Metin Türkay Metin Türkay, Erdal Aydin (Eds.)
Proceedings of the 31th European Symposium on Computer Aided Process Engineering
June 6th to 9th, 2021, Istanbul, Turkey. 

# Input transformation for linearization, decoupling and disturbance rejection with application to steam networks

Cristina Zotică[a] and Sigurd Skogestad[a]

[a]*Department of Chemical Engineering, Norwegian University of Science and Technology, Sem Sælands vei 4, Trondheim, 7491, Norway*

## Abstract

Pressure-flow networks are inherently multivariable, coupled and nonlinear systems that are not straightforward handled by conventional PID controllers. In this work we analyze the control problem for a steam distribution network in the framework of a recent proposed method, that gives linearization and decoupling, together with a systematic design for nonlinear feedforward control for perfect disturbance rejection.

**Keywords:** steam networks, linearization, decentralized control, feedforward

## 1. Introduction

Steam networks are are used to produce and transfer steam as utility for downstream processes such as distillation column, paper machines, reactors etc. Pressure-flow networks are inherently highly coupled system where a large and fast disturbance such as a shutdown or start-ups of a consumers becomes a large disturbance both on the generation and demand side for the other consumers. In addition, the dynamics of steam generators are much slower compared to the dynamics of the steam network. Therefore, to be able to respond fast to load changes, control of the network pressure is commonly implemented in industry (Majanne, 2005).

Both decentralized and centralized control methods for steam networks are presented in the literature. The work by (Bertrand and Mcavoy, 1986) presents a solution based on PI-controllers that has good performance for disturbance rejection. The work by (Kristoffersen et al., 2014) implements model predictive control (MPC) combined with real time optimization approach to increase the energy efficiency. The work by (Majanne, 2005) compares the performance of PI-controllers and MPC, and the MPC outperforms due to its ability of handling coupled systems.

In this work, we apply the method by (Zotică et al., 2020), which transforms a nonlinear system into a first-order linear decoupled system with no effect from disturbances. In addition, we extend the method by explaining how to select the new introduced tuning parameter. Similar methods have been proposed. Feedback linearization linearizes the input-state map of a nonlinear affine in the inputs system (Isidori et al., 1981; Khalil, 1992). Input-output linearization linearizes only a part of the system (Henson and Seborg, 1997). Active disturbance rejection control introduces an observer to estimate unmeasured states and disturbances (Huang and Xue, 2014). The work by Lee et al. (2016) combines an extended high-gain observer for unmeasured states and uncertainty together with dynamic inversion that inverts the model using a fast inner loop I-controller. All of these methods give a chains of integrators, which brings additional limitations for control, and it is not very robust as in some cases, the input might move in the opposite direction initially.

## 2. New method for input transformation

Figure 1 shows the block diagram for the new proposed method. The key assumptions are that we have the same number of inputs ($u$) and outputs ($y$) and that we can measure the disturbances ($d$). In the example, we will also allow for some measured states ($T_L$, $T_T$) to be treated as measured disturbances. We define a new input ($v$) from the model equations as a function of the physical input ($u$), disturbances ($d$) and output ($y$) with the objective of transforming the original nonlinear system into a linear first-order system. The resulted system is also decoupled and therefore, we use SISO-controllers to control $y$ by using $v$. We find the physical input $u$ by solving as set of nonlinear algebraic equations that give $u$ as a function of $v, y, d$. Consider a general nonlinear system given by Eq. 1.

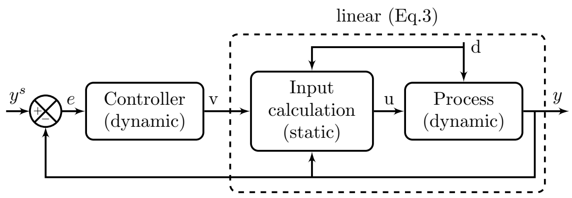



Figure 1: Block diagram for transformed inputs for linearization, decoupling and perfect disturbance rejection

$$\frac{dy}{dt} = f(y,u,d) \tag{1}$$

We define the transformed input $v$ as given in Eq. 2.

$$v = f(y,u,d) - Ay \tag{2}$$

where, $A$ is a new tuning parameter,which we discuss in Section 2.1.

By introducing the input $v$, the new system becomes first-order, linear, decoupled and with no effect from disturbances, as shown in Eq.3.

$$\frac{dy}{dt} = v - Ay \tag{3}$$

### *2.1. New tuning parameter A*

One way to select $A$ is such that nominally the positive feedback from $y$ to $v$ is small. Therefore, for each output $y_i$, we may select $A_i$ as the diagonal elements of the Jacobian of $f_i(y,u,d)$ with respect to the output $y_i$ evaluated at the nominal operating conditions, $A = \text{diag}\left(\frac{\partial f(y,u,d)}{\partial y}\Big|^*\right)$. We may also select a larger A to speed-up the response, or smaller to slow it down. Note that selecting $A = 0$ gives an integrating process similarly to feedback linearization for a model as given in Eq. 1.

### 2.2. Input calculation

The input calculation solves Eq. 2 with respect to the inputs $u$ given controller outputs $v$, outputs $y$ and disturbances $d$. For some processes, we may also require to measure some of the states $x$, see the example in Section 3. If there is no explicit solution of inverting Eq. 2, we may use a numerical algebraic solver, or simply an I-controller in a fast inner loop. The second method can be applied to systems with singularities in the transformation.

### 2.3. Controller tuning

We tune the SISO-controllers based on the SIMC tuning rules in Eq. 4 (Skogestad, 2003).

$$K_c = \frac{1}{k}\frac{\tau}{\tau_C + \theta} = \frac{1}{\tau_C + \theta} \tag{4a}$$

$$\tau_I = \min\left(\tau, 4(\tau_C + \theta)\right) \tag{4b}$$

where, $K_C$ is the proportional gain, $\tau_I$ is the integral time, $k$ is the process gain, $\tau$ is the open-loop time constant, $\theta$ is the delay and $\tau_C$ is the desired closed-loop time constant. Note that from Eq. 3, $k = \tau = \frac{1}{A}$.

## 3. Case study: steam network

Figure 2 shows the system we analyze within the new proposed method. It is composed of a high-pressure header (i.e. pipelines that physically connect the steam generators and consumers), a turbine and a low-pressure header. High-pressure steam is produced at a pressure $p_0$ by burning fuel in a boiler. Note that we do not include the boiler in our analysis. The high-pressure steam is supplied as utility to one high pressure consumer with receiving pressure $p_{HC}$. The remaining steam is expanded to lower pressure steam, either through a fixed-speed back pressure turbine connected to the electric grid to produce electricity or through a valve that bypass the turbine ($z_{TB}$). Note that in this case the fixed-speed turbine is not a degree of freedom available for operation. The low-pressure steam is supplied as utility to two consumers, with receiving pressures $p_{LC1}$ and $p_{LC2}$ respectively.

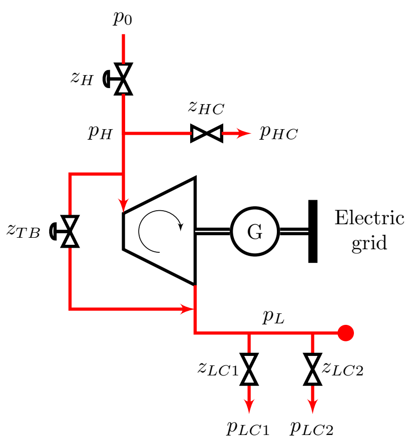



Figure 2: Process flowsheet of the steam network with two pressure headers (high and low) considered in this work.

The manipulated variables are $u = [z_H \; z_{TB}]$ (i.e. the supply of high pressure steam and the turbine bypass). The controlled variables are $y = [p_H \; p_L]$ (i.e. pressure in the high and low pressure headers). The main disturbances are the high pressure steam supply ($p_0$) and the consumers demand of high and low pressure steam, given by changes at the receivers pressure or of the valve positions ($z_{HC}$, $z_{LC1}$ and $z_{LC2}$), i.e. $d = [p_0 \; z_{HC} \; z_{LC1} \; z_{LC2} \; p_{HC} \; p_{LC1} \; p_{LC2}]$.

### *3.1. Nominal operating conditions for the steam network*

Table 1 shows the nominal operating conditions, which are representative of a typical steam network found in a chemical plant. Here, $V$ is the volume of the two headers.

Table 1: Nominal operating conditions

| Variable | $p_0$ | $p_H$ | $p_{HC}$ | $p_L$ | $p_{LC1}$ | $p_{LC2}$ | $z$ | $T_H$ | $T_L$ | $V_H$ | $V_L$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Value | 42 | 40 | 38 | 7 | 6 | 5 | 0.5 | 380 | 200 | 1 | 5 |
| Unit | bar | bar | bar | bar | bar | *bar* | – | °C | °C | m$^3$ | m$^3$ |

### *3.2. Model*

We assume ideal gas, constant specific heat capacity, no pressure losses and perfect mixing in both pressure headers. Assuming isothermal conditions in the high pressure header ($T_H$ constant), the dynamic mass balance in pressure form becomes Eq. 5.

$$\frac{dp_H}{dt} = \frac{RT_H}{V_H}(q_H - q_HC - q_TB - q_T) \stackrel{\text{def}}{=} f_H \tag{5}$$

where $q_j$ is the molar flow though a valve.

The low pressure header is not isothermal because work is extracted in the turbine, and therefore the mass and energy balance become coupled. The energy balance in temperature form is given in Eq.6.

$$\frac{dT_L}{dt} = \frac{RT_L}{V_L p_L}\left(q_{TB}(T_H - T_L) + q_T(T_T - T_L)\right) \tag{6}$$

The mass balance in pressure form is given in Eq. 7.

$$\frac{dp_L}{dt} = \frac{R}{V_L}(q_{TB}T_H + q_T T_T - q_{LC1}T_L - q_{LC2}T_L) \stackrel{\text{def}}{=} f_L \tag{7}$$

We assume isentropic expansion in the turbine and that there are no constraints for the power supplied to the electric grid. Therefore the temperature at the turbine outlet ($T_T$) is computed from Eq.8.

$$T_T = T_H\left(\frac{p_L}{p_H}\right)^{\frac{\gamma-1}{\gamma}} \tag{8}$$

where $\gamma$ is the heat ratio capacity for steam.

To model the molar flows through valves, we use a valve equation with a linear valve characteristic (Eq.9).

$$q_i = C_{v,i} z_i \sqrt{|p_{in}^2 - p_{out}^2|}, \forall i \in (H, HC, TB, LC) \tag{9}$$

where $C_{v,i}$ is the valve coefficient, $z_i$ is the valve opening, $p_{in}$ and $p_{out}$ are the pressures before and after the valve respectively.

To model the molar flow through the turbine, we assume a constant mass flow coefficient ($\phi$), equivalent to a chocked turbine (Eq. 10).

$$q_T = \phi \frac{p_H}{\sqrt{T_H}} \quad (10)$$

### 3.3. Input transformation

The new input $v = [v_H \ v_L]$ is defined by applying Eq. 2 resulting in Eq. 11. We assume that the measurements for $T_H$, $T_L$ and $T_T$ are available.

$$v_i = f_i - A_i p_i \quad \forall i = (H, L) \quad (11)$$

where $A_i = \frac{\partial f_j}{\partial p_i}, \forall i = (H, L)$, evaluated at the nominal conditions from Table 1.

The new system in Eq.12 is linear, decoupled and has perfect disturbance rejection.

$$\frac{dp_i}{dt} = v_i - A_i y_i \quad \forall i = (H, L) \quad (12)$$

### 3.4. Input calculation

We find the unknown variable $u = [z_H \ z_{TB}]$ by solving the system of linear equations (Eq. 13) resulted from rewriting Eq. 11.

$$\begin{bmatrix} z_H \\ z_{TB} \end{bmatrix} = \begin{bmatrix} \alpha & -\beta \\ 0 & \beta \end{bmatrix}^{-1} \begin{bmatrix} \frac{V_H}{RT_H}(v_H + A_H p_H) + q_{HC} + q_T \\ \frac{V_L}{R}(v_L + A_L p_L) - q_T T_T + (q_{LC1} + q_{LC2}) T_L \end{bmatrix} \quad (13)$$

with $\alpha = C_{vH}\sqrt{|p_0^2 - p_H^2|}$, $\beta = C_{vTB} T_H \sqrt{|p_H^2 - p_L^2|}$, the flows $q$ calculated from Eq. 9.

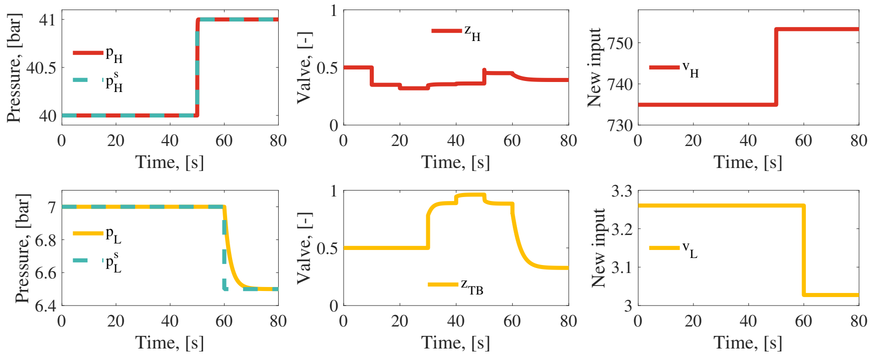
Note that from Eq. 6, $T_L$ depends on $u$, therefore it is not a true disturbance. However, the use of a measured $T_L$ in the input transformation is not a problem in this case because the dynamics from the inputs $u$ to the outputs $y$ have a stable inverse (have no RHP-zeros), hence the inverse generated by the input transformation will be stable.

### 3.5. Simulation results

Figure 3 shows the responses for disturbance rejection and setpoint changes for $y = [p_H \, p_L]$ (Figure 3a), $u = [z_h \, z_{TB}]$ (Figure 3b) and $v = [v_H \, v_L]$ (Figure 3c) to $p_0 = 42$ bar at time $t = 10s$, $p_{HC} = 39$ bar at time $t = 20s$, $p_{LC1} = 5.5$ bar at time $t = 30s$, $p_{LC2} = 1$ bar at time $t = 40s$, $p_H^s = 39$ bar at time $t = 50s$ and $p_L^s = 6$ bar at time $t = 60s$. We tune the PI-controllers with $\tau_C = -\frac{1}{2A}$, which are only used for setpoint changes. The results in Figure 3 show a decoupled process with perfect disturbance rejection.

## 4. Discussion

The calculation block is inherently a nonlinear feedforward controller, and therefore we do not need the feedback control in Figure 1 (typically a PI-controlled) as long as we have a perfect model and measurements. Setpoint changes can be handled by directly changing the setpoint for $v$. However, in a real plant we will always have unmeasured disturbances and unmodelled dynamics, and we need the outer PI-controller loop.




(a) Controlled variables (b) Physical inputs (c) Transformed inputs

Figure 3: Simulation results for disturbance rejection and setpoint changes

## 5. Conclusion

Steam networks are interactive systems, where the main task of the control system is to reject disturbances either on the steam generation or demand side. We design the control structure by using a new method for input transformation that gives decoupling and perfect disturbance rejection both dynamically and at steady-state (Eq. 12), which makes it a good fit for the control structure of a steam network, as seen in Figure 3. The method also transforms a nonlinear system into a first-order linear by introducing a new tuning parameter $A$.

**Acknowledgements:** This publication has been partly funded by HighEFF - Centre for an Energy Efficient and Competitive Industry for the Future, an 8-years' Research Centre under the FME-scheme (Centre for Environment-friendly Energy Research, 257632). The authors gratefully acknowledge the financial support from the Research Council of Norway and user partners of HighEFF.

## References

C. R. Bertrand, T. J. Mcavoy, 1986. Short-Cut Analysis of Pressure Control in Steam Headers. In: 1986 American Control Conference. pp. 1757–1763.

M. A. Henson, D. E. Seborg (Eds.), 1997. Nonlinear Process Control. Prentice-Hall.

Y. Huang, W. Xue, 2014. Active disturbance rejection control : Methodology and theoretical analysis. ISA Transactions 53 (4), 963–976.

A. Isidori, A. J. Krener, C. Gori-Giorgi, M. Salvatore, apr 1981. Nonlinear decoupling via feedback: A differential geometric approach. IEEE Transactions on Automatic Control 26 (2), 331–345.

H. K. Khalil, 1992. Nonlinear systems. Macmillan, New York.

T. T. Kristoffersen, D. Snarheim, L. Imsland, M. S. Govatsmark, 2014. Optimal and Robust Production of High Pressure Steam. In: 2014 European Control Conference. IEEE, Strasbourg, pp. 491–497.

J. Lee, R. Mukherjee, H. K. Khalil, 2016. Output feedback performance recovery in the presence of uncertainties. Systems & Control Letters 90, 31–37.

Y. Majanne, 2005. Model predictive pressure control of steam networks. Control Engineering Practice 13, 1499–1505.

S. Skogestad, 2003. Simple analytic rules for model reduction and PID controller tuning. Journal of Process Control 13 (4), 291–309.

C. Zotică, N. Alsop, S. Skogestad, 2020. Transformed Manipulated Variables for Linearization, Decoupling and Perfect Disturbance Rejection. IFAC-PapersOnLine.