Simplified dynamic models for control of riser slugging in offshore oil production

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Simplified dynamic models for control of riser slugging in offshore oil production

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Abstract

Elaborated models such as those used for simulation purposes (e.g. OLGA simulator) cannot be used for model-based control design. The focus of this paper is on deriving simple dynamical models with few state variables that capture the essential behaviour for control. We propose a new simplified dynamic model for severe slugging flow in pipeline-riser systems. The proposed model, together with five other simplified models found in the literature, are compared with results from the OLGA simulator. The new model can be extended to other cases, and we consider also a well-pipeline-riser system. The proposed simple models are able to represent the main dynamics of severe slugging flow and compare well with OLGA simulations and experiments.

Keywords: Oil production, two-phase flow, severe slugging, riser slugging, dynamical model

1 Introduction

Severe slugging flow regimes usually occur in pipeline-riser systems that transport oil and gas mixture from the seabed to the surface. Such flow regimes, also referred to as "riser slugging", are characterised by severe flow and pressure oscillations. Slugging problems have also been observed in gas-lifted oil wells where two types of instabilities, casing heading and density wave instability, have been reported (Hu and Golan (2003)).

Slugging has been recognised as a serious problem in offshore oilfields, because the irregular flow caused by slugging can cause serious operational problems for the downstream surface facilities (e.g. overflow of inlet separators). Therefore, effective ways to handle or remove riser slugging are needed, and many efforts have been made in order to prevent such occurrences (Courbot (1996), Havre et al. (2000)). The conventional solution is to partially close the top-side choke valve (choking), but this may reduce the production rate especially for fields where the reservoir pressure is relatively low. Therefore, a solution that guarantees stable flow together with the maximum possible production rate is desirable.

Fortunately, automatic feedback control has been shown to be an effective strategy to eliminate the slugging problem. As shown in Figure 1, the top-side choke valve is usually used as the manipulated variable to regulate (control) the riser base pressure (P_{rb}) at a given pressure set-point (P_{set}). Such a system is referred to as 'anti-slug control' and it aims at stabilising the flow in the pipeline at the operating conditions that, without control, would lead to riser

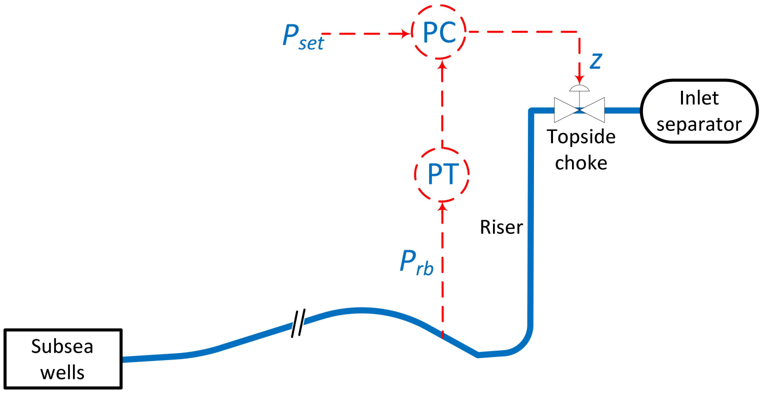


Figure 1: Preventing slug flow by control of riser base pressure

slugging. Different anti-slug control strategies have tested in practice (Havre et al. (2000)) and experimentally (Godhavn et al. (2005)).

OLGA is a commercial multiphase simulator widely used in the oil industry (Bendlksen et al. (1991)). When the development of this simulator was initiated in the 80's, one of motivations was to study the dynamics of slugging flow regimes at offshore oilfields. There have been some research on riser slugging using the OLGA simulator to test anti-slug control (Fard et al. (2006)), but for controllability analysis and controller design a simpler dynamical model of the system is desired.

The focus of this paper is on deriving the simple dynamical models which capture the essential behaviour for control. For control, it is more important to capture the main dynamics for the onset of slugging, not the slugging itself. The aim is to avoid the slug flow regime and, instead, operate at a steady (non-slug) flow regime. Therefore, the shape and length of the slugs are not the main concerns in this modeling.

Five simplified dynamical models for the pipeline-riser systems were found in the literature. The "Storkaas model" (Storkaas and Skogestad (2003)) is a three-dimensional state-space model which has been used for controllability analysis (Storkaas and Skogestad (2007)). The "Eikrem model" is a four dimensional state-space model (Tuvnes (2008), Eikrem (2008)). Another simplified model, referred to as the "Kaasa model" (Kaasa et al. (2008)), only predicts the pressure at the bottom of the riser. The "Nydal model" (Martins Da Silva et al. (2010)) is the only model that includes friction in the pipes. The most recently published simplified model is the "Di Meglio model" (Di Meglio et al. (2009), Di Meglio et al. (2010)). In addition, we present a new four-state model which includes useful features of the other five models. The six models are simulated in the time domain and compared to results from the more detailed OLGA model in the following five aspects, listed in order of importance:

- Critical valve opening for onset of slugging
- Frequency of oscillations at the critical point (onset of slugging)
- Dynamical response to a step change in the valve opening (non-slug regime)
- Steady-state pressure and flow rate values (non-slug regime)
- Maximum and minimum (pressure and flow rate) of the oscillations (slug regime)

For simplicity, we consider two-phase flow (gas phase and bulk liquid phase) in this paper. The bulk liquid phase can include oil and water phases when we have a three phase systems. In this case, the density and other parameters for the liquid phase would be the averages of the parameters for the oil and water phases. This simplification does not have any significant effect on the main dynamics of slugging.

The simplified models are also analysed linearly in the frequency domain where we consider the location of unstable poles and important unstable (Right-Half-Plane, RHP) zeros in the model. The results presented in this paper have been partially presented by Jahanshahi and Skogestad (2011), and Jahanshahi et al. (2012). In the present paper, we compare the new model to the experiments and we extend it to a well-pipeline-riser system.

This paper is organised as follows. In Section 2, we present our new simplified model for pipeline-riser systems. Then, In Section 3, we compare the proposed model and the other simple models to the results from the OLGA simulator and experiments. Finally, in Section 4, we extend this model to well-pipeline-riser systems and compare the extended model to OLGA simulations.

2 New simplified four-state model

2.1 Mass balance equations for pipeline and riser

For the new simplified model, consider the schematic presentation of the system in Figure 2. The four differential equations in the proposed model are simply the mass conservation law for the gas and liquid phases in the pipeline and riser sections:

$$\frac{\mathsf{d}\,(m_{\mathrm{G}})_{\mathrm{p}}}{\mathsf{d}t} = (w_{\mathrm{G}})_{\mathrm{in}} - (w_{\mathrm{G}})_{\mathrm{rb}} \tag{1a}$$

$$\frac{\mathsf{d}\,(m_{\mathrm{L}})_{\mathrm{p}}}{\mathsf{d}t} = (w_{\mathrm{L}})_{\mathrm{in}} - (w_{\mathrm{L}})_{\mathrm{rb}} \tag{1b}$$

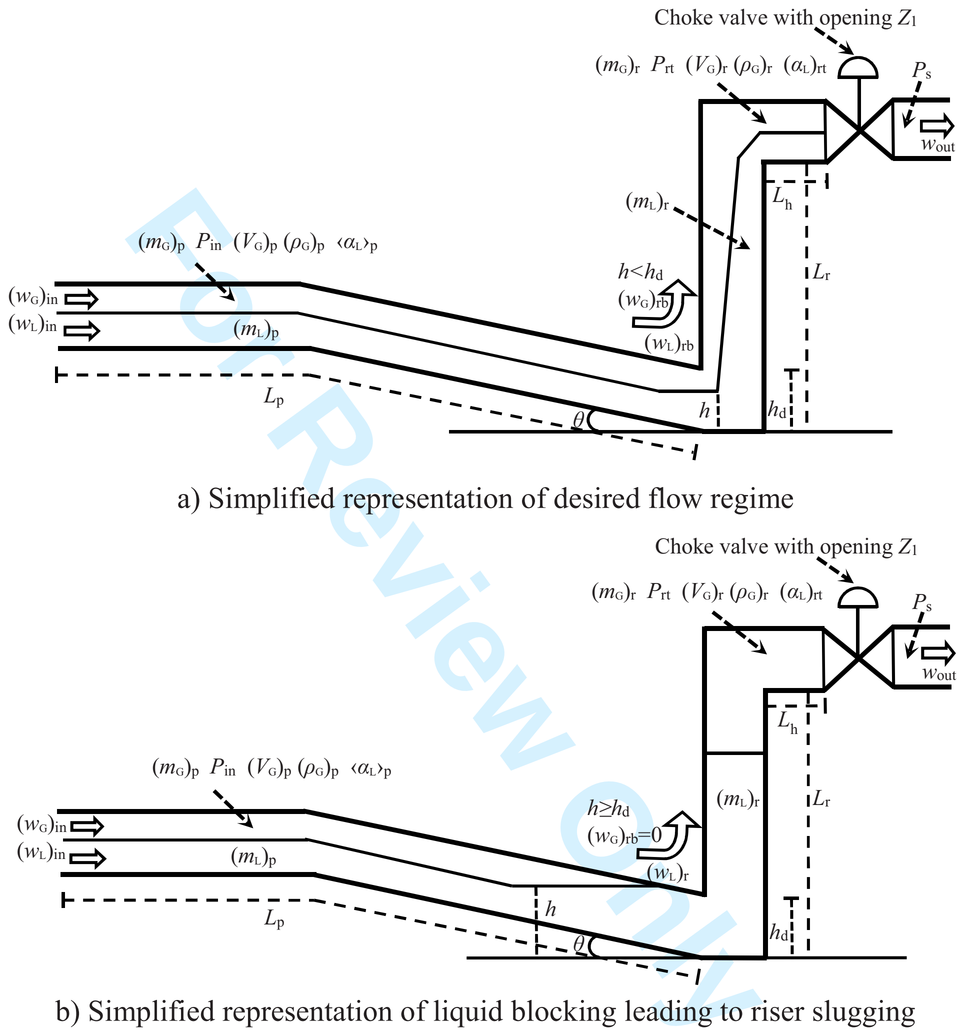
$$\frac{\mathsf{d}\,(m_{\mathrm{G}})_{\mathrm{r}}}{\mathsf{d}t} = (w_{\mathrm{G}})_{\mathrm{rb}} - (w_{\mathrm{G}})_{\mathrm{out}} \tag{1c}$$

$$\frac{\mathsf{d}\,(m_{\mathrm{L}})_{\mathrm{r}}}{\mathsf{d}t} = (w_{\mathrm{L}})_{\mathrm{rb}} - (w_{\mathrm{L}})_{\mathrm{out}} \tag{1d}$$

The four state variables in the model are:

- $(m_{\mathrm{G}})_{\mathrm{p}}$: mass of gas in pipeline [kg]
- $(m_{\mathrm{L}})_{\mathrm{p}}$: mass of liquid in pipeline [kg]
- $(m_{\mathrm{G}})_{\mathrm{r}}$: mass of gas in riser [kg]
- $(m_{\mathrm{L}})_{\mathrm{r}}$: mass of liquid in riser [kg]

The time derivatives of the state variables appear on the left-hand side of the state equations (1a)-(1d). w [kg/s] on the right-hand side denotes mass flow rate, and the subscripts 'in', 'rb' and 'out' stand for 'inlet', 'riser base' and 'outlet' respectively. The flow rates on the right-hand side are calculated by the additional model equations described below.



b) Simplified representation of liquid blocking leading to riser slugging

Figure 2: Pipeline-riser system with important parameters

2.2 Boundary conditions

Inlet conditions

In equations (1a) and (1b), $(w_{\mathrm{G}})_{\mathrm{in}}$ and $(w_{\mathrm{L}})_{\mathrm{in}}$ are the inlet gas and liquid mass flow rates. They are assumed to be constant in the four state model, but the inlet boundary conditions are changed in the extended model to make the inlet flow pressure-driven.

Outlet conditions

We consider a constant pressure (separator pressure, P_{s}) as the outlet boundary condition, and a simple choke valve equation determines the outflow of the two-phase mixture.

$$w_{\mathrm{out}} = C_{\mathrm{v1}} f(z_1) \sqrt{\rho_{\mathrm{rt}} \max(P_{\mathrm{rt}} - P_{\mathrm{s}}, 0)}, \tag{2}$$

Here $0 < z_1 < 1$ is the normalized valve opening (we use the 'capital' Z_1 when the valve opening is given in percentage, $0 < Z_1 < 100$) and $f(z_1)$ is the characteristic equation of the valve. In our simulations, a linear valve is used, i.e. $f(z_1) = z_1$, but this should be changed for other valves. The individual outlet mass flow rates of liquid and gas are calculated as follows,

$$(w_{\mathrm{L}})_{\mathrm{out}} = (\alpha_{\mathrm{L}}^{\mathrm{m}})_{\mathrm{rt}}\, w_{\mathrm{out}}, \tag{3}$$

$$(w_{\mathrm{G}})_{\mathrm{out}} = [1 - (\alpha_{\mathrm{L}}^{\mathrm{m}})_{\mathrm{rt}}]\, w_{\mathrm{out}}. \tag{4}$$

Here, $(\alpha_{\mathrm{L}}^{\mathrm{m}})_{\mathrm{rt}}$ [kg/kg] is the liquid mass fraction at top of the riser, which is given by

$$(\alpha_{\mathrm{L}}^{\mathrm{m}})_{\mathrm{rt}} = \frac{(\alpha_{\mathrm{L}})_{\mathrm{rt}}\, \rho_{\mathrm{L}}}{(\alpha_{\mathrm{L}})_{\mathrm{rt}}\, \rho_{\mathrm{L}} + [1 - (\alpha_{\mathrm{L}})_{\mathrm{rt}}]\, (\rho_{\mathrm{G}})_{\mathrm{r}}}. \tag{5}$$

Note the subscript 'rt' to define 'riser top', and just 'r' for 'riser'. The liquid volume fraction, $(\alpha_{\mathrm{L}})_{\mathrm{rt}}$ [m^3/m^3] is calculated by equation (39). The density of the two-phase mixture at top of the riser in equation (2) is

$$\rho_{\mathrm{rt}} = (\alpha_{\mathrm{L}})_{\mathrm{rt}}\, \rho_{\mathrm{L}} + [1 - (\alpha_{\mathrm{L}})_{\mathrm{rt}}]\, (\rho_{\mathrm{G}})_{\mathrm{r}}. \tag{6}$$

2.3 Pipeline model

The liquid volume fraction, α_{L}, in the pipeline section is given by the liquid mass fraction, α_{L}^{m}, and densities of the two phases (Brill and Beggs (1991)):

$$\alpha_{\mathrm{L}} = \frac{\alpha_{\mathrm{L}}^{\mathrm{m}}/\rho_{\mathrm{L}}}{\alpha_{\mathrm{L}}^{\mathrm{m}}/\rho_{\mathrm{L}} + (1 - \alpha_{\mathrm{L}}^{\mathrm{m}})/\rho_{\mathrm{G}}}$$

The average liquid mass fraction in the pipeline section is assumed to be given by the inflow boundary condition:

$$\langle \alpha_{\mathrm{L}}^{\mathrm{m}} \rangle_{\mathrm{p}} = \frac{(w_{\mathrm{L}})_{\mathrm{in}}}{(w_{\mathrm{L}})_{\mathrm{in}} + (w_{\mathrm{G}})_{\mathrm{in}}}$$

Throughout this paper, $\langle . \rangle$ denotes the average operator. The average liquid volume fraction in the pipeline is then

$$\langle \alpha_{\mathrm{L}} \rangle_{\mathrm{p}} = \frac{\langle \rho_{\mathrm{G}} \rangle_{\mathrm{p}}\, (w_{\mathrm{L}})_{\mathrm{in}}}{\langle \rho_{\mathrm{G}} \rangle_{\mathrm{p}}\, (w_{\mathrm{L}})_{\mathrm{in}} + \rho_{\mathrm{L}}\, (w_{\mathrm{G}})_{\mathrm{in}}}. \tag{7}$$

The gas density $\langle \rho_{\mathrm{G}} \rangle_{\mathrm{p}}$ is calculated based on the nominal pressure (steady-state) in the pipeline by assuming ideal gas,

$$\langle \rho_{\mathrm{G}} \rangle_{\mathrm{p}} = \frac{P_{\mathrm{in,nom}} M_{\mathrm{G}}}{R T_{\mathrm{p}}} \tag{8}$$

Here, $P_{\mathrm{in,nom}}$, which itself depends on $\langle \alpha_{\mathrm{L}} \rangle_{\mathrm{p}}$, is calculated from a steady-state solution of the combined equations for the overall model. By using (8) and constant (nominal) inflow rates, we get $\langle \alpha_{\mathrm{L}} \rangle_{\mathrm{p}}$ in (7) as a constant parameter.

The cross-sectional area of the pipeline is $A_{\mathrm{p}} = (\pi/4)\, D_{\mathrm{p}}^2$, where D_{p} is the diameter of the pipeline, and the volume of the pipeline is $V_{\mathrm{p}} = A_{\mathrm{p}} L_{\mathrm{p}}$. When gas and liquid are distributed homogeneously along the pipeline, the average mass of liquid in the pipeline is

$$\langle m_{\mathrm{L}} \rangle_{\mathrm{p}} = \rho_{\mathrm{L}} V_{\mathrm{p}} \langle \alpha_{\mathrm{L}} \rangle_{\mathrm{p}}. \tag{9}$$

With this assumption, the level of liquid in the pipeline at the low-point is given approximately by $\langle h \rangle \approx h_{\mathrm{d}} \langle \alpha_{\mathrm{L}} \rangle_{\mathrm{p}}$ where $h_{\mathrm{d}} = D_{\mathrm{p}}/cos(\theta)$ is the pipeline opening at the riser base and θ is the pipe inclination with respect to horizontal ($0° \leq \theta < 90°$) at the low-point. More precisely, we use in the model

$$\langle h \rangle = K_{\mathrm{h}} h_{\mathrm{d}} \langle \alpha_{\mathrm{L}} \rangle_{\mathrm{p}} \tag{10}$$

where K_{h} is a correction factor around unity which can be used to fine-tune the model. If the liquid content of the pipeline increases by $\Delta\,(m_{\mathrm{L}})_{\mathrm{p}}$, it starts to fill up the pipeline from the low-point. A length of pipeline equal to ΔL will be occupied by only liquid, where $\Delta\,(m_{\mathrm{L}})_{\mathrm{p}} = (m_{\mathrm{L}})_{\mathrm{p}} - \langle m_{\mathrm{L}} \rangle_{\mathrm{p}} = \Delta L A_{\mathrm{p}} (1 - \langle \alpha_{\mathrm{L}} \rangle_{\mathrm{p}}) \rho_{\mathrm{L}}$ and the level of liquid in the pipeline becomes $h = \langle h \rangle + \Delta L \sin(\theta)$ or

$$h = \langle h \rangle + \left(\frac{(m_{\mathrm{L}})_{\mathrm{p}} - \langle m_{\mathrm{L}} \rangle_{\mathrm{p}}}{A_{\mathrm{p}} (1 - \langle \alpha_{\mathrm{L}} \rangle_{\mathrm{p}}) \rho_{\mathrm{L}}} \right) \sin(\theta). \tag{11}$$

Thus, the level of liquid in the pipeline, h, can be written as a function of liquid mass in the pipeline $(m_{\mathrm{L}})_{\mathrm{p}}$ which is a state variable of the model. The remaining parameters in (11) are constants.

The pipeline gas density is $(\rho_{\mathrm{G}})_{\mathrm{p}} = (m_{\mathrm{G}})_{\mathrm{p}} / (V_{\mathrm{G}})_{\mathrm{p}}$, where the volume occupied by gas in the pipeline is $(V_{\mathrm{G}})_{\mathrm{p}} = V_{\mathrm{p}} - (m_{\mathrm{L}})_{\mathrm{p}} / \rho_{\mathrm{L}}$. The pressure at the inlet of the pipeline, assuming ideal gas, is

$$P_{\mathrm{in}} = \frac{(\rho_{\mathrm{G}})_{\mathrm{p}} R T_{\mathrm{p}}}{M_{\mathrm{G}}}. \tag{12}$$

We consider only the liquid phase when calculating the friction pressure loss in the pipeline (Brill and Beggs (1991)).

$$(\Delta P_f)_{\mathrm{p}} = \frac{\lambda_{\mathrm{p}} \rho_{\mathrm{L}} \langle U_{\mathrm{sL}} \rangle_{\mathrm{p}}^{2} L_{\mathrm{p}}}{2 D_{\mathrm{p}}} \tag{13}$$

Here, λ_{p} is the friction factor of the pipeline. The correlation developed by Drew et al. (1932) for turbulent flow in smooth wall pipes is used as the friction factor in the pipeline.

$$\lambda_{\mathrm{p}} = 0.0056 + 0.5\,(N_{\mathrm{Re}})_{\mathrm{p}}^{-0.32} \tag{14}$$

This is also the equation recommended by Dukler et al. (1964) in their horizontal two-phase flow correlation (Brill and Beggs (1991)). Here, the Reynolds number is

$$(N_{\mathrm{Re}})_{\mathrm{p}} = \frac{\langle \rho_{\mathrm{m}} \rangle_{\mathrm{p}} \langle U_{\mathrm{m}} \rangle_{\mathrm{p}} D_{\mathrm{p}}}{\langle \mu_{\mathrm{m}} \rangle_{\mathrm{p}}}. \tag{15}$$

In equation (15), the average mixture density is given by

$$\langle \rho_{\mathrm{m}} \rangle_{\mathrm{p}} = \langle \alpha_{\mathrm{L}} \rangle_{\mathrm{p}} \rho_{\mathrm{L}} + [1 - \langle \alpha_{\mathrm{L}} \rangle_{\mathrm{p}}] \left(\rho_{\mathrm{G}} \right)_{\mathrm{p}} , \tag{16}$$

and $\langle \mu_{\mathrm{m}} \rangle_{\mathrm{p}}$ is the average mixture viscosity,

$$\langle \mu_{\mathrm{m}} \rangle_{\mathrm{p}} = \langle \alpha_{\mathrm{L}} \rangle_{\mathrm{p}} \mu_{\mathrm{L}} + [1 - \langle \alpha_{\mathrm{L}} \rangle_{\mathrm{p}}] \mu_{\mathrm{G}} . \tag{17}$$

$\langle U_{\mathrm{m}} \rangle_{\mathrm{p}} = \langle U_{\mathrm{sL}} \rangle_{\mathrm{p}} + \langle U_{\mathrm{sG}} \rangle_{\mathrm{p}}$ is the average mixture velocity in the pipe where

$$\langle U_{\mathrm{sG}} \rangle_{\mathrm{p}} = \frac{(w_{\mathrm{G}})_{\mathrm{in}}}{(\rho_{\mathrm{G}})_{\mathrm{p}} A_{\mathrm{p}}}, \tag{18}$$

$$\langle U_{\mathrm{sL}} \rangle_{\mathrm{p}} = \frac{(w_{\mathrm{L}})_{\mathrm{in}}}{\rho_{\mathrm{L}} A_{\mathrm{p}}}. \tag{19}$$

2.4 Riser model

The total volume of the riser is

$$V_{\mathrm{r}} = A_{\mathrm{r}} (L_{\mathrm{r}} + L_{\mathrm{h}}), \tag{20}$$

where $A_{\mathrm{r}} = (\pi/4)\, D_{\mathrm{r}}^2$ is the cross-sectional area of the riser. The volume occupied by gas in the riser is

$$(V_{\mathrm{G}})_{\mathrm{r}} = V_{\mathrm{r}} - (m_{\mathrm{L}})_{\mathrm{r}} / \rho_{\mathrm{L}}, \tag{21}$$

and the density of gas in the riser is $(\rho_{\mathrm{G}})_{\mathrm{r}} = (m_{\mathrm{G}})_{\mathrm{r}} / (V_{\mathrm{G}})_{\mathrm{r}}$. Then, the pressure at the top of the riser from ideal gas law becomes

$$P_{\mathrm{rt}} = \frac{(\rho_{\mathrm{G}})_{\mathrm{r}} R T_{\mathrm{r}}}{M_{\mathrm{G}}}. \tag{22}$$

The average liquid volume fraction in the riser is

$$\langle \alpha_{\mathrm{L}} \rangle_{\mathrm{r}} = \frac{(m_{\mathrm{L}})_{\mathrm{r}}}{(V_{\mathrm{r}} \rho_{\mathrm{L}})}, \tag{23}$$

and the average density of the mixture inside the riser is

$$\langle \rho_{\mathrm{m}} \rangle_{\mathrm{r}} = \frac{(m_{\mathrm{G}})_{\mathrm{r}} + (m_{\mathrm{L}})_{\mathrm{r}}}{V_r}. \tag{24}$$

The friction loss in the riser is

$$(\Delta P_f)_{\mathrm{r}} = \frac{\lambda_{\mathrm{r}} \langle \rho_{\mathrm{m}} \rangle_{\mathrm{r}} \langle U_{\mathrm{m}} \rangle_{\mathrm{r}}^2 (L_{\mathrm{r}} + L_{\mathrm{h}})}{2 D_{\mathrm{r}}}, \tag{25}$$

where the friction factor λ_{r} is computed from an explicit approximation of the implicit Colebrook-White equation (Haaland (1983)):

$$\frac{1}{\sqrt{\lambda_{\mathrm{r}}}} = -1.8 \log_{10} \left[\left(\frac{\epsilon / D_{\mathrm{r}}}{3.7} \right)^{1.11} + \frac{6.9}{(N_{\mathrm{Re}})_{\mathrm{r}}} \right] \tag{26}$$

where the Reynolds number for the flow in the riser is

$$(N_{\mathrm{Re}})_{\mathrm{r}} = \frac{\langle \rho_{\mathrm{m}} \rangle_{\mathrm{r}} \langle U_{\mathrm{m}} \rangle_{\mathrm{r}} D_{\mathrm{r}}}{\langle \mu_{\mathrm{m}} \rangle_{\mathrm{r}}} \tag{27}$$

The average mixture velocity in the riser is $\langle U_{\mathrm{m}}\rangle_{\mathrm{r}} = \langle U_{\mathrm{sL}}\rangle_{\mathrm{r}} + \langle U_{\mathrm{sG}}\rangle_{\mathrm{r}}$ where

$$\langle U_{\mathrm{sL}}\rangle_{\mathrm{r}} = \frac{(w_{\mathrm{L}})_{\mathrm{in}}}{\rho_{\mathrm{L}} A_{\mathrm{r}}}, \tag{28}$$

$$\langle U_{\mathrm{sG}}\rangle_{\mathrm{r}} = \frac{(w_{\mathrm{G}})_{\mathrm{in}}}{(\rho_{\mathrm{G}})_{\mathrm{r}} A_{\mathrm{r}}}. \tag{29}$$

In equation (27), $\langle \mu_{\mathrm{m}}\rangle_{\mathrm{r}}$ is the average mixture viscosity,

$$\langle \mu_{\mathrm{m}}\rangle_{\mathrm{r}} = \langle \alpha_{\mathrm{L}}\rangle_{\mathrm{r}}\mu_{\mathrm{L}} + [1 - \langle \alpha_{\mathrm{L}}\rangle_{\mathrm{r}}]\mu_{\mathrm{G}}. \tag{30}$$

2.5 Gas flow model at riser base

As illustrated in Figure 2(b), when the liquid level in the pipeline section exceeds the openings of the pipeline at the riser base ($h > h_{\mathrm{d}}$), the liquid blocks the low-point and the gas flow rate $(w_{\mathrm{G}})_{\mathrm{rb}}$ at the riser base is zero,

$$(w_{\mathrm{G}})_{\mathrm{rb}} = 0, \quad h \geq h_{\mathrm{d}} \tag{31}$$

When the liquid is not blocking at the low-point ($h < h_{\mathrm{d}}$ in Figure 2(a)), the gas will flow from the volume $(V_{\mathrm{G}})_{\mathrm{p}}$ to $(V_{\mathrm{G}})_{\mathrm{r}}$ with a mass rate $(w_{\mathrm{G}})_{\mathrm{rb}}$ [kg/s] which is assumed to be given by an"orifice equation" (e.g. Skogestad (2009)):

$$(w_{\mathrm{G}})_{\mathrm{rb}} = K_{\mathrm{G}} A_{\mathrm{G}} \sqrt{(\rho_{\mathrm{G}})_{\mathrm{p}}\, \Delta P_{\mathrm{G}}}, \quad h < h_{\mathrm{d}} \tag{32}$$

where

$$\Delta P_{\mathrm{G}} = P_{\mathrm{in}} - (\Delta P_f)_{\mathrm{p}} - P_{\mathrm{rt}} - \langle \rho_{\mathrm{m}}\rangle_{\mathrm{r}} g L_{\mathrm{r}} - (\Delta P_f)_{\mathrm{r}}\,. \tag{33}$$

The free area for gas, A_{G}, can be calculated precisely using trigonometric functions (Storkaas and Skogestad (2003)), but for simplicity, a quadratic approximation is used in this model,

$$A_{\mathrm{G}} = \begin{cases} A_{\mathrm{p}}\left(\frac{h_{\mathrm{d}}-h}{h_{\mathrm{d}}}\right)^2, & \text{if} \quad h < h_{\mathrm{d}}; \\ 0, & \text{if} \quad h \geq h_{\mathrm{d}}. \end{cases} \tag{34}$$

2.6 Liquid flow model at riser base

The liquid mass flow rate at the riser base is also described by an orifice equation:

$$(w_{\mathrm{L}})_{\mathrm{rb}} = K_{\mathrm{L}} A_{\mathrm{L}} \sqrt{\rho_{\mathrm{L}} \Delta P_{\mathrm{L}}}, \tag{35}$$

where

$$\Delta P_{\mathrm{L}} = P_{\mathrm{in}} - (\Delta P_f)_{\mathrm{p}} + \rho_{\mathrm{L}} g h - P_{\mathrm{rt}} - \langle \rho_{\mathrm{m}}\rangle_{\mathrm{r}} g L_{\mathrm{r}} - (\Delta P_f)_{\mathrm{r}} \tag{36}$$

and

$$A_{\mathrm{L}} = A_{\mathrm{p}} - A_{\mathrm{G}} \tag{37}$$

2.7 Phase distribution model at outlet choke valve

In order to calculate the mass flow rates of the individual phases as given in equations (2)-(6), the phase distribution at top of the riser must be known. The liquid volume fraction at top of the riser, $(\alpha_\mathrm{L})_\mathrm{rt}$, can be calculated by the entrainment model proposed by Storkaas and Skogestad (2003), but their entrainment equations are complicated which make the model stiff for numerical solvers. Instead, we use the fact that in a vertical gravity-dominant two-phase pipeline there is approximately a linear relationship between the pressure and the liquid volume fraction. This has also been observed in OLGA simulations. In addition, the pressure gradient is assumed constant along the riser for the desired non-slugging flow regimes, which then gives that the liquid volume fraction gradient is constant, i.e. $\frac{\partial(\alpha_\mathrm{L})_\mathrm{r}}{\partial y}$ = constant. It then follows that the average liquid volume fraction in the riser is

$$\langle\alpha_\mathrm{L}\rangle_\mathrm{r} = \frac{(\alpha_\mathrm{L})_\mathrm{rt} + (\alpha_\mathrm{L})_\mathrm{rb}}{2} \tag{38}$$

Here, $\langle\alpha_\mathrm{L}\rangle_\mathrm{r}$ is given by equation (23), and $(\alpha_\mathrm{L})_\mathrm{rb}$ is determined by the flow area of the liquid phase at the riser base (low-point) as $(\alpha_\mathrm{L})_\mathrm{rb} = A_\mathrm{L}/A_\mathrm{p}$. Therefore, the liquid volume fraction at the top of the riser becomes

$$(\alpha_\mathrm{L})_\mathrm{rt} = 2\langle\alpha_\mathrm{L}\rangle_\mathrm{r} - (\alpha_\mathrm{L})_\mathrm{rb} = \frac{2\,(m_\mathrm{L})_\mathrm{r}}{V_\mathrm{r}\rho_\mathrm{L}} - \frac{A_\mathrm{L}}{A_\mathrm{p}} \tag{39}$$

3 Comparison of models

3.1 OLGA test case and reference model

In order to study the dominant dynamic behavior of a typical, yet simple riser slugging problem, we consider the test case for severe slugging used in the OLGA simulator. The geometry of the system is given in Figure 3. The pipeline diameter is 0.12 m and its length is 4300 m. Starting from the inlet, the first 2000 m of the pipeline is horizontal and the remaining 2300 m inclines downwards with a 1° angle. This gives a 40.14 m descent and creates a low point at the end of the pipeline. The riser is a vertical 300 m pipe with a diameter of 0.1 m. A 100 m horizontal section with the same diameter as that of the riser connects the riser to the outlet choke valve. The feed into the system is nominally constant at 9 kg/s, with $(w_\mathrm{L})_\mathrm{in} = 8.64$ kg/s (oil) and $(w_\mathrm{G})_\mathrm{in} = 0.36$ kg/s (gas). The separator pressure (P_s) after the choke valve, is nominally constant at 50.1 bar. This leaves the choke valve opening Z_1 as the only control degree of freedom (manipulated variable) in the system. The model constants for the OLGA case are given in Table 1.

For the present case study, the critical value of the relative valve opening for the transition between a stable non-oscillatory flow regime and riser slugging is $Z_1^* = 5\%$. This is illustrated by the OLGA simulations in Figure 4 which show the inlet pressure, topside pressure and outlet flow rate, with the valve openings of 4% (no slug), 5% (transient) and 6% (riser slugging).

3.2 Model fitting

The four main tuning parameters for the new pipeline-riser model are:

- K_h: correction factor for level of liquid in pipeline (eq. 10)
- C_{v1}: production choke valve constant (eq. 2)
- K_G: coefficient for gas flow through low point (eq. 32)

Table 1: Model constants for pipeline-riser OLGA case

Symbol	Description	Value	Unit
R	Universal gas constant	8314	J/(kmol.K)
g	Gravity	9.81	m/s^2
μ_{L}	Liquid viscosity	1.43×10^{-4}	Pa.s
μ_{G}	Gas viscosity	1.39×10^{-5}	Pa.s
ϵ	Pipe roughness	2.80×10^{-5}	m
ρ_{L}	Liquid density	832.2	kg/m^3
M_{G}	Gas molecular weight	20	gr
T_{p}	Pipeline temperature	337	°K
V_{p}	Pipeline volume	48.63	m^3
D_{p}	Pipeline diameter	0.12	m
L_{p}	Pipeline length	4300	m
T_{r}	Riser temperature	298.3	°K
V_{r}	Riser volume	3.14	m^3
D_{r}	Riser diameter	0.1	m
L_{r}	Riser length	300	m
L_{h}	Length of horizontal section	100	m
P_{s}	Separator pressure	50.1	bar
$(w_{\mathrm{L}})_{\mathrm{in}}$	Inlet liquid mass flow	8.64	kg/s
$(w_{\mathrm{G}})_{\mathrm{in}}$	Inlet gas mass flow	0.36	kg/s

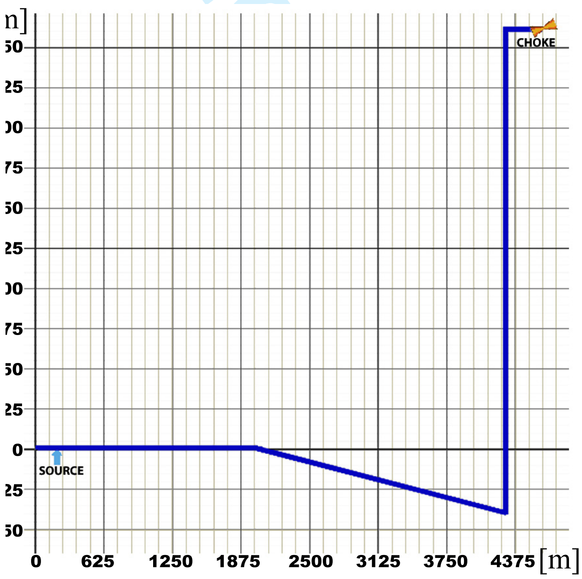


Figure 3: Geometry of OLGA pipeline-riser test case

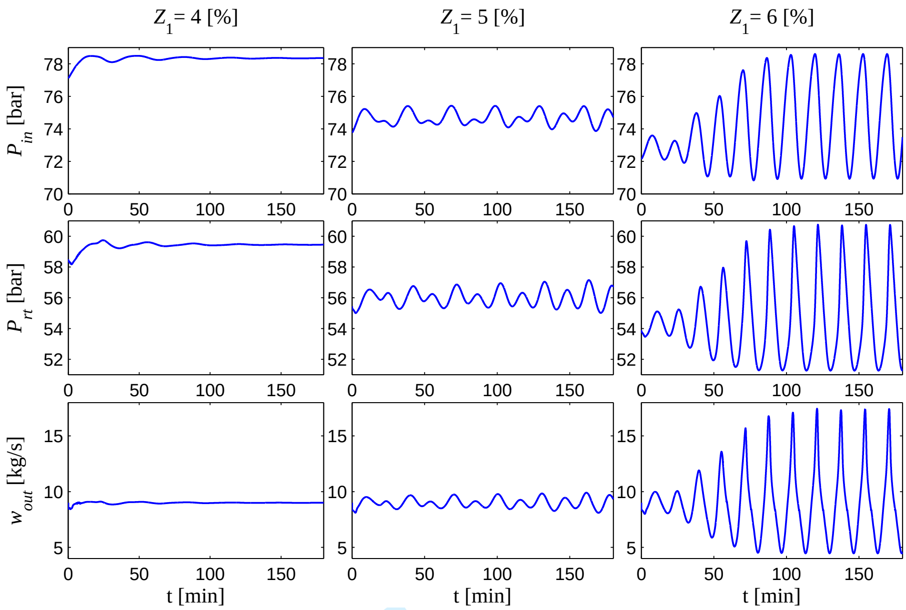


Figure 4: Simulations of OLGA test case for different valve openings

Table 2: Tuning parameters for pipeline-riser model fitted to OLGA simulations

Symbol	Description	Value
K_{h} [–]	Correction factor for level of liquid in pipeline	0.700
K_{G} [–]	Coefficient for gas flow through low point	3.87×10^{-2}
K_{L} [–]	Coefficient for liquid flow through low point	1.64×10^{-1}
C_{v1} [m^2]	Production choke valve constant	1.12×10^{-2}

- K_{L}: coefficient for liquid flow through low point (eq. 35)

The tuning parameters can be adjusted to fit the model to numerical data from a given pipeline-riser system. The six different simplified models were simulated in Matlab and their tuning parameters were adjusted to match the OLGA reference model simulations. We believe that the obtained tuning parameters for all the models result in the best possible fit. The four primary tuning parameters of the new model fitted to the OLGA case are given in Table 2.

We performed the model fitting by inverting the model equations to back-calculate the parameter values, and finally some manual adjustments. By this approach, we always obtain parameter values which give a single physically meaningful solution to the model. We avoided using optimization routines for this purpose, because the different model fitting criteria, mentioned in the introduction, make it very difficult to formulate and solve such an optimization problem. A more systematic approach has been proposed by Di Meglio et al. (2010) for tuning the Di Meglio model, but this approach did not work well for the present case study.

3.3 Comparison of models with OLGA simulations

The results are summarized in Table 3 which shows the error (in %) of various model fitting criteria. Our most important criterion for the model fitting is the critical value of the valve

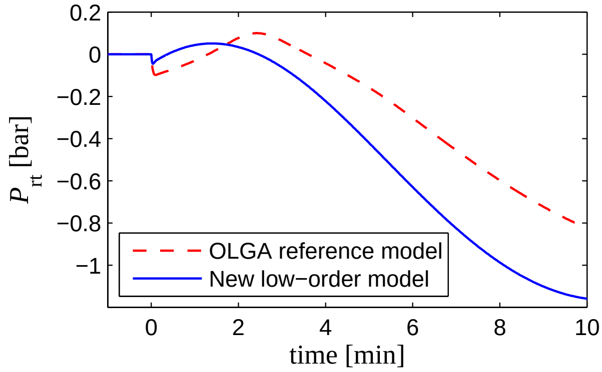


Figure 5: Step response of pressure at top of riser

opening (Z_1^*) and all the models were made to match this value. Next, we looked at the oscillation frequency at this point (T_c) which could not be matched exactly by Eikrem and Nydal models. For the present case study, we have $Z_1^* = 5\%$ and $T_c = 15.6\,[\text{min}]$.

3.3.1 Frequency of oscillations

All models were linearised at the critical operating point, $Z_1^* = 5\%$. The period of oscillations at this operating point is related to poles of the linear models. Most of the models give a pair of complex conjugate poles, $s = \pm\omega_c i = \pm 0.0067i$. Note that the critical frequency is

$$\omega_c = \frac{2\pi}{T_c}, \quad \frac{2\pi}{15.6\,[\text{min}]\;60\,[\text{s/min}]} = 0.0067\,\text{s}^{-1}$$

The exceptions are the Eikrem and Nydal models which are not able to get the right period time (15.6 [min]), and consequently they result in different poles at $Z_1^* = 5\%$.

3.3.2 Step response

Figure 5 shows the pressure response at the top of the riser to a step change in the valve opening from $Z_1 = 4\%$ to $Z_1 = 4.2\%$ for the OLGA reference model and the new model. Step responses of the OLGA model has one undershoot and one overshoot. The amplitudes of the overshoot and undershoot for different simplified models are given in Table 3 in the form of errors from those of the OLGA model. The inverse response (overshoot) corresponds to the Right-Half-Plane zeros near the imaginary axis which are also given for $Z_1 = 5\%$ in Table 3.

3.3.3 Bifurcation diagrams

Similar to the simulations shown in Figure 4, we simulate the models for different valve openings, covering the whole operation ranges of the valve, to get the bifurcations diagrams as shown in Figure 6. Bifurcations diagrams demonstrate the transition from non-slug to slugging flow at the critical valve opening. For $Z_1 > 5\%$, the steady-state (central line) is an unstable equilibrium for the system, like an inverted pendulum, but it can be stabilised using feedback control. The steady-sate behaviour of the new model and also the minimum and maximum of the oscillations

Table 3: Comparison of different simplified models to OLGA reference case

Parameters	OLGA Simulation	Storkaas Model	Eikrem Model	Kaasa Model	Nydal Model	Di Meglio Model	New Model
State equations	Many	3 diff. 1 alg.	4 diff.	3 diff.	4 diff.	4 diff.	4 diff.
Complexity	Complicated	Average	Simple	Very simple	Average	Simple	Average
Tuning parameters	—	5	3	7	3	5	4
RHP zeros of P_{rt} at $Z = 5\%$	—	0.0146	0.006 + 0.005i 0.006 − 0.005i	—	0.0046	0.019 + 0.034i 0.019 − 0.034i	0.0413 0.0126
Values from OLGA simulator		**Error of simplified models when compared to OLGA**					
Critical valve opening	**5%**	0 (0%)	0 (0%)	0 (0%)	0 (0%)	0 (0%)	0 (0%)
Period [min] at $Z = 5\%$	**15.6**	0 (0%)	26.15 (168%)	0 (0%)	15.97 (102%)	0 (0%)	0 (0%)
Step response of P_{rt}							
undershoot	**-0.098**	0.10 (99%)	0.08 (78%)		0.12 (89%)	0.07 (68%)	0.05 (54%)
overshoot	**0.198**	0.16 (80%)	0.18 (89%)	—	0.10 (50%)	0.17 (84%)	0.10 (51%)
at t=10 min	**-0.824**	0.43 (53%)	0.62 (75%)		0.25 (30%)	0.47 (58%)	0.33 (41%)
Steady-state							
P_{in} [bar]	**68.22**	1.9 (2.7%)	4.4 (6.46%)	—	1.77 (2.6%)	0.70 (1%)	0.21 (0.32%)
P_{rb} [bar]	**66.76**	—	—	0.02 (0.04%)	—	—	—
P_{rt} [bar]	**50.10**	0.01 (0.02%)	0.01 (0.02%)	—	0.01 (0.02%)	0.01 (0.02%)	0.01 (0.02%)
w_{out} [kg/s]	**9.00**	0 (0%)	0 (0%)	—	2.90 (32%)	0 (0%)	0 (0%)
Minimum							
P_{in} [bar]	**63.50**	2.7 (4.3%)	9 (14.2%)	—	8.3 (13%)	2.6 (4.1%)	2.0 (3.1%)
P_{rb} [bar]	**62.08**	—	—	2.57 (4.1%)	—	—	—
P_{rt} [bar]	**50.09**	4e-4 (8e-4%)	5e-4 (9e-4%)	—	0.41 (0.82%)	0.003 (.006%)	4e-4 (8e-4%)
w_{out} [kg/s]	**0.791**	0.55 (69%)	0.14 (17%)	—	0.79 (100%)	3.3 (405%)	0.55 (69%)
Maximum							
P_{in} [bar]	**75.83**	1.4 (1.8%)	1.89 (2.5%)	—	1.00 (1.3%)	1.3 (1.7%)	1.9 (2.6%)
P_{rb} [bar]	**74.55**	—	—	0.55 (.075%)	—	—	—
P_{rt} [bar]	**50.14**	1.5 (3%)	0.95 (1.9%)	—	1.09 (2.2%)	0.71 (0.35%)	0.1 (0.2%)
w_{out} [kg/s]	**31.18**	80 (278%)	57 (200%)	—	13.3 (43%)	22 (77%)	2.0 (7.0%)

are compared to those of the OLGA model in Figure 6. In order to have a quantitative comparison, deviations of the different simplified models from the OLGA reference model for fully open valve ($Z_1 = 100\%$) are summarised in Table 3.

Note that the steady-state behaviour is much more important for control purpose than the minimum and maximum oscillatory (slugging) pressure values. Another example is riding a bicycle where we want to control the bicycle at its upright position which is naturally unstable. However, when the bicycle falls down to the ground, it is too late to control it. Hence, a model for the fallen bicycle is not useful for control.

3.3.4 Comparison summary

As seen in Table 3, there is a trade-off between model complexity and the number of tuning parameters used to match the actual process data. Generally, models with simple structures can achieve a good fit to complicated physical processes by excess use of fitting parameters. A famous example for this is least square fitting of an elephant (Wei (1975)). Here, simple models like the Kaasa and the Di Meglio models use seven and five tuning parameters, respectively, to get a good fit. However, finding the parameter values is difficult. The Nydal model and the Eikrem model (with three parameters) are also simple, but they are not able to match the OLGA simulations because of few tuning parameters. The new model (with four parameters) is somewhat more complicated, but is able to give a good match with relatively few tuning parameters. Also, as opposed to the other simplified models, the new model does not require adjusting any physical property of the system, such as volume of gas in the pipeline.

The new model and the De Meglio model show similar accuracy in prediction of the steady-state and also minimum and maximum values, but dynamically the new model is closer to the OLGA simulations.

3.4 Comparison with experiments

The experiments were performed on a laboratory setup for anti-slug control at the Chemical Engineering Department of NTNU. Figure 7 shows a schematic presentation of the laboratory setup. The pipeline and the riser are made from flexible pipes with 2 cm inner diameter. The length of the pipeline is 4 m, and it is inclined with a 15° angle. The height of the riser is 3 m. A buffer tank is used to simulate the effect of a long pipe with the same volume, such that the total resulting length of pipe would be about 70 m. Other model constants are given in Table 4, and the four tuning parameters of the model fitted to the experiments are given in Table 5.

The topside choke valve is used as the input for control. The separator pressure after the topside choke valve is nominally constant at atmospheric pressure. The feed into the pipeline is assumed to be at constant flow rates, 4 litre/min of water and 4.5 litre/min of air. With these boundary conditions, the critical valve opening where the system switches from stable (non-slug) to oscillatory (slug) flow is at $Z_1^* = 15\%$.

In addition, we developed a new OLGA case with the same dimensions and boundary conditions as the experimental set-up. The bifurcation diagrams are shown in Figure 8 where simplified model (thin solid lines) is compared to the experiments (bold solid lines) and the OLGA model (dashed lines). In Figure 8, the system has a stable (non-slug) flow when the topside valve opening Z_1 is smaller than 15%, and it switches to slugging flow conditions for $Z_1 > 15\%$.

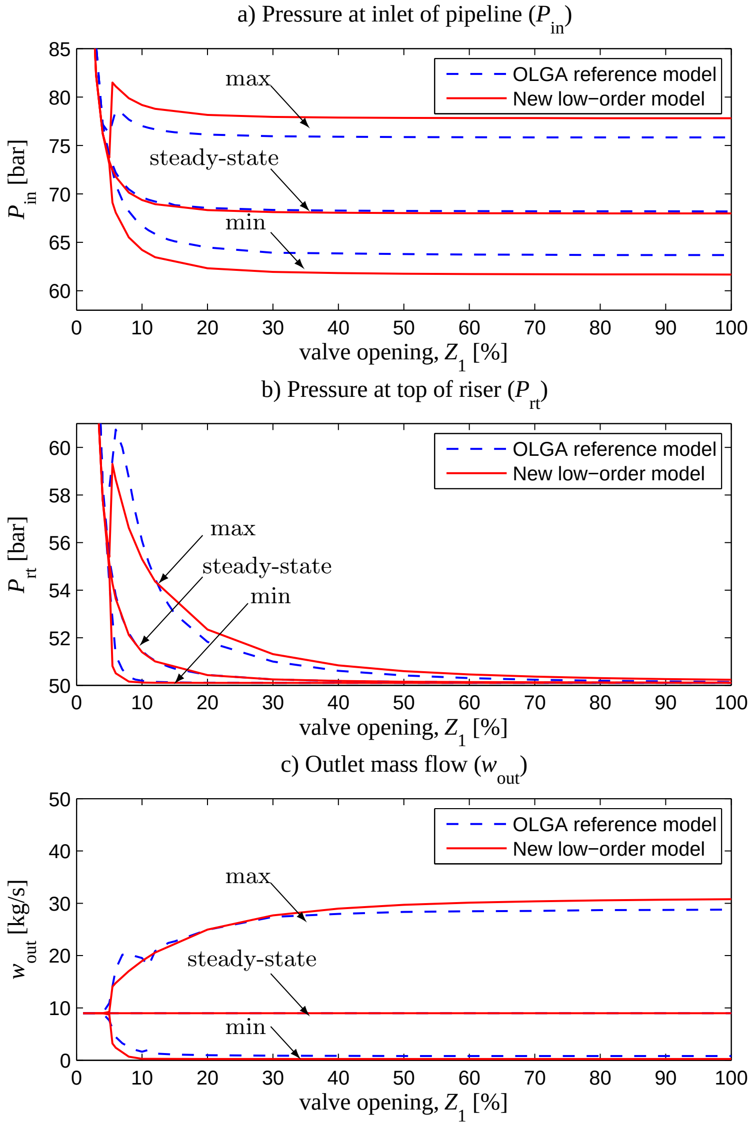


Figure 6: Bifurcation diagrams of simplified pipeline-riser model (solid lines) compared OLGA reference model (dashed)

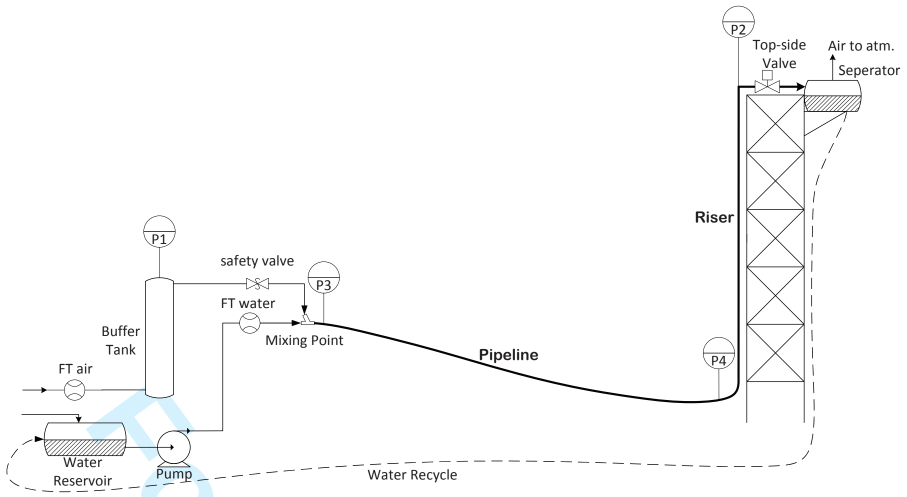


Figure 7: Experimental setup

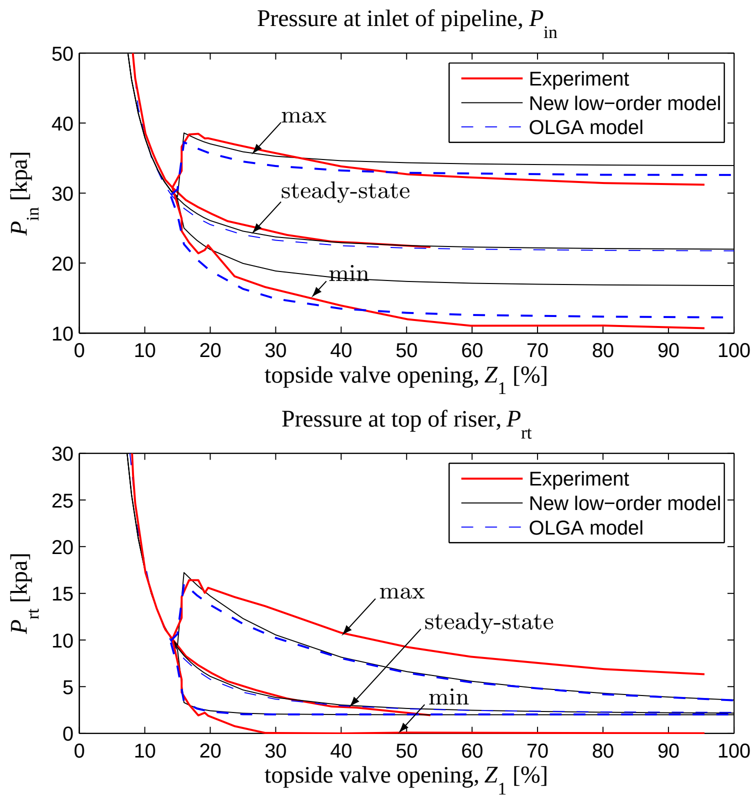


Figure 8: Bifurcation diagrams of simplified pipeline-riser model (thin solid lines) compared to experiments (thick red solid lines) and OLGA simulations (dashed blue lines)

Table 4: Model constants for small-scale experimental setup

Symbol	**Description**	**Value**	**Unit**
μ_L	Liquid viscosity	8.90×10^{-4}	Pa.s
μ_G	Liquid viscosity	1.81×10^{-5}	Pa.s
ϵ	Pipe roughness	1.00×10^{-6}	m
ρ_L	liquid density	1000	kg/m^3
M_G	Gas molecular weight	18	gr
T_p	Pipeline temperature	288	°K
V_p	Pipeline volume	0.0219	m^3
D_p	Pipeline diameter	0.02	m
L_p	Pipeline length	69.71	m
T_r	Riser temperature	288	°K
V_r	Riser volume	0.001	m^3
D_r	Riser diameter	0.02	m
L_r	Riser length	3	m
L_h	Length of horizontal section	0.2	m
P_s	Separator pressure	1.013	bar

Table 5: Tuning parameters for pipeline-riser model fitted to experiments

Symbol	**Description**	**Value**
K_h [–]	Correction factor for level of liquid in pipeline	1.00
K_G [–]	Coefficient for gas flow through low point	1.42×10^{-2}
K_L [–]	Coefficient for liquid flow through low point	1.90×10^{-1}
C_{v1} [m^2]	Production choke valve constant	2.39×10^{-4}

4 Well-Pipeline-Riser System

In the pipeline-riser model described above, constant gas and liquid flow rate were used as inlet boundary conditions. In order to study effect the of pressure-driven inflow which is more physically correct, we add an oil well and assume a constant reservoir pressure as the boundary condition (see Figure 9).

4.1 Simplified six-state model

We add two state variables, the mass of gas and mass of liquid inside the oil well, to the pipeline-riser system in (1a)–(1d) to obtain a six-state model. The two additional state equations are as follows,

$$\frac{\mathsf{d}\,(m_{\mathrm{G}})_{\mathrm{w}}}{\mathsf{d}t} = \left(\frac{\eta}{\eta+1}\right) w_{\mathrm{r}} - (w_{\mathrm{G}})_{\mathrm{wh}}\,, \tag{40}$$

$$\frac{\mathsf{d}\,(m_{\mathrm{L}})_{\mathrm{w}}}{\mathsf{d}t} = \left(\frac{1}{\eta+1}\right) w_{\mathrm{r}} - (w_{\mathrm{L}})_{\mathrm{wh}}\,, \tag{41}$$

where η is the average mass ratio of gas and liquid produced from the reservoir which is assumed to be a known parameter of the well. $(w_{\mathrm{G}})_{\mathrm{wh}}$ and $(w_{\mathrm{L}})_{\mathrm{wh}}$ are the mass flow rates of gas and liquid at the well-head. The production mass rate w_{r} [kg/s] from the reservoir to the well is assumed to be described by a linear Inflow Performance Relationship (IPR).

$$w_{\mathrm{r}} = C_{\mathrm{PI}} \max(0, P_{\mathrm{res}} - P_{\mathrm{bh}}), \tag{42}$$

where C_{PI} [kg/(sPa)] is the mass productivity constant of the well, P_{res} is the reservoir pressure, which can be assumed constant in a short period of time (e.g. few months), and P_{bh} is the flowing bottom-hole pressure of the well,

$$P_{\mathrm{bh}} = P_{\mathrm{wh}} + \langle\rho_{\mathrm{m}}\rangle_{\mathrm{w}} g L_{\mathrm{w}} + \Delta P_{fw}. \tag{43}$$

Here, $(\Delta P_f)_{\mathrm{w}}$ is the pressure loss due to friction in the well, which is assumed to be given as

$$(\Delta P_f)_{\mathrm{w}} = \frac{\lambda_{\mathrm{w}} \langle\rho_{\mathrm{m}}\rangle_{\mathrm{w}} \langle U_{\mathrm{m}}\rangle_{\mathrm{w}}{}^2 L_{\mathrm{w}}}{2D_{\mathrm{w}}}. \tag{44}$$

We get the friction factor of the well using the same correlation as for the riser by Haaland (1983).

$$\frac{1}{\sqrt{\lambda_{\mathrm{w}}}} = -1.8\log_{10}\left[\left(\frac{\epsilon/D_{\mathrm{w}}}{3.7}\right)^{1.11} + \frac{6.9}{(N_{\mathrm{Re}})_{\mathrm{w}}}\right] \tag{45}$$

The Reynolds number for the flow in the well is

$$(N_{\mathrm{Re}})_{\mathrm{w}} = \frac{\langle\rho_{\mathrm{m}}\rangle_{\mathrm{w}} \langle U_{\mathrm{m}}\rangle_{\mathrm{w}} D_{\mathrm{w}}}{\langle\mu_{\mathrm{m}}\rangle_{\mathrm{w}}}, \tag{46}$$

and the average mixture viscosity in the well is

$$\langle\mu_{\mathrm{m}}\rangle_{\mathrm{w}} = \langle\alpha_{\mathrm{L}}\rangle_{\mathrm{w}}\mu_{\mathrm{L}} + [1 - \langle\alpha_{\mathrm{L}}\rangle_{\mathrm{w}}]\mu_{\mathrm{G}}, \tag{47}$$

where $\langle\alpha_{\mathrm{L}}\rangle_{\mathrm{w}}$ is the average liquid volume fraction inside the well, $\langle\alpha_{\mathrm{L}}\rangle_{\mathrm{w}} = (m_{\mathrm{L}})_{\mathrm{w}}/V_{\mathrm{w}}\rho_{\mathrm{L}}$. The average mixture velocity in the well is

$$\langle U_{\mathrm{m}}\rangle_{\mathrm{w}} = \frac{4\langle w_{\mathrm{nom}}\rangle}{\pi D_{\mathrm{w}}{}^2 \langle\rho_{\mathrm{m}}\rangle_{\mathrm{w}}}, \tag{48}$$

where $\langle w_{\mathrm{nom}}\rangle$ is a priori know nominal flow rate of the well, and the average density of the two-phase mixture is

$$\langle\rho_{\mathrm{m}}\rangle_{\mathrm{w}} = \frac{(m_{\mathrm{G}})_{\mathrm{w}} + (m_{\mathrm{L}})_{\mathrm{w}}}{V_w}. \tag{49}$$

The density of the gas phase in the well is

$$(\rho_{\mathrm{G}})_{\mathrm{w}} = \frac{(m_{\mathrm{G}})_{\mathrm{w}}}{V_{\mathrm{w}} - (m_{\mathrm{L}})_{\mathrm{w}}/\rho_{\mathrm{L}}}, \tag{50}$$

then the pressure at the well-head, assuming ideal gas, becomes

$$P_{\mathrm{wh}} = \frac{(\rho_{\mathrm{G}})_{\mathrm{w}} R T_{\mathrm{wh}}}{M_{\mathrm{G}}}. \tag{51}$$

In order to calculate the liquid volume fractions at the top of the well, we use the same assumptions as for the phase fraction of the riser in Section 2.7.

$$(\alpha_{\mathrm{L}})_{\mathrm{wt}} = 2K_\alpha \langle\alpha_{\mathrm{L}}\rangle_{\mathrm{w}} - (\alpha_{\mathrm{L}})_{\mathrm{wb}} \tag{52}$$

In this case, because of the high pressure at the bottom-hole, the fluid from the reservoir is saturated (Ahmed (2006)) and liquid volume fraction at the bottom is $(\alpha_{\mathrm{L}})_{\mathrm{wb}} = 1$. $K_\alpha \approx 1$ is a tuning parameter which can be used for model fitting purpose. The gas mass fraction at top of the well is then

$$(\alpha_{\mathrm{G}}^{\mathrm{m}})_{\mathrm{wt}} = \frac{[1 - (\alpha_{\mathrm{L}})_{\mathrm{wt}}](\rho_{\mathrm{G}})_{\mathrm{w}}}{(\alpha_{\mathrm{L}})_{\mathrm{wt}}\rho_{\mathrm{L}} + [1 - (\alpha_{\mathrm{L}})_{\mathrm{wt}}](\rho_{\mathrm{G}})_{\mathrm{w}}}. \tag{53}$$

Density of mixture at top of the well:

$$\rho_{\text{wt}} = (\alpha_{\text{L}})_{\text{wt}}\,\rho_{\text{L}} + [1 - (\alpha_{\text{L}})_{\text{wt}}]\,(\rho_{\text{G}})_{\text{w}}. \tag{54}$$

Mass flow rate of the mixture at the well-head:

$$w_{\text{wh}} = C_{\text{v2}} f(z_2)\sqrt{\rho_{\text{wt}}\max(P_{\text{wh}} - P_{\text{in}}, 0)}, \tag{55}$$

where P_{in} is the pressure at the inlet of the pipeline which is given by equation (12) in the pipeline-riser model, Section 2.3. The flow rates of gas and liquid phases from the well-head are as follows.

$$(w_{\text{G}})_{\text{wh}} = (\alpha_{\text{G}}^{\text{m}})_{\text{wt}}\,w_{\text{wh}}. \tag{56}$$

$$(w_{\text{L}})_{\text{wh}} = [1 - (\alpha_{\text{G}}^{\text{m}})_{\text{wt}}]\,w_{\text{wh}}. \tag{57}$$

Flow rates of gas and liquid phases into the pipeline are respectively

$$(w_{\text{G}})_{\text{in}} = (w_{\text{G}})_{\text{wh}} + d_1, \tag{58}$$

$$(w_{\text{L}})_{\text{in}} = (w_{\text{L}})_{\text{wh}} + d_2, \tag{59}$$

where d_1 and d_2 are assumed to represent disturbances from the other production wells in the network. Usually, multiple production wells are connected to a subsea manifold, and their products are combined and transported through a shared pipeline. In this paper, we have considered only one oil well. This can easily be extended for multiple oil wells in a network.

4.2 Comparison with OLGA simulations

In the OLGA reference test case introduced in Section 3.1, constant inflow rates were assumed. We modified the OLGA reference model by connecting an oil well to the inlet of the pipeline as shown in Figure 9. The oil well is vertical, has a depth of 3000 m, and it has the same inner diameter as for the pipeline, 0.12 m. The reservoir pressure is constant at 230 bar. The parameters related to the pipeline and the riser are same as for the OLGA reference model. The constants related to the additional well section are given in Table 6.

The well-pipeline-riser model includes two additional tuning parameter C_{v2}, the valve constant of the subsea choke valve, and K_α in equation (52). Hence, we have six tuning parameters in the simple well-pipeline-riser model. Numerical values for the tuning parameters are given in Table 7. The resulting bifurcation diagrams of the simple model are compared to the modified OLGA model in Figure 10. The simple model could predict the steady-state and the bifurcation point with a good accuracy. Figure 10.b shows that the inlet mass flow is increasing by opening the topside choke valve. This is because of pressure-driven nature of the flow.

Table 6: Model constants for well section

Symbol	Description	Value	Unit
P_{res}	Reservoir pressure	320	bar
C_{PI}	Mass productivity constant	2.75×10^{-6}	kg/(s.Pa)
$\langle w_{\mathrm{nom}} \rangle$	Well nominal mass flow	9	kg/s
η	Mass gas oil ratio	0.04	–
T_{w}	Well temperature	369	°K
V_{w}	Well volume	33.93	m^3
D_{w}	Well diameter	0.12	m
L_{w}	Well depth	3000	m
ϵ	Well roughness	2.80×10^{-5}	m

Table 7: Tuning parameters for well-pipeline-riser model fitted to OLGA simulations

Symbol	Description	Value
K_{h} [–]	Correction factor for level of liquid in pipeline	0.60
K_{G} [–]	Coefficient for gas flow through low point	3.49×10^{-2}
K_{L} [–]	Coefficient for liquid flow through low point	6.55×10^{-1}
C_{v1} [m^2]	Production choke valve constant	1.26×10^{-2}
K_{α} [–]	Liquid fraction correction factor	0.96
C_{v2} [m^2]	Wellhead choke valve constant	3.30×10^{-3}

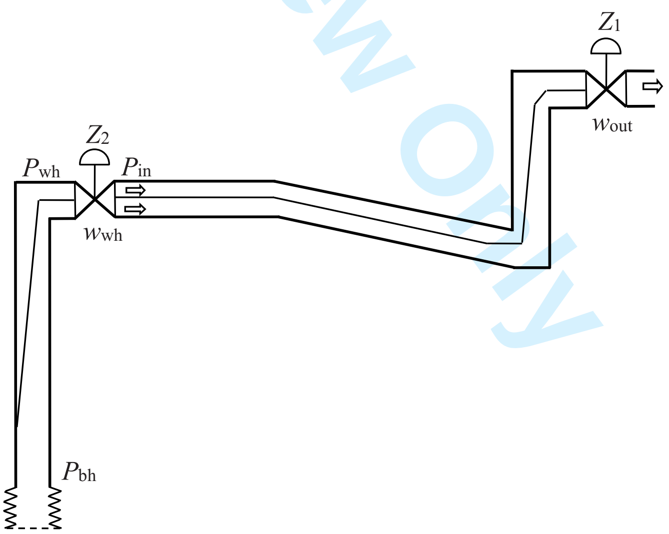


Figure 9: Schematic presentation of well-pipeline-riser system

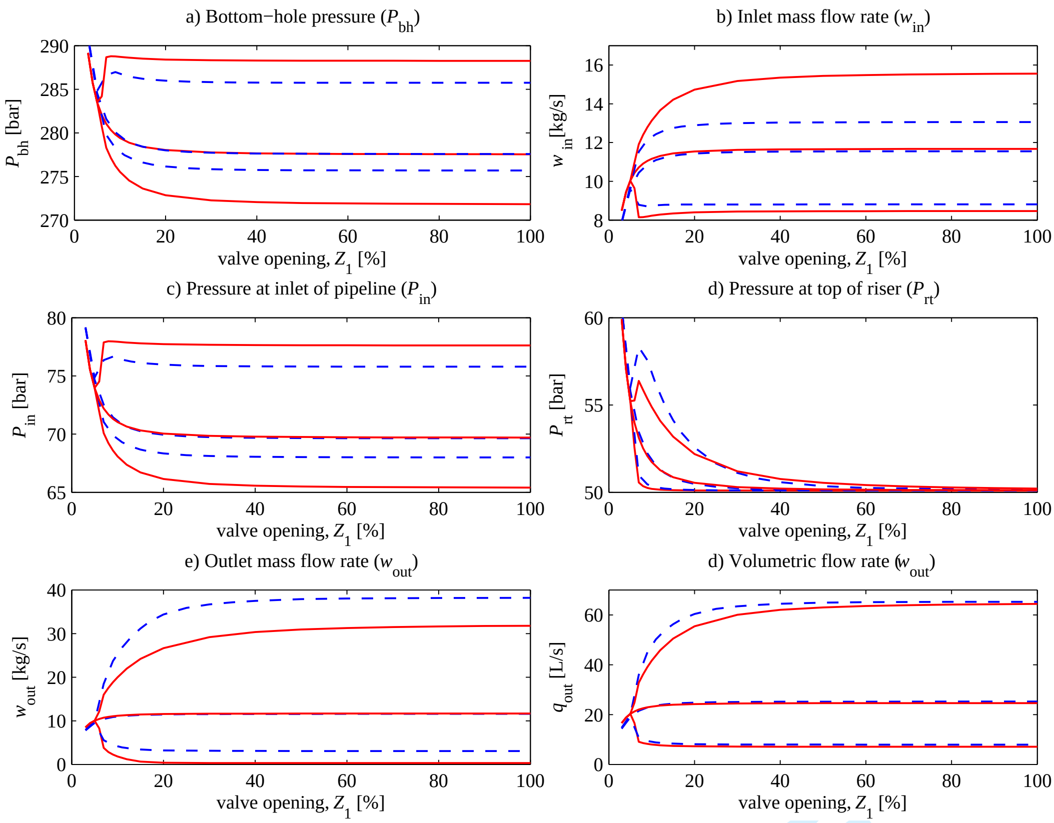


Figure 10: Bifurcation diagrams of simplified well-pipeline-riser model (solid lines) compared to OLGA reference model (dashed lines)

5 Conclusions

We have proposed a new simplified dynamic model for severe-slugging flow in pipeline-riser systems. The new model and five other models from the literature have been compared with a test case in the OLGA simulator. Furthermore, we verified the new model experimentally. The new model compares well with the OLGA simulations and the experiments.

Finally, we extended the four-state model to a well-pipeline-riser system by adding two states. The extended model was compared well to an OLGA test case.

The Matlab codes for the models are available at home page of Sigurd Skogestad (Jahanshahi (2012)).

The proposed four-state model captures the main dynamics of the severe-slugging flow regime, and the extended six-state model shows the (pressure driven) well inflow behaviour. These simplified models are useful for controllability analysis of the system in order to design a structure (e.g. choosing suitable controlled variable) for anti-slug control systems. The proposed simplified models are also of great importance for dynamic optimisation of the oil production networks.

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