Dynamic compensation of static estimators from Loss method

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Motivation

Goal: Correct the dynamic behaviour of a variable which is calculated by combining different measurements

Area of application

1. Self-optimizing control
   - Active constraint $c = \mathbf{H}y$ (c is a physical variable)
   - Combination of measurements $c = \mathbf{H}y$ (c is not a physical variable) [1]
2. Static soft-sensor
   - Estimation of a primary variable by combining different measurements with different weights $\hat{y} = \mathbf{H}y$

[1] Alstad et al., Optimal measurement combinations as controlled variables, J.Proc. Control, 19, 1,38-148,2009

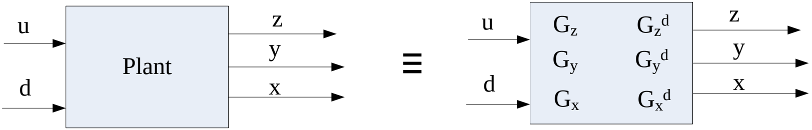
1. Introduction
   - Loss method: closed-loop estimation
   - Distillation case-study
   - Results
2. Dynamic compensation of static estimators
   - Cascade control
   - Selection of subset of measurements
   - Filtering
     - Optimization of LP filter parameters
     - Explicit solution for the filter problem
3. Concluding remarks

Loss Method

OBJECTIVE

The main objective is to find a linear combination of measurements such that keeping these constant indirectly leads to nearly accurate estimation with a small loss L in spite of unknown disturbances, d, and measurement noise, n^x.

$$\min_{\mathbf{H}} \|e\|_2 = \|\mathrm{y} - \hat{\mathrm{y}}\|_2$$



Assumption: Linear models for the primary variables y, measurements x, and secondary variables z

$$y = \mathbf{G}_y u + \mathbf{G}_y^d d \qquad x = \mathbf{G}_x u + \mathbf{G}_x^d d \qquad z = \mathbf{G}_z u + \mathbf{G}_z^d d$$

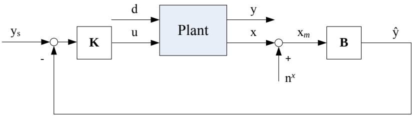
$$\mathbf{G}_y = \left(\frac{\partial y}{\partial u}\right)_d, \quad \mathbf{G}_y^d = \left(\frac{\partial y}{\partial d}\right)_u \quad \mathbf{G}_x = \left(\frac{\partial x}{\partial u}\right)_d, \quad \mathbf{G}_x^d = \left(\frac{\partial x}{\partial d}\right)_u \quad \mathbf{G}_z = \left(\frac{\partial z}{\partial u}\right)_d, \quad \mathbf{G}_z^d = \left(\frac{\partial z}{\partial d}\right)_u$$

The actual measurements x_m, containing measurement noise n^x is

$$x_m = x + n^x$$

The linear estimator is of the form

$$\hat{y} = \mathbf{H} x_m$$



$$\min_{\mathbf{H}} \left\| \mathbf{H} \left[\begin{array}{cc} \mathbf{F}\mathbf{W}_d & \mathbf{W}_{n^x} \end{array} \right] \right\|_F$$
$$\text{s.t. } \mathbf{H}\mathbf{G}_x = \mathbf{G}_y$$

If $\tilde{\mathbf{F}} = \left[\begin{array}{cc} \mathbf{F}\mathbf{W}_d & \mathbf{W}_{n^x} \end{array} \right]$ is full rank, **which is always the case if we include independent measurement noise**, then [3]

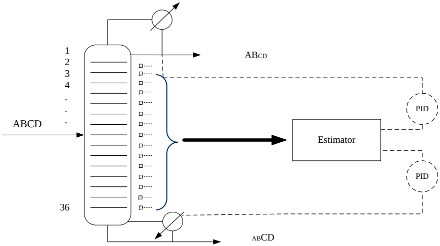
$$\mathbf{H}_4^T = \left(\tilde{\mathbf{F}}\tilde{\mathbf{F}}^T\right)^{-1} \mathbf{G}_x \left(\mathbf{G}_x^T \left(\tilde{\mathbf{F}}\tilde{\mathbf{F}}^T\right)^{-1} \mathbf{G}_x\right)^{-1} \mathbf{G}_y$$

[2] Ghadrdan et al., A new class of static estimators. Ind. Eng. Chem. Res., 2013, 52 (35), 12451-12462

[3] Alstad et al. (2009), Optimal measurement combinations as controlled variables, J. Proc. Control, 19 (1), 138-148

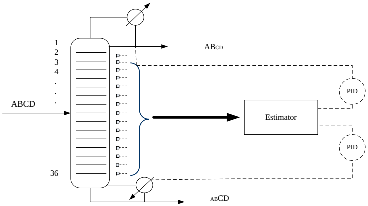
Distillation case-study

- components
  - A- Methanol
  - B- Ethanol
  - C- Propanol
  - D- *n*-Butanol
- 4-component system
- Thermodynamics: Wilson

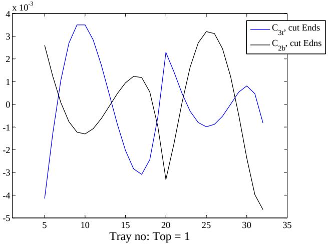


- Objective: Estimate compositions from combination of temperaure measurements

H values



$$\begin{bmatrix} \Delta\hat{y}_1 \\ \Delta\hat{y}_2 \end{bmatrix} = H \times \begin{bmatrix} \Delta T_5 \\ \Delta T_6 \\ \Delta T_7 \\ \Delta T_8 \\ \vdots \\ \Delta T_{32} \end{bmatrix}$$



Monitoring the composition estimated when single temperature loops are closed ("Open-loop estimation (S1)")

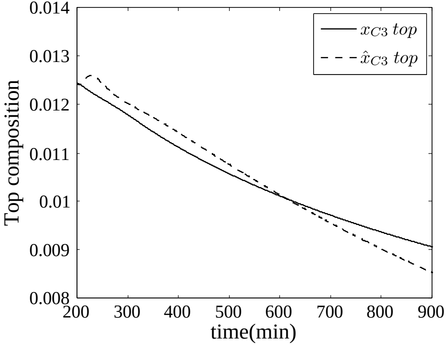


Figure : Top estimate with -1% change in boilup

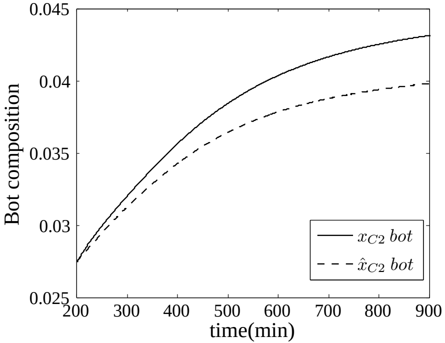
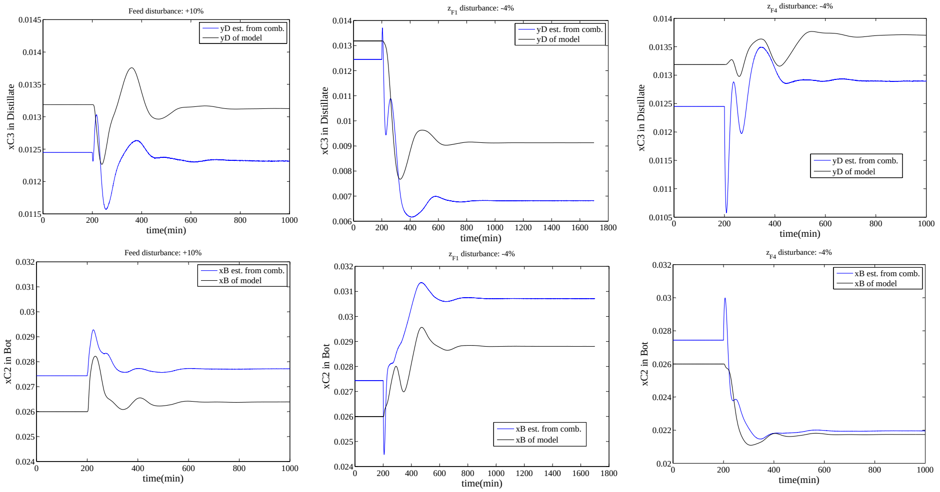


Figure : Bot. estimate with -1% change in boilup

Monitoring the composition estimated when single temperature loops are closed ("Open-loop estimation (S3)")



We studied 3 approaches:

- **Cascade Control**:
  The idea is to close a fast inner loop based on a measurement with no RHP-zero and adjust the setpoint on a time scale slower than the RHP-zero.

- **Use of measurements from the same section of the process**:
  It is less likely to get RHP-zero if the dynamic behavior of the measurements are similar. However, this gives a larger steady-state error.

- **Filters**:
  Low-pass filters will keep the system optimal at steady state. The filtered measurements are $\hat{\mathrm{y}} = \mathbf{H}\mathbf{H}_F\mathrm{u}$

Example

$$\mathbf{G}_x = \begin{bmatrix} \frac{1}{3s+1} \\ \frac{1}{s+1} \end{bmatrix}$$

and the optimal matrix **H** is

$$\mathbf{H} = \begin{bmatrix} 2 & -1 \end{bmatrix}$$

the transfer function from u to ŷ is

$$\mathbf{G} = \mathbf{H}\mathbf{G}_x = \frac{2}{3s+1} - \frac{1}{s+1} = \frac{1-s}{(3s+1)(s+1)} \approx \frac{e^{-1.5s}}{3.5s+1}$$

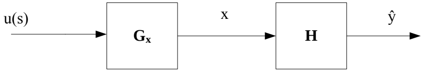
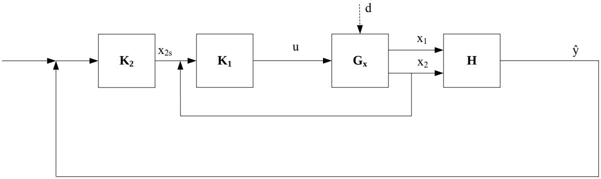


Figure : Block diagram of the estimation

Theorem

Cascade (inner-loop) can not move the zero of $\mathbf{HG}_x$

Proof.



The expression for the estimated primary variable is

$$\hat{y} = h_1 x_1 + h_2 x_2$$

where

$$x_1 = g_1 u, \; x_2 = g_2 u$$

and $u = k(x_{2s} - x_2)$

So,

$$\mathrm{x}_1 = \frac{g_1}{g_2}\mathrm{x}_2, \ \mathrm{x}_2 = \frac{kg_2}{1+kg_2}\mathrm{x}_{2s}$$

The transfer function from x_{2s} to $\hat{\mathrm{y}}$ is

$$\hat{\mathrm{y}} = (h_2 + h_1\frac{g_1}{g_2})\frac{kg_2}{1+kg_2}x_{2s}$$

The term $(h_1g_1 + h_2g_2)$, which includes the RHP zero, is unchanged. □

Selection of subset of measurements

To improve the dynamic controllability: Put structural constraints on the measurements [4]

This is done to

- reduce the time delay between the MVs to CVs,
- have measurements of the same dynamics to avoid inverse response.

In our example: Choose measurement from one side of the column
Drawback: Less accurate compared to the option where we use all the measurements

[4] Yelchuru, R. and Skogestad, S. (2011). Optimal controlled variable selection with structural constraints using MIQP formulations. In Proceedings of the 18th IFAC World Congress Milano, 4977-4982

Filtering

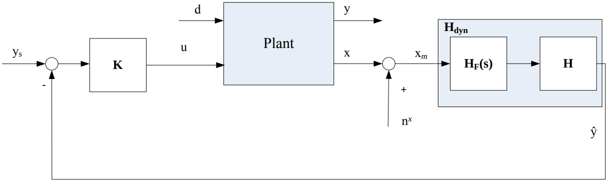


Figure : Block diagram of the estimation system including filter ($\mathbf{H_F}$)

$$\mathbf{H}_F = \begin{bmatrix} \frac{1}{\tau_{F1}s+1} & 0 \\ 0 & \frac{1}{\tau_{F2}s+1} \end{bmatrix}$$

$$\mathbf{H}_F(0) = I$$

Example

$$\mathbf{G}_x = \begin{bmatrix} \frac{1}{3s+1} \\ \frac{1}{s+1} \end{bmatrix}$$

and the optimal matrix $\mathbf{H}$ is

$$\mathbf{H} = \begin{bmatrix} 2 & -1 \end{bmatrix}$$

Some Filters:

$$\mathbf{H}_{F1} = \begin{bmatrix} \frac{1}{s+1} & 0 \\ 0 & \frac{1}{3s+1} \end{bmatrix}$$

$$\mathbf{H}_{F2} = \begin{bmatrix} 1 & 0 \\ 0 & \frac{s+1}{3s+1} \end{bmatrix}$$

$$\mathbf{H}_{F3} = \begin{bmatrix} \frac{3s+1}{s+1} & 0 \\ 0 & 1 \end{bmatrix}$$

The filtered transfer function will be

$$\mathbf{H}_{dyn1}\mathbf{G}_x = \frac{1}{(3s+1)(s+1)} \approx \frac{e^{-0.5s}}{3.5s+1}$$

$$\mathbf{H}_{dyn2}\mathbf{G}_x = \frac{1}{3s+1}$$

$$\mathbf{H}_{dyn3}\mathbf{G}_x = \frac{1}{s+1}$$

Example

$$\mathbf{G}_x = \begin{bmatrix} \frac{1}{3s+1} \\ \frac{1}{s+1} \end{bmatrix}$$

and the optimal matrix **H** is

$$\mathbf{H} = \begin{bmatrix} 2 & -1 \end{bmatrix}$$

Some Filters:

$$\mathbf{H}_{F4} = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{3s+1} \end{bmatrix}$$

The filtered transfer function will be

$$\mathbf{H}_{dyn4}\mathbf{G}_x = \frac{2s+1}{(3s+1)(s+1)} \approx \frac{0.83e^{-0.25s}}{1.25s+1}$$

Example

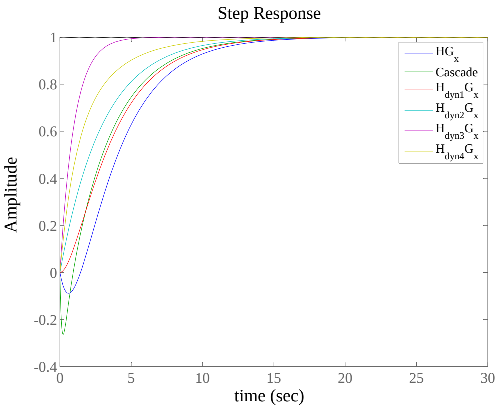


Figure : Step response for different cases

Using Lead-lag compensators, we can make the response as fast as we want.

Distillation case-study

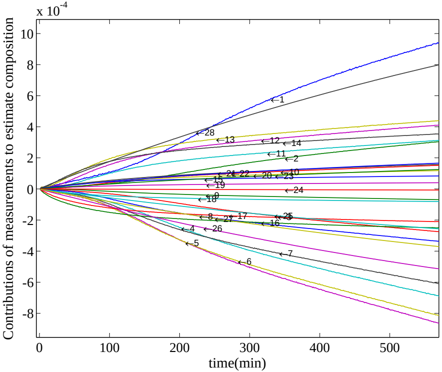


Figure : $\mathbf{HG}_x(t)$ with -1% change in boilup and constant Reflux ratio

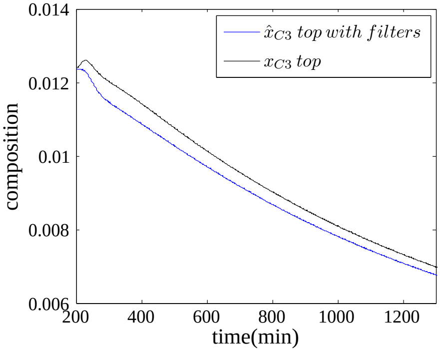


Figure : Estimated composition (tf $= \mathbf{HG}_x$) and filtered estimated composition (tf $= \mathbf{HH}_F\mathbf{G}_x$) where there are filters only on 6th, 16th and 17th measurements

Optimizing filters

$$\min_{\mathbf{H_F}} \left\| \mathbf{G}_{ref} - \mathbf{H}\mathbf{H}_F\mathbf{G}_x \right\|$$

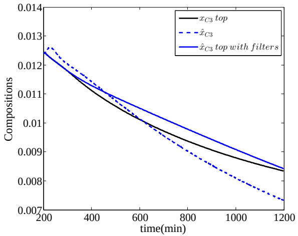
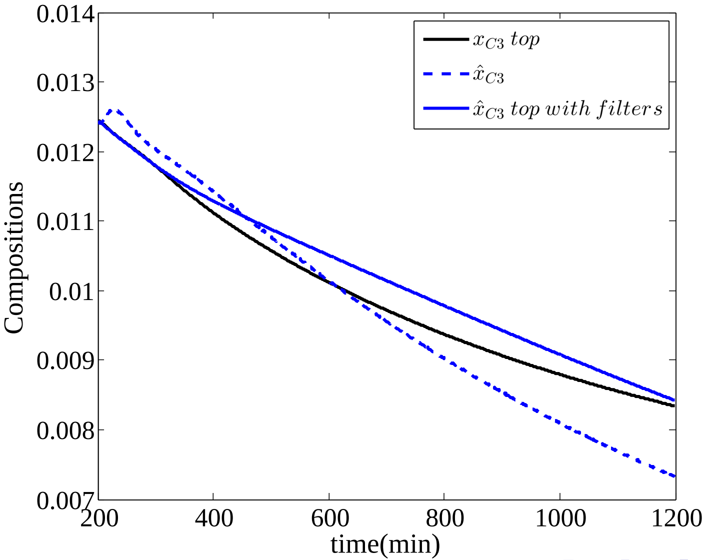


Figure : Real composition, Estimated composition ($\mathbf{HG}_x$) and filtered estimated composition ($\mathbf{HH}_F\mathbf{G}_x$) where filters are optimized for first 100 min. assuming $G_{ref} = G_{u \to y_1}$

Stage no.	H for top comp.	τ_F
5	-0.0043	1039.8
6	-0.0013	1338.3
7	0.0012	33.6
8	0.0028	514.1
9	0.0037	1209.1
10	0.0036	55.0
11	0.0029	1211.4
12	0.0018	1589.6
13	0.0004	554.6
14	-0.0010	1976.2
15	-0.0021	909.4
16	-0.0030	1424.3
17	-0.0032	466.6
18	-0.0025	1640.2
19	-0.0006	278.2
20	0.0023	19.8
21	0.0015	8.3
22	0.0005	1577.0
23	-0.0003	484.9
24	-0.0008	1158.8
25	-0.0010	1026.5
26	-0.0009	925.0
27	-0.0005	860.4
28	0.0000	1992.7
29	0.0006	868.7
30	0.0009	1404.7
31	0.0005	831.4
32	-0.0009	477.4

Optimizing filters



Explicit solution for the filter problem

Approach:

Convert the model matching problem to Nehari problem

$$\|\mathbf{T}_1 - \mathbf{T}_2\mathbf{Q}\|_\infty \;\Rightarrow\; \|\mathbf{R} - \mathbf{X}\|_\infty = \|\Gamma_R\| \leq 1$$

- An optimal $\mathbf{Q}$ exists if the ranks of the two matrices $\mathbf{T}_2(j\omega)$ and $\mathbf{T}_3(j\omega)$ are constant for all $0 < \omega < \infty$ [5].

Our problem:

$$\min_{\mathbf{H_F}} \|\mathbf{G}_{ref} - \mathbf{H}\mathbf{H}_F\mathbf{G}_x\|_\infty$$

Define: $\mathbf{H}_{dyn} = \mathbf{H}\mathbf{H}_F$

[5] Francis, B. (1987). Lecture Notes in Control and Information Sciences: A Course in H∞ Control Theory. Springer-Verlag.

Matrix-valued case

Lemma

Let $\mathbf{U}$ *be an inner matrix and define* $\mathbf{E} = \begin{bmatrix} \mathbf{U}^{\sim} \\ \mathbf{I} - \mathbf{U}\mathbf{U}^{\sim} \end{bmatrix}$, *then,*
$\|\mathbf{EG}\|_{\infty} = \|\mathbf{G}\|_{\infty}$

Proof.

It suffices to show that $\mathbf{E}^{\sim}\mathbf{E} = \mathbf{I}$ □

Lemma

Suppose $\mathbf{F}$ *and* $\mathbf{G}$ *are matrices with no poles on imaginary axis with equal number of columns. If* $\left\| \begin{bmatrix} \mathbf{F} \\ \mathbf{G} \end{bmatrix} \right\|_{\infty} < \gamma$ *then* $\|\mathbf{G}\|_{\infty} < \gamma$ *and* $\|\mathbf{F}\mathbf{G}_o^{-1}\|_{\infty} < 1$

Theorem

(i) $\alpha = \inf\{\gamma : \|\mathbf{Y}\|_\infty < \gamma, \text{dist}(R, \mathrm{RH}_\infty) < 1\}$

(ii) *Suppose* $\gamma > \alpha$, $\mathbf{G}, \mathbf{X} \in \mathrm{RH}_\infty$

$$\|R - \mathbf{X}\|_\infty \leq 1$$

Then $\|\mathbf{T}_1 - \mathbf{T}_2\mathbf{Q}\|_\infty \leq \gamma$

Proof.

(i) Let

$$\gamma_{inf} = \inf\{\gamma : \|\mathbf{Y}\|_\infty < \gamma, \text{dist}(R, \mathrm{RH}_\infty) < 1\}$$

choose $\varepsilon > 0$ and then choose γ such that $\alpha + \varepsilon > \gamma > \alpha$. Then there exist $\mathbf{Q}$ in RH_∞ such that

$$\|\mathbf{T}_1 - \mathbf{T}_2\mathbf{Q}\|_\infty < \gamma$$

From Lemma 1 we have:

$$\left\| \begin{bmatrix} \mathbf{U}_i^\sim \\ \mathbf{I} - \mathbf{U}_i\mathbf{U}_i^\sim \end{bmatrix} (\mathbf{T}_1 - \mathbf{T}_2\mathbf{Q}) \right\|_\infty \leq \gamma$$

This is equivalent to $\left\| \begin{bmatrix} \mathbf{U}_i^{\sim}\mathbf{T}_1 - \mathbf{U}_o\mathbf{Q} \\ \mathbf{Y} \end{bmatrix} \right\|_\infty < \gamma$

This implies from Lemma 2 that

$$\|\mathbf{Y}\|_\infty < \gamma$$

$$\|\mathbf{U}_i^{\sim}\mathbf{T}_1\mathbf{Y}_o^{-1} - \mathbf{U}_o\mathbf{Q}\mathbf{Y}_o^{-1}\|_\infty < 1$$

The latter inequality implies $\text{dist}\left(R, \mathbf{U}_o \mathrm{RH}_\infty \mathbf{Y}_o^{-1}\right) < 1$

$\mathbf{U}_o$ is right-invertible in RH_∞ and $\mathbf{Y}_o$ is invertible in RH_∞. So, (26) gives

$$\text{dist}\left(R, \mathrm{H}_\infty\right) < \text{dist}\left(R, \mathrm{RH}_\infty\right) < 1$$

□

The general algorithm to obtain $\mathbf{Q}$ is as follows

Step 1 Compute $\mathbf{Y}$ and $\|\mathbf{Y}\|_\infty$

Step 2 Find an upper bound α_1 for α ($\|\mathbf{T}_1\|_\infty$ is the simplest choice)

Step 3 Select a trial value for γ in the interval $(\|\mathbf{Y}\|_\infty, \alpha_1]$

Step 4 Compute $\mathbf{R}$ and $\|\Gamma_R\|$. Then $\|\Gamma_R\| < 1$ iff $\alpha < \gamma$. Change the value of γ correspondingly to meet this criteria

Step 5 Find a minimal realization of $\mathbf{R}$: $\mathbf{R}(s) = [A, B, C, 0]$

Step 6 Solve the lyapunov equations to find controllability and observability gramians and set $\mathbf{N} = (\mathbf{I} - \mathbf{L}_o \mathbf{L}_c)^{-1}$

Step 7 Set

$$\mathbf{L}_1(s) = \left[\begin{array}{c|c} \mathbf{A} & -\mathbf{L}_c\mathbf{N}\mathbf{C}^T \\ \hline \mathbf{C} & \mathbf{I} \end{array}\right]$$

$$\mathbf{L}_2(s) = \left[\begin{array}{c|c} \mathbf{A} & \mathbf{N}^T\mathbf{B} \\ \hline \mathbf{C} & 0 \end{array}\right]$$

$$\mathbf{L}_3(s) = \left[\begin{array}{c|c} -\mathbf{A}^T & \mathbf{N}\mathbf{C}^T \\ \hline -\mathbf{B}^T & 0 \end{array}\right]$$

$$\mathbf{L}_4(s) = \left[\begin{array}{c|c} -\mathbf{A}^T & \mathbf{N}\mathbf{L}_o\mathbf{B}^T \\ \hline \mathbf{B}^T & \mathbf{I} \end{array}\right]$$

Step 8 Select $\mathbf{Y}$ in RH_∞ with $\|\mathbf{Y}\|_\infty \leq 1$ (for example $\mathbf{Y} = 0$) and set $\mathbf{X} = \mathbf{R} - (\mathbf{L}_1\mathbf{Y} + \mathbf{L}_2)(\mathbf{L}_3\mathbf{Y} + \mathbf{L}_4)$

Example

$$\mathbf{G}_x = \begin{bmatrix} \frac{1}{3s+1} \\ \frac{1}{s+1} \end{bmatrix}, \quad \mathbf{G}_{ref} = \frac{1}{0.5s+1}$$

and the optimal matrix $\mathbf{H}$ is

$$\mathbf{H} = \begin{bmatrix} 2 & -1 \end{bmatrix}$$

$$\mathbf{H}_F = \begin{bmatrix} \frac{2.011s^5+5.971s^4+0.5466s^3-7.412s^2-1.321s+0.2051}{s^6+3.699s^5-0.418s^4-13.02s^3-7.559s^2+1.578s+0.4668} \\ \frac{3.017s^4+11.27s^3+9.929s^2-1.597s-1.026}{s^6+3.699s^5-0.418s^4-13.02s^3-7.559s^2+1.578s+0.4668} \end{bmatrix}$$

- A weighting transfer function should be included to make $\mathbf{H}_F(0) = I$

What should $\mathbf{G}_{ref}$ be?

In the case of estimation

- First option: actual values from simulation
- The estimate can be even faster (since $\hat{y}$ is being controlled)

One idea is to specify a first-order transfer function with the smallest time constant in the process as the desired transfer function from inputs to the estimates.

For our case: $\mathbf{G}_{ref} = \frac{1}{\tau_{int} s+1}$
Internal time constants can be found from changing the two inputs boilup and reflux rate at the same time such that the external flows remain constant.

$$\Delta y_D = \left(\frac{-0.06\,\mathrm{e}^{-2s}}{740s+1}\right)\Delta V$$

$$\Delta x_B = \left(\frac{-0.067\,\mathrm{e}^{-0.33s}}{137s+1}\right)\Delta V$$

Concluding remarks

- Extra dynamic compensation is necessary when measurements with different dynamics are combined
  - Cascade will not remove the RHP zero, but helps with rejecting disturbance
  - Choosing measurements with similar dynamics might help to avoid dynamic problems
  - Filtering fast dynamic measurements will help remove the inverse response
- Explicit solution comes from converting model matching problem to Nehari problem
- The filter matrix gets big as the number of measurements increase.
- A weight function should be considered to weaken the effect of filter at steady-state

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Thanks for your attention