Transformed Manipulated Variables for Perfect Decoupling and Disturbance Rejection

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Outline

Objective

Find new manipulated variables

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- transform a nonlinear process into a first order system

1. Introduction

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- give perfect decoupling

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Related alternatives

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Drawbacks of previous methods

- accurate process inverse $\Rightarrow$ lack of robustness to model uncertainty

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- cannot explicitly handle process constraints

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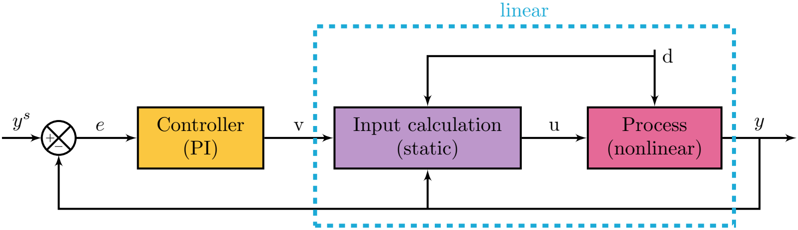
- accurate process inverse $\Rightarrow$ lack of robustness to model uncertainty
- non-robust decoupling control $\Rightarrow$ difficult to extend to MIMO
- cannot explicitly handle process constraints
- state measurement
- only for minimum phase systems $\Rightarrow$ no RHP-zeros and time delay
- give a chain of integrators
- not for static systems

Input transformation

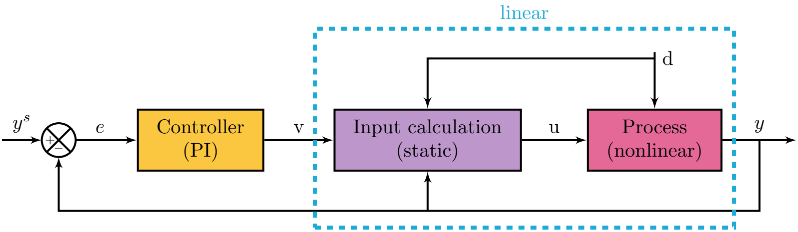


y	$\in \mathbb{R}^{n_y}$	outputs	d	$\in \mathbb{R}^{n_d}$	disturbances
u	$\in \mathbb{R}^{n_u}$	original inputs	e	$\in \mathbb{R}^{n_y}$	error
v	$\in \mathbb{R}^{n_u}$	new transformed inputs	y^s	$\in \mathbb{R}^{n_y}$	setpoint

Assumptions

2. New proposed method

Input transformation



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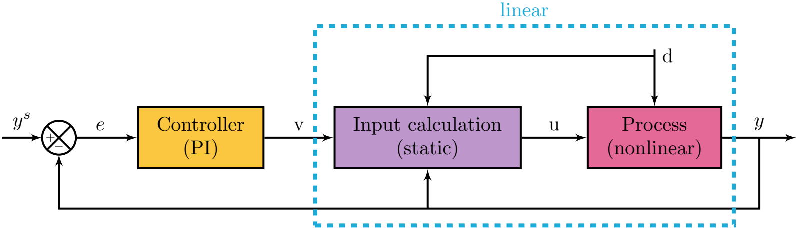
Assumptions

- *as many* outputs (i.e. differential equations) as inputs ($n_y = n_u$)

Input transformation



y	$\in \mathbb{R}^{n_y}$ outputs	d	$\in \mathbb{R}^{n_d}$ disturbances
u	$\in \mathbb{R}^{n_u}$ original inputs	e	$\in \mathbb{R}^{n_y}$ error
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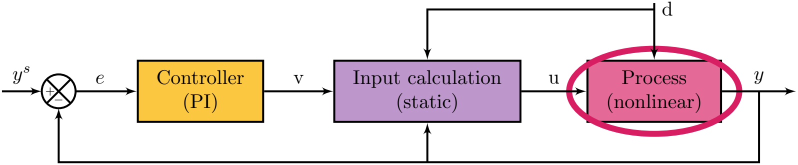
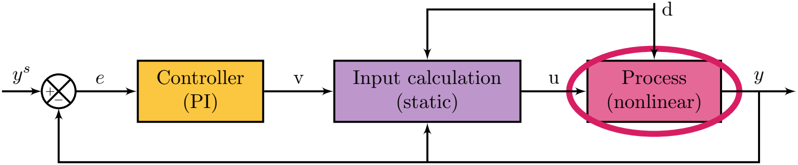
Assumptions

- *as many* outputs (i.e. differential equations) as inputs ($n_y = n_u$)
- all disturbances can be measured

Process



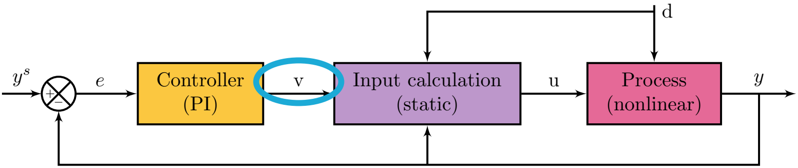
$y \in \mathbb{R}^{n_y}$ (outputs) $u \in \mathbb{R}^{n_u}$ (original inputs) $d \in \mathbb{R}^{n_d}$ (disturbances)



$y \in \mathbb{R}^{n_y}$ (outputs) $u \in \mathbb{R}^{n_u}$ (original inputs) $d \in \mathbb{R}^{n_d}$ (disturbances)

Model: $\frac{dy}{dt} = f(y, u, d)$

Transformed inputs



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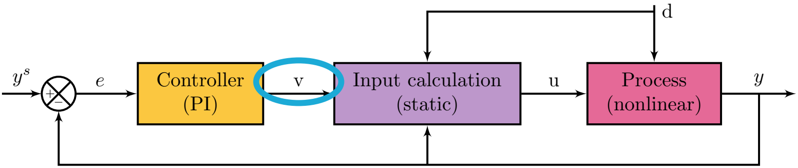

Input Transformations

- simple input transformation (feedback linearization) $\Rightarrow$ integrating system

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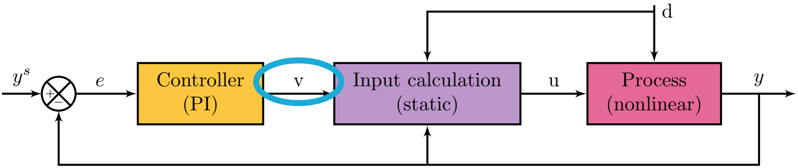

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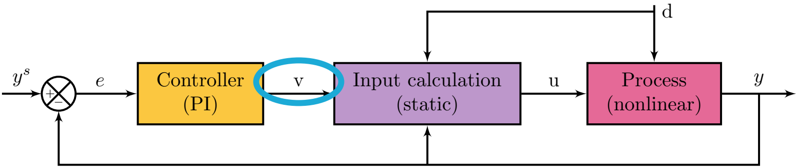

Input Transformations

- simple input transformation (feedback linearization) ⇒ integrating system
- refined input transformation ⇒ first-order system

System: two $MV = [u_1\ u_2]$, two $CV = [y_1\ y_2]$ and a disturbance vector d

Simple input transformation

Model

$$\frac{dy_1}{dt} = f_1'(u_1, u_2, d, y_1, y_2) \quad \frac{dy_2}{dt} = f_2'(u_1, u_2, d, y_1, y_2)$$

Simple input transformation

Model

$$\frac{dy_1}{dt} = f_1'(u_1, u_2, d, y_1, y_2) \quad \frac{dy_2}{dt} = f_2'(u_1, u_2, d, y_1, y_2) \tag{1}$$

Define the new transformed inputs (v') as the RHS

$$v_1' = f_1'(u_1, u_2, d, y_1, y_2)$$
$$v_2' = f_2'(u_1, u_2, d, y_1, y_2)$$

Simple input transformation

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Two decoupled linear integrating systems independent of disturbances.

Simple input transformation

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Two decoupled linear integrating systems independent of disturbances.

- cannot handle static systems (e.g. perfect mixing without accumulation)
- only for integrating systems (e.g. level control)

Refined input transformation

New tuning parameter τ_0 and reformulated model

$$\tau_{01}\frac{dy_1}{dt} + y_1 = f_1(u_1, u_2, d, y_1, y_2)$$
$$\tau_{02}\frac{dy_2}{dt} + y_2 = f_2(u_1, u_2, d, y_1, y_2)$$

Refined input transformation

New tuning parameter τ_0 and reformulated model

$$\tau_{01}\frac{dy_1}{dt} + y_1 = f_1(u_1, u_2, d, y_1, y_2) \tag{2a}$$

$$\tau_{02}\frac{dy_2}{dt} + y_2 = f_2(u_1, u_2, d, y_1, y_2) \tag{2b}$$

Define the new transformed inputs (v) as the RHS

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Define the new transformed inputs (v) as the RHS

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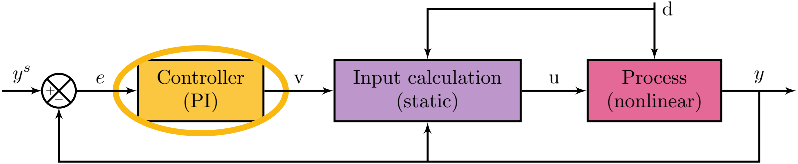
Two decoupled first-order systems independent of disturbances.

τ_0 - free to choose. *intuitively* keep it close to the original system dynamics

Key idea: use decentralized SISO controllers for controlling $y = [y_1\ y_2]$ using $v = [v_1\ v_2]$ as inputs.



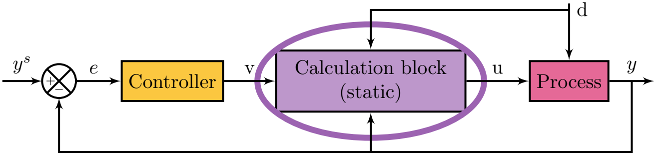
Calculation block

Simple transformation

$\frac{dy}{dt} = v'$
$u = f'^{-1}(v', d, y)$
$\Rightarrow$ implicit nonlinear state feedback

Refined transformation

$\tau_0 \frac{dy}{dt} + y = v$
$u = f^{-1}(v, d, y)$
$\Rightarrow$ avoid implicit nonlinear state feedback if v is independent of y



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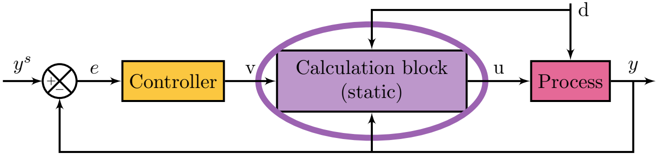
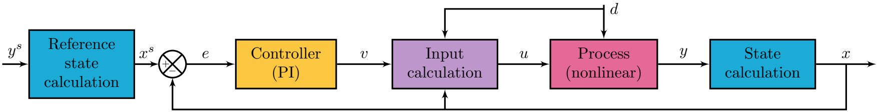


- algebraic solver
- numerical solver
- PI-controller (inner slave loop)

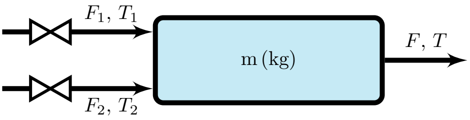
Output Transformation



$y \in \mathbb{R}^{n_y}$ (outputs) $\quad u \in \mathbb{R}^{n_u}$ (original inputs)
$x \in \mathbb{R}^{n_x}$ (states) $\quad d \in \mathbb{R}^{n_d}$ (disturbances)

- measurement nonlinearity (e.g. pH, density)
- static calculation block: $x = h^{-1}(y)$

Control of flow and temperature in a mixing process



MVs (original inputs):

$$u_1 = F_1 \text{ [kg/s]}$$
$$u_2 = F_2 \text{ [kg/s]}$$

CVs (outputs):

$$y_1 = F \text{ [kg/s]}$$
$$y_2 = T \text{ [°C]}$$

DVs (disturbances):

$$d_1 = T_1 \text{ [°C]}$$
$$d_2 = T_2 \text{ [°C]}$$

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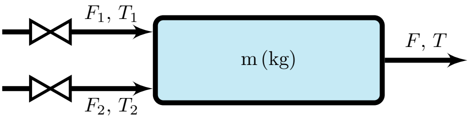

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$$d_1 = T_1 \text{ [°C]}$$
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Objective: find new transformed inputs (new MVs), v_1 and v_2

$\Rightarrow$ decoupling and perfect disturbance rejection.

Mixing process. Transformed inputs

Mass balance (static) $F = \underbrace{F_1 + F_2}_{v_1}$

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Energy balance (dynamic)

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$$m\frac{dT}{dt} = F_1(T_1 - T) + F_2(T2 - T)$$

Mixing process. Transformed inputs

Mass balance (static) $F = \underbrace{F_1 + F_2}_{v_1}$

Energy balance (dynamic)

$$m\frac{dT}{dt} = F_1(T_1 - T) + F_2(T2 - T)$$

$$\tau_0\frac{dT}{dt} + T = \underbrace{\frac{\tau_0}{m}(F_1T_1 + F_2T2) + \left(1 - \frac{\tau_0}{\tau_r}\right)T}_{v_2}$$

$\tau_r = \frac{m}{F_1+F_2}$

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$\tau_r = \frac{m}{F_1+F_2}$

Tuning parameter τ_0

if $\tau_0 = \tau_r \Rightarrow v_2$ is independent of y.

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$\tau_r = \frac{m}{F_1+F_2}$

New system:

$$y_1 = v_1$$

$$\tau_0\frac{dy_2}{dt} + y_2 = v_2$$

Tuning parameter τ_0

if $\tau_0 = \tau_r \Rightarrow v_2$ is independent of y.

Mixing process. Calculation block

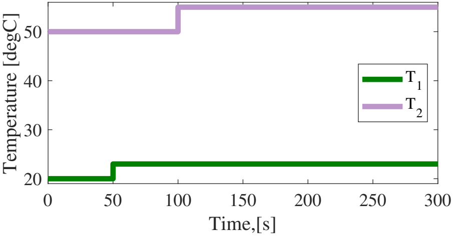
Algebraic solver

Solve for u_1 and u_2:

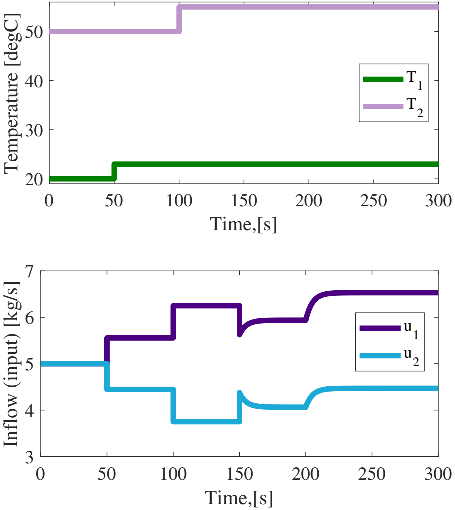
$$v_1 = F_1 + F_2$$
$$v_2 = \frac{\tau_0}{m}(F_1 T_1 + F_2 T_2) + \left(1 - \frac{\tau_0}{\tau_r}\right) T$$

Given inputs v_1 and v_2, outputs $y_1 = F$ and $y_2 = T$, and disturbances $d_1 = T_1$ and $d_2 = T_2$.

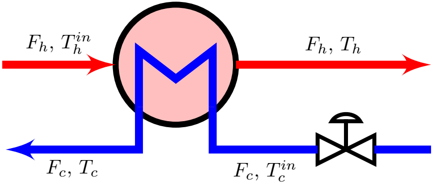
Mixing process. Closed loop responses



Mixing process. Closed loop responses



Control of hot outlet temperature of a heat exchanger



MVs (original inputs):

$$u = F_c \text{ [kg/s]}$$

CVs (outputs):

$$y = T_h \text{ [°C]}$$

DVs (disturbances):

$$d_1 = T_c^{in} \text{ [°C]}$$
$$d_2 = T_h^{in} \text{ [°C]}$$
$$d_3 = F_h \text{ [kg/s]}$$

Heat exchanger. Input transformation

Objective: find transformed input (new MV), v
$\Rightarrow$ perfect disturbance rejection at steady-state.

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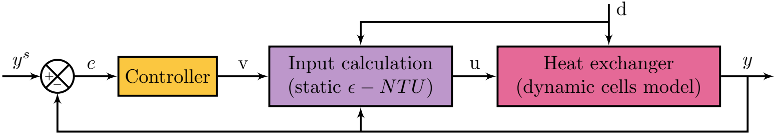

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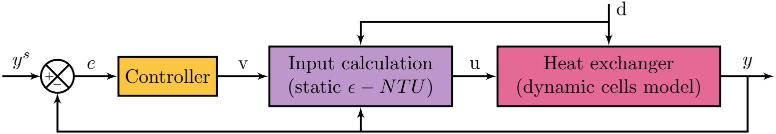

Energy balance (static $\epsilon - NTU$)

$$T_h = \underbrace{(1-\epsilon_h)T_h^{in} + \epsilon_h T_c^{in}}_{v}$$

with $\epsilon_h = \epsilon_h(u, d_1, d_2, d_3)$

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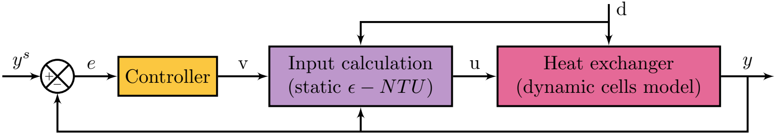

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with $\epsilon_h = \epsilon_h(u, d_1, d_2, d_3)$

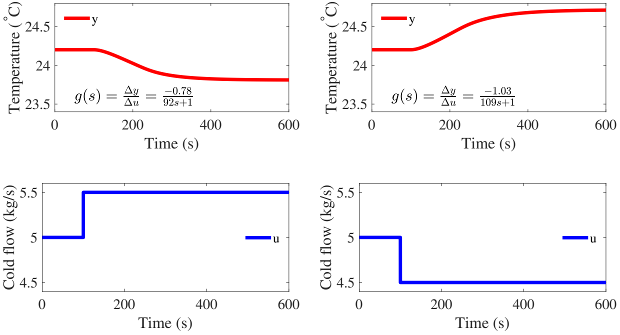
New system: $y = v$

Tuning parameter: $\tau_0 = 0$

Consider actual dynamics with a static transformation and dynamic process

Heat exchanger. Open loop responses

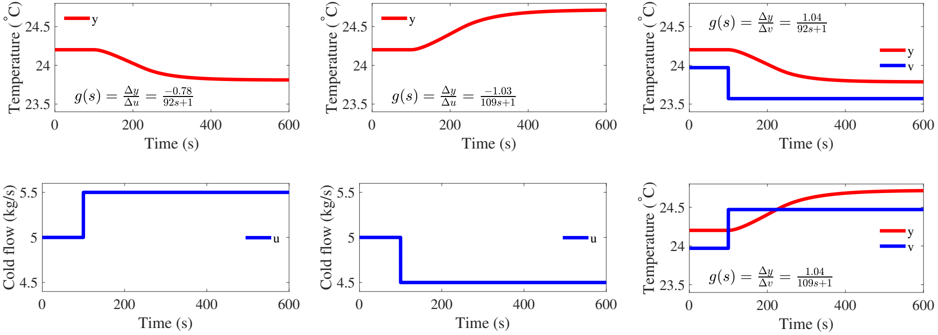
Original system



Heat exchanger. Open loop responses

Original system

Transformed system



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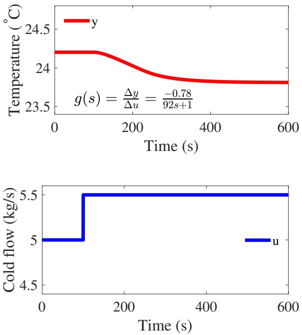
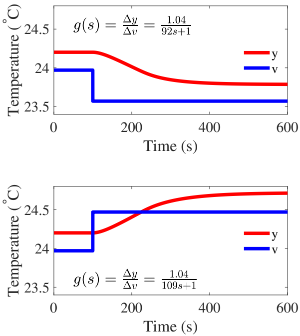
Original system



Different steady-state gain

Transformed system



Same steady-state gain

Heat exchanger. Controller. Calculation block

3 different controllers implemented

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Heat exchanger. Controller. Calculation block

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1. feedback only with a PI-controller ($K_C = -1.32$, $\tau_I = 109$);
2. transformed and feedback with a PI-controller ($K_C = 1.31$, $\tau_I = 109$);
3. transformed only (feedforward only).

Calculation block. Numerical solver

Solve for u

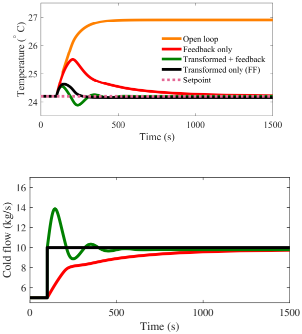
$$v = (1 - \epsilon_h) T_h^{in} + \epsilon_h T_c^{in}$$

with

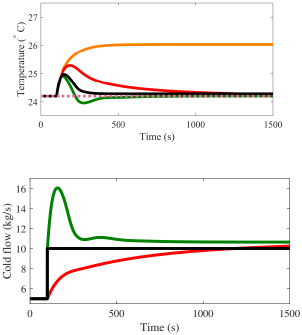
$$\epsilon_h = \epsilon_h(u, d_1, d_2, d_3)$$

Given input v, and disturbance $d_1 = T_c^{in}$,$d_2 = T_h^{in}$,$d_3 = F_h$.

Disturbance rejection



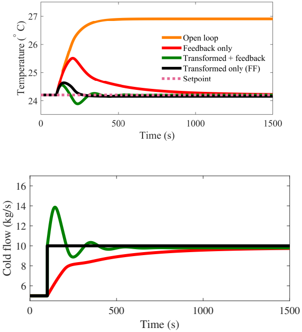
$\Delta d_3 = 0.6\,\mathrm{kg/s}$



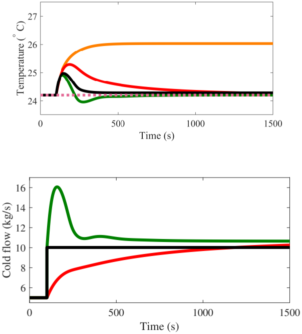
$\Delta d_1 = 2\,^\circ\mathrm{C}$

Heat exchanger. Closed loop responses

Disturbance rejection

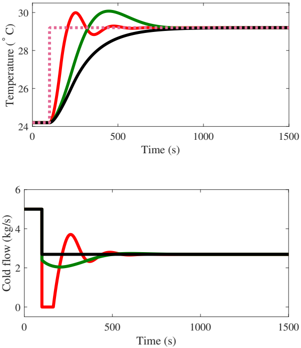


$$\Delta d_3 = 0.6\,\mathrm{kg/s}$$



$$\Delta d_1 = 2\,^{\circ}\mathrm{C}$$

Setpoint change



$$\Delta T_h^s = 5\,^{\circ}\mathrm{C}$$

4. Conclusions

Transformed inputs

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- feedforward disturbance rejection
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- generalized theory for input and output transformations

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