Transformed Manipulated Variables for Linearization, Decoupling and Perfect Disturbance Rejection

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Abstract: The objective of this work is to find new transformed manipulated variables (MVs) for nonlinear systems which linearize and decouple the system, and give perfect disturbance rejection (at least at steady-state). The proposed new input transformation is more general than feedback linearization in that it also allows for multiple-inputs multiple-outputs (MIMO) systems, disturbances, a more general class of models, and introduces a tuning parameter τ_0. The key idea is to use decentralized SISO controllers for the output y using the new transformed inputs v as MVs. The SISO controllers give v, and a nonlinear calculation block solves algebraic equations which explicitly gives the original input u as a function of the controller output v, output y and disturbances d. The calculation block also handles decoupling, and feedforward action from the disturbance d. This new procedure can be applied both for static and dynamic processes, which is typical in process control.

Keywords: linearization, decentralized control, decoupling problems, process control, disturbance rejection

1. INTRODUCTION

Different techniques for controlling nonlinear systems have been presented in the literature, including: linear controllers designed using a linearized model around an operating point (e.g. PID controllers, linear model predictive control etc.); adaptive control (Åström and Wittenmark, 2008); nonlinear model predictive control (Rawlings et al., 2017) or nonlinear control (e.g. feedback linearization (Isidori, 1989; Khalil, 2015; Nijmeijer and van der Schaft, 1990a), input-output linearization (Henson and Seborg, 1997), disturbance decoupling (Huijberts et al., 1991), input decoupling (Isidori et al., 1981; Balchen et al., 1988; Nijmeijer and van der Schaft, 1990b) elementary nonlinear decoupling (Balchen, 1998) etc.).

The objective of this work is to find new manipulated variables that transform a nonlinear process into a linear one (preferably first order), give decoupling and perfect disturbance rejection. The literature presents a few approaches with similar objectives (often with different names) though with different methodologies, and we discuss a few of them. *Feedback linearization* received a large interest in the control literature starting with the differential geometry approach introduced by the work of Isidori et al. (1981) and peaking in the work of Isidori (1989), Nijmeijer and van der Schaft (1990a) and Khalil (2015). A comprehensive overview and analysis for nonlinear process control is presented in the work of Henson and Seborg (1991). However, to the best of the authors' knowledge this methods have not been practically implemented for chemical processes. We will explain briefly why they are not used for chemical processes.

Feedback linearization works by transforming a nonlinear $n'th$ order system into a new system described by a chain of r (i.e. relative degree with $r \leq n$) integrators, therefore linear and controllable.

Input-output linearization is another nonlinear technique similar to feedback linearization, but applied to systems for which the state-inputs equations cannot be linearized (usually with $r < n$ and RHP-zero). It partly linearizes the system, that is, it linearizes the output-input behaviour, while keeping some nonlinear state-input equations (Isidori, 1989; Henson and Seborg, 1997). Feedback linearization is more suitable for stabilization purposes, whereas input-output linearization can be applied for systems for which the output is specified *a priori* which makes it more appropriate for process control applications (Henson and Seborg, 1997).

The main limitations of these linearization methods are:

- lack of robustness to model uncertainty as it requires an accurate process inverse;
- difficult to extend to multivariate systems as it needs a type of non-robust decoupling control;

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- cannot explicitly handle process constraints;
- all the states must be available for measurement, or can be estimated;
- inability to deal with uncertainty in RHP-zeros and time delays.

These limitations may be acceptable for mechanical systems which inherently have few states that can be easily measured or estimated. Moreover, mechanical system are often integrating processes, and thus transforming them into a chain of integrators does not necessarily bring additional control limitations. However, this is rarely the case for most process control applications, and this is arguably the reason feedback linearization is yet to be implemented in chemical processes.

Elementary nonlinear decoupling on the other hand, generates a directly invertible system based on designing of a property transformation of the state x and generating an input u such that the property transformation has the desired rate of change. For systems with relative degree one, it turns a $n'th$ order system into one linear integrator (Balchen, 1998).

2. METHODOLOGY

The principle of our proposed method is shown in the block diagram in Fig. 1, where, y is the outputs vector, u is the original inputs vector (manipulated variables MVs), v is the new inputs (transformed MVs) vector, d is the disturbance vector, e is error vector, y^s is the setpoint vector.

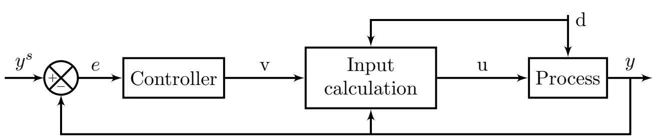


Fig. 1. Proposed method for linearization, decoupling and perfect disturbance rejection.

In Fig. 1, a decentralized PI-controller computes the transformed input v, and the original input u is back-calculated by numerically solving a set of algebraic equations with given outputs y and disturbances d. We discuss the structure of each block in Fig.1 in the following.

2.1 Assumptions

- *as many* outputs (i.e. differential equations) as inputs (i.e. $n_y = n_u$);
- all disturbances can be measured.

Thus, we can handle low-order systems, but this is often the case in process control applications. We present two cases:

(1) Simple input transformation (that gives an integrating process, similar to feedback linearization)
(2) Refined input transformation (that introduces a new tuning parameter τ_0 to give a first-order process).

2.2 Simple input transformation

Assume that we can write the nonlinear dynamic model as shown in Eq. 1 (for simplicity, we consider two controlled variables (CVs), y_1 and y_2, two MVs, u_1 and u_2, and a disturbance vector d without compromising the generality of the method).

$$\frac{dy_1}{dt} = f_1'(u_1, u_2, d, y_1, y_2) \tag{1a}$$

$$\frac{dy_2}{dt} = f_2'(u_1, u_2, d, y_1, y_2) \tag{1b}$$

We follow the idea of the classical nonlinear control method of feedback linearization, and introduce two new transformed input variables (v_1' and v_2') in Eq. 2 (input functions) which simply are the right hand side of the differential Eq. 1.

$$v_1' = f_1'(u_1, u_2, d, y_1, y_2) \tag{2a}$$

$$v_2' = f_2'(u_1, u_2, d, y_1, y_2) \tag{2b}$$

We then have two decoupled linear integrating systems, Eq. 3, which also are independent of disturbances.

$$\frac{dy_1}{dt} = v_1' \tag{3a}$$

$$\frac{dy_2}{dt} = v_2' \tag{3b}$$

With v' as the controller outputs (or transformed inputs to the process), this is a linear decoupled system for which controller design in principle is straightforward. We assume that d is measured, so that the physical input u can be back-calculated from v' using a calculation block. However, Eq. 3 is a set of integrating systems, and integrating systems are not easy to control.

Limitations. The above approach cannot handle static systems. More generally, it will not work well for cases where the original dynamics are very fast, because we are replacing any dynamics by an integrating system by introducing an implicit feedback through the variable transformation in Eq. 2. In general, integrating systems are difficult to control, so the transformation used in feedback linearization may introduce unnecessary limitations. As mentioned below, we will use it for integrating processes only.

2.3 Refined input transformation

Because of the mentioned limitations of the simple input transformation, we rewrite the model Eq. 1 slightly and introduce the new tuning parameter τ_0. The reason is to transform the process into a *first-order* system instead of an integrating one. To do this, we assume that we can write the nonlinear model with the outputs (CVs) separated from the other variables as follows (for simplicity we consider two CVs, y_1 and y_2, two MVs, u_1 and u_2, and a disturbance vector d, without reducing the generality of the method):

$$\tau_{01}\frac{dy_1}{dt} + y_1 = f_1(u_1, u_2, d, y_1, y_2) \tag{4a}$$

$$\tau_{02}\frac{dy_2}{dt} + y_2 = f_2(u_1, u_2, d, y_1, y_2) \tag{4b}$$

Comparing Eq. 4 with Eq. 1, we see that $f_1 = \tau_{01}f_1' + y_1$ and $f_2 = \tau_{02}f_2' + y_2$. We introduce two new transformed input variables (input functions) as the right hand side of Eq. 4, yielding Eq. 5.

$$v_1 = f_1(u_1, u_2, d, y_1, y_2) \tag{5a}$$

$$v_2 = f_2(u_1, u_2, d, y_1, y_2) \tag{5b}$$

where we assume that d is measured.

We then have two decoupled linear systems, both first-order and independent of disturbances, as shown in Eq. 6.

$$\tau_{01}\frac{dy_1}{dt} + y_1 = v_1 \tag{6a}$$

$$\tau_{02}\frac{dy_2}{dt} + y_2 = v_2 \tag{6b}$$

2.4 Controller design

The key idea is now to use decentralized SISO controllers (Eq. 7) for controlling $y = [y_1\ y_2]^T$ using $v = [v_1\ v_2]^T$ as MVs.

$$u(t) = K_c\left(e(t) + \frac{1}{\tau_I}\int_0^t e(t)dt\right) \tag{7}$$

where K_C is the proportional gain and τ_I is the integral time. To tune the PI-controller we may use a systematic tuning method such as the SIMC tuning rules (Skogestad, 2003). For a transformed system in the form of Eq. 6, we calculate the tuning parameters for a PI-controller with Eq. 8.

$$K_C = \frac{1}{k}\frac{\tau}{\tau_C + \theta} = \frac{\tau_0}{\tau_C + \theta} \tag{8a}$$

$$\tau_I = \min(\tau_0, 4(\tau_C + \theta)) \tag{8b}$$

where, for the transformed system, $k = 1$ (process gain), $\tau = \tau_0$ (time constant) and θ is the time delay.

For static systems (i.e. $\tau_0 = 0$ in Eq. 6) we use a pure I-controller given by Eq. 9.

$$K_I = \lim_{\tau\to 0}\frac{K_C}{\tau_I} = \frac{1}{k}\frac{1}{\tau_C + \theta} = \frac{1}{\tau_C + \theta} \tag{9}$$

where K_I is the integral gain.

2.5 Calculation block

For finding the actual u from v, we use a static calculation block that inverts Eq. 5, resulting in Eq. 10, where we assume f^{-1} exists. When we write the equations in the form Eq. 4 rather than Eq. 1, we will quite often find that v in Eq. 5 is independent of the outputs y, at least nominally. Thus, solving Eq. 5 with respect to u, frequently avoids the implicit nonlinear static feedback resulting from solving Eq. 2 with respect to u.

$$u = f^{-1}(v, y, d) \tag{10}$$

Eq. 10 can be solved algebraically (i.e. explicitly) or numerically (i.e. by using a numeric solver or by using a fast inner loop PI-controller).

2.6 New tuning parameter τ_0

The time constants τ_0 are tuning parameters. The question now is how should we choose them? One way of selecting them is to keep them close to the original systems dynamics to minimize the implicit feedback from the output y to the new input v. For a static model equation we have that the time constant is zero, e.g. $\tau_{01} = 0$, and $y_1 = v_1$. Note that this means that the output from the controller (v_1) is equal to the CV (y_1). On the other hand, a pure integrating system, such as a liquid level, would correspond to an infinite time constant and we write the model on the original form in Eq. 3, that is, $dy_1/dt = v_1'$ with $v_1' = f_1'$.

2.7 Comparison with feedback linearization

As indicated, the approach in Eq. 1 and Eq. 2, where we introduce new inputs v' and end up with an integrating system (Eq. 3), is closely related to the classical nonlinear control method of feedback linearization which considers SISO systems of the form (here written for the case with one differential equation, that is, $n = 1$):

$$\frac{dy}{dt} = f(y) + g(y)u \tag{11}$$

With $n = 1$ the new input is $v = f(y) + g(y)u$ and we end up with an integrating system $dy/dt = v$. This is identical to Eq. 1,2 and 3, except that we in Eq. 1 allow for a more general right hand side than in Eq. 11. The proposed new input transformation is more general than feedback linearization in that it also allows for MIMO systems, disturbances, a more general class of models and introduces a tuning parameter τ_0. The limitation with the new approach compared to feedback linearization is that we must have a low-order system (with as many number of inputs u as number of outputs y).

2.8 Output transformation

The principle of both input and output transformation is illustrated in Fig. 2. For some chemical processes, the nonlinearities arise from state-measurement relationship, e.g. pH, or density measurement. Therefore, we may want to introduce an output transformation, which is also a static calculation block. In addition, the structure (i.e. algebraic equations) of the input calculation block is fixed from the start and an output transformation block may make the system more robust. For such systems, we may want to introduce new outputs transformation that linearizes the input (v)- state (x) behaviour, where the choice of x is a degree of freedom.

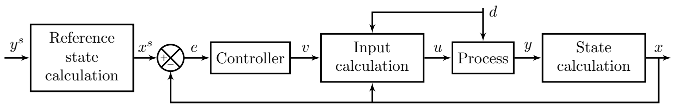


Fig. 2. Input and output transformation for linearization, decoupling and perfect disturbance rejection.

3. SIMULATION CASE STUDIES

We apply the proposed method from Section 2 to two simulation examples:

(1) Control of flow and temperature in a mixing process with both slow temperature and fast mass dynamics;
(2) Control of the hot stream temperature of a heat exchanger using a static model to derive the input transformation and construct the calculation block, and a different dynamic lumped model for the real process (plant model mismatch).

3.1 Case 1. Control of flow and temperature in a mixing process

Fig. 3 shows the mixing process with two inflows and one outflow that we are analyzing.

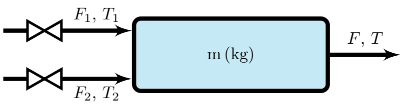


Fig. 3. Mixing process with two original MVs ($u_1 = F_1$ and $u_2 = F_2$) and two CVs ($y_1 = F$ and $y_2 = T$).

The original inputs of the process are the two inlet flows: $u_1 = F_1$ (kg/s); $u_2 = F_2$ (kg/s). The outputs are the outlet flow F and temperature T: $y_1 = F$ (kg/s) $y_2 = T$ (°C). The main disturbances that we consider are the temperature of the two inlet flows: $d_1 = T_1$ (°C); $d_2 = T_2$ (°C).

Following the procedure from Section 2, we find the transformed inputs v_1 and v_2 with the objective of decoupling and perfect disturbance rejection.

Assuming constant m holdup, and fast mixing, the mass balance (static) is given by Eq. 12.

$$F = F_1 + F_2 \tag{12}$$

Assuming constant and equal heat capacity c_P, and after substituting the mass balance (Eq. 12), the dynamic energy balance can be rearranged as given by Eq. 13.

$$m\frac{dT}{dt} = F_1(T_1 - T) + F_2(T_2 - T) \tag{13}$$

Introducing the new tuning parameter τ_0 gives Eq. 14 (similar with Eq. 4).

$$\tau_0\frac{dT}{dt} + T = \frac{\tau_0}{m}(F_1T_1 + F_2T_2) + \big(1 - \frac{\tau_0}{\tau_r}\big)T \tag{14}$$

where $\tau_r = \frac{m}{F_1+F_2}$ is the residence time (s).

We define the transformed inputs v_1 and v_2 as the right hand side of Eq. 12 and Eq.14 as shown in Eq. 15.

$$v_1 = F_1 + F_2 \tag{15a}$$

$$v_2 = \frac{\tau_0}{m}(F_1T_1 + F_2T2) + \left(1 - \frac{\tau_0}{\tau_r}\right)T \tag{15b}$$

Note that if $\tau_0 = \tau_r^*$, then v_2 is independent of $y_2 = T$, at least nominally.

With the new transformed input, v_1 and v_2, the new system is given by Eq. 16, which represents two decoupled processes with no effect from disturbances.

$$y_1 = v_1 \tag{16a}$$

$$\tau_0\frac{dy_2}{dt} + y_2 = v_2 \tag{16b}$$

In transfer function matrix form, from input $v = [v_1 \quad v_2]^T$ and disturbances $d = [d_1 \quad d_2]^T$ to outputs $y = [F \quad T]^T$, the system can be rewritten as given in Eq. 17.

$$y(s) = G(s)v(s) + G_d(s)d(s) \tag{17}$$

with

$$G(s) = \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{\tau_0 s+1} \end{pmatrix}, \quad G_d(s) = 0$$

Calculation block. Algebraic solver The calculation block solves Eq. 15 for u_1 and u_2 given inputs v_1 and v_2, outputs $y_1 = F$ and $y_2 = T$ and disturbances d. In this case, this a linear system, Eq. 18. We select the new tuning parameter equal to the nominal residence time, i.e. $\tau_0 = \tau_r^* = 10\,\text{s}$.

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ \frac{\tau_0}{m}(T_1 - T) & \frac{\tau_0}{m}(T_2 - T) \end{bmatrix}^{-1} \begin{bmatrix} v_1 \\ v_2 - T \end{bmatrix} \tag{18}$$

Note that the matrix inverted in Eq. 18 loses rank when $T_1 = T_2$. However, physically, the temperature control of the mixing is clearly not possible when both inlet stream have equal temperature.

Simulation results Table 1 shows the nominal operating conditions (marked with $*$) for the mixing process. Note that at nominal conditions the two inputs are equal ($F_1^* = F_2^*$), which makes the process highly coupled and difficult to control using conventional PID-controllers.

Table 1. Case 1 nominal operating conditions

Variable	F_1^*	F_2^*	F^*	T_1^*	T_2^*	T^*	m
Value	5	5	10	20	50	35	100
Unit	kg/s	kg/s	kg/s	°C	°C	°C	kg

We use an I-controller for controlling $y_1 = F$ using v_1 with $K_I = 0.2$, and a PI-controller for controlling $y_2 = T$ using v_2 with $K_C = 4$ and $K_I = 0.2$. We use $\tau_C = 5\,\text{s}$ for both controllers.

Fig. 4 shows the closed-loop response for $y_1 = F$, $u_1 = F_1$ and $u_2 = F_2$ (left) and $y_2 = T$ (right) to a step increase in disturbance $d_1 = T_1$ of $3\,°C$ at time 50 s, and a step increase in disturbance $d_2 = T_2$ of $5\,°C$ at time 100 s, followed by a setpoint change in $y_2^s = T^s$ of $1\,°C$ at time 150 s and a setpoint change in $y_1^s = F^s$ of $1\,\text{kg/s}$ at time 200 s.

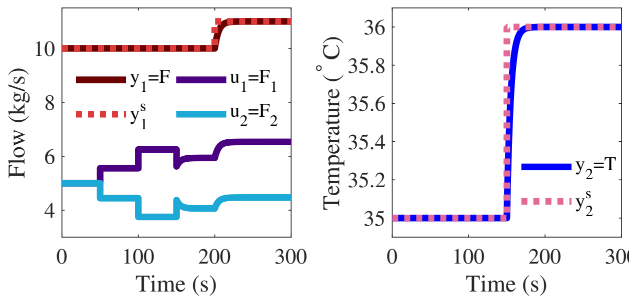


Fig. 4. Closed-loop response for $y_1 = F$ (dark red), $y_2 = T$ (dark blue) and $u_1 = F_1$ (purple) and $u_2 = F_2$ (light blue) to disturbance step changes of $\Delta d_1 = 3°C$ at time 50 s and $\Delta d_2 = 5°C$ at time 100 s followed by setpoint changes of $\Delta y_2^s = 1\,°C$ at time 150 s and $\Delta y_1^s = 1\,\text{kg/s}$ at time 200 s.

The simulation results in Fig. 4 show a decoupled process with perfect disturbance rejection (dynamically and at steady-state).

3.2 Case 2. Control of hot outlet temperature of a heat exchanger

Simple and accurate steady-state models of chemical processes are often available, whereas simple dynamic models are either not available, or are not good enough. For this reason, we analyze within our proposed method a countercurrent heat exchanger example for which a simple and good model is only available at steady-state (ϵ-NTU method).

Fig. 5 shows the countercurrent heat exchanger that we are analyzing, with one original input (cold side flow, $u = F_c$) and one output (hot side outlet temperature, $y = T_h$).

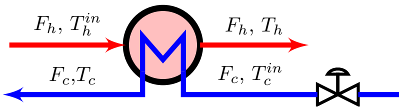


Fig. 5. Heat exchanger with one original MV ($u = F_c$) and one CV ($y = T_h$).

The original input of the process is the cold side flow:$u = F_c$ (kg/s).The process output is the hot side outlet temperature T_h: $y = T_h$ (°C). The main disturbances are the hot side flow and the inlet temperatures of the two flows: $d_1 = T_c^{in}$ (°C); $d_2 = T_h^{in}$ (°C); $d_3 = F_h$ (kg/s).

Fig. 6 shows the block diagram of the proposed method applied to the heat exchanger. The static model (ϵ-NTU method) is used to derive the new input transformation, which is then applied to a dynamic lumped cells model.

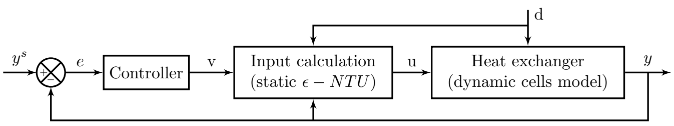


Fig. 6. Block diagram of the proposed method for the heat exchanger using a static model for the input calculation block, and a dynamic model for the process.

Following the procedure from Section 2, we find the transform inputs v with the objective of linearization and perfect disturbance rejection. We use a static model ϵ-NTU (number of transfer units) (Welty, 2008), Eq. 19.

$$T_h = (1 - \epsilon_h)T_h^{in} + \epsilon_h T_c^{in} \tag{19a}$$

$$T_c = e_c T_h^{in} + (1 - e_c)T_c^{in} \tag{19b}$$

$$\epsilon_c = \frac{1 - exp(-NTU(C-1))}{C - exp(-NTU(C-1))} \tag{19c}$$

$$\epsilon_h = \epsilon_c C \tag{19d}$$

$$C = \frac{F_c c_{p_c}}{F_h c_{p_h}} \tag{19e}$$

$$NTU = \frac{UA}{F_c c_{p_c}} \tag{19f}$$

We define the transformed inputs v as the right hand side of Eq. 19a resulting in Eq. 20.

$$v = (1 - \epsilon_h)T_h^{in} + \epsilon_h T_c^{in} \tag{20}$$

That is, at steady-state we have $y = v$, but this does not hold dynamically because the system is not static.

Calculation block. Algebraic solver The calculation block numerically solves Eq. 20 for u given inputs v and disturbances d. Note that we are using $v = y = T_h$ in the calculation block, and select the new tuning parameter to $\tau_0 = 0$ because we use a static heat exchanger map to derive the transformed input.

Dynamic cells heat exchanger model The process is given by the dynamic lumped model given in Eq. (21), where the heat exchanger is discretized in space in $N = 100$ cells. The boundary conditions for cell $i = 1$ is $T_h^0 = T_h^{in}$, and for cell $i = N$ is $T_c^{N+1} = T_c^{in}$. With infinite cells, the dynamic model has the same steady-state as the static model. Wall capacities are neglected.

$$\frac{dT_c^i}{dt} = \frac{F_c}{\rho_c V_c^i}(T_c^{i+1} - T_c^i) + \frac{UA(T_h^i - T_c^i)}{N\rho_c V_c^i c_{p_c}} \tag{21a}$$

$$\frac{dT_h^i}{dt} = \frac{F_h}{\rho_h V_h^i}(T_h^{i-1} - T_h^i) + \frac{UA(T_h^i - T_c^i)}{N\rho_h V_h^i c_{p_h}} \tag{21b}$$

$$\forall i \in 1 \dots N$$

where c is the cold side, h is the hot side, V is the volume, U is the heat transfer coefficient, A is the heat transfer area, ρ is density and c_p is specific heat.

The nominal operating conditions (marked with $*$) are shown in Table 2.

Table 2. Case 2 nominal operating conditions

Variable	F_c^*	F_h^*	T_h^s	T_c^{in*}	T_h^{in*}	U	A	V
Value	5	5	24.2	20	70	150	90	0.45
Unit	kg/s	kg/s	°C	°C	°C	$\frac{W}{m^2 °C}$	m^2	m^3

Open loop responses The open loop responses for the original and transformed systems are shown in Figs. 7 and 8 respectively, together with fitted first-order plus time delay transfer functions.

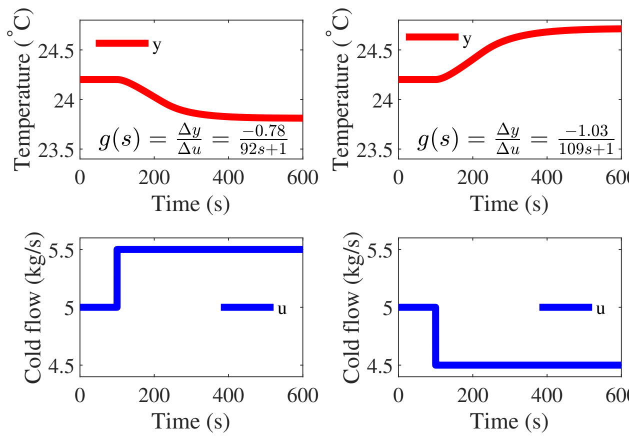


Fig. 7. Open loop response for $y = T_h$ to a step change in the physical input of $\Delta u = 0.5$ kg/s (left) and $\Delta u = -0.5$ kg/s (right) at time 100 s.

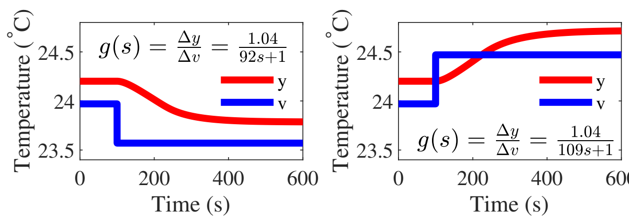
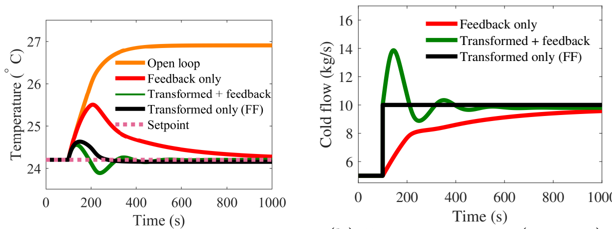


Fig. 8. Open loop response for $y = T_h$ to a step change in the transformed input of $\Delta v = -0.4\,°C$ (left) and $\Delta v = 0.5\,°C$(right) at time 100 s.

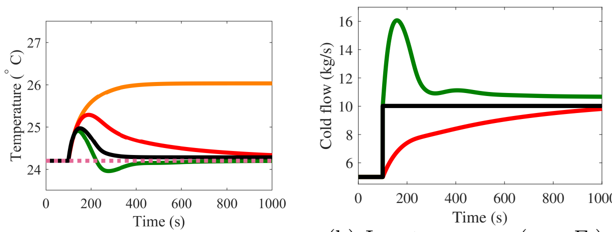
For the transformed system the steady-state gains in both directions are equal, whereas for the original system they are not. If the models used for the transformation and dynamic simulation were identical, we would have (1) a process gain of 1 (rather than 1.04) and (2) $y = v$ at steady state (so the read and blue lines should start and end at the same value in Fig. 8).

Closed loop responses We compare three controllers: (1) feedback only with a PI-controller tuned based on the open loop response from Fig. 7 (right), with a closed loop time constant $\tau_C = \theta = 40\,\mathrm{s}$, $K_C = -1.32$ and $\tau_I = 109$; (2) transformed and feedback with a PI-controller tuned based on the open loop response from Fig. 8, with a closed loop time constant $\tau_C = \theta = 40\,\mathrm{s}$, $K_C = 1.31$ and $\tau_I = 109$; (3) transformed only (feedforward only). Fig. 9 and Fig. 10 show the disturbance rejection response for $y = T_h$ and $u = F_c$ to a step change $\Delta d_3 = 0.6\,\mathrm{kg/s}$ at time 100 s and $\Delta d_1 = 2\,°C$ respectively. Fig. 11 shows the response for $y = T_h$ and $u = F_c$ to a step change in the setpoint of $\Delta T_h^s = 5\,°C$ at time 100 s.



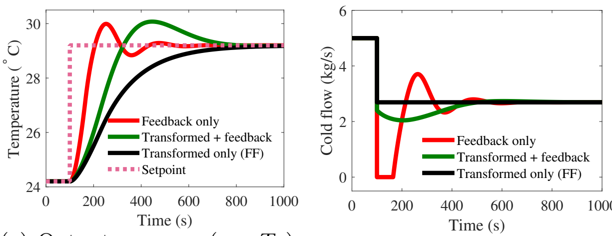
(a) Output response ($y = T_h$) (b) Input response ($u = F_c$)

Fig. 9. Disturbance rejection: $\Delta d_3 = 0.6\,\mathrm{kg/s}$ at time 100 s.



(a) Output response ($y = T_h$) (b) Input response ($u = F_c$)

Fig. 10. Disturbance rejection: $\Delta d_1 = 2\,°C$ at time 100 s.



(a) Output response ($y = T_h$) (b) Input response ($u = F_c$)

Fig. 11. Setpoint change of $\Delta T_h^s = 5\,°C$ at time 100 s.

The response for disturbance rejection (Figs. 9 and 10) for transformed + feedback (green) is not perfect dynamically because we apply a static transformation to a dynamic process and perfect disturbance rejection can only happen at steady-state, yet it is better than feedback only (red) and transformed only (black). The steady-state offset for transformed only is due to the process gain mismatch (Fig. 8). The overshoot from Figs. 9(a) and 10(a) is caused by the feedforward and feedback parts independently correcting at the same time. The nonlinearity in the input transformation makes the response for setpoint change (Fig. 11) appear less aggressive for the transformed system. It may seem that the feedback controller is not very helpful when we use transformed variables, but it is needed to handle measurement errors for the disturbances, and to handle differences between the model and the real system, both nominally (which we had in this case, although it was quite small) and due to changes over time, for example, due to heat exchanger fouling which changes the value of the parameter U.

4. CONCLUSION

The main contribution is the introduction of the new tuning parameter τ_0 that transforms a general nonlinear process into a first order system (Eq. 4) instead of an integrating system as in feedback linearization (Eq. 1). The method also gives decoupling, perfect disturbance rejection (at least at steady-state), and can be applied for pure static processes. It can be applied when the original system is dynamic, yet only a static model is available, in which case the transformed system (given by $y = v$ at steady state, but not dynamically) is linear, decoupled and independent of disturbances only at steady state.

REFERENCES

Balchen, J.G. (1998). *Elementary Nonlinear Decoupling (END), A General Approach to Model Based Control of Nonlinear Multivariable Processes*, 275–310. Springer Netherlands, Dordrecht.

Balchen, J.G., Lie, B., and Solberg, I. (1988). Internal Decoupling in Non-Linear Process Control. *Modeling, Identification and Control*, 9(9), 137–148.

Henson, M.A. and Seborg, D.E. (1991). Critique of exact linearization strategies for process control. *Journal of Process Control*, 1(3), 122–139.

Henson, M.A. and Seborg, D.E. (eds.) (1997). *Nonlinear Process Control*. Prentice-Hall.

Huijberts, H.J.C., Nijmeijer, H., and van der Wegen, L.L.M. (1991). *Dynamic Disturbance Decoupling for Nonlinear Systems: The Nonsquare and Noninvertible Case*, 243–252. Birkhäuser Boston, Boston, MA.

Isidori, A. (1989). *Nonlinear Control Systems*. Springer-Verlag Berlin Heidelberg GmbH, second edition.

Isidori, A., Krener, A.J., Gori-Giorgi, C., and Salvatore, M. (1981). Nonlinear decoupling via feedback: A differential geometric approach. *IEEE Transactions on Automatic Control*, 26(2), 331–345.

Khalil, H.K. (2015). *Nonlinear systems*. Pearson, Harlow, England, third edition.

Nijmeijer, H. and van der Schaft, A. (1990a). *Feedback Linearization of Nonlinear Systems*, 161–192. Springer New York, New York, NY.

Nijmeijer, H. and van der Schaft, A. (1990b). *The Input-Output Decoupling Problem*, 223–250. Springer New York, New York, NY.

Rawlings, J.B., Mayne, D.Q., and Diehl, M.M. (2017). *Model Predictive Control: Theory, Computation, and Design*. Nob Hill Publishing, second edition.

Skogestad, S. (2003). Simple analytic rules for model reduction and PID controller tuning. *Journal of Process Control*, 13(4), 291–309.

Åström, K.J. and Wittenmark, B. (2008). *Adaptive Control*. Dover Publications, New York, second edition.

Welty, J.R. (2008). *Fundamentals of momentum, heat and mass transfer*. Wiley, Hoboken, N.J, 5th edition.