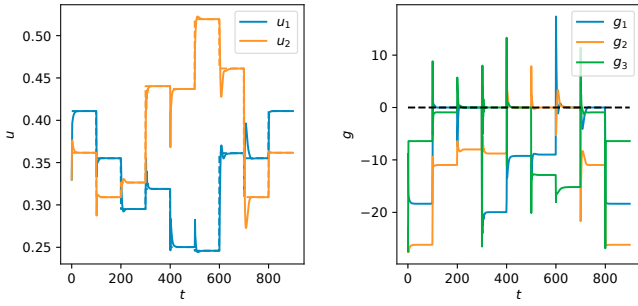


Fig. 6. Test of primal-dual control framework over case study 1, along with optimal inputs (dashed).

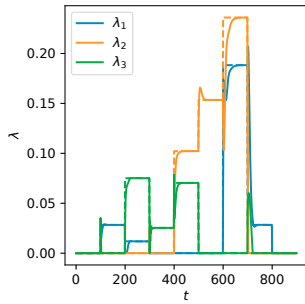


Fig. 7. Lagrange multiplier estimates from the primal-dual control framework in case study 1, along with optimal multiplier values (dashed).

#### 4. CASE STUDY 2: TWO DISTILLATION COLUMNS IN SEQUENCE

Another system studied in this work has been described by Jacobsen and Skogestad (2012), and it consists of two distillation columns in series, as presented in Figure 8. The inlet stream, composed of three components A, B, and C, is fed into the first column, with the goal of separating the most volatile component with minimal purity specification  $x_A$ . The bottom product is then fed into the second column, which generates distillate and bottom products with minimal purity specifications  $x_B$  and  $x_C$ , respectively. In addition, there are constraints related to maximum boilup of the columns,  $V_1$  and  $V_2$ . The operational goal is to optimize plant economics, with costs related to feed and vapor consumption, and profit from selling the products. The steady-state optimization problem can then be written as:

$$\begin{aligned}
 \min_u \quad & J = p_F F + p_V (V_1 + V_2) - p_A D_1 - p_B D_2 - p_C B_2 \\
 \text{s.t.} \quad & g_1 = x_{A,min} - x_A \leq 0 \\
 & g_2 = x_{B,min} - x_B \leq 0 \\
 & g_3 = x_{C,min} - x_C \leq 0 \\
 & g_4 = V_1 - V_{1,max} \leq 0 \\
 & g_5 = V_2 - V_{2,max} \leq 0
 \end{aligned} \tag{6}$$

The vector of manipulated variables for operation are the internal flows of the columns  $u = [L_1, L_2, V_1, V_2]$ , and the possible disturbances are  $d = [F, p_V]$ , namely the feed to the first column and the steam generation price. The

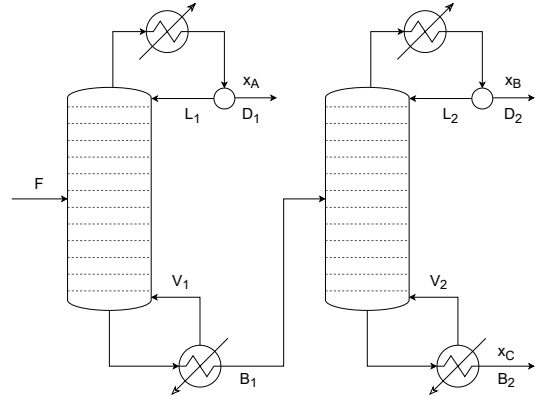


Fig. 8. Scheme of two distillation columns in sequence, based on Jacobsen and Skogestad (2012)

system is considered to be at steady state at all times, meaning that all changes in the operating conditions are quickly accommodated by the system. This assumption imposes the limitation that this control structure operates in a slower timescale than that of the process, which is a common practice when dealing with supervisory control.

In the considered operating range, there are 8 possible active constraint regions. These regions are described in Table 4. Unlike case study 1, calculating gradient projections for each region becomes more complicated, as the system constraints have a nonlinear relationship with the inputs. Therefore, even though constraint control can be achieved by using a selector-based logic, control of the unconstrained degrees of freedom requires an elaborate and interconnected logic, and is therefore deemed outside of the scope of this paper.

Table 4. Active constraints per region for case study 2

Region number	Active constraints
I	$x_B$
II	$x_B, x_A$
III	$x_B, V_1$
IV	$x_B, x_A, x_C$
V	$x_B, x_A, V_1$
VI	$x_B, V_1, V_2$
VII	$x_B, x_A, V_1, V_2$
VIII	$x_B, x_A, x_C, V_1$

The performance of primal-dual optimizing control in this system is presented in Figure 9, and the corresponding Lagrange multiplier estimates are shown in Figure 10. As in the previous case study, the optimal values for the plant inputs and estimated Lagrange multipliers are attained at steady state, and any constraint violation is corrected.

#### 5. CONCLUSION

The presented case studies illustrate the complexity of optimal operation problems when there is switching of active constraint regions. We explore the concept of ideal controlled variables for optimal operation, and the close link between steady-state plant gradients and optimal operation. It is assumed that these gradients are available for the control structure, which can be burdensome especially in cases where these depend on the plant states. However,

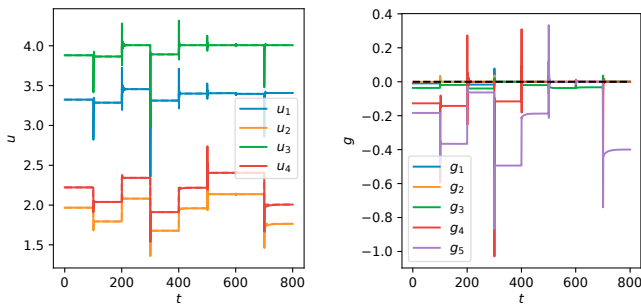


Fig. 9. Test of primal-dual control framework over case study 2, along with optimal inputs (dashed).

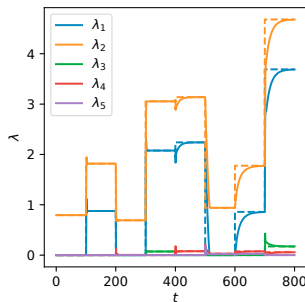


Fig. 10. Lagrange multiplier estimates from the primal-dual control framework in case study 2, along with optimal multiplier values (dashed).

as this layer is meant to operate in the slower timescales, it can be assumed that the system is settled, and gradient estimation is possible. This can be done by using a model, with further correction by the plant data. In this case, the use of an incorrect model is not detrimental, as the inclusion of biases in the gradients do not impose hard constraints on the control problem. The performance of this control structure may be improved even further by making use of optimization results, as one can retrieve estimated gradients and Lagrange multipliers from these calculations.

The implementation of simple feedback structures that deal with constraints and allow for optimal operation was accomplished. The choice of which structure is favored lies in the nature of the system at hand. If there is a low number of constraints, and pairing can be done to contemplate all regions, a simple structure consisting of selectors is to be considered, as in this case constraints are kept more effectively under control. However, no big loss was noticed with the implementation of indirect constraint control mediated by the control of the KKT conditions. The latter is also not affected by the combinatorial nature of the number of active constraint regions, and by the concern of adequate pairing between variables. The use of primal-dual feedback optimizing control is therefore deemed promising in the handling of constrained systems, and further evaluation of this strategy is encouraged, especially in terms of controller tuning and accounting for process dynamics. The further study of control structures that deal with active constraint switching in fast timescales remains relevant, bearing in mind that the studied primal-

dual framework requires a timescale separation between the primal and dual control layers.

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## REFERENCES

- Dirza, R., Skogestad, S., and Krishnamoorthy, D. (2021). Optimal resource allocation using distributed feedback-based real-time optimization. *IFAC-PapersOnLine*, 54(3), 706–711.
- François, G., Srinivasan, B., and Bonvin, D. (2005). Use of measurements for enforcing the necessary conditions of optimality in the presence of constraints and uncertainty. *Journal of Process Control*, 15(6), 701–712.
- Gros, S., Srinivasan, B., and Bonvin, D. (2009). Optimizing control based on output feedback. *Computers & Chemical Engineering*, 33(1), 191–198.
- Jacobsen, M.G. and Skogestad, S. (2012). Active constraint regions for optimal operation of distillation columns. *Industrial & engineering chemistry research*, 51(7), 2963–2973.
- Jäschke, J. and Skogestad, S. (2011). Nco tracking and self-optimizing control in the context of real-time optimization. *Journal of Process Control*, 21(10), 1407–1416.
- Jäschke, J. and Skogestad, S. (2014). Optimal operation of heat exchanger networks with stream split: Only temperature measurements are required. *Computers & chemical engineering*, 70, 35–49.
- Krishnamoorthy, D. (2021). A distributed feedback-based online process optimization framework for optimal resource sharing. *Journal of Process Control*, 97, 72–83.
- Krishnamoorthy, D. and Skogestad, S. (2019). Online process optimization with active constraint set changes using simple control structures. *Industrial & Engineering Chemistry Research*, 58(30), 13555–13567.
- Krishnamoorthy, D. and Skogestad, S. (2022). Real-time optimization as a feedback control problem—a review. *Computers & Chemical Engineering*, 107723.
- Marchetti, A., de Avila Ferreira, T., Costello, S., and Bonvin, D. (2020). Modifier adaptation as a feedback control scheme. *Industrial & Engineering Chemistry Research*, 59(6), 2261–2274.
- Marchetti, A., Chachuat, B., and Bonvin, D. (2009). Modifier-adaptation methodology for real-time optimization. *Industrial & engineering chemistry research*, 48(13), 6022–6033.
- Nocedal, J. and Wright, S. (2006). *Numerical optimization*. Springer Science & Business Media.
- Reyes-Lúa, A., Zotică, C., and Skogestad, S. (2018). Optimal operation with changing active constraint regions using classical advanced control. *IFAC-PapersOnLine*, 51(18), 440–445.
- Roberts, P.D. and Williams, T. (1981). On an algorithm for combined system optimisation and parameter estimation. *Automatica*, 17(1), 199–209.
- Skogestad, S. (2000). Plantwide control: The search for the self-optimizing control structure. *Journal of process control*, 10(5), 487–507.