Nonlinear model-based control of two-phase flow in risers by feedback linearization

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Abstract: Active control of the production choke valve is the recommended solution to prevent severe slugging flow conditions at offshore oilfields. The slugging flow constitutes an unstable and highly nonlinear system; the gain of the system changes drastically for different operating points. Although PI and PID controllers are the most widely used controllers in the industry, they need to be re-tuned for different operating conditions. The focus of this paper is to design a model-based nonlinear controller in order to counteract nonlinearities of the system. Feedback linearization based on the riser-base pressure and the topside pressure was used for the control design. Stability and convergence of the closed-loop system was verified in theory as well as by experiments on a test rig.

Keywords: Oil production, anti-slug control, nonlinear cascade systems, stabilizing control.

1. INTRODUCTION

The oscillatory flow condition in offshore multi-phase pipelines is undesirable and an effective solution is needed to suppress it. One way to prevent this behaviour is reducing the opening of the top-side choke valve. However, this conventional solution increases the back pressure of the valve, and it reduces the production rate from the oil wells. The recommended solution to avoid slugging flow regime while maintaining the maximum possible production rate is active control of the topside choke valve. The control system used for this purpose is called anti-slug control. This control system uses measurements such as pressure and flow rate as the controlled variables and the topside choke valve is the main manipulated variable (Storkaas and Skogestad (2007)).

Anti-slug control systems tend to become unstable after some time, because of large inflow disturbances or plant changes. In this work, we consider the nonlinearity of the system as a source of plant change at different operating conditions. The nonlinearity can be counteracted by gain-scheduling of PID controllers or by model-based nonlinear controllers. The focus of this paper is on nonlinear model-based control by feedback linearization.

A back-stepping design has been used by Kaasa et al. (2007) and Kaasa et al. (2008) for nonlinear control of slug flow in risers. Di Meglio et al. (2010a) proposed a partially linearizing feedback controller that uses the mass of liquid in the riser for state feedback. Di Meglio et al. (2010b) used a simple nonlinear Luenberger-type observer to estimate the state variable needed for the controller. However, the *separation principle* does not hold for nonlinear systems in general, and the closed-loop observer/controller is not guaranteed to be stable for all conditions. Jahanshahi et al. (2013) showed that a nonlinear observer fails in closed-loop when using the subsea pressure as the measurement. In this work, we propose a nonlinear controller that uses the measurements of the system directly without using any observer. Then, we will look into controllability limitations of the system when using the top-side pressure measurement with the proposed nonlinear controller.

This paper is organized as follows. A simplified model for sever-slugging is introduced in Section 2. In Section 3 we represent the system as a cascade connection of two subsystems and discuss the stability of the cascade. The stabilizing feedback control is designed in Section 4 and the experiments are presented in Section 5. Next, controllability limitations are discussed in Section 6. Finally, the main conclusions are summarized in Section 7.

2. SIMPLIFIED DYNAMICAL MODEL

2.1 Summary of original model

A four-state simplified model for the severe-slugging flow has been proposed by Jahanshahi and Skogestad (2011). The state variables of this model are as follows:

- m_{gp}: mass of gas in pipeline [kg]
- m_{lp}: mass of liquid in pipeline [kg]
- m_{gr}: mass of gas in riser [kg]
- m_{lr}: mass of liquid in riser [kg]

The four state equations of the model are

$$\dot{m}_{gp} = w_{g,in} - w_g \quad (1)$$

$$\dot{m}_{lp} = w_{l,in} - w_l \quad (2)$$

$$\dot{m}_{gr} = w_g - \alpha w \quad (3)$$

$$\dot{m}_{lr} = w_l - (1-\alpha) w \quad (4)$$

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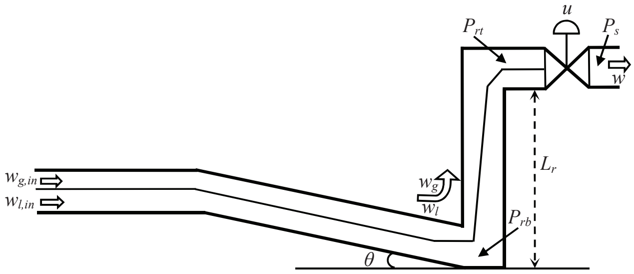

Fig. 1. Schematic presentation of system

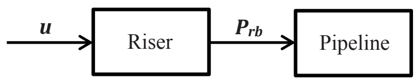

Fig. 2. Pipeline-riser system as a cascaded connection of two subsystems

Fig. 1 shows a schematic presentation of the model. The inflow rates of gas and liquid to the system, $w_{g,in}$ and $w_{l,in}$, are assumed to be constant. The flow rates of gas and liquid from the pipeline to the riser, w_g and w_l, are determined by pressure drop across the riser-base where they are described by virtual valve equations. The outlet mixture flow rate, w, is determined by the opening percentage of the top-side choke valve, u, which is the manipulated variable of the control system. The different flow rates and the gas mass fraction, α, in the equations (1)-(4) are given by additional model equations provided by Jahanshahi and Skogestad (2011).

2.2 Cascade system structure

In order to analyse a complicated system, we can separate it into subsystems and analyse the individual subsystems and their interconnecting relationships. As illustrated in Fig. 2, we represent the pipeline-riser system by a cascaded connection of two subsystems.

The input to the "Riser" subsystem is the choke valve opening, u, and the output is the pressure at the riser-base, P_{rb}, which is also the input to the "Pipeline" subsystem.

3. STABILITY ANALYSIS OF CASCADE SYSTEM

First, we consider the "Pipeline" subsystem with the riser-base pressure, P_{rb}, as its input. Different phenomena can account for the flow instability in pipeline-riser systems. The pipeline-riser system may have an unstable oil well as the inlet boundary, also the density-wave instability in long risers can happen. Here, we consider only the riser-slugging instability.

Hypothesis 1. If the riser-slugging is the destabilizing dynamic of the pipeline-riser system, then the Pipeline subsystem with the riser-base pressure, P_{rb}, as its input is "input-to-state stable".

We investigated input-to-state stability of the pipeline subsystem by a simulation test shown in Fig. 3. The riser-base pressure in this simulation is 19.7 kPa. This pressure is corresponding to 50% opening of the top-side valve for which the pipeline-riser system is unstable. However, the pipeline subsystem separated from the riser is always

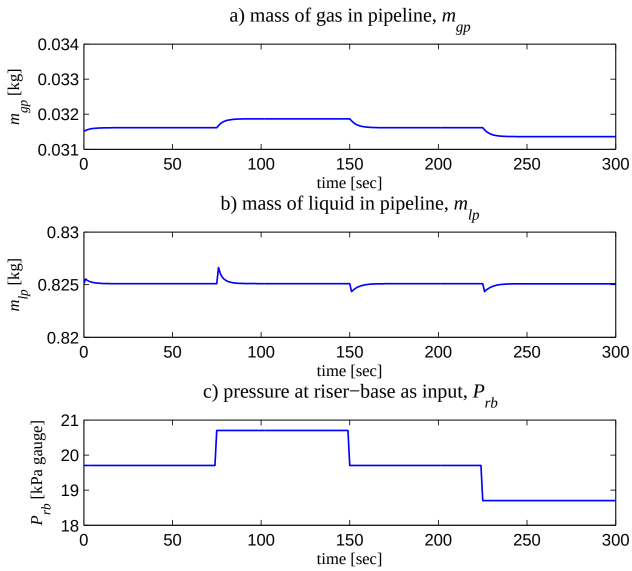

Fig. 3. Simulation test of pipeline subsystem

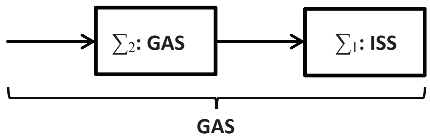

Fig. 4. Stability of a cascaded system

stable. In addition, the local exponential stability can be verified by looking at eigenvalues. The above hypothesis is reasonably correct, because the riser-base pressure is a recommended candidate controlled variable to stabilize the system (Jahanshahi et al. (2012)). This means when the riser-base pressure has small variations, the whole system become stabilized which follows $\mathcal{L}_2$-gain stability from P_{rb} to state variables of the pipeline subsystem.

We can now state the following proposition:

Proposition 2. Let hypothesis 1 holds. If the Riser subsystem becomes globally asymptotically stable under a stabilizing feedback control, then the pipeline-riser system is globally asymptotically stable.

Proof: We use conditions for stability of the cascaded systems as stated by *Corollary 10.5.3* of Isidori (1999). As shown in Fig. 4, if Σ_1 is input-to-state stable (ISS) and origin of Σ_2 is globally asymptotically stable (GAS), then origin of the cascaded system Σ_1 and Σ_2 is globally asymptotically stable (GAS). Therefore, if *Hypothesis 1* holds, the proposition is verified.

□

In the next section, we design a stabilizing feedback control for the Riser subsystem.

4. STABILIZING FEEDBACK CONTROL

We use feedback linearization to design a control law. For simplicity, the outlet mass flow rate, w, is used as a virtual control input. First, we transform the state equations and write a model in 'normal form' based on the two measurements, pressure at the riser-base and pressure at top of the riser.

4.1 Transforming state equations

Two state variables govern the dynamics of the Riser subsystem. We start with writing the model in the new coordinates $(x_3, x_4) = (m_{gr}, m_{lr} + m_{gr})$. We get the following system:

$$\dot{x}_3 = w_g - \alpha w \tag{5}$$

$$\dot{x}_4 = w_g + w_l - w \tag{6}$$

The two measurements are $y_1 = P_{rb}$ and $y_2 = P_{rt}$. From the ideal gas law we get

$$y_2 = \frac{a x_3}{b + x_3 - x_4}, \tag{7}$$

where $a = RT_r\rho_l/M_G$ and $b = \rho_l V_r$ are the model parameters (Jahanshahi and Skogestad (2011)). The pressure drop over the riser is sum of the hydrostatic head and the friction term:

$$y_1 = y_2 + c x_4 + F_r, \tag{8}$$

where $c = gL_r/V_r$ and F_r is the friction in the riser that depends on constant inflow rates and other constant parameters. Differentiating and rearranging the equations gives the system equations in y coordinates.

$$\dot{y}_1 = (w_g + w_l)\left[F(y) + c\right] - \left[F(y) + c\right] w \tag{9}$$

$$\dot{y}_2 = (w_g + w_l)F(y) - F(y)w, \tag{10}$$

where

$$F(y) = c\left(1 - \frac{y_2}{a}\right)\frac{a\alpha + y_2(1-\alpha)}{bc - (y_1 - y_2 - F_r)}. \tag{11}$$

Since $\rho_l > \rho_g$, we have that $a > y_2$. In addition, $bc = \rho_l g L_r$ is the hydrostatic pressure when the riser is full of liquid which is larger than the gravity term in normal operation $(y_1 - y_2 - F_r)$. Thus, the physical domain of the system outputs is defined as follows.

$$\mathcal{D} = \{(y_1, y_2) | y_2 + bc + F_r > y_1 > y_2 > 0\} \tag{12}$$

The numerator in equation (11) is also positive, therefore, $F(y) > 0, \forall (y_1, y_2) \in \mathcal{D}$. In addition, It can be shown that $F(y)$ is strictly increasing in y_1 and strictly decreasing in y_2.

The transformation $T : \mathcal{S} \to \mathcal{D},\ y = T(x)$ in (7) and (8) where $y = (y_1, y_2)^T$, $x = (x_3, x_4)^T$ and

$$T(x) = \begin{bmatrix} \frac{ax_3}{b+x_3-x_4} + cx_4 + F_r \\ \frac{ax_3}{b+x_3-x_4} \end{bmatrix} \tag{13}$$

is a diffeomorphism on $\mathcal{S} = \{(x_3, x_4) | x_3 > 0, x_4 - x_3 < b\}$, because both $T(x)$ and $T^{-1}(y)$ exist and are continuously differentiable. $b = \rho_l V_r$ is the mass of a volume of liquid equal to volume of the riser, hence $b > m_{lr} = x_4 - x_3$ in the normal operation of the system where $x_3 > 0$.

For simplicity we will use the following assumption:
Assumption 1. We use gas and liquid inflow rates to the system to calculate the gas mass fraction, α. Although it is different from the original model, it is the same at steady-state:

$$\alpha = \frac{w_{g,in}}{w_{g,in} + w_{l,in}}. \tag{14}$$

4.2 Input-output linearization

By choosing $h(x) = y_1$, with $\xi = y_1 - \bar{y}_1$ and $\eta = y_2 - \bar{y}_2$, where $\bar{y}_1$ and $\bar{y}_2$ are steady-state values, we can write the system in a normal form:

$$\dot{\xi} = (w_g + w_l)\left[F(\xi,\eta) + c\right] - \left[F(\xi,\eta) + c\right] w \tag{15}$$

$$\dot{\eta} = (w_g + w_l)F(\xi,\eta) - F(\xi,\eta)w, \tag{16}$$

The feedback controller

$$w = \frac{-1}{F(\xi,\eta) + c}\left(-(w_g + w_l)\left[F(\xi,\eta) + c\right] + v\right).$$

reduces equation (15) to $\dot{\xi} = v$ and choosing $v = -K_1\xi$ gives

$$\dot{\xi} = -K_1\xi, \tag{17}$$

where $K_1 > 0$ results in *Exponential Stability* of ξ dynamics. By inserting the control law (equation (17)) into equation (16) we get

$$\dot{\eta} = \frac{F(\xi,\eta)}{F(\xi,\eta) + c} v = -\mathcal{F}K_1\xi, \tag{18}$$

where

$$0 < \underline{\mathcal{F}} < \mathcal{F} = \frac{F(\xi,\eta)}{F(\xi,\eta) + c} < \overline{\mathcal{F}} < 1. \tag{19}$$

Since $0 < \mathcal{F} < 1$ and $\xi \to 0$ exponentially fast, η will therefore remain bounded. This is partial stabilization of the system with respects to ξ (Vorotnikov (1997)).

□

Assumption 2. In order to make the control law realizable, we replace $w_g + w_l$ by inflow rates to the system, $w_{g,in} + w_{l,in} = w_{in}$.

$$w = \frac{w_{in}(F(y) + c) + K_1(y_1 - \bar{y}_1)}{F(y) + c} \tag{20}$$

The final control signal to the valve is

$$u = sat\left(\frac{w}{C_v\sqrt{\rho_{rt}(y_2 - P_s)}}\right), \tag{21}$$

where C_v and P_s are the choke valve constant and the separator pressure, respectively. F_r and ρ_{rt} are calculated based on the two measurements y_1 and y_2 and model parameters (see Appendix A).

By choosing $h(x) = y_2$, we can design a control law that linearizes equation (16). Although we are using the both y_1 and y_2 in $F(y)$, we use only y_2, the topside pressure, for feedback in the linear part of the controller. The resulting controller is

$$w = \frac{1}{F(y)}\left(w_{in}F(y) + K_2(y_2 - \bar{y}_2)\right). \tag{22}$$

The final control signal to the valve is same as equation (21). However, for feedback linearization we need the system to be *minimum phase*, but the linearized 4-state model constitutes two *Right-Half-Plane zeros* from the valve position (input) to the top-side pressure (output) (Jahanshahi et al. (2012)). Although we cannot prove stabilization of the system using the controller in (22), we will try it in experiments.

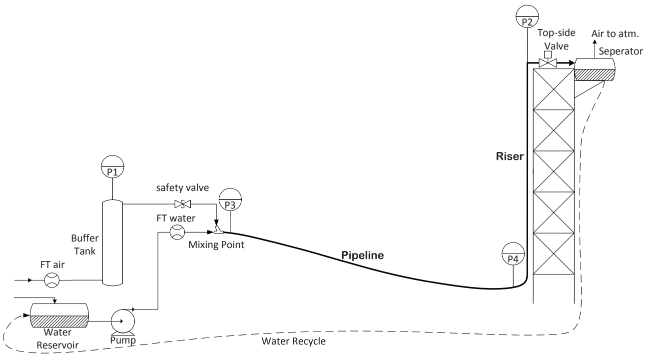

Fig. 5. Schematic diagram of experimental setup

4.3 Stability of composite system

After designing a stabilizing feedback control for the "Riser" subsystem using feedback linearization, we can consider the complete pipeline-riser system as a partially linear composite system.

$$\dot{x} = f(x, \xi), \; x \in \mathbb{R}^2, \; \xi \in \mathbb{R} \tag{23}$$

$$\dot{\xi} = -K_1 \xi, \tag{24}$$

where $f(x, \xi)$ represents dynamics of the "Pipeline" subsystem. We can check the conditions for stabilization of the composite system as stated by Saberi et al. (1989) and Jankovic et al. (1996):

"A linear controllable-nonlinear asymptotically stable cascade system is globally stabilizable by smooth dynamic state feedback if (a) the linear subsystem is right-invertible and weakly minimum-phase, (b) the only variables entering the nonlinear subsystem are the outputs and the zero dynamics corresponding to this output."

We use *Hypothesis 1* for stability of the nonlinear (pipeline) subsystem. The two conditions (a) and (b) are satisfied when using the pressure at the riser-base as the output. The riser-base pressure is minimum-phase and it is the output which enters the nonlinear subsystem as shown in Fig. 2. Therefore, the composite system is *stabilizable* on the domain $\mathcal{D}$ by using the riser-base pressure as the controlled output. However, the top-side pressure is not minimum-phase and this output does not enter the nonlinear subsystem.

5. EXPERIMENTAL RESULTS

5.1 Experimental Setup

The experiments were performed on a laboratory setup for anti-slug control at the Chemical Engineering Department of NTNU. Fig. 5 shows a schematic presentation of the laboratory setup. The pipeline and the riser are made from flexible pipes with 2 cm inner diameter. The length of the pipeline is 4 m, and it is inclined with a 15° angle. The height of the riser is 3 m. A buffer tank is used to simulate the effect of a long pipe with the same volume, such that the total resulting length of pipe would be about 70 m.

The topside choke valve is used as the input for control. The separator pressure after the topside choke valve is nominally constant at atmospheric pressure. The feed into the pipeline is assumed to be at constant flow rates, 4

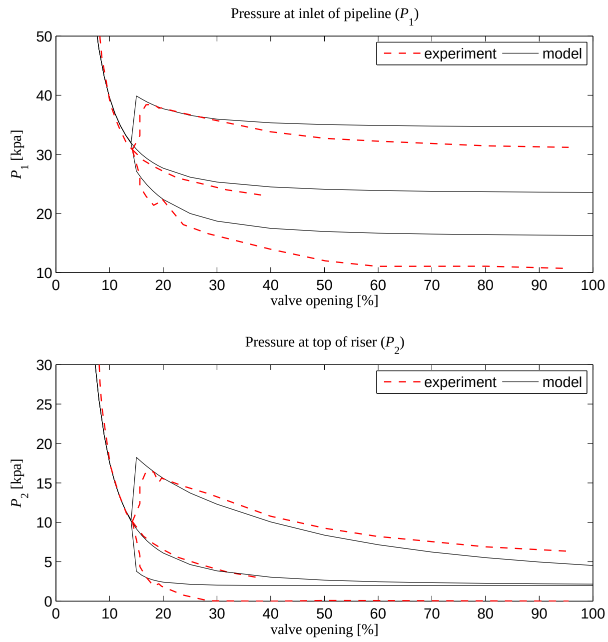

Fig. 6. Bifurcation diagrams of simplified model (solid) compared to experiments (dashed)

$litre/min$ of water and 4.5 $litre/min$ of air. With these boundary conditions, the critical valve opening where the system switches from stable (non-slug) to oscillatory (slug) flow is at $Z^* = 15\%$ for the top-side valve.

Bifurcations diagrams, describing the steady-state and the dynamics of this system, are used to fit the model to the experimental rig (Storkaas and Skogestad (2007)). Fig. 6 shows the bifurcation diagrams of the simplified model (solid lines) compared to the those of the experiments (dashed lines). The system has a stable (non-slug) flow when the valve opening Z is smaller than 15%, and it switches to slugging flow conditions for larger valve openings. The minimum and maximum of the oscillations of the slugging together with the steady-state (in the middle) are shown in Fig. 6.

The desired steady-state (middle line) in slugging condition ($Z > 15\%$) is unstable, but it can be stabilized by using control. The slope of the steady-state line is the static gain of the system, $G = \partial y/\partial u = \partial P_1/\partial Z$. As the valve opening increase this slope decreases, and the gain finally approaches to zero. This makes control of the system with large valve openings very difficult. The controller should keep the loop-gain ($L = KG$, where G is the process gain and K is the controller gain) constant in order to stabilize the system over the whole range of operation.

5.2 Results

The pressure measurements at the riser-base and at top of the riser are very noisy because of air bubbles and hydrodynamic slugs which have much faster dynamics compared to the sever-slugging dynamics. In order to reduce the noise effect on the control signal, we used a second order low-pass filter. The experiment result using the riser-base pressure for feedback-linearization with 30% of the valve opening is shown in Fig. 7. The low-pass filter

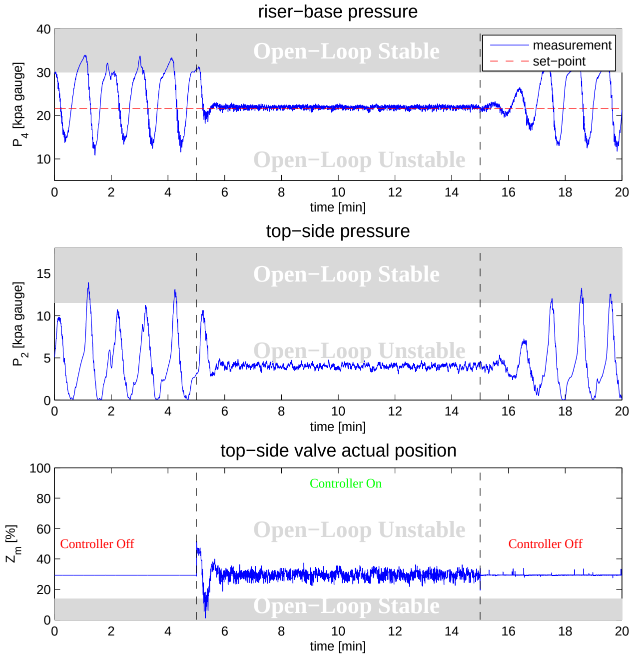

Fig. 7. Experimental result of using riser-base pressure for feedback linearization with 30% valve opening

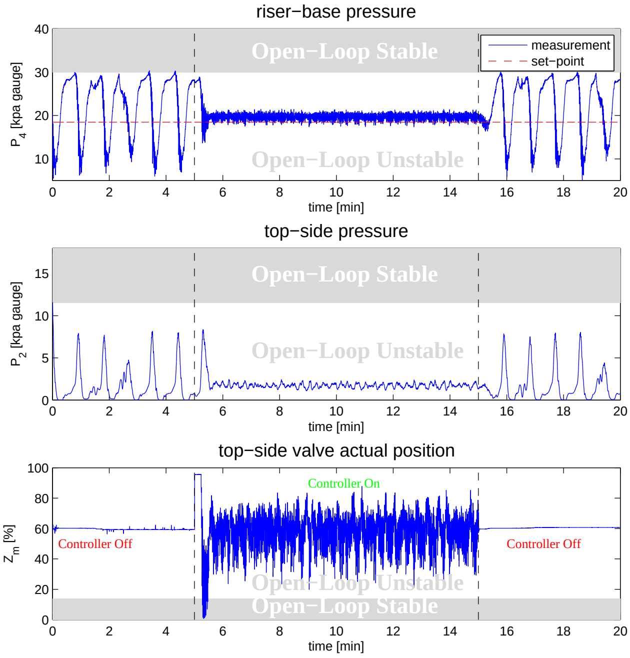

Fig. 8. Experimental result of using riser-base pressure for feedback linearization with 60% valve opening

was used for this case and the system could be stabilized without much noise effect.

It is desired to open the top-side choke valve as much as possible to get the maximum production. For example, one experiment was performed with 60% of valve opening as shown Fig. 8. We did not need to re-tune the controller; the control law generates a larger proportional gain to stabilize the system. On the other hand, the low-pass filter adds a time-lag to the control loop, and it limits the controllability. The extra time-lag from the low-pass filter destabilises the closed-loop system when using the large control gain. Therefore, we can not filter out the noise completely; we should keep the system stable in expense of accepting measurement noises.

We could stabilize the system using the controller in (22) which has the top-side pressure in the linear part. The maximum achievable valve opening for this case as illustrated in Fig. 9 is 20%. This result is same as using a nonlinear observer and state feedback (Jahanshahi et al. (2013)).

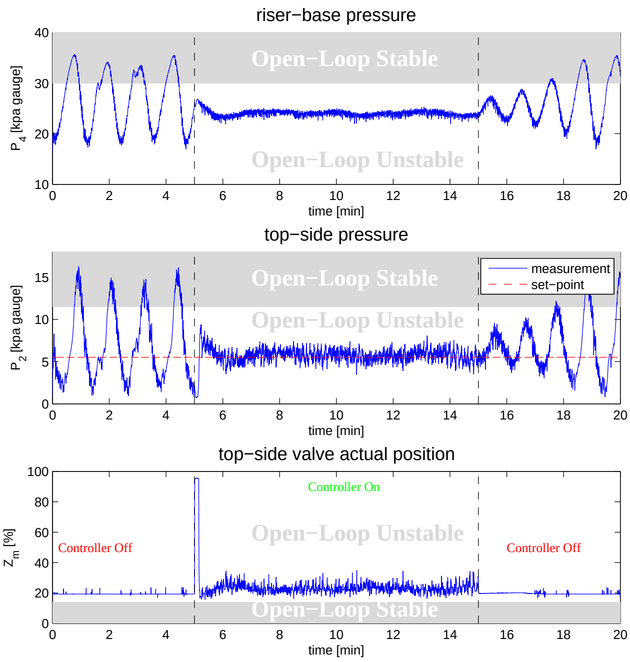

Fig. 9. Experimental result of using top-side pressure for feedback linearization with 20% valve opening

6. CONTROLLABILITY LIMITATIONS

When using the riser-base pressure for feedback linearization the control law counteracts the nonlinearity of the system and we are able to stabilize the system for very large valve openings. The only limitation regarding the riser-base pressure is that with large valve openings the gain of the system decreases drastically and the controller generates a very large proportional gain to stabilize the system. In this situation, the controller can not differentiate between noises and the unstable dynamics. The controller amplified the noises and the control signal is very aggressive as shown in Fig. 8. One problem is how fast our valve can follow the control command signal, another problem is saturation of the valve.

By using the top-side pressure, linear controllers (PID, LQG, $\mathcal{H}_{\text{inf}}$) are not able to stabilize the system, but the nonlinear control law based on feedback linearization could stabilize the system. However, we could stabilize the system using the top-pressure in a very limited range (20% valve opening). Although the nonlinear controller law can counteract the nonlinearity, the fundamental limitations regarding the *non-minimum phase* dynamics (Skogestad and Postlethwaite (2005)) are still in place.

7. CONCLUSION

A nonlinear model-based controller was proposed for anti-slug control. The proof of convergence was shown in theory and experiments. The controller was able to stabilize the system up to very large valve openings without re-tuning.

The advantage of the proposed controller over the previous works is that it directly uses two pressure measurements at the riser-base and at the riser top, not the state variable. We do not have to deal with observers and hope for the *the separation principle* to work.

Further, we showed controllability limitations of the system when using the riser-base pressure and the top pressure for the feedback linearization design. The fundamental limitation related to using the riser-base pressure is small gain of the system with large valve openings. In addition to nonlinearity, the top-side pressure has still the limitation regarding *non-minimum phase* dynamics which can not be by-passed by any control solution.

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Appendix A. CALCULATION OF FRICTION AND DENSITY

Gas mass fraction:

$$\alpha = \frac{w_{g,in}}{w_{g,in} + w_{l,in}} \tag{A.1}$$

Gas density:

$$\rho_g = \frac{y_2 M_G}{R T_r} \tag{A.2}$$

Liquid volume fraction:

$$\alpha_{lt} = \frac{\rho_g w_l}{\rho_g w_l + \rho_l w_g} \tag{A.3}$$

Mixture density at top of riser:

$$\rho_{rt} = \alpha_{lt} . \rho_l + (1 - \alpha_{lt}) \rho_g \tag{A.4}$$

Liquid superficial velocity:

$$\bar{U}_{sl} = \frac{w_{l,in}}{\rho_l A_r} \tag{A.5}$$

Gas superficial velocity:

$$\bar{U}_{sg} = \frac{w_{g,in}}{\rho_g A_r} \tag{A.6}$$

Mixture velocity:

$$\bar{U}_m = \bar{U}_{sl} + \bar{U}_{sg} \tag{A.7}$$

Average density in riser:

$$\bar{\rho} = \frac{y_1 - y_2}{c L_r A_r} \tag{A.8}$$

Reynolds number of flow in riser:

$$Re = \frac{\bar{\rho} \bar{U}_m D_r}{\mu} \tag{A.9}$$

An explicit approximation of the implicit Colebrook-White equation proposed by Haaland (1983) is used as the friction factor in the riser.

$$\frac{1}{\sqrt{\lambda}} = -1.8 \log_{10} \left[\left(\frac{\epsilon / D_r}{3.7} \right)^{1.11} + \frac{6.9}{Re} \right] \tag{A.10}$$

Pressure loss due to friction in riser:

$$F_r = \frac{\alpha_{lt} \lambda \bar{\rho} \bar{U}_m^2 L_r}{2 D_r} \tag{A.11}$$