Simple method for parameter identification of a nonlinear Greitzer compressor model ⋆

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Abstract: This work introduces a simple method to estimate state variables and identify parameters of a nonlinear dynamic Greitzer compressor model. The observer is based upon an Extended Kalman Filter, which estimates the dynamic states as well as a subset of parameters. In a Monte-Carlo-fashioned approach, the remaining set of parameters are then identified by minimizing an objective function representing the error between the measured variables and their estimates. The developments are demonstrated in numerical simulations.

Keywords: Parameter identification; estimation; observability

1. INTRODUCTION

The problem of combined state estimation and parameter identification in dynamic systems can be solved by many methodologies. These include particle filters, but also the widely utilized augmented (extended) Kalman filter. Nevertheless, this work should also be seen in the context of the wide field of system identification (Ljung, 1999). In many engineering applications it is often not possible to determine the values of model parameters exactly. This holds for models that have been derived from first principles, but also for empirically derived models. Furthermore, parameters may change over operating time of a technical system. Hence, determination of these parameters is crucial to obtain an accurate representation of the system in model-based controllers and observers.

There are many publications, which address the topic of combined state and parameter estimation and identification. Pure state estimation for linear dynamic systems is mostly solved by applying Luenberger-style observers and Kalman filters. For nonlinear systems, the principles of Luenberger observers and Kalman filters can be utilized by steady linearization of the system dynamics around the present operating point. Furthermore, extensive research efforts have led to the development of nonlinear state-observers solving the problem for specific classes of nonlinear systems (Arcak and Kokotovic, 2001; Rajamani, 1998; Raghavan and Hedrick, 1994; Esfandiari and Khalil, 1992). For all of the aforementioned methods, observability or at least detectability must be ensured, may it be in the linear case as introduced by Kalman or the nonlinear case, as e.g. described in Marino and Tomei (1995).

In an early work on the topic of combined state and parameter estimation, Cox (1964) describes a real-time approach for discrete-time linear systems. In Schön (2006), combined state and parameter estimation for differential-algebraic equations is presented. The author utilizes marginal particle filters and sequential Monte-Carlo-simulations to solve the estimation problem. An algorithm for linear systems in observer canonical form, where parameters denote entries in the system matrix A and the input matrix B is presented in Ding (2014).

Parameter identification and estimation for nonlinear systems is often solved my means of least-squares comparison of measured data and model candidates. Development of frameworks for parameter estimation in nonlinear systems by synchronization of measured data and a candidate model is introduced in Creveling (2008) and Abarbanel et al. (2009). Instead of using least squares, the authors utilize conditional Lyapunov exponents as a measure to determine the coupling between data and model. Rao et al. (2008) present a real-time algorithm for simultaneous estimation of parameters and identification of anomaly patterns using escort probabilities. A least-squares support vector machine approach for parameter identification of dynamical system is introduced in Mehrkanoon et al. (2012). The authors claim that the method is applicable for both time invariant and time-varying dynamics.

A standard way for combined state and parameter estimation is the implementation of extended Kalman filters by augmenting the dynamics of the original system with parameter dynamics. Again, observability has to be ensured, now for the augmented system dynamics. In this paper, we present a simple approach for combined state estimation and parameter identification for nonlinear systems in the case if this augmentation of the dynamic system leads to an unobservable system. The drawback of the method's

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simplicity is the rather high computational cost, since one simulation has to be conducted for each parameter combination. However, the big advantage is that the method can be applied offline, if measurements are available that satisfy properties of persistence of excitation. Without this property, it might not be possible to reconstruct parameters from the measurement data at all.

The remainder of this paper is structured as follows: In Section 2 we introduce the methodology. Section 3 presents the case study, where at first the mathematical model is introduced, followed by simulation results. The paper is concluded in Section 4.

2. METHODOLOGY

The combined estimation of state variables and parameters utilizing augmented (extended) Kalman filters is quite mature and can be considered a standard way to solve problems of this kind. However, it only works for systems that provide full observability after introducing the parameters as new state variables and for given output functions. In some cases it might be enough to rely on detectability, i.e. that all unobservable modes are stable. In the following we only consider full observability as a criterion.

We consider state space models in the form

$$\dot{x} = f(x, u) + w(t),$$
$$y = h(x) + v(t),$$

where we assume that $h(x)$ is a linear function that can be expressed in the well-known linear output relation Cx with output matrix C. Furthermore, $w(t)$ and $v(t)$ represent process and measurement noises, with their covariance matrices $Q = \text{cov}\{w(t)w^T(t)\}$ and $R = \text{cov}\{v(t)v^T(t)\}$. No covariance is considered between $w(t)$ and $v(t)$.

In order to apply an EKF, the nonlinear state equations $\dot{\hat{x}} = f(\hat{x}, u)$ are linearized in order to obtain the Jacobian

$$A = \left.\frac{\partial f_i}{\partial \hat{x}_j}\right|_{\hat{x}_s}. \tag{1}$$

Thereby it is important to keep the linearization points $\hat{x}_s$ variable, such that the Jacobian can be updated subject to the current operating point of the process, hence $A = A(\hat{x}_s)$. This state- and hence time-varying Jacobian is then used to solve the differential matrix Riccati equation and calculate the time-varying Kalman feedback gain $K(t)$

$$\dot{P}(t) = A(t)P(t) + P(t)A^T(t) - K(t)CP(t) + Q,$$
$$K(t) = P(t)C^T R^{-1}$$

to track the real process and estimate the unmeasurable state variables. In case of parameter estimation, the parameter dynamics $\dot{\hat{p}}$ are included as new state variables in a quasi-stationary fashion

$$\dot{\hat{x}} = f(\hat{x}, \hat{p}, u),$$
$$\dot{\hat{p}} = 0.$$

This implies that the Jacobian (1) will increase from initially $A_{(n\times n)}$ to $A_{((n+\rho)\times(n+\rho))}$, where n is the dimension of the system and ρ represents the number of parameters. In addition, the output matrix will have to become adapted accordingly. This is typically done by adding zeros to the respective entries in the new state variables $\hat{p}$, since these are not measurable, hence $C_{ext} = \left[C \; 0_{(n\times\rho)}\right]$. Ultimately, the observer dynamics for the augmented system with $\hat{x}_{ext} = [\hat{x} \; \hat{p}]^T$ are given as

$$\dot{\hat{x}}_{ext} = f_{ext}(\hat{x}_{ext}, u) + K(t)(y - C_{ext}\hat{x}_{ext}). \tag{2}$$

2.1 Observability

Observability is defined as a measure indicating the reconstruction of unmeasurable internal states of a system by only using external outputs. It must thereby hold that the well-known observability matrix linking the output matrix C and the system matrix A has full rank. In addition, there exist observability measures particularly for nonlinear systems, as e.g. stated in Marino and Tomei (1995). It is likely that systems initially providing full observability might lose this property by adding the full set of parameters $\mathcal{P}$ as additional state variables. Nevertheless, by only adding a subset $\mathcal{P}_s \in \mathcal{P}$, the full observability property might still hold. It is essential to test for the maximum allowable subset $\overline{\mathcal{P}_s}$ for which full observability is still ensured. Full observability should hold for all admissible and feasible operating points $\hat{x}_s$, meaning that even in the case of a state- and hence time-varying Jacobian the observability matrix should still have full rank.

2.2 Determination of unobservable parameters

Determination of the real values of the unobservable parameters $\hat{p}_{u_i} \in \mathcal{P}_u = \mathcal{P} \setminus \overline{\mathcal{P}_s}$ is achieved in a Monte-Carlo-fashioned approach. By defining bounds for feasible parameter values $\underline{\hat{p}_{u_i}}$ and $\overline{\hat{p}_{u_i}}$ and increments between these bounds, we run simulations for all feasible parameter combinations. Ultimately, by minimizing an objective function, one can find the real values for the unobservable parameters. Since y is the only information available from the real plant, a feasible objective function that can be used in this context is the difference between the measurements and their respective estimates, and hence

$$\underset{\mathcal{P}_u}{\text{minimize}} \quad J = \|y - C_{ext}\hat{x}_{ext}\|_1 = \sum_{t=0}^{t_{sim}} |y - C_{ext}\hat{x}_{ext}| \tag{3}$$

$$\text{subject to} \quad (2),$$
$$\underline{\hat{p}_{u_i}} \le \hat{p}_{u_i} \le \overline{\hat{p}_{u_i}},$$

where we chose the 1-norm as objective function in (3), since it provides steeper gradients for small estimation errors compared to e.g. the 2-norm. However, other norms, combinations of norms and functions of norms might be applicable too, depending on the system at hand.

In order to estimate and identify parameters, some models require persistent excitation until the parameters have converged. This is often achieved by sinusoidal reference signals. Other systems naturally provide persistent excitation since some operating points lead to limit cycle oscillations of state variables. In this work we only consider systems that have cyclic and oscillatory response, either caused by the dynamics themselves or sinusoidal excitation.

2.3 Algorithm

Algorithm 1 is held very basic and generically describes the single steps that are required to run the simulations.

Algorithm 1 Identification Algorithm

1: Initialization
 - Design the extended Kalman filter (EKF)
 - Find $\overline{\mathcal{P}_s}$ for which the model is still fully observable
 - Define lower as well as upper bounds and increments between these bounds for all $\hat{p}_{u_i} \in \mathcal{P}_u$
 - Choose tuning matrices Q and R for the EKF
 - If necessary: Pre-filter noisy measurements

2: Loop
 - Define one *for-loop* for each $\hat{p}_{u_i} \in \mathcal{P}_u$
 - Run one simulation for all each feasible combination within $\mathcal{P}_u$
 - Compute the value of the objective function (3) and stack values

3: Evaluation
 - Find parameter combination that gives the smallest norm → candidates for real parameter values
 - Re-run simulation with minimizing parameter combination

4: Verification
 - Oscillations in estimates of the EKF? → Parameter combination most likely not optimal
 - Change bounds and / or increments for $\hat{p}_{u_i}$
 - Re-tune EKF and / or potentially change the objective function
 - Continue at step 2

3. CASE STUDY

Here, we present a case study for which the methodology proposed in Section 2 delivers satisfactory results, even in the presence of added measurement noise. This is demonstrated with numerical simulations.

3.1 Mathematical Model

We use a two-state Greitzer compressor model in combination with a close-coupled valve (CCV). A nonlinear state-estimation approach for the model has been described in Backi et al. (2013). The following dimensionless equations hold for the compressor with the CCV, where the state variables have been transformed to the origin as equilibrium point (Gravdahl and Egeland, 1997)

$$\begin{aligned} \dot{\psi} &= \frac{1}{B}\left(\phi - \Phi\left(\psi\right)\right), \\ \dot{\phi} &= B\left(\Psi_c\left(\phi\right) - \psi - u\right). \end{aligned} \tag{4}$$

The input u denotes the pressure drop across the CCV,

$$\Phi\left(\psi\right) = \gamma\left(\operatorname{sgn}(\psi+\psi_0)\sqrt{|\psi+\psi_0|} - \sqrt{\psi_0}\right)$$

describes the throttle characteristics and

$$\Psi_c\left(\phi\right) = -k_3\phi^3 - k_2\phi^2 - k_1\phi,$$

indicates the compressor characteristics.

The variable ϕ represents the non-dimensional mass flow $\left(\phi = \frac{\dot{m}}{\rho\, U A_c}\right)$, whereas ψ defines the non-dimensional pressure $\left(\psi = \frac{p}{0.5\,\rho\,U^2}\right)$. Note furthermore that $\operatorname{sgn}(0) = 0$, that time has been normalized by the Helmholtz frequency, thus $\tau = t\,\omega_H$, and that the pressure drop across the CCV typically only provides positive pressure differences, meaning that $u \geq 0$. This is based on the fact that the CCV should be fully opened in the equilibrium point. However, it can be operated as an initially throttled valve, which will lower the performance of the overall compression system.

For the parameters, it holds that $B = \frac{U}{2a_s}\sqrt{\frac{V_p}{A_c L_c}} > 0$, where U is the compressor blade tip speed, a_s is the speed of sound, V_p is the plenum volume, A_c is the flow area and L_c is the length of ducts and compressor. It holds furthermore that $k_1 = \frac{3H\phi_0}{2W^2}\left(\frac{\phi_0}{W} - 2\right)$, $k_2 = \frac{3H}{2W^2}\left(\frac{\phi_0}{W} - 1\right)$ and $k_3 = \frac{H}{2W^3}$ with $H > 0$, $W > 0$ and $\phi_0 > 0$.

The operating point ψ_0 can be calculated via the compressor characteristics in initial coordinates (before transformation to the origin)

$$\psi_0(\phi_0) = \psi_{0_c} + H\left[1 + \frac{3}{2}\left(\frac{\phi_0}{W} - 1\right) - \frac{1}{2}\left(\frac{\phi_0}{W} - 1\right)^3\right] \tag{5}$$

and the throttle gain via $\gamma = \frac{\phi_0}{\sqrt{\psi_0}}$. Be advised that certain operating points (ψ_0, ϕ_0) lead to a limit cycle (surge).

We set $\psi = x_1$, $\phi = x_2$ and express the parameters k_1, k_2 and k_3 in terms of H and W, reducing the number of parameters by one. Ultimately, the system (4) can be rewritten as

$$\begin{aligned} \dot{x}_1 &= \frac{1}{B}\left[x_2 - \gamma\left(\operatorname{sgn}(x_1+\psi_0)\sqrt{|x_1+\psi_0|} - \sqrt{\psi_0}\right)\right], \\ \dot{x}_2 &= B\Bigg[-\frac{H}{2W^3}\,x_2^3 - \frac{3H}{2W^2}\left(\frac{\phi_0}{W} - 1\right)x_2^2 \\ &\qquad - \frac{3H\phi_0}{2W^2}\left(\frac{\phi_0}{W} - 2\right)x_2 - x_1 - u\Bigg], \end{aligned} \tag{6}$$

where ψ_0 is defined in (5). The only state assumed measurable is the pressure, hence $y = x_1$ and $C = [1\ 0]$.

3.2 Observer Design

The observer will now be designed subject to the model (6). The parameters that are assumed unknown or uncertain are H, W and B. Hence the extended observer dynamics are

$$\begin{aligned} \dot{\hat{x}}_1 &= \frac{1}{\hat{B}}\left[\hat{x}_2 - \gamma\left(\operatorname{sgn}(\hat{x}_1+\hat{\psi}_0)\sqrt{\left|\hat{x}_1+\hat{\psi}_0\right|} - \sqrt{\hat{\psi}_0}\right)\right], \\ \dot{\hat{x}}_2 &= \hat{B}\Bigg[-\frac{\hat{H}}{2\hat{W}^3}\,\hat{x}_2^3 - \frac{3\hat{H}}{2\hat{W}^2}\left(\frac{\phi_0}{\hat{W}} - 1\right)\hat{x}_2^2 \\ &\qquad - \frac{3\hat{H}\phi_0}{2\hat{W}^2}\left(\frac{\phi_0}{\hat{W}} - 2\right)\hat{x}_2 - \hat{x}_1 - u\Bigg], \\ \dot{\hat{H}} &= 0, \quad \dot{\hat{W}} = 0, \quad \dot{\hat{B}} = 0, \end{aligned} \tag{7}$$

where $\hat{\psi}_0$ is defined in (5), but now with the estimates $\hat{H}$ and $\hat{W}$. The Jacobian for the observer dynamics (7) is

$$A = \left.\begin{bmatrix} -\dfrac{\gamma\,\operatorname{sgn}(\hat{x}_1+\hat{\psi}_0)}{2\hat{B}\sqrt{\left|\hat{x}_1+\hat{\psi}_0\right|}} & \dfrac{1}{\hat{B}} & A_{13} & A_{14} & A_{15} \\ -\hat{B} & A_{22} & A_{23} & A_{24} & A_{25} \\ & & \mathbf{0}_{(3\times5)} & & \end{bmatrix}\right|_{\hat{x}_s}, \tag{8}$$

where the entries A_{ij} are listed in Appendix A for completeness. Checking the observability condition, we see that the above system is not fully observable for the full set $\mathcal{P} = \{\hat{H}, \hat{W}, \hat{B}\}$, yet by defining $\overline{\mathcal{P}_s} = \{\hat{H}\}$, hence $\mathcal{P}_u = \{\hat{W}, \hat{B}\}$, we gain full observability with $C_{ext} = [1\ 0\ 0]$. The Jacobian (8) reduces to the upper left 3–by–3 matrix.

Nevertheless, the Jacobian (8) allows for a pre-investigation of the parameters $\hat{W}$ and $\hat{B}$. As pointed out before, $\hat{W}$ is not allowed to be zero, the same holds for $\hat{B}$. Both can be inferred from (A.1)–(A.2). Investigating the square-roots in the first two equations in (A.1), we see that the radicand

$$\psi_{0_c} + \hat{H}\hat{\alpha} \qquad (9)$$

must be positive. Solving for $\hat{W}$, we can find a lower bound for $\hat{W}$ as a function of ψ_{0_c} and ϕ_0, which both are assumed to be known, and ultimately $\hat{H}$, which is estimated by the EKF. The radicand of the other square-root $\left|\hat{x}_1 + \psi_{0_c} + \hat{H}\hat{\alpha}\right|$ is the absolute value of the radicand (9) plus the estimated state variable $\hat{x}_1$. Unfortunately, this radicand can become zero in the case if $-\hat{x}_1$ equals (9). However, knowing the range of the measured variable x_1 and assuming that its estimate $\hat{x}_1$ converges fairly fast to the measured value, one can still obtain a bound for $\hat{W}$ in the same fashion as for (9).

3.3 Simulations

All simulations were performed in open-loop, meaning $u = 0$, with a fixed-step solver of step size 0.01 in MATLAB Simulink. Although the problem is formulated in continuous-time, evaluation of performance was conducted in quasi-discrete-time since the system was discretized in a consistent way using the fixed-step solver. The oscillations in the state variables x_1 and x_2 are due to the operating point in the surge area.

No added noise: As an illustrative example, we present a noise-free case. As can be seen in Figure 1, the ranges of the parameters W and B can be narrowed down successfully until a near optimum is reached. Figure 2 presents the estimates of the two state variables and the parameter H for the obtained optimal values of W and B, where the errors between real value and estimate go to zero.

Added noise to the measurement of x_1: In this simulation case, we added noise to the measurement, which was band-limited white noise with a sample time of 0.01, a noise power of 10^{-5} dB and a seed of [23341]. Figure 3 demonstrates that the ranges of W and B can be narrowed down successfully. However, the distance to the optimal point is larger than for the noise-free case, in particular for B, since W can be obtained correctly. Figure 4 shows the estimates for x_1, x_2 and H for the obtained optimal values of W and B. As can be seen, estimation is not perfect since there exist small oscillations, especially for $\hat{H}$, which is an indicator for not having found the best values for W and / or B.

Filtered measurement: To demonstrate that pre-filtering of the noisy measurement is advisable, we implemented a moving average filter for the noisy measurement introduced above. The filter utilized the respective 30 previous and subsequent measurement points to calculate the average. Comparing Figure 5 to the simulations in Section 3.3.2, it can be seen that the optimal values for W and B are much closer to the real values. Figure 6 shows the estimation of states and H, which are still in an acceptable range, even for the poor identified value of B. In Figure 7 we demonstrate the difference between the noisy measurement, its filtered representation and the estimate. Furthermore, the parts of the noise that could not be filtered are shown in the bottom plot.

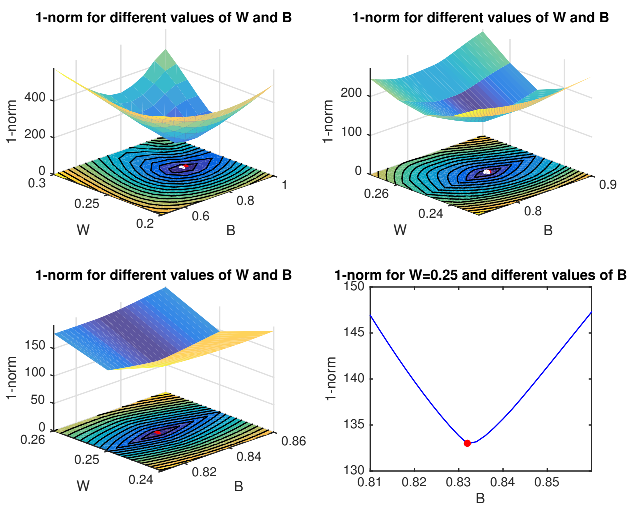

Fig. 1. Simulations for different ranges of W and B. The red dots display the obtained minima, whereas the white / black dots display the real value.

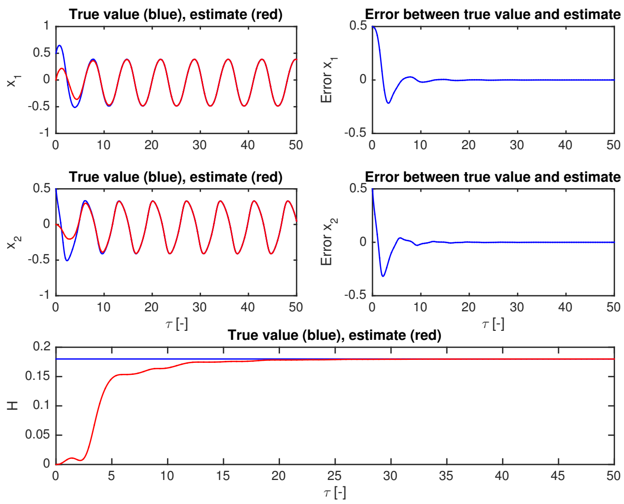

Fig. 2. Real (blue) and estimated (red) states x_1, x_2 and H for the best obtained minima of W and B.

4. DISCUSSION

We presented a method for combined state and parameter estimation and identification for general nonlinear systems. The basic idea is to utilize an extended Kalman filter in combination with a minimizing criterion to identify unknown parameter values.

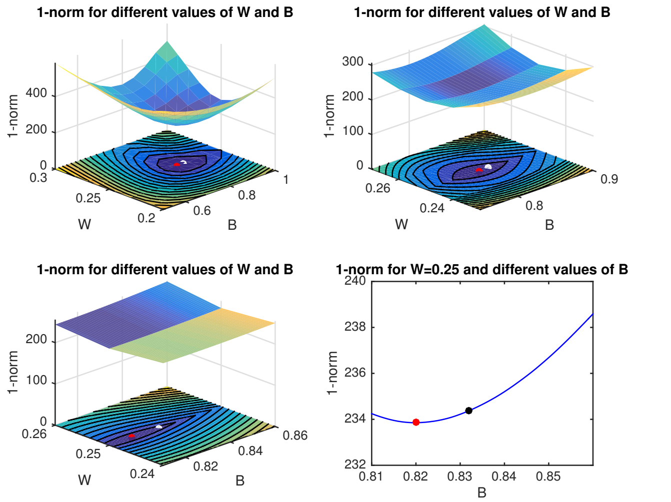

Fig. 3. Simulations for different ranges of W and B. The red dots display the obtained minima, whereas the white / black dots display the real value.

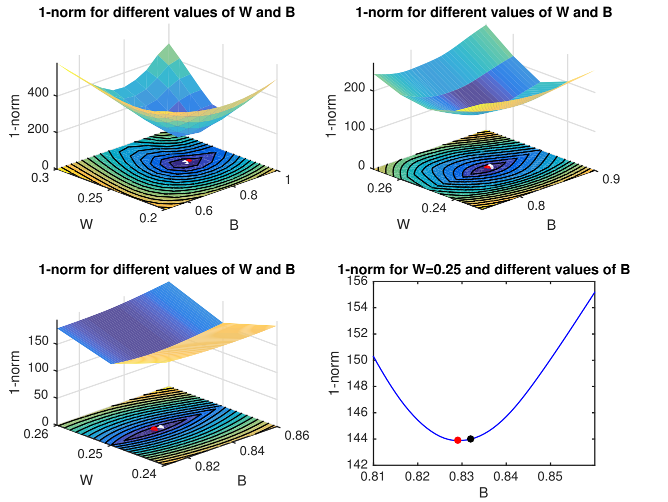

Fig. 5. Simulations for different ranges of W and B. The red dots display the obtained minima, whereas the white / black dots display the real value.

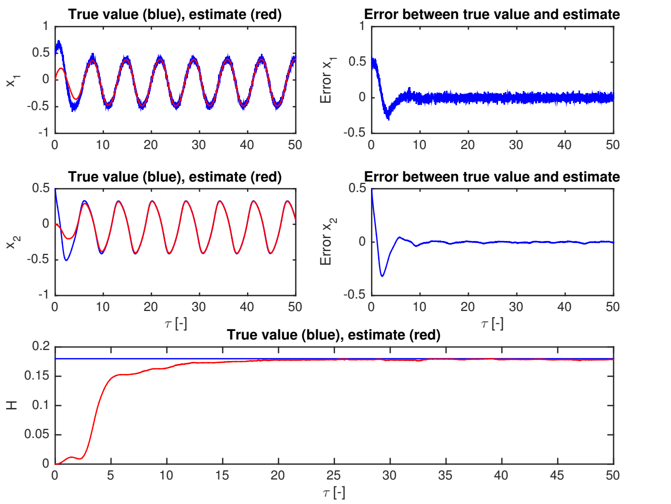

Fig. 4. Real (blue) and estimated (red) states x_1, x_2 and H for the best obtained minima of W and B for added measurement noise.

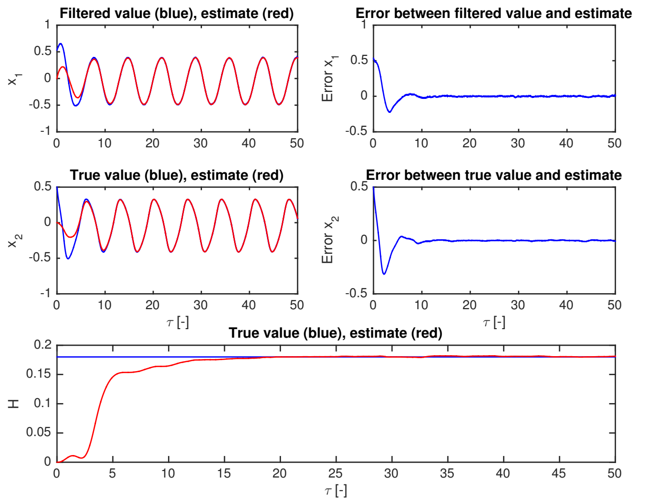

Fig. 6. Real (blue) and estimated (red) states x_1, x_2 and H for the best obtained minima of W and B for the filtered measurement signal.

From the simulation results it can be inferred that pre-processing of the measurement signals is crucial in order to increase performance of the proposed method. In particular, filtering the signal to remove high-frequency parts of the noise increases performance. In this work, we used a moving average filter, for which the number of included data points should be carefully chosen; for large numbers of data points the amplitude of the filtered signal will not reach that of the original signal. This holds for many filters that could be implemented here, such as standard low-pass filters or the Wiener filter. However, unlike e.g. low-pass filters, moving average filters do not necessarily introduce delay to the signal.

The tuning of the EKF has a large impact on the convergence time and hence on the required simulation time in general. In order to obtain good estimates of the unobservable parameters $\hat{p}_u$, a certain minimum simulation time is needed. However, if the proposed method is used in an offline manner with measurements of limited duration, fast convergence of the EKF is crucial.

The method can also be used if the set $\mathcal{P}_s$ is empty, and hence no (parameter) states are added to the EKF. Nevertheless, the identification of parameters in a dynamic way is computationally more efficient and reduces the overall computational cost, since not the full set of parameters $\mathcal{P}$ has to be identified in a Monte-Carlo fashion. Therefore, the EKF should always utilize the maximum allowable set of parameters $\overline{\mathcal{P}_s}$ in its formulation.

Like for all dynamic systems, persistence of excitation is crucial to enable the identification of model parameters. In the case study presented in this paper, this was not an issue since the operating point of the compression system was chosen in the surge area.

Future work includes a generalization of the obtained results to systems that are not inherently marginally

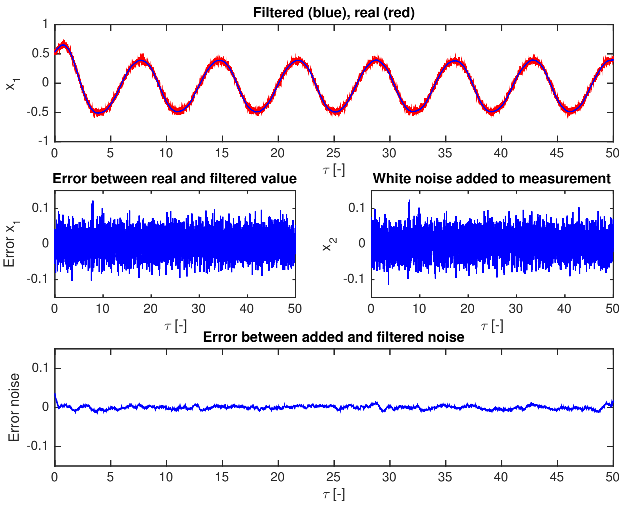

Fig. 7. Real and filtered state x_1 and residual noise

stable, but are excited by e.g. sinusoidal reference signals. In addition, the authors are working on improvements for the algorithm, since the EKF will still give a reasonably good estimate of the measured state even if the parameters are not correctly estimated, but in a close region around the real values. Hence, it is possible that a non-optimal estimate might give a lower objective function value than the real value.

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Appendix A. JACOBIAN ENTRIES

$$A_{13} = -\frac{\gamma}{2\hat{B}}\left[\frac{\operatorname{sgn}\left(\hat{x}_1+\psi_{0_c}+\hat{H}\hat{\alpha}\right)\hat{H}\hat{\alpha}}{\sqrt{\left|\hat{x}_1+\psi_{0_c}+\hat{H}\hat{\alpha}\right|}} - \frac{\hat{\alpha}}{\sqrt{\psi_{0_c}+\hat{H}\hat{\alpha}}}\right],$$

$$A_{14} = -\frac{\gamma}{2\hat{B}}\left[\frac{\operatorname{sgn}\left(\hat{x}_1+\psi_{0_c}+\hat{H}\hat{\alpha}\right)\hat{H}\hat{\beta}}{\sqrt{\left|\hat{x}_1+\psi_{0_c}+\hat{H}\hat{\alpha}\right|}} - \frac{\hat{H}\hat{\beta}}{\sqrt{\psi_{0_c}+\hat{H}\hat{\alpha}}}\right],$$

$$A_{15} = -\frac{1}{\hat{B}^2}\left[\hat{x}_2 - \gamma\left(\operatorname{sgn}(\hat{x}_1+\hat{\psi}_0)\sqrt{\left|\hat{x}_1+\hat{\psi}_0\right|} - \sqrt{\hat{\psi}_0}\right)\right],$$

$$A_{22} = \hat{B}\left[-\frac{3\hat{H}\hat{x}_2^2}{2\hat{W}^3} - \frac{3\hat{H}\hat{x}_2}{2\hat{W}^2}\left(\frac{\phi_0}{\hat{W}}-1\right) - \frac{3\hat{H}\phi_0}{2\hat{W}^2}\left(\frac{\phi_0}{\hat{W}}-2\right)\right],$$

$$A_{23} = \hat{B}\left[-\frac{\hat{x}_2^3}{2\hat{W}^3} - \frac{3\hat{x}_2^2}{2\hat{W}^2}\left(\frac{\phi_0}{\hat{W}}-1\right) - \frac{3\phi_0\hat{x}_2}{2\hat{W}^2}\left(\frac{\phi_0}{\hat{W}}-2\right)\right],$$

$$A_{24} = \frac{3\hat{B}\hat{H}\hat{x}_2}{\hat{W}^3}\left[\frac{\hat{x}_2^2+\phi_0\hat{x}_2+\phi_0^2}{2\hat{W}} + \left(\frac{\phi_0}{\hat{W}}-1\right)\hat{x}_2 + \left(\frac{\phi_0}{\hat{W}}-2\right)\phi_0\right],$$

$$A_{25} = -\frac{\hat{H}\hat{x}_2^3}{2\hat{W}^3} - \frac{3\hat{H}\hat{x}_2^2}{2\hat{W}^2}\left(\frac{\phi_0}{\hat{W}}-1\right) - \frac{3\hat{H}\phi_0\hat{x}_2}{2\hat{W}^2}\left(\frac{\phi_0}{\hat{W}}-2\right) - \hat{x}_1 - u, \tag{A.1}$$

where

$$\hat{\alpha} = -\frac{1}{2} + \frac{3}{2}\frac{\phi_0}{\hat{W}} - \frac{1}{2}\left(\frac{\phi_0}{\hat{W}}-1\right)^3, \quad \hat{\beta} = \frac{3}{2}\frac{\phi_0}{\hat{W}^2}\left(\left(\frac{\phi_0}{\hat{W}}-1\right)^2-1\right) \tag{A.2}$$

Appendix B. SIMULATION PARAMETERS

A_c	flow area	0.01 m^2
B	B-Parameter	≈ 0.832
H	coefficient	0.18
L_c	length of ducts and compressor	3 m
U	compressor blade tip speed	80 m s^{-1}
V_p	plenum volume	1.5 m^3
W	coefficient	0.25
a_s	speed of sound	340 m s^{-1}
ψ_0	operating point for ψ, respective x_1	0.533
ϕ_0	operating point for ϕ, respective x_2	0.3
ψ_{0_c}	constant of the compressor characteristics	0.3

Kalman Filter tunings

$$Q_O = \operatorname{diag}\left(10^{-3},\ 10^{-1},\ 10^{-1}\right),$$
$$R_O = 10^{-1}$$