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Optimal selection of sensor network and backed-off operating point based on economics

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Abstract— **This paper discusses the simultaneous selection of measurements and economic backed off operating point when the nominal optimal operating point is constrained. However, operation at this point becomes infeasible due to uncertainties. In this regard, we recently proposed an optimization formulation that determines the backed-off point to ensure feasibility assuming accurate measurement of the states is available and disturbance as the only source of uncertainty. In the present work, we extend the formulation to partial state information case and also determine the optimal set of measurements for economical operation. The formulation also finds a suitable multivariable controller to achieve economic benefits. The final formulation is a mixed integer non-linear program. Here, we present a branch and bound type solution such that a two stage iterative problem is solved at each branching step. Finally, the proposed approach is demonstrated in an evaporator system.**

Keywords: Process control, sensor networks, multivariable control, convex optimization

I. Introduction

Optimal operating point of a chemical process is determined using a non-linear optimizer and it is often constrained. However, process plants are typically operated at the more conservative operating point to ensure safe operation of the plant. Owing to the developments in control theory, the process plants could be operated more aggressively and closer to the constraints to increase profitability while ensuring safe operation. Therefore, the notion of back-off is highly useful in determining the dynamically feasible and profitable operating point. Recently[1], we presented an optimization formulation to determine the economic backed-off operating point such that feasibility is ensured under dynamic conditions of the plant. In our previous work, we presented the back-off point selection problem based on continuous-time model. We assumed disturbance as the only source of uncertainty and it is characterized by Gaussian white noise process with zero mean and known variance. Furthermore, we assumed full state feedback ($u = Lx$). In the current study, we consider partial state information case ($u = L\hat{x}$) which considers measurement error as an additional source of uncertainty. Thus, the loss we incur in backing off from the active constraints consists of two components: First, the loss due to disturbances which could be partially recovered by a suitable controller design and second, the loss due to measurement error which could be partially recovered using the state estimator. The performance of the state estimator depends critically on the chosen sensors. Hence, the problem of measurement selection is an important task to achieve optimal operation. Therefore, the current study focuses on addressing the issue of simultaneous selection of measurements and economic backed off operating point.

Sensor network plays a vital role in the optimal operation of a chemical process. The problem of measurement selection has been addressed in several frameworks such as fault diagnosis[2], process monitoring[3] and control[4]. In the current study, we limit our discussion on sensor selection from a control viewpoint. In this case, the problem of measurement selection is finding a set of controlled variables. In general, in order to achieve optimal operation, the set of active constraints are controlled and an additional set of controlled variables are determined for the remaining unconstrained degrees of freedom which is the idea of self-optimizing control[4]. However, operating the plant at active constraints is difficult due to uncertainties (such as disturbances, measurement noise, modeling errors, etc.) affecting the process. Therefore, we need to back-off from the active constraints to handle these uncertainties. In this context, the problem of deciding on right choice of measurements plays a crucial role in determining the economic back-off point as there is a loss because of measurement error. Several authors have addressed the role of process economics on control structure selection ([5], [6]). They used back-off as an economic measure to quantify the operational loss and determine the best set of controlled variables or measurements.

In the next section, we formulate the economic back-off selection problem for a partial state information case. Next, convex relaxations of the constraints are presented and a solution methodology is proposed. Finally, the proposed formulation is exemplified using an evaporator system.

II. Problem Formulation

In this section, we develop an optimization formulation that determines the optimal steady state (backed-off) operating point such that the process dynamics remain feasible under uncertain conditions for the prescribed confidence limit. Also, we need to determine the sensor network that results in a minimum economic loss.

A. Economic Back-off

Consider the state variables $x_0 \in \mathbf{R}^{\mathbf{n_x}}$, manipulated inputs $u_0 \in \mathbf{R}^{\mathbf{n_u}}$ and disturbances $d_0 \in \mathbf{R}^{\mathbf{n_d}}$. Generally, the

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economically optimal point is determined by solving a non-linear optimization problem which minimizes the negated profit function J subject to the process model g and performance bounds h,

$$\min_{x_0,u_0} \quad J(x_0, u_0, d_{nom}) \tag{1a}$$

$$s.t. \quad g(x_0, u_0, d_{nom}) = 0 \tag{1b}$$

$$h(x_0, u_0, d_{nom}) \leq 0 \tag{1c}$$

This is a steady state optimization problem solved for the nominal values of the disturbance variables d_{nom}. And, the optimal values of the states and manipulated inputs are denoted by x_0^* and u_0^*. However, if some of the bound constraints are active then there might be violation of constraints for some values of disturbance variables. Therefore, we need to ensure dynamic feasibility for all possible disturbances. One possible solution is to push the optimal operating point (back-off) inside the feasible region such that the system dynamics are feasible and the economic loss due to backing off is minimum. Thus, back off is defined as,

$$\begin{aligned} Back-off = |&Actual\ steady\ state\ operating\ point \\ &-Nominally\ optimal\ steady\ state\ operating\ point| \end{aligned} \tag{2}$$

In order to guarantee dynamic feasibility, the above optimization problem should include differential constraints. As a result, we have a dynamic optimization problem which is computationally demanding ([7], [8]). Alternately by characterizing the disturbance using a Gaussian random process (with zero mean and known variances), we can describe the dynamic operating region as a covariance ellipsoid provided the system is linear time invariant, full state information and the feedback control law is given by $u = Lx$ [9].

Defining deviation variables $(\tilde{x}, \tilde{u}, \tilde{d})$ with respect to the optimal point and linearization of the process model (1b) around the nominally optimal operating point (x_0^*, u_0^*, d_{nom}) results in

$$\dot{\tilde{x}} = A\tilde{x} + B\tilde{u} + G\tilde{d} \tag{3}$$

where A, B and G are the partial derivative of g evaluated at (x_0^*, u_0^*, d_{nom}). At steady state, the process model can be expressed in terms of backed-off variables $(\tilde{x}_{ss}, \tilde{u}_{ss})$ as

$$0 = A\tilde{x}_{ss} + B\tilde{u}_{ss} \tag{4}$$

There is no term corresponding to disturbance because of zero mean assumption and this expression defines the set of feasible backed-off operating points. Our objective is to determine this backed-off operating point that minimizes the loss in profit while ensuring the dynamics. With the linearized process model and linear controller (i.e., $u = Lx$), we can describe the dynamics around the back-off point using closed loop steady state covariance matrix of the state vector ($\Sigma_x := \lim_{t\to\infty} \mathbf{E}[x(t)^T x(t)]$) which is a symmetric positive semi-definite solution to the Lyapunov equation

$$(A + BL)\Sigma_x + \Sigma_x(A + BL)^T + G\Sigma_d G^T = 0 \tag{5}$$

where Σ_d is the covariance of the disturbance input. Similar development of the output signal z yields,

$$\Sigma_z = (Z_x + Z_u L)\Sigma_x(Z_x + Z_u L)^T + Z_d\Sigma_d {Z_d}^T \tag{6}$$

where Z_x, Z_u and Z_d are the partial derivative of h evaluated at $(x_0^*, u_0^*, \overline{d}_0)$. Using the idea of transformation of variables and relaxation arguments, we can write the above non-linear matrix equalities as Linear Matrix Inequalities (LMIs). Hence, the final formulation of the economic back-off selection problem follows[1]:

$$\min {J_x}^T \tilde{x}_{ss} + {J_u}^T \tilde{u}_{ss} + \tilde{u}_{ss}^T J_{uu} \tilde{u}_{ss} \tag{7a}$$

$$s.t.\ 0 = A\tilde{x}_{ss} + B\tilde{u}_{ss} \tag{7b}$$

$$\tilde{z}_{ss} = Z_x\tilde{x}_{ss} + Z_u\tilde{u}_{ss} \tag{7c}$$

$$(AX + BY) + (AX + BY)^T + G\Sigma_d G^T \prec 0 \tag{7d}$$

$$\begin{bmatrix} Z - Z_d\Sigma_d {Z_d}^T & Z_x X + Z_u Y \\ (Z_x X + Z_u Y)^T & X \end{bmatrix} \succeq 0 \tag{7e}$$

$$P = Z^{1/2} \tag{7f}$$

$$\begin{bmatrix} -\tau_i - {h_i}^T \tilde{z}_{ss} - t_i & \frac{\alpha}{2}{h_i}^T P \\ (\frac{\alpha}{2}{h_i}^T P)^T & \tau_i I \end{bmatrix} \succeq 0; \tag{7g}$$

where $\tilde{x}_{ss}$, $\tilde{u}_{ss}$, $\tilde{z}_{ss}$, $\tau_i > 0$, $Y = LX^{-1}$, $X = \Sigma_x \succeq 0$, $Z = \Sigma_z \succeq 0$ and $P \succeq 0$ are the decision variables. For complete derivation of the above formulation, the reader is referred to [1]. It is important to note in the above formulation that we assumed full state information. In other words, state variables are measured accurately. However, measurements contain errors and this uncertainty contributes to a further loss. Therefore, we need to find a set of measurements that minimizes the operational loss. Hence, the focus of this article is to extend the formulation to a partial state information case and also find the optimal sensor network from the set of possible measurements.

B. Sensor Placement

Consider the measurement vector $y = Cx + v$ where the measurement error vector, v is a zero mean, normally distributed variables with diagonal covariance matrix $\Sigma_v (= \mathbb{E}[vv^T])$. It is well known that Kalman filter is an optimal state estimator. Thus, we use the Kalman filter to estimate the states from the set of available measurements. Also, we assume the measurement errors are independent and uncorrelated which represents a diagonal Σ_v. Let us denote $Q = \Sigma_v^{-1} = diag(\frac{q_i}{\sigma_{v,i}^2})$ where q_i is a binary variable (0 or 1) denoting that the particular variable is unmeasured or measured respectively. And, $\sigma_{v,i}^2$ is the corresponding variance. It is important to note that an unmeasured variable ($q_i = 0$) can also be statistically inferred as a sensor with infinite variance. This definition of Q helps us to address the sensor placement problem with q_i as decision variables.

In order to describe the system dynamics for the partial state information case with disturbance variances Σ_d and variance of the measurement noise Σ_v, the steady state covariance of the signal z is given by

$$\begin{aligned} \Sigma_z \quad &= (Z_x + Z_u L)(\Sigma_x - \Sigma_e)(Z_x + Z_u L)^T + Z_x\Sigma_e Z_x^T \\ &\qquad + Z_w\Sigma_d Z_w^T \end{aligned} \tag{8}$$

where Σ_x and Σ_e are the positive semi definite solutions to

$$A\Sigma_x+\Sigma_x A^T+BL(\Sigma_x-\Sigma_e)+(\Sigma_x-\Sigma_e)B^T L^T+G\Sigma_d G^T=0 \tag{9}$$

and

$$A\Sigma_e+\Sigma_e A^T-\Sigma_e C^T Q C\Sigma_e+G\Sigma_d G^T=0 \tag{10}$$

Theorem $\exists$ stabilizing L, $\Sigma_x \succeq 0$, $\Sigma_e \succeq 0$ and Σ_z s.t. $A\Sigma_x+\Sigma_x A^T+BL(\Sigma_x-\Sigma_e)+(\Sigma_x-\Sigma_e)B^T L^T+G\Sigma_d G^T=0$, $A\Sigma_e+\Sigma_e A^T-\Sigma_e C^T Q C\Sigma_e+G\Sigma_d G^T=0$, $\Sigma_z=(Z_x+Z_u L)(\Sigma_x-\Sigma_e)(Z_x+Z_u L)^T+Z_x\Sigma_e Z_x^T+Z_w\Sigma_w Z_w^T$ and $\sigma_{z,i}^2 \leq \overline{z}_i^2$, $i=1,\ldots n_z$, if and only if $\exists$ Y, $X \succ 0$, $W \succ 0$ and $\sigma_{z,i}^2$ s.t. $(AX+BY)+(AX+BY)^T+G\Sigma_d G^T \prec 0$, $\begin{bmatrix} C'QC-A'W-WA & WG \\ (WG)^T & \Sigma_d^{-1} \end{bmatrix} \succ 0$, $\begin{bmatrix} Z-Z_d\Sigma_d {Z_d}^T & Z_xX+Z_uY & 0 \\ (Z_xX+Z_uY)^T & X & I \\ 0 & I & W \end{bmatrix} \succ 0$ and $z_i \leq \overline{z}_i^2$. For proof, the reader is referred to the original article [10].

Now the simultaneous economic back-off and measurement selection problem is reformulated in terms of LMI constraints as :

$$min\ {J_x}^T\tilde{x}_{ss}+{J_u}^T\tilde{u}_{ss}+\tilde{u}_{ss}^T J_{uu}\tilde{u}_{ss} \tag{11a}$$

$$s.t.\ A\tilde{x}_{ss}+B\tilde{u}_{ss}=0 \tag{11b}$$

$$\tilde{z}_{ss}=Z_x\tilde{x}_{ss}+Z_u\tilde{u}_{ss} \tag{11c}$$

$$(AX+BY)+(AX+BY)^T+G\Sigma_d G^T \prec 0 \tag{11d}$$

$$\begin{bmatrix} Z-Z_d\Sigma_d {Z_d}^T & Z_xX+Z_uY & 0 \\ (Z_xX+Z_uY)^T & X & I \\ 0 & I & W \end{bmatrix} \succeq 0 \tag{11e}$$

$$\begin{bmatrix} C'QC-A'W-WA & WG \\ (WG)^T & \Sigma_d^{-1} \end{bmatrix} \succeq 0 \tag{11f}$$

$$P=Z^{1/2} \tag{11g}$$

$$\begin{bmatrix} -\tau_i-{h_i}^T\tilde{z}_{ss}-t_i & \frac{\alpha}{2}{h_i}^T P \\ (\frac{\alpha}{2}{h_i}^T P)^T & \tau_i I \end{bmatrix} \succeq 0; \tau_i>0 \tag{11h}$$

where $\tilde{x}_{ss}$, $\tilde{u}_{ss}$, $\tilde{z}_{ss}$, W, Y, $X \succeq 0$, $Z \succeq 0$, $P \succeq 0$ and τ_i are the continuous decision variables. Also, recall $Q=diag(\frac{q_i}{\sigma_v^2})$ where q_i is a binary decision variable. Hence, the final formulation is a mixed integer nonlinear programme (MINLP). It is important to note that the last constraint (11h) in based on the explicit ellipsoid representation of the dynamics (covariance) constrained by the polytope. This helps us to find the feasible backed off operating point such that the system dynamics defined by the LMI constraints (11d) - (11f) are satisfied. The non-linearity (and hence non convexity) in the formulation is due to (11g). Therefore, we need a specialized solution technique to solve this non-convex problem which will be addressed in the next section.

In the above formulation, quadratic term for inputs denotes the economic penalty for backing off the inputs from the nominal optimal value. In other words, it penalizes the excess use of the available unconstrained degrees of freedom. And, it is important to include this term in the cost function to get meaningful back-off points when there exists some unconstrained degrees of freedom. This situation arises when the number of manipulated inputs is greater than the number of active constraints. Thus, we need second order information on inputs ($J_{uu} \succeq 0$) which can be obtained numerically by perturbing the unconstrained inputs. Note that this cost function considers only the steady state effect on economics. Since the disturbances are assumed to be Gaussian and zero mean, this implies that the cost accounts only for the nominal steady state value of disturbances. Furthermore, we design an optimal stabilizable controller such that the back-off point selected is close to the optimal operating point.

III. SOLUTION METHODOLOGY

The formulation of simultaneous selection of economic back-off and measurements results in a mixed integer non-linear program. The integer decision variables is a result of sensor placement problem. First, let us consider the relaxed problem where the binary decision variables are considered to be continuous in the range 0 - 1. Now, the problem is still non convex due to the non linearity in $P=Z^{1/2}$. In this regard, we presented a simple two stage iterative procedure for a full state information case that reduce the variability of the economically important (i.e., active constrained) variables by progressively increasing the variability of the economically unimportant variables at each iteration. At each stage of the iteration, we solve a convex problem. In this work, we adapt the solution technique to handle the partial state information case with relaxed integer constraints.

The basic idea in the two stage approach is to first determine the feasible dynamic operating region (solution of stage 1) and then determine the back-off point (solution of stage 2) corresponding to the dynamic region [1]. From this solution, we can determine the departure from the true optimal point by defining the parameter $\delta_{i,j}$

$$\delta_{i,j}=\frac{distance\ of\ variable\ i\ from\ its\ closest\ bound}{distance\ of\ variable\ j\ from\ its\ closest\ bound} \tag{12}$$

which is used to create bounds for the individual variances and are updated upon successive iteration to find the economic back-off point. This parameter δ is initialized to zero before start of the algorithm and are updated on further iterations.

A. Stage 1

In the first stage, our objective is to determine the smallest (in terms of trace) feasible ellipsoid Z and a suitable multivariable controller L.

$$\begin{array}{ll} \underset{X\succeq 0, \Sigma_z \succeq 0, Y}{min} & Tr(Z) \\ \text{s.t.} & (AX+BY)+(AX+BY)^T+G\Sigma_d G^T \prec 0 \\ & \begin{bmatrix} Z - Z_d \Sigma_d Z_d^T & Z_x X + Z_u Y & 0 \\ (Z_x X + Z_u Y)^T & X & I \\ 0 & I & W \end{bmatrix} \succeq 0 \\ & \begin{bmatrix} C^T Q C - A^T W - WA & WG \\ (WG)^T & \Sigma_d^{-1} \end{bmatrix} \succeq 0 \\ & \sigma_{z,i}^2 < \frac{1}{4\alpha^2}\,(\tilde{z}_{max,i} - \tilde{z}_{min,i})^2; i = 1 \cdots n_z \\ & \sigma_{z,i}^2 > \frac{\delta_{i,j}^2}{\alpha^2}\sigma_{z,j}^2; i = 1, j-1, j+1, n_z \end{array}$$

The output of the stage 1 after first iteration is a feasible closed loop operating region. Since no economic information is used in the objective function, the resulting controller and output covariance matrix might not be economically optimal. If the solution is infeasible, then there is no feasible solution to the original problem for the assumed uncertainty. Note that the integer variables in Q are relaxed and hence the sub problem is a semi-definite program which is known to be convex and could be solved for global optimality.

B. Stage 2

In the second stage, the covariance ellipsoid Z is used to determine the closest possible back-off point ($\tilde{z}_{ss}$) to the OOP (x_0^*, u_0^*, d_{nom}) such that the dynamics lie in the feasible space. To achieve this, we first compute $P = Z^{1/2}$ which is used to determine the center of the ellipsoid such that the ellipsoid is within the constraints polytope.

$$\begin{array}{ll} \underset{\tilde{x}_{ss}, \tilde{u}_{ss}, \tilde{z}_{ss}}{min} & {J_x}^T \tilde{x}_{ss} + {J_u}^T \tilde{u}_{ss} + \tilde{u}_{ss}^T J_{uu} \tilde{u}_{ss} \\ \text{s.t.} & 0 = A\tilde{x}_{ss} + B\tilde{u}_{ss} \\ & \tilde{z}_{ss} = Z_x \tilde{x}_{ss} + Z_u \tilde{u}_{ss} \\ & \begin{bmatrix} -\tau_i - {h_i}^T \tilde{z}_{ss} - t_i & \frac{\alpha}{2} {h_i}^T P \\ (\frac{\alpha}{2} {h_i}^T P)^T & \tau_i I \end{bmatrix} \succeq 0; \\ & \tau_i \succeq 0;\ i = 1, \cdots 2n_z \end{array}$$

This sub problem is a convex program. The back-off point obtained at the first iteration might not be economically optimal because of non-optimal Z. However, this BOP is used to create bounds and update the parameter δ and resolve Stage 1. It is to be noted that P is not a decision variable since Z is known from first stage.

In general, the result of the above iterative procedure might result in non integer solutions to the binary variables q_i. Hence, we can use the traditional branch and bound type of algorithms to solve for integer variables where the two stage iterative procedure described above is used at each branching step. The proposed solution scheme could be implemented using YALMIP, a freely available software for solving semi-definite problems[11].

IV. Evaporation Process

The proposed approach for simultaneously selecting the back-off operating point and measurements is applied to the evaporation process of [12]. Figure 1 depicts the forced-circulation evaporation process where the concentration of the feed stream is increased by evaporating the solvent through a vertical heat exchanger with circulated liquor.

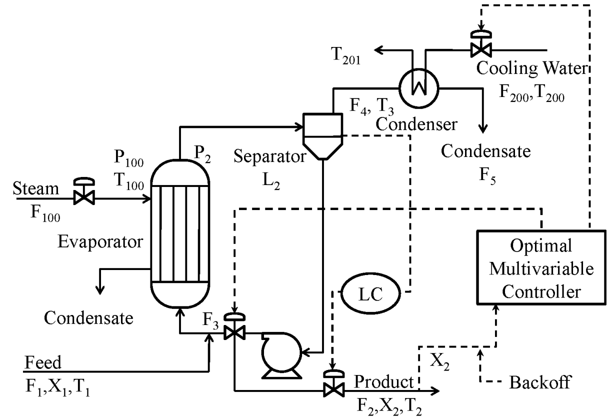

Fig. 1. Evaporator system

The overhead vapor is condensed by the use of process heat exchanger. The details of the mathematical model can be found in [12]. The separator level is assumed to be perfectly controlled using the exit product flow rate F_2 which also eliminates the integrating nature of the state. The economic objective is to maximize the operational profit [\$/h], formulated as a minimization problem of the negative profit [13]. The first three terms of (13) are utility costs relating to steam, coolant and pumping respectively. The fourth term is the raw material cost, whereas the last term is the product value.

$$J = 600F_{100} + 0.6F_{200} + 1.009(F_2 + F_3) + 0.2F_1 - 4800F_2 \quad (13)$$

The process has the following constraints related to product specification, safety, and design limits:

$$X_2 \geq 35\% \quad (14)$$

$$40\ kPa \leq P_2 \leq 80\ kPa \quad (15)$$

$$P_{100} \ \leq 400\ kPa \quad (16)$$

$$0\ kg/min \leq F_{200} \leq 400\ kg/min \quad (17)$$

$$0\ kg/min \leq F_1 \leq 20\ kg/min \quad (18)$$

$$0\ kg/min \leq F_3 \leq 100\ kg/min \quad (19)$$

Nominal operating point. The nominal steady state values are obtained by solving a nonlinear optimization problem for the nominal values of disturbances and the profit is found to be $J = \$693.41/h$ and the nominal values can be found in [1]. At the nominal optimal point, there are two active constraints: product composition, $X_2 = 35\%$ and steam pressure, $P_{100} = 400\ kPa$. And, the corresponding Lagrange multipliers are 229.36 $\$/\%\ h$ and -0.096685 $\$/kPa\ h$ respectively.

Degree of freedom analysis. The process model has seven degrees of freedom. Inlet conditions of the feed (flow rate, composition, temperature) and inlet temperature of the condenser are considered as disturbances (i.e., $d = [F_1\ X_1\ T_1\ T_{200}]^T$). There are three manipulated inputs, $u = [F_3\ P_{100}\ F_{200}]^T$. The disturbance range is assumed to be 10% variation of the nominal value (i.e., $\Sigma_d =$

TABLE I

NOMINAL AND BACK-OFF OPERATION

Variables	Units	Nominal value	Closed loop back-off	
			FSI case (7)	PSI case(11)
		States		
X_2	%	35.00	35.26	35.428
P_2	kPa	56.15	56.10	56.067
		Inputs		
F_3	kg/min	27.70	27.78	27.833
P_{100}	kPa	400.00	400	400
F_{200}	kg/min	230.57	232.71	234.22
Profit	$\$/h$	693.41	634.76	595.18

$diag([1\ 0.25\ 16\ 6.25])^2$) and the set of active constraints do not change in the whole range of disturbances . It is important to note that there is one unconstrained degrees of freedom.

Linearized steady state model. Linear approximation of the process model at the nominal optimum yields,

$$A = \begin{bmatrix} -0.16709 & -0.17185 \\ -0.013665 & -0.043132 \end{bmatrix};$$

$$B = \begin{bmatrix} 0.44083 & 0.04217 & 0 \\ 0.062976 & 0.0060243 & -0.0016249 \end{bmatrix};$$

$$G = \begin{bmatrix} -1.2211 & 0.5 & 0.031818 & 0 \\ 0.039837 & 0 & 0.0045455 & 0.03665 \end{bmatrix}$$

and the C matrix denoting all possible measurements (i.e., $y = [X_2\ P_2\ T_2\ T_3]^T$) is given by

$$C^T = \begin{bmatrix} 1 & 0 & 0.5616 & 0 \\ 0 & 1 & 0.3126 & 0.507 \end{bmatrix}$$

and the measurement error is considered to be $\Sigma_v = diag([0.01\ 0.01\ 0.01\ 0.01])^2$. The performances z are defined by the matrices,

$$Z_x = [I_{2\times 2}|0_{2\times 3}]^T; Z_u = [0_{3\times 2}|I_{3\times 3}]^T; Z_d = [0_{4\times 5}]^T$$

and the bound constraints written in the form of $h_i{}^T \tilde{z}_{ss} + t_i \leq 0$ are obtained from the rows of the matrix H and elements of vector t, $H = [I_{5\times 5}| - I_{5\times 5}]^T$; $t = [-5\ \ -23.849\ \ -72.299\ 0\ -169.43\ 0\ -16.151\ -27.701\ -200\ -230.57]^T$. The linearized negative profit function is

$$J_x = [-293.23\ \ -526.8]^T; J_u = [1368.9\ 130.85\ 0.6]^T$$

As the input P_{100} is constrained, the quadratic penalty is included only for the other inputs and the numerical perturbation of inputs F_3 and F_{200} yield,

$$J_{uu} = \begin{bmatrix} 4.4953 & 0.00010226 \\ 0.00010226 & 0.0052699 \end{bmatrix}$$

Results. The amount of necessary back off to remain feasible for 10% variation in the nominal disturbances (Full state information case) is tabulated in Table I. Also, the economic back-off required for the partial state information case is tabulated. Since steam pressure (P_{100}) is a input variable and constrained at the optimal solution, it can be set at its optimal value without backing off. This could be easily recognized from zero back-off in the table. On the

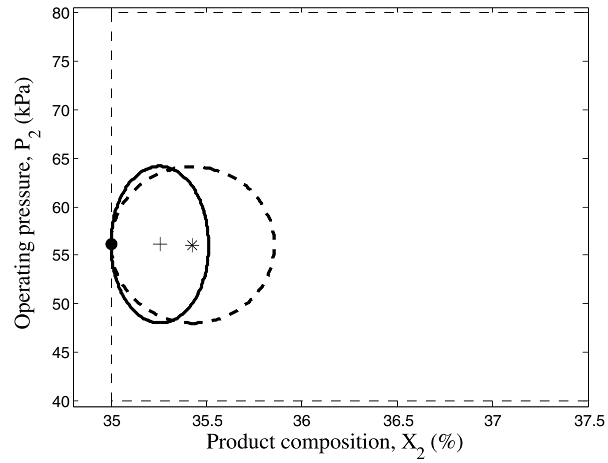

Fig. 2. Product composition vs operating pressure. a) Continuous line : FSI case b) Dashed line : PSI case

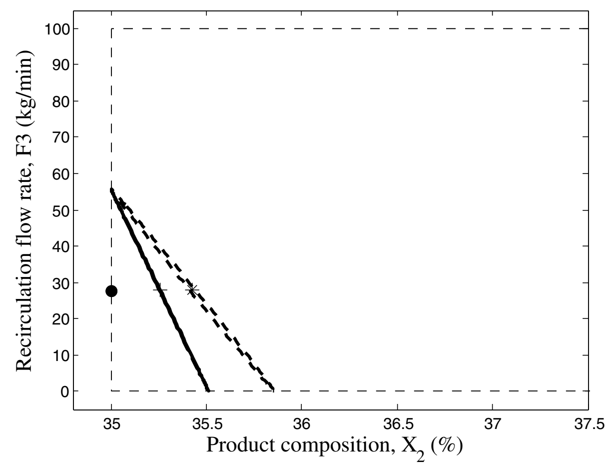

Fig. 3. Product composition vs recirculation flow rate

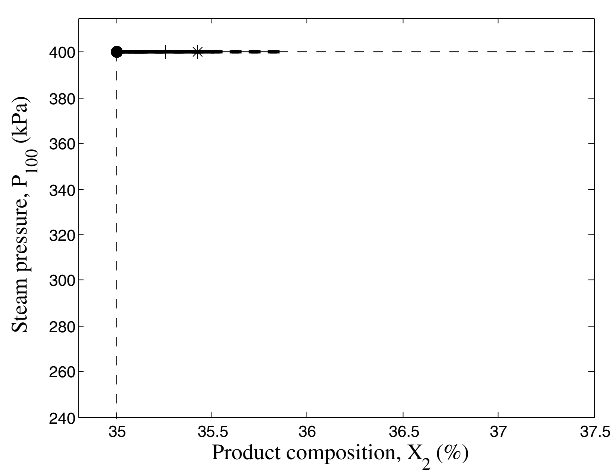

Fig. 4. Product composition vs steam pressure

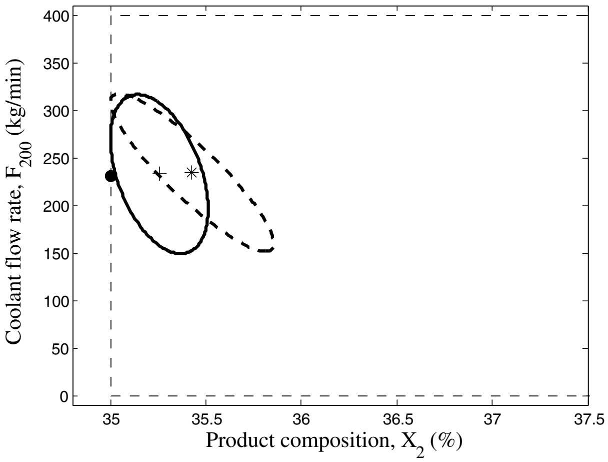

Fig. 5. Product composition vs coolant flow rate

TABLE II
MEASUREMENT SELECTION

Sensor Network	Variance	Loss, $\$/h$
X_2, T_2	{0.01,0.01}	98.226*
X_2, P_2	{0.01,0.01}	103.62
X_2, T_3	{0.01,0.01}	103.63
P_2, T_2	{0.01,0.01}	139.06
T_3, T_2	{0.01,0.01}	140.08
T_3, P_2	{0.01,0.01}	1556.2
X_2, T_2	{0.1,0.1}	304.29
X_2, P_2	{0.1,0.1}	324.17
T_2, P_2	{0.1,0.1}	436.21
X_2, T_2	{0.1,0.01}	161.29
X_2, P_2	{0.1,0.01}	321.68
X_2, T_3	{0.1,0.01}	323.33

*optimal solution obtained using YALMIP[11]

other hand, product exit composition X_2 requires significant back-off for the assumed disturbances. It is important to note that the lagrange multiplier for X_2 is very high (has a value of 229.36 $\$/\%\ h$) and hence even a small variation in product composition will result in a very high loss. The dynamic operating region for the FSI and PSI cases are shown as ellipses in Figures 2-5. The center of the ellipse denotes economic back-off solution. For PSI case, the loss obtained for operating the evaporator at this backed off operating point is $\$98.226/h$ which corresponds to the achievable profit of $\$595.18/h$. In other words, the loss we incur to ensure feasible operation with 95% confidence interval is $\$98.226/h$. The multivariable feedback controller ($u = L\hat{x}$) to be implemented to operate the system profitably is

$$L = \begin{bmatrix} -64.97 & 0.7556 \\ -0.0585 & 0.0007 \\ -171.4 & 28.45 \end{bmatrix} \tag{20}$$

This controller gain could be used to find the objective function weights of Model Predictive Control using the inverse optimality results[10]. Table II gives the loss for different set of measurements obtained by enumeration and the minimum loss network obtained using Yalmip[11]. From the enumerated list, it can be inferred that we need to measure product composition more precisely to minimize the loss. The sensor network $\{X_2, T_2\}$ is obtained by solving the relaxed problem. Since the solution to this problem resulted in integer solution to the binary variables, branch and bound technique is not used in this case. However in the absence of concentration measurements, we need precise measurements of $\{P_2, T_2\}$ which however results in an additional loss of $\$40.838/h$. This loss in the absence of concentration measurement could be attributed to the error in estimating the concentration variable. This feature illustrates the importance of sensor network design for optimal operation.

V. Conclusion

In this work, we addressed the economic back-off operating point selection problem for partial state information case where both disturbances and measurement errors are considered as uncertainties. The formulation also yields a multivariable controller which when implemented to operate the evaporation process at the determined economic back-off operating point will ensure feasible and profitable operation. Furthermore, we obtained the optimal set of measurements from the formulation that result in minimal loss.

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