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# Optimal operation of energy storage in buildings: Use of the hot water system

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ABSTRACT

We consider the optimal operation of energy storage in buildings with focus on the optimization of an electric water heating system. The optimization objective is to minimize the energy costs of heating the water, with the requirement that we should satisfy the uncertain demand at any time. The main complications in this problem are the time varying nature of the electricity price and the unpredictability of the future water demand. In this paper we use the water heating system as an example for formulating a general framework which could easily be applied to similar problems with energy storage capacity. Feasibility and optimality are discussed and the main points are illustrated in the simulation case studies.



## 1. Introduction

Recently, considerable attention has been paid to renewable energy sources like wind turbines and photovoltaic parks. These alternative energy sources suffer a major drawback, however, due to their strong dependence on uncontrolled and varying weather conditions. This is an important limitation since the energy production is expected to cover the demand at any given time.

A possible approach for handling these fluctuations in the production is to shift the consumer load to periods where a lot of energy can be produced cheaply. This is referred to as demand side load management [1]. Field tests in the USA have demonstrated that such an optimization of domestic energy consumption can significantly reduce load peaks [2,3]. This can be achieved by manipulating the energy price according to demand information and weather forecasts. Electricity consumers are thus encouraged to consume electricity more prudently in order to minimize their electric bill. The dynamic energy pricing for demand load management is in itself a non-trivial problem and it is currently an active research area. The interested reader may check the literature [4–6] for more information, as this problem is outside the scope of this work.

Local energy storage in such setting provides several benefits for the consumer without having to adjust their consumption pattern. In particular, it enables

1. Higher peak capacity. For example, one may heat extra hot water in the morning to make sure there is enough water for everyone to have a shower.
2. Taking advantage of varying energy price. Energy can be purchased when prices are low and it can be used when the prices are high. (Since human users typically have a weak response to energy prices [7], automatically controlled consumers are better suited for a variable pricing scenario.)
3. Taking advantage of favourable outdoor conditions (e.g. cooling at night or heating during the day).

Energy storage devices could include hot water tanks, batteries, ice banks, liquid nitrogen, thermal storage building mass thermal capacity and compressed air storage [7].

Recently, there has been significant research activity around the problem of optimal usage of energy storing devices. For example, in [8] the problem of optimizing the end-consumer energy storage policies is considered. The proposed idea is to charge batteries when the electricity price is low and use the stored energy when the price is high. The authors show that the optimal policy has a simple structure based on two threshold levels: if the battery level is below a certain lower threshold value, the optimal policy is to charge it as close to the lower threshold value as possible. If the

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battery level lies above some upper threshold value then it is optimal to use the stored energy from the battery instead of purchasing from the grid. The difficulty lies in the computation of the optimal threshold levels which are a function of the varying energy price. However, analytical results can be derived for a few simplified cases, e.g. assuming perfect efficiency for charging and discharging the battery.

Ericson [9] presented results from a large scale Norwegian project where load control was applied on domestic hot water heaters. The main idea was to disconnect the water heaters from the electricity grid during peak hours in order to reduce the peak load. Electrical consumption of 475 households were investigated over a six month period from November 2003 to May 2004. The results show significant peak shavings in consumption during disconnection of hot water heaters. However, the researchers observed a considerable increased consumption after the reconnection of the heaters, which may have the adverse consequence of causing a new peak in the system.

Henze et al. [10] consider the optimization of the cooling system in commercial buildings. The authors propose shifting the thermal load by precooling the buildings structure at night, in addition to using *active* storage means such as ice thermal storage. The ultimate goal is to take advantage of ambient conditions and of real-time pricing to maximize the energy cost savings. The simulations show that the cost savings and on-peak demand reductions can be substantial (up to 57% and 50%, respectively) if a good model and accurate weather predictions are used.

Many recent contributions use model predictive control (MPC) solutions for this problem. In [11] a MPC controller is used to minimize a multi-objective function which trades off energy cost and comfort level in a dynamic real-time pricing scenario. They show that there is a good potential for savings compared to traditional control strategies. Not surprisingly, it is shown that the energy cost increases as the comfort level increases.

In this paper, we focus on the optimization of an electric water heating system which provides hot water for domestic usage. The optimization objective is to minimize the energy costs while obeying some operating constraints. The main idea is to use the heat capacity of the water tank to *store energy* in times when electric power is cheap and use it to match the demand when energy is expensive. The main contribution of this paper is to provide a systematic comparison between different strategies to operate the system. The idea here is to have a better understanding of the potential benefits of using energy storage in this problem. A comparison of the various strategies will be presented. We will distinguish between the following cases:

- *Ideal case*, where the optimal solution is computed assuming perfect knowledge of the future demand. This is a theoretical limit which cannot be achieved in practice, unless the future demand is known exactly.
- *Maximum storage policy*, where we maintain maximum storage in the tank at all times. This is achieved by fixing the tank temperature setpoint $T_s$ and tank volume setpoint $V_s$ at their maximum allowed value. This is the safest policy in terms of avoiding constraint violation caused by unforeseen high demand as it minimizes the risk of not having enough hot water.
- *Simple variable storage policy*, an intuitive money saving strategy in which we buy and store as much energy as possible during the night to be used during the day. The idea is to activate a 'storage mode' during night, when we set the energy storage setpoint $E_s$ to its maximum, and a 'saving mode' during the day when we set $E_s$ to a lower value. This policy is analogous to the work of [8], where the setpoint $E_s$ plays the role of the switching threshold discussed in that paper.
- *Optimal variable storage policy*, where the temperature setpoint $T_s$ and tank volume setpoint $V_s$ are updated at every time step using a moving horizon optimization (MHO) approach. The optimization algorithm relies on a simple forecast model to predict the future demand. A detailed derivation of this method is presented in an accompanying paper [12].

Additional contributions of this paper include:

1. A detailed general problem formulation which may also be suitable for different applications involving dynamic optimization, energy storage and variable energy prices.
2. Guidelines about implementation strategies including control structures.

The paper is organized as follows: Section 2 presents the process modelling; Section 3 formulates the optimal control problem; in Section 4, insights into the implementation strategies are given; in Section 5 we detail different strategies for control and optimization of the system; Section 6 details a simulation study comparing various approaches. Section 7 presents a discussion on the subject and Section 8 concludes the paper.

## 2. Process model for hot water storage tank

The process we are dealing with consists of a heater which provides hot water for domestic usage. A simplified process flow scheme is shown in Fig. 1 where the important notation is presented. The system includes a cold water source, a thermally insulated tank, a heating coil with adjustable power and control valves that regulate the cold water inflow $q_{in}$ and the hot water outflow $q_{out}$. A somewhat unusual feature of this system is that the hot water that leaves the tank ($q_{out}$) is mixed with a cold water stream ($q_{cw}$) from the same water source. This extra mixer is to allow extra flexibility and implies that the water in the storage

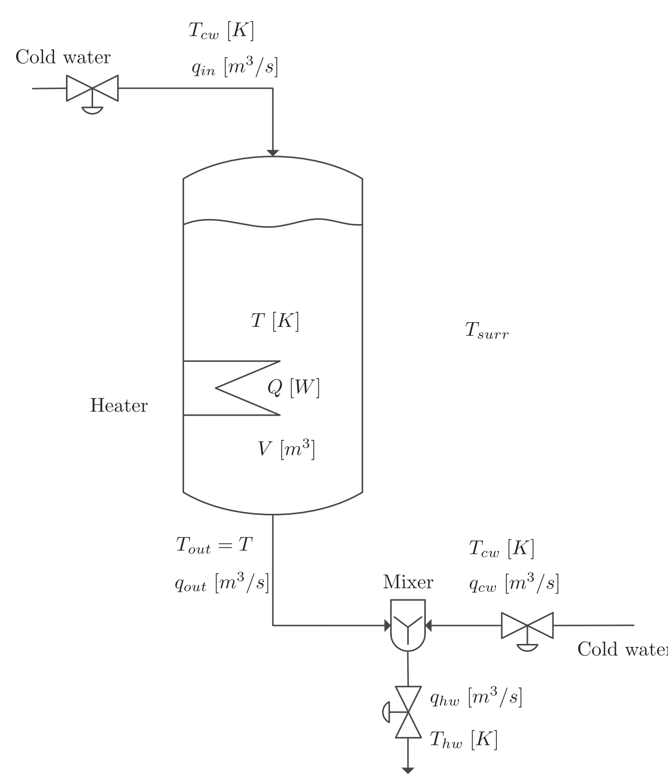


**Fig. 1.** Simplified process flow scheme.

tank ($T_{out} = T$) can be heated to a higher temperature than the hot water to the consumer ($T_{hw}$).

We defined the tank as our control volume and derive a dynamic model from mass and energy balances for the water in the tank. The mass balance for the tank is

$$\frac{d(\rho V)}{dt} = \rho_{in} q_{in} - \rho_{out} q_{out} \quad [\text{kg/s}] \tag{1}$$

where $V\,[\text{m}^3]$ is the volume of the tank. We will assume constant fluid density ($\rho = \rho_{in} = \rho_{out}$). Assuming constant pressure and no mechanical work and neglecting kinetic and potential energy, the energy balance for the tank becomes (e.g. [13])

$$\frac{dH}{dt} = h_{in} - h_{out} + Q - Q_{loss} \quad [\text{J/s}] \tag{2}$$

where $Q_{loss}$ [J/s] is the heat loss to the surroundings, $H$ [J] is the enthalpy of the system, $h_{in}$ [J/s] and $h_{out}$ [J/s] is the enthalpy of the streams and $Q$ [J/s] is the added power. The standby heat loss from the heater to the surroundings is

$$Q_{loss} = UA(T - T_{surr}) \tag{3}$$

where $UA$ [W/K] is the heat transfer constant and $T_{surr}$ is the temperature of the surroundings. Assuming constant heat capacity $c_p$, no phase change and perfect mixing ($T_{out} = T$), the enthalpies are given by ([13])

$$H = \rho V c_p (T - T_{ref}) \quad [\text{J}] \tag{4}$$

$$h_{in} = \rho q_{in} c_p (T_{cw} - T_{ref}) \quad [\text{J/s}] \tag{5}$$

$$h_{out} = \rho q_{out} c_p (T - T_{ref}) \quad [\text{J/s}] \tag{6}$$

where $T_{ref}$ [K] is a fixed reference temperature, $q\,[\text{m}^3/\text{s}]$ is the flowrate and $T_{cw}$ is the temperature of the cold water from the network. Combining Eqs. (1) and (2), with the assumption of constant $c_p$ and $\rho$, the dynamic model of the tank becomes

$$\frac{dV}{dt} = q_{in} - q_{out} \tag{7a}$$

$$\frac{dT}{dt} = \frac{1}{V}\left[q_{in}(T_{cw} - T) + \frac{Q - Q_{loss}}{\rho c_p}\right] \tag{7b}$$

where $T$ is the tank water temperature and $T_{cw}$ is the temperature of the inlet flow. Note that $T_{ref}$ drops out of the equations.

Similarly, we may write mass and energy balances for the mixer system, which is assumed to be a static process. The mass balance is given by

$$q_{hw} = q_{out} + q_{cw} \tag{8}$$

The steady sate energy balance for the mixer is given by

$$h_{hw} = h_{cw} + h_{out} \quad [\text{J/s}] \tag{9}$$

where $h_{cw}$ and $h_{hw}$ are the enthalpies of the cold and hot water streams, which are defined as

$$h_{cw} = \rho q_{cw} c_p (T_{cw} - T_{ref}) \tag{10}$$

$$h_{hw} = \rho q_{hw} c_p (T_{hw} - T_{ref}) \tag{11}$$

Rearranging (9) gives the hot water temperature

$$T_{hw} = \frac{q_{out} T + q_{cw} T_{cw}}{q_{hw}} \tag{12}$$

### 2.1. Energy storage and demand

In this subsection we introduce some terms that will be useful for analysis. We define the energy stored in the tank relative to the current cold water supply temperature ($T_{cw}$) as

$$E = \rho c_p V (T - T_{cw}) \quad [\text{J}] \tag{13}$$

We define the power demand at any given time as

$$Q_{\text{demand}} = \rho c_p q_{hw} (T_{hw} - T_{cw}) \quad [\text{J/s}] \tag{14}$$

This is the power we would need to supply at any given time if there were no energy storage. Upon combining (2) and (9) we obtain the overall energy balance of the combined tank-mixer system

$$\frac{dH}{dt} = h_{in} + h_{cw} - h_{hw} + Q - Q_{loss} \tag{15}$$

By introducing $E$ and $Q_{\text{demand}}$, the energy balance can be written in the following alternative form (see Appendix A for derivation)

$$\frac{dE}{dt} = Q - Q_{\text{demand}} - Q_{loss} - \rho V c_p \frac{dT_{cw}}{dt} \tag{16}$$

which will be useful for analysis of the system. Note that for the case with constant cold water supply temperature ($(dT_{cw}/dt) = 0$), which is also assumed in the case study, we simply get

$$\frac{dE}{dt} = Q - Q_{\text{demand}} - Q_{loss} \tag{17}$$

that is, the change in the stored energy is the difference between the current heating ($Q - Q_{loss}$) and current use $Q_{\text{demand}}$. It is also relevant to define the maximum energy capacity as

$$E_{\max} = \rho c_p V_{\max} (T_{\max} - T_{cw}) \tag{18}$$

and the minimum energy amount that needs to be satisfied at all times as

$$E_{\min} = \rho c_p V_{\min} (T_{\min} - T_{cw}) \tag{19}$$

The bounds for volume and temperature $V_{min}$, $V_{max}$, $T_{min}$ and $T_{max}$ are discussed in Section 3.2. For analysis it will be helpful to define the scaled stored energy

$$\bar{E}(t) = \frac{E(t) - E_{\min}}{E_{\max} - E_{\min}} \tag{20}$$

which lies between $\bar{E}_{\min} = 0$ and $\bar{E}_{\max} = 1$.

## 3. Problem formulation

### 3.1. Independent variables

#### 3.1.1. Control degrees of freedom

From Fig. 2, the system has four independent variables, namely, $Q$, $q_{cw}$, $q_{hw}$ and $q_{in}$. However, as discussed next, two of these variables ($q_{hw}$ and $q_{cw}$) are used to satisfy demand requirements on the hot water flow and temperature, respectively. The remaining two degrees of freedom (decision variables) for control and optimization are the power input $Q$ and the cold water inlet flow $q_{in}$.

#### 3.1.2. Disturbances

The hot water flow rate, $q_{hw}$, and the hot water temperature setpoint, $T_{hw,s}$ are set by the user and are considered disturbances from a control point of view. We assume perfect temperature control ($T_{hw} = T_{hw,s}$) whenever feasible. By 'whenever feasible' we mean whenever the tank temperature is above the delivery setpoint, $T \geq T_{hw,s}$. In this case, the flows $q_{out}$ and $q_{cw}$ are given by (8) and (12) with $T_{hw} = T_{hw,s}$. For the case when $T < T_{hw,s}$, we cannot

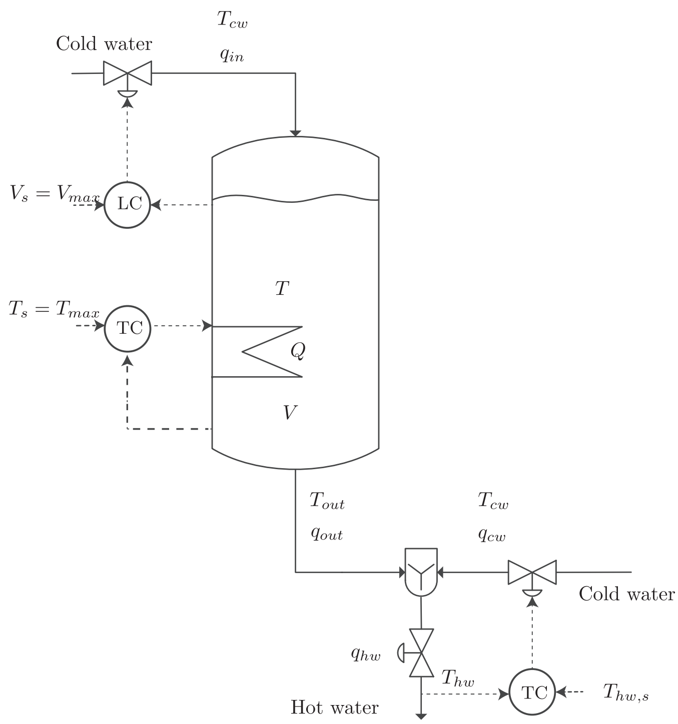


**Fig. 2.** Proposed control structure when $T_s = T_{max}$ and $V_s = V_{max}$ (simple policy (28) during night).

achieve the desired setpoint and we set $q_{cw} = 0$ in order to maximize the delivery temperature $T_{hw}$, and we get $q_{out} = q_{hw}$ and $T_{hw} = T$.

The main disturbances for the optimization are related to the user demand ($T_{hw,s}$ and $q_{hw}$), the cooling water temperature ($T_{cw}$) and the price of electricity ($p$) and can be regarded as stochastic variables. The cold water temperature ($T_{cw}$) can vary significantly in the long term (e.g. from summer to winter), but this variation is not important in our time scale (which is from minutes to hours). From an operational point of view, the effect of changes in both $q_{hw}$ or $T_{hw,s}$, is a change in the power demand $Q_{demand}$. Therefore, for simplicity, we here assume that $T_{hw,s}$ is constant and consider disturbances in the hot water demand, $q_{hw}$. In the case study we assume $T_{cw}$ is constant.

### 3.2. Constraints

The operation of the system should respect constraints related to physical limitations, safety and specifications. Firstly, in terms of inputs, the heating power and water inflows are limited, so that

$$0 \leq Q \leq Q_{\max} \tag{21}$$

$$0 \leq q_{in} \leq q_{\max} \tag{22}$$

In terms of output constraints, the temperature of the water should be bounded above ($T_{max}$) for safety reasons and indirectly bounded below ($T_{hw,s}$) to guarantee that the desirable temperature of the hot water is achievable. Naturally, the volume is bounded by the size of the tank. Therefore, we have

$$T_{\min} \leq T \leq T_{\max} \tag{23}$$

$$V_{\min} \leq V \leq V_{\max} \tag{24}$$

where $T_{min} = T_{hw,s}$.

### 3.3. Optimal control problem formulation

In its simplest form, the objective we would like to minimize is the future energy cost

$$J = \int_{t_0}^{t_f} p(t) Q \, dt \tag{25}$$

where $p(t)$ is the time-varying electricity prices, $Q(t)$ is the power we buy at time $t$, $t_0$ is the initial time and $t_f$ is the final time. In addition, we want to satisfy the operational constraints (21)–(24) and we have to satisfy the process dynamics (7). Notice that the process dynamics introduce nonlinearity into the optimization problem, which makes the problem more difficult to solve.

## 4. Insights into the optimal solution

In this section we will present some properties of the solution that can be used to simplify the optimization problem. In addition, these insights will be used to derive simple implementation strategies for this system.

### 4.1. Ideal liquid level

When a target for the energy storage $E$ is specified (e.g. by an optimization algorithm) a decision on the appropriate values for $T$ and $V$ that achieve the given energy storage needs to be made. This is because of the non-uniqueness in the energy storage $E = \rho c_p V(T - T_{cw})$ in terms of temperature and volume. In practice, to reduce the heat loss, we want to keep the temperature $T$ as low as possible, which for a given energy storage ($E$) is achieved by maximizing the tank filling. We then have the following important insight:

> *For a given energy storage E it is optimal to keep the liquid V in the tank as large as possible to minimize energy losses.*

This means that we will keep $V = V_{max}$ as long as the temperature in the tank $T$ is above the hot water setpoint $T_{hw,s}$. When the temperature $T$ reaches $T_{hw,s}$ we may have to reduce the refilling cold water, which means that $V$ will drop below $V_{max}$. However, note that for safety reasons we always have to keep $V \geq V_{min}$.

### 4.2. Initial condition and terminal state constraint

The electricity price tends to be at its lowest during night and it typically peaks in the morning when the demand is high. The hot water demand profiles show a similar behaviour, with demand peaking early in the morning. Based on this observation we have the following insight

> *It is optimal to have maximum energy stored ($V = V_{max}$ and $T = T_{max}$) late in the night.*

This is an important insight because it means that we can always consider a 24 h optimization horizon, even when the actual horizon ($t_f$) is longer. For an optimization horizon of 24 h this implies that we should have

$$T(t_0) = T(t_0 + 24\,\text{h}) = T_{\max} \tag{26}$$

$$V(t_0) = V(t_0 + 24\,\text{h}) = V_{\max} \tag{27}$$

where the initial time $t_0$ should be appropriately chosen. For example, it could be some time after midnight. This suggests that the optimization of every 24-h interval may be performed independently because the terminal constraints decouple the optimization problems of two consecutive days.

## 5. Solution approaches

In this section we will describe in more details the different approaches that will be compared.

### 5.1. The ideal solution

The solution to the optimization problem (25) requires the characterization of future price $p(t)$ and demand $q_{hw}(t)$ for the horizon of interest. In the ideal case, we assume perfect knowledge of the future demand $q_{hw}$. The term *ideal* refers to the fact that this assumption is generally not satisfied and this solution should be regarded as a theoretical limit. The results obtained in this case are very useful to benchmark the performance of any other policy.

### 5.2. Maximum storage policy

This is the simplest policy, where we maintain maximum energy storage in the tank at all times. This is achieved by fixing the tank temperature setpoint $T_s$ and tank volume setpoint $V_s$ at their maximum allowed value. The control structure that can be used is shown in Fig. 2. This is the safest policy in terms of avoiding constraint violation caused by unexpectedly high demand, but it does not seek to reduce the electricity costs.

### 5.3. Simple variable storage policy

The observations and insight presented in Section 4.2 suggests a simple money-saving strategy in which we attempt to store as much energy as possible during the night. Therefore, the idea is to manipulate the energy storage setpoint $E_s$ using a simple time-based feedforward rule

$$\text{Simple policy}: \quad \begin{cases} E_s = E_{\max} & \text{during night (storage mode)} \\ E_s = E_{\min} + E_{\text{backoff}} & \text{during day (saving mode)} \end{cases} \tag{28}$$

where the positive constant $E_{\text{backoff}}$ is a backoff from the constraint to reduce the risk of frequent constraint violation caused by large demand during the day. The backoff should be adjusted such that the amount of constraint violations is acceptable for the given case. Using $E_s = E_{max}$ at night accomplishes two things: takes advantage of more favourable electricity prices at night and it anticipates for a high consumer demand in the morning. The main problem here is to determine the most beneficial times to switch between the setpoints. One approach is to use historical price data to compute the time interval with the lowest price in average. Such time interval should be long enough to ensure a full tank with maximum temperature before entering the 'saving mode'.

#### 5.3.1. Simple policy: temperature and volume setpoints

For a practical implementation using control it is convenient to know what the simple policy (28) means in terms of temperature and volume setpoints. During 'storage mode' we obviously have $T_s = T_{max}$ and $V_s = V_{max}$. On the other hand, during 'saving mode' we need to use the insight in Section 4.1 to compute the setpoints which for small backoff ($E_{\text{backoff}}$) becomes

$$T_s = T_{hw,s}$$
$$V_s = \frac{E_{\min} + E_{\text{backoff}}}{\rho c_p (T_{hw,s} - T_{cw})} = V_{\min} + V_{\text{backoff}}$$

where the constant $V_{\text{backoff}} = (E_{\text{backoff}})/(\rho c_p(T_{hw,s} - T_{cw}))$ is non-negative. When the backoff $E_{\text{backoff}}$ is large enough we will have $V_s = V_{max}$ and the temperature setpoint needs to be greater than the lower bound ($T_s > T_{hw,s}$).

After switching from one mode to another, there will always be a transient period where we are not meeting the energy storage setpoint $E_s$. During the transition from storing to saving mode (night to day) we don't want to add any power $Q$ because of the high price in this period. In addition, to reduce heat losses we should let the temperature $T$ drop to the setpoint $T_s$ before we start reducing the volume to the new setpoint. This can be easily achieved by keeping $V_s = V_{max}$ while $T > T_{hw,s}$ and only when the temperature reaches the new setpoint ($T = T_{hw,s}$) we switch the volume setpoint to the desired 'saving mode' level ($V_s = V_{min} + V_{\text{backoff}}$).

On the other hand, during the transition from day to night operation we should first increase the tank level until $V = V_{max}$ and then we start increasing $T$ to reduce losses. This can be achieved by setting $Q = Q_{max}$ while using the water refilling to keep $T = T_{hw,s}$ until the tank is full ($V = V_{max}$). At this point then switch to the structure in Fig. 2 for the night operation.

#### 5.3.2. Simple policy: implementing insights using control

The policy during 'storage mode' can be implemented using feedback control:

- Use the water refilling ($q_{in}$) to keep $V$ at constant setpoint $V_{max}$.
- Use the power input ($Q$) to control $T$ at constant setpoint $T_{max}$.

Fig. 2 depicts the control structure for this case. During 'saving mode' (after the transient) a similar structure can be used. However, for large disturbances the temperature controller might saturate ($Q = Q_{max}$) and it is not advantageous to have $V = V_s = V_{min} + V_{\text{backoff}}$ as it will force $T$ to drop below $T_{hw,s}$ and we should let $V$ drop. A simple way to achieve this is to use split range control as shown in Fig. 3. The basic idea is that a single controller uses both the power input $Q$ and the level setpoint $V_s$ to control the temperature when the setpoint is $T_s = T_{hw,s}$. The temperature controller computes a virtual control action $u$ which is translated to values for $Q$ and $V_s$ according to a defining function as depicted in Fig. 4. When the volume setpoint reaches the lower bound $V_s = V_{min}$ the temperature control is lost and $T$ drops below $T_{hw,s}$.

### 5.4. Optimal variable storage policy

The main idea here is to use a moving horizon optimization (MHO) scheme to solve the problem. At each time sample a model-based dynamic optimization problem with horizon $h$ is solved using the information that is currently available. However, only the first portion of the optimal profile corresponding to $t \in [t_0, t_0 + \Delta t]$ is implemented, where $t_0$ is the initial time and $\Delta t$ is the sample time. Within this framework we find two different implementation philosophies:

- **Single layer strategy** in which optimization and control are integrated; Here, optimization problem (25) is solved using a moving horizon approach and the optimal inputs ($q_{in}$ and $Q$) are re-computed directly (by the optimizer) at every time sample. This is, in theory, the optimal approach. However, it requires high computational power as the full optimization problem needs to be solved at every time sample. In the literature this strategy is sometimes called economic model predictive control [14].
- **Two level strategy** where the optimization problem is decomposed in two simpler problems where the economic objectives are decoupled from the control objetives.

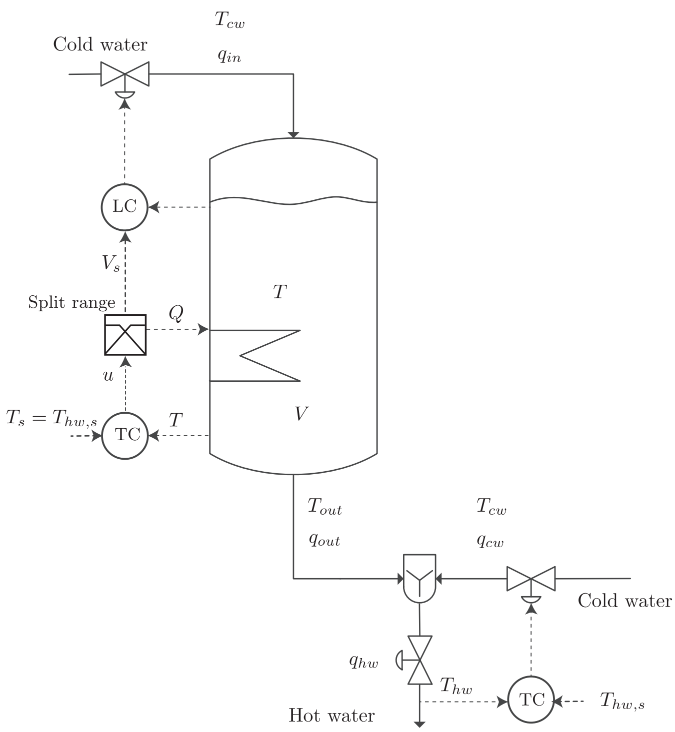


**Fig. 3.** Split range control structure, used when $T_s = T_{hw,s}$ and $V_s$ may vary between $V_{min} + V_{\text{backoff}}$ and $V_{min}$; see Fig. 4. (simple policy (28) during day). The lower temperature controller, where extra cold water is mixed in, is not active during normal day operation. In the transition period between night (Fig. 2) and day (Fig. 3) operation, we set $Q = 0$ and first let $T_{out}$ drop from $T_{max}$ to $T_{hw,s}$ (with $V_s = V_{max}$) and then set $q_{in} = 0$ and let $V$ drop from $V_{max}$ to $V_{min} + V_{\text{backoff}}$.

The second strategy is our preferred and is the approach used in this paper. The basic idea is to decompose the overall problem of economic optimization and control (Eq. (25)) into simpler subproblems by using a cascade feedback structure. In this scheme the bottom layer is a regulatory control layer that follow the set-points specified by an optimizer in the upper layer. In our problem the regulatory control layer is similar to that of Fig. 2 and the task of the optimizer is to update the setpoints $T_s$ and $V_s$. Our idea is to write a simplified optimization problem in terms of the energy

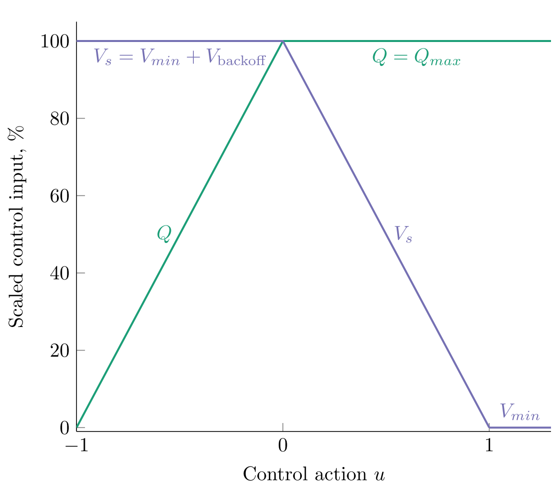


**Fig. 4.** Simple representation of a split-range control where both the power $Q$ and the level setpoint $V_s$ are used to control the temperature when the setpoint is $T_s = T_{hw}$. Note it is not necessary to have $u = 0$ as the point where you switch; this value may be used to equalize the scaled process gains for the two inputs.

storage $E$ only and then translate the optimal energy level $E_{opt}$ into setpoints $V_s$ and $T_s$.

A detailed derivation of this method is presented in the accompanying paper [12]. In [12] it is shown that the remaining optimization problem for the upper layer can be written as a simple linear program (LP) which can be solved very efficiently at a low computational cost.

An important factor for the success of this optimization scheme is to have relevant information about the user demand and the future price. An idea is to construct a demand model based on the empirical distribution of hot water consumption for every time step using historical data. This model can be updated online as new measurements become available, making it possible to adapt to new consumption patterns when necessary. However, our simulation studies suggest that even simpler models (e.g. assume constant demand) can give satisfactory results if the optimization problem is resolve frequently enough. For simplicity, we will assume the electricity price is known 24h in advance.

As in the simple policy (28), it may be necessary to include a backoff $E_{\text{backoff}}$ from the constraint in order to reduce the probability of breaking the constraints due to large disturbances. The idea is to shift the current desired energy level $E_s$ (computed by the optimizer) if it is too close from the boundary $E_{min}$ so that $E_s \geq E_{min} + E_{\text{backoff}}$.

## 6. Case Study

In this section we will show a simulation example of the methodology presented in the previous section. The idea here is to have a better understanding of the potential benefits of using energy storage in this problem. A comparison of the various strategies will be presented.

### 6.1. Electricity prices

For simulation and optimization we used the electricity price data available in the archives of Nord Pool [15] spot market. A sample of the electricity price for the first 10 days of February, 2012 in Trondheim, Norway is shown in Fig. 5. Although Norway currently does not use real-time pricing for the end-user, the spot prices provide a reasonable real-time pricing estimates. The resolution of the price data is one hour.

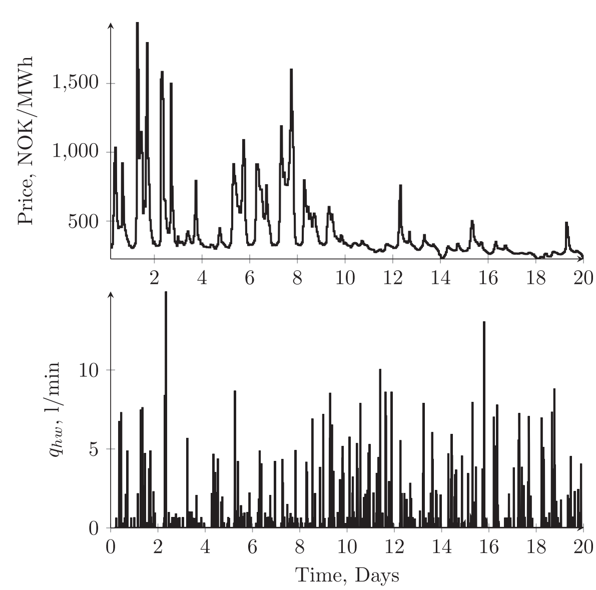


**Fig. 5.** Electricity price and hot water demand.

### 6.2. Realistic hot water demand

For a realistic comparison, we emulate hot-water flow demand ($q_{hw}$) profiles based on the empirical probability distributions published by [16]. The consumption profiles have a resolution of one minute and correspond to a single family house with a mean load volume of 350 L per day. An example of a consumption profile is depicted in Fig. 5, where twenty unique hot-water profiles were generated. For simplicity, we will assume constant temperature setpoint $T_{hw,s}$ and cold water temperature $T_{cw}$.

### 6.3. Heat loss

The typical heat loss from a domestic hot water tank is approximately 0.1 kWh/h at a temperature of 75 °C [9]. For a room temperature $T_{surr}$ = 25 °C it follows from (3) that the heat transfer constant ($UA$) for domestic water heaters is approximately

$$UA = \frac{Q_{loss}}{T - T_{surr}} = \frac{0.1}{50} = 0.002 \quad [\text{kW/K}] \tag{29}$$

which we assume constant throughout the simulations. Additional important parameter values for our case study are presented in Table 1.

### 6.4. Ideal case

To compute the ideal solution, we discretize the optimal control problem using orthogonal collocation in a simultaneous approach [17]. In this approach, the differential equations are converted to algebraic ones by orthogonal collocation which should be satisfied only at the solution of the optimization problem [17]. The key characteristic here is that both states and manipulated variables profiles are approximated, with the same accuracy, by orthogonal polynomials, resulting in a large scale nonlinear programming problem (NLP). An interesting characteristic of this method is that it can efficiently handle problems with constraints on states and control inputs.

We formulate the problem in Matlab and solve it using the sparse NLP solver SNOPT [18]. This solver employs a sparse SQP algorithm with quasi-Newton approximations to the Hessian. Gradient information is obtained using a symbolic differentiation approach. The interface between Matlab and SNOPT is handled by the optimization environment TOMLAB.

The optimization for every day is carried out independently, where we consider the initial time $t_0$ = 4 h in the morning and a horizon $h$ = 24 h. The tank is always initially full ($E(t_0) = E_{max}$) and we impose the terminal constraint $E(t_f) = E_{max}$.

**Table 1**
Parameters for case study.

| Parameter | Description | Value | Unit |
|---|---|---|---|
| $Q_{max}$ | Maximum power | 5 | kW |
| $Q_{min}$ | Minimum power | 0.0 | kW |
| $T_{max}$ | Temperature upper bound | 90 | °C |
| $V_{max}$ | Volume upper bound | 150 | l |
| $V_{min}$ | Volume lower bound | 50 | l |
| $T_{cw}$ | Cold water temperature | 5 | °C |
| $T_{hw}^{sp}$ | Hot water temperature | 50 | °C |
| $c_p$ | Heat capacity of the water | 4.19 | kJ/kg/K |
| $UA$ | Heat transfer constant | 0.002 | kW/K |
| $T_{surr}$ | Room temperature | 25 | °C |

### 6.5. Simple variable storage policy

The main decision in the design of the simple policy (28) is the time to switch the temperature setpoints. The duration of the 'storage mode' period ($\Delta t_{night}$) should be long enough to ensure that maximum storage energy ($E = E_{max}$) can always be reached at the end of the interval. This value depends on the size of the tank, the maximum water inflow rate and the installed electric power. After determining the minimum duration $\Delta t_{night}$ we can use historical price data to determine the period of the day with duration $\Delta t_{night}$ with the lowest price in average. In this case study we have chosen to activate the 'storage mode' only from 2 am to 6 am. In this example, we have chosen the backoff level $E_{\text{backoff}} = 0.2(E_{max} - E_{min})$, which corresponds to $V_{\text{backoff}}$ = 50 l.

### 6.6. Optimal variable storage policy

As in the ideal solution, we include the terminal constraint $E(t_f) = E_{max}$ into our optimization problem. This suggests a shrinking horizon approach where the optimization horizon $h$ is periodically decreased according to

$$h_k = h_{k-1} - \Delta t \tag{30}$$

where $\Delta t$ is the time between two consecutive optimizations. When $h_k = \Delta t$ we have to reset it to the initial horizon $h_0$. The initial horizon is chosen as $h_0$ = 24 h. The electricity price changes every hour so we discretize the optimization problem with sample time $\Delta t_0$ = 1 h. Note that $\Delta t$ may differ from the time between consecutive optimizations $\Delta t$. In that case, we may need to vary the size of the first step of the discretized problem in order to synchronize with price variations. To estimate hot water consumption we used the simplest model where the predicted flow $\hat{q}_{hw}$ is assumed constant throughout the day. Here we have chosen $\hat{q}_{hw} = 0.2431$ L/min to match the daily average consumption of 350 L/day. The backoff level $E_{\text{backoff}}$ was chosen the same as in the simple policy (28).

### 6.7. Simulation results

Figs. 7 and 8 show a comparison between the costs achieved by the various strategies when subjected to the disturbances in price and demand shown in Fig. 5. The figure includes the result for the optimal variable storage policy with time between consecutive optimizations $\Delta t$ = 5 min. The first thing to notice from these results is that all methods give substantial savings compared to the maximum storage policy. The optimal variable storage approach results in close-to-ideal performance even with very limited information about the demand available to the optimizer. More remarkable is the performance of the simple variable storage policy. Without any optimization algorithm or information about the price or demand it is able to give results comparable to that of the optimization-based approach. This finding should encourage practitioners to implement such simple polices to manage their energy storage units even when limited resources for advanced computer control are available.

The behaviour of the different methods can be analysed by looking at the tank volume and temperature in Fig. 9. We chose to show only the first three days to facilitate the visualization. Information about price and demand for the first three days is repeated in Fig. 6. The simple variable storage policy, the optimal variable storage and the ideal solution show very similar temperature trends. However, they differ in terms of volume. During the day, both the optimal and the simple variable storage policy try to keep the volume above a certain level $V_s = V_{min} + V_{\text{backoff}}$ which is function of the backoff $E_{\text{backoff}}$. On the other hand, the

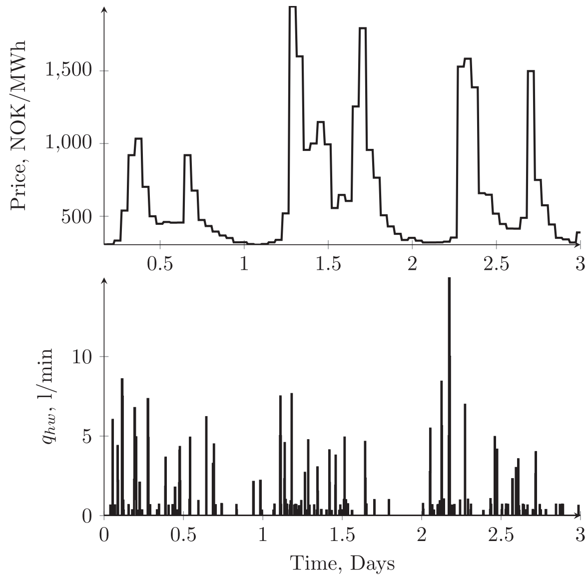


**Fig. 6.** Electricity price and hot water demand. First three days.

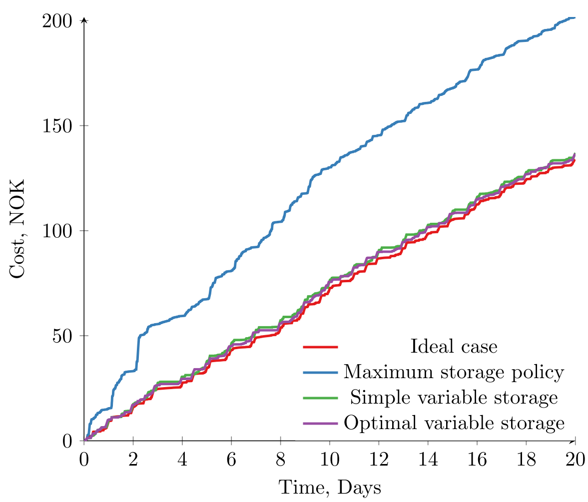


**Fig. 7.** Accumlated cost for the different strategies.

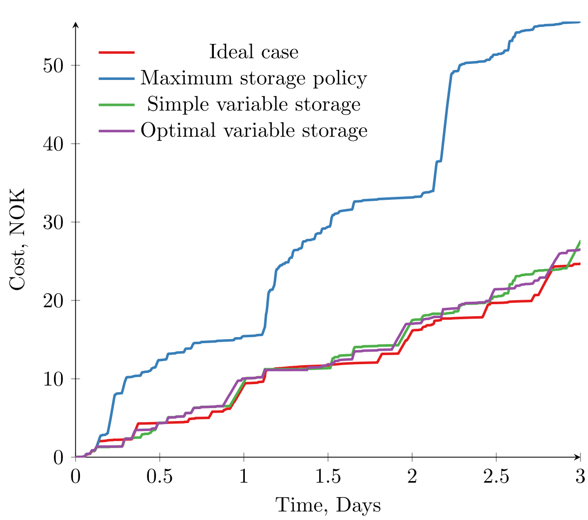


**Fig. 8.** Accumlated cost for the different strategies. First three days.

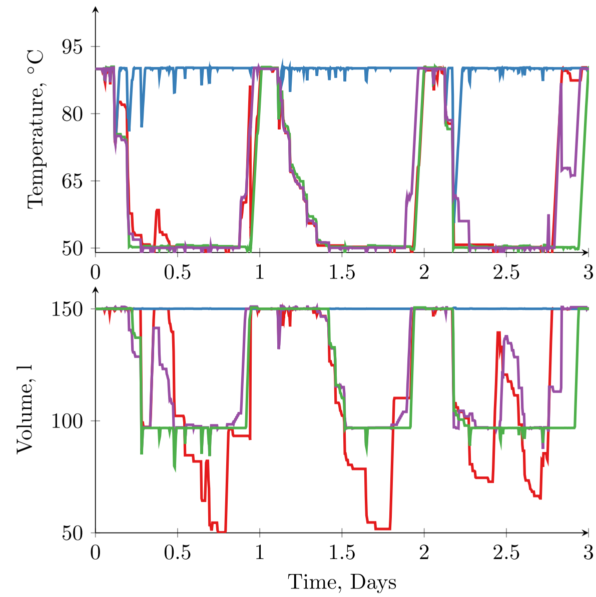


**Fig. 9.** Tank volume and temperature. Red lines: ideal case. Green lines: simple variable storage policy. Blue lines: maximum storage policy. Violet lines: optimal variable storage policy. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

ideal case is able to bring the volume close to the limit $V_{min}$ without violating constraints.

The information of temperature and volume can be summarized by the energy levels $E$ given by the different approaches, as shown in Fig. 10. The scaling is done according to (20) to ease the analysis. Because of the perfect knowledge of the demand the ideal solution is able to take maximum advantage of price variations by letting the energy levels drop close to minimum. This is in contrast with the simple and the optimal variable storage policies, which enforce an additional buffer to ensure feasibility. In addition, the knowledge of future price allows the optimization-based approaches to buy cheaper energy in advance. This behaviour can be exemplified in the first and third days as seen in

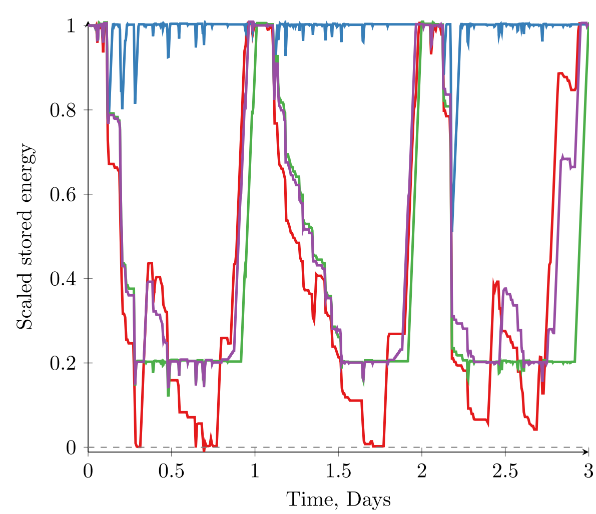


**Fig. 10.** Energy level $E$ (scaled) currently stored in the tank for the different strategies. The scaling is such that $E_{max}$ when scaled equals one and $E_{min}$ scaled equals zero. Red line: ideal case. Green line: simple variable storage policy. Blue line: maximum storage policy. Violet line: optimal variable storage policy. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Fig. 10. Nonetheless, all strategies show similar behaviour during the night, when they seek to maximize the stored energy.

## 7. Discussion

### 7.1. Choice of the method

We presented several strategies for operation of the water heater aiming at taking advantage of the flexibility given by the energy storage. The simple variable storage policy (28) gives big savings compared to the maximum storage policy, with performance comparable to that of more complex optimization-based approaches. However, the question of whether or not the increased complexity and computational load in the optimal variable storage policy is justified will depend on the specific case. In scenarios in which the time intervals with the lowest price vary considerably from day to day we can expect the optimization-based method to be more beneficial. This is because the performance of the simple policy (28) is sensitive to our definition of 'day' or 'night'.

The ideal solution has two fundamental advantages compared to other approaches: knowledge of the future price, which allows it to buy cheaper energy in advance; and its perfect knowledge of the demand, which makes it possible to operate closer to feasibility limits. Although perfect knowledge of future demand is not realistic for this problem, there might be cases where the demand is more predictable, for instance, when the demand is linked to a contract between supplier and consumer.

### 7.2. Power consumption

Fig. 11 shows the hourly average of 20 days of electric power consumption. The figure shows the reduction of electricity consumption during the peak hours by using an appropriate strategy. Notice that the total consumption is equal in both cases, but in the ideal case we are able to shift the load to a more beneficial period. During the peak hours in the morning (from 6 am to 10 am) we are able to reduce the average consumption from 1.3 kW to 0.43 kW. This indicates that the flexibility given by the thermal energy storage capacity of water heaters can be a good allied for reducing the peak demand in the electricity grid.

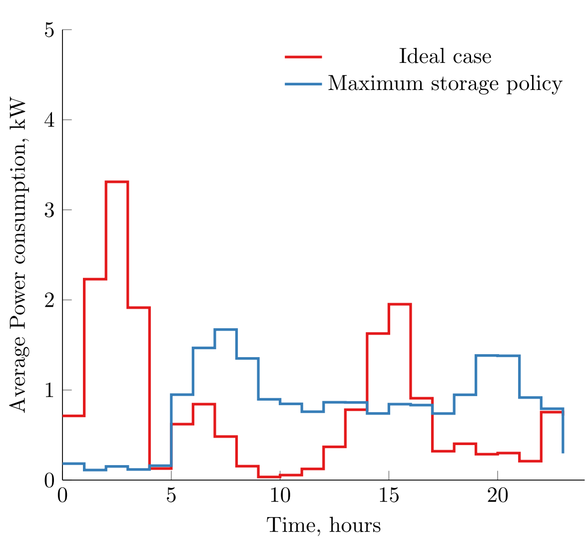


**Fig. 11.** Power consumption by hour of the day. Average of 100 days. Comparison between ideal and base cases. The figure shows the reduction of electricity consumption during the peak hours (morning and afternoon).

### 7.3. Design considerations: benefits of increasing storage size

The amount of savings that can be achieved strongly depends on the storage capacity. Ideally we would like to have enough capacity so that all the demand during high price period can be supplied with energy purchased at the lowest price. The electricity price may show large variations within a day where the price is usually the lowest during the first hours. Therefore, an appropriate tank capacity should exceed the average daily consumption of the household.

In order to study this aspect of the problem we have computed ideal solutions to the optimization problem for a specific day where we varied the maximum tank capacity ($V_{max}$) from 60 to 600 L but kept the same price and demand profiles. The chosen price and demand profiles represent the first day shown in Fig. 5. Fig. 12 depicts the optimal savings with respect to the maximum storage policy for different capacities. The savings obtained by the simple variable storage policy are also shown. The saving is defined as follows

$$\text{Saving} = \frac{J^*_{\text{base}} - J^*_{\text{method}}}{J^*_{\text{base}}} \times 100 \tag{31}$$

where $J^*_{\text{base}}$ is the cost (including the penalty as in (25)) for the maximum storage policy and $J^*_{\text{method}}$ is the cost for a particular approach.

The study showed that there is a substantial benefit in increasing the tank size. For small tank capacities (below 100 L) the simple variable storage policy performs worse than the maximum storage policy because of frequent constraint violations. In these cases the simple policy is not storing enough energy to handle the demand variations. However, for large capacities the simple policy performs very well and eventually becomes optimal.

Another interesting conclusion is that the savings flatten out for capacities above a certain level. The tank size after which the savings flattens out depends on the size of the demand for a given day. The total demand for this period in terms of energy is given by

$$E_{\text{demand}} = \int_{t_0}^{t_0+24} Q_{\text{demand}}(t)dt \tag{32}$$

For the ideal case there is not benefit in increasing the tank size when $E(t_0) = E_{\text{demand}} + E_{min}$. If we assume $E(t_0) = E_{max}$ we can compute the ideal tank volume using $V_{opt} = (E_{\text{demand}} + E_{min})/(\rho c_p(T_{max} - T_{cw}))$. For this particular day $V_{opt} = 272$ L. The ideal tank size for the simple variable storage policy can be computed in a similar way. The only difference is that we need to take the

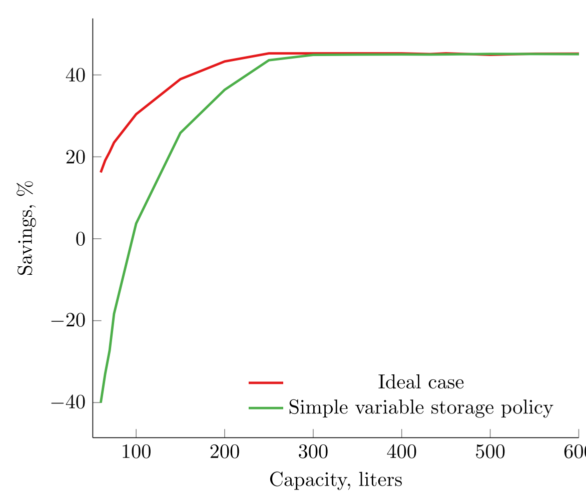


**Fig. 12.** Trade-off study showing the benefits of increased storage capacity for a specific day. The savings are relative to the maximum storage policy. The nominal capacity for the case study is 150 L.

backoff $E_{\text{backoff}}$ into consideration so $V_{opt} = (E_{\text{demand}} + E_{min} + E_{\text{backoff}})/(\rho c_p(T_{max} - T_{cw}))$. In this case the ideal tank size for the simple policy is $V_{opt} = 297$ L.

### 7.4. Alternative applications

The methodology and insights presented in this paper could help solving other problems involving energy storage. An example is a district heating system with storage capacity. Although these systems are typically closed, in which the volume of the heating medium is constant, there might be a possibility of manipulating volume and temperature at different parts of the system if more than one storage tanks are in place. This would allow a straightforward application of the strategies presented here.

The simple policy (28) or the optimal variable storage policy could be directly used to control home batteries, which can help electricity consumers avoid paying peak rates.

### 7.5. Comments on the modelling assumptions

In the derivation of the dynamic model we made use of the simplifying assumption of perfect mixing in the tank. This assumption is unlikely to hold for domestic hot water systems, where vertical thermal stratification is often observed [19,20]. Nevertheless, this assumption does not affect the actual results in terms of operating policies. This is because the standby losses to through the walls are approximately the same whether we consider an homogeneous temperature or a vertical temperature of the water in the tank. In addition, the economical performance depends mainly on the relation between the power that we supply ($Q$) and remove ($Q_{demand}$) and the current energy stock ($E$), and is not affected by the temperature distribution in the tank.

## 8. Conclusion

In this paper we discussed the optimal operation of the water heater system. We aimed at presenting a problem formulation that is sufficiently general to be used in similar problems that include energy storage and variable energy price and uncertain demand. The goal is to exploit the flexibility given by the energy storage capacity to take advantage of varying electricity prices. We showed that the economical benefits of energy storage can be large for the consumer, and they increase with the storage capacity. In addition, we showed that such strategies can help reducing the power consumption during peak hours, which will benefit the electricity producers. We presented several alternatives strategies for operation of the system and we showed that simple policies can give very good performance when compared to more complex, optimization based approaches. This finding should encourage practitioners to implement such simple polices to manage their energy storage units even when very limited resources are available.

## Acknowledgements

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## Appendix A. Derivation of the alternative energy balance

The energy balance of the combined tank-mixer system is

$$\frac{dH}{dt} = h_{in} + h_{cw} - h_{hw} + Q - Q_{loss} \tag{A.1}$$

For sake of simplicity, we will use the notation $RHS \triangleq h_{in} + h_{cw} - h_{hw} + Q - Q_{loss}$ and $LHS \triangleq (dH/dt)$.

The left hand side can be expanded as

$$\frac{1}{\rho c_p} LHS = (T - T_{ref})\frac{dV}{dt} + V\frac{dT}{dt} \tag{A.2}$$

By adding and subtracting $(T_{cw}(dV/dt) + V(dT_{cw}/dt))$ to the right hand side of (A.2) we obtain

$$\frac{1}{\rho c_p} LHS = \underbrace{(T - T_{cw})\frac{dV}{dt} + V\frac{d(T - T_{cw})}{dt}}_{(1/\rho c_p)dE/dt} + (T_{cw} - T_{ref})\frac{dV}{dt} + V\frac{dT_{cw}}{dt} \tag{A.3}$$

The right hand side of (A.1) is written as

$$\frac{1}{\rho c_p}(\text{RHS}) = q_{in}(T_{cw} - T_{ref}) + q_{cw}(T_{cw} - T_{ref}) - q_{hw}(T_{hw} - T_{ref}) + \frac{Q - Q_{loss}}{\rho c_p} \tag{A.4}$$

Adding and subtracting $q_{hw}(T_{cw} - T_{ref})$ to the right hand side of (A.4), after some rearrangements yields

$$\frac{1}{\rho c_p}(RHS) = (T_{cw} - T_{ref})\overbrace{(q_{in} + q_{cw} - q_{hw})}^{dV/dt} - \overbrace{q_{hw}(T_{hw} - T_{cw})}^{Q_{\text{demand}}/\rho c_p} + \frac{Q - Q_{loss}}{\rho c_p} \tag{A.5}$$

where we have introduced the mass balance:

$$\frac{dV}{dt} = q_{in} + q_{cw} - q_{hw} \tag{A.6}$$

Finally, by equating (A.5) and (A.3) we get the alternative form of the energy balance:

$$\frac{dE}{dt} = Q - Q_{\text{demand}} - Q_{loss} - \rho V c_p \frac{dT_{cw}}{dt} \tag{A.7}$$

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