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Control Structure Selection for Optimal Operation of a Heat Exchanger Network

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***Abstract*—We consider the control structure design for a heat exchanger network (HEN), where a stream is split into parallel lines which are heated individually before they are merged together again. The objective is to find a control structure which maximizes the final temperature. We consider two scenarios, where (Scenario 1) the flow rates and the heat transfer coefficients are considered as disturbances, and (Scenario 2) where the hot stream temperatures are treated as additional disturbances. In both scenarios it is found that controlling linear measurement combinations gives very good performance, and that including flow measurements in the combinations gives little advantage over using only combinations of temperature measurements.**

***Index Terms*—Control structure selection, Self-optimizing control, Heat exchanger networks, Optimization**

I. Introduction

With growing markets and limited natural resources, it is increasingly necessary to use the available resources in the best possible way. In many cases, this can be directly translated into re-using energy. Important tools for re-using energy are heat exchanger networks (HENs), which are operated such that a maximum amount of energy is transferred from one set of process streams to another.

It has been shown in [1], [2], that if there are no stream splits in the heat exchanger network, then optimal operation of the HEN can be considered and modelled as a linear program (LP), where the optimal operation point is always at constraints. In particular this means that the available degrees of freedom should be used for

1) keeping required target temperatures at their setpoints
2) meeting active constraints (i.e. fully opening or closing of some bypasses and utilities)

Since the stream parameters such as temperatures and flow rates change under operation, the task of achieving optimal operation can be re-formulated as finding the set of active constraints corresponding to the current operating conditions.

However, when streams are split, the optimal operating point will generally no longer be at constraints; and the optimal split will not remain constant because of disturbances such as changing temperatures of the streams, changing flow rates, or change of heat transfer properties due to fouling.

One approach for adjusting the split optimally is to periodically solve a numerical optimization problem to find the optimal setpoint values for some set of controlled variables (real-time optimization (RTO)) [3]. A good choice of controlled variables (CVs) will reduce the necessity to update the setpoints, while a poor choice will require frequent setpoint updates to remain close to optimality.

The RTO approach relies heavily on a good process model and the ability to solve the optimization problem online. Furthermore it is necessary to obtain good estimates of the model parameters. All these factors render the online-optimization approach relatively expensive.

Therefore, it is desirable to find controlled variables, which do not have to be updated often. A control structure which, in spite of varying disturbances, gives an acceptable loss without the need to adjust the setpoints for the controlled variables is called "self-optimizing" [4]. In such a control structure the setpoints of the controlled variables need to be updated very infrequently, or not even at all. For a recent overview of available methods for control structure design using the self-optimizing control ideas, we refer to [5] and the literature cited therein.

The contribution of this work is to study the effect of different self-optimizing control structures for a HEN with stream split, and in particular to evaluate the necessity of relative expensive flow measurements. Our paper is structured such that in the next section we present some basic concepts for finding self-optimizing variables. In Section III we introduce the heat exchanger network, and Section IV contains the simulation scenarios and results. Our paper is closed with a discussion and conclusions in Section V.

II. Controlled variable selection

In this paper we apply local controlled variable selection procedures from self-optimizing control [4], [6], [7]. We assume that optimal operation corresponds to minimizing a scalar cost function, and for local optimality around the nominal optimum, it can be approximated [8] as

$$\min_{\Delta u} J(\Delta u, \Delta d) := [\Delta u^T\ \Delta d^T] \begin{bmatrix} J_{uu} & J_{ud} \\ J_{ud}^T & J_{dd} \end{bmatrix} \begin{bmatrix} \Delta u \\ \Delta d \end{bmatrix}. \quad (1)$$

Here, J is the scalar approximation of the nonlinear cost function, $\Delta u \in \mathbb{R}^{n_u}$ and $\Delta d \in \mathbb{R}^{n_d}$ denote the inputs and the disturbances in deviation variables, respectively. $J_{uu} = \frac{\partial^2 J}{\partial u^2}$ denotes the positive definite Hessian of the cost function with respect to the degrees of freedom u, and $J_{ud} = \frac{\partial^2 J}{\partial u \partial d}$, $J_{dd} = \frac{\partial^2 J}{\partial d^2}$ denote second order partial derivatives with respect to $[u, d]^T$ and $[d, d]^T$, respectively.

The model is linearized around the nominal optimum, such that

$$\Delta y = G^y \Delta u + G_d^y \Delta d + \Delta n^y, \quad (2)$$

where Δy denotes the n_y-dimensional measurement vector, $G^y = \frac{\partial y}{\partial u}$ and $G_d^y = \frac{\partial y}{\partial d}$ are gain matrices of appropriate sizes, evaluated at the nominal operating point. Finally, $\Delta n^y \in \mathbb{R}^{n_y}$ denotes the measurement noise or implementation error.

A. Exact local method

The goal is to find a set of controlled variables (CV)

$$\Delta c = H \Delta y, \quad (3)$$

where $\Delta c \in \mathbb{R}^{n_u}$, and H is a measurement selection or combination matrix of size $n_u \times n_y$, such that controlling the CVs at constant setpoints, $\Delta c = 0$, results in acceptable loss

$$L = J(H, \Delta d, \Delta n^y) - J_{opt}(d) \quad (4)$$

in spite of varying disturbances Δd and noise Δn^y. In [6] it is shown that solving the convex optimization problem

$$\begin{aligned} &\min_H ||HY||_F \\ &\text{s.t.} \\ &HG^y = J_{uu}^{1/2} \end{aligned} \quad (5)$$

leads to a H which locally minimizes the average and worst case loss. Here $||\cdot||_F$ denotes the Frobenius norm,

$$Y = [FW_d \quad W_{n^y}] \quad (6)$$

denotes the augmented optimal sensitivity matrix where $F = \partial y^{opt} / \partial d$, and the matrices W_d and W_{n^y} denote diagonal scaling matrices of appropriate dimensions, with

$$\Delta d = W_d d' \quad \text{and} \quad \Delta n^y = W_{n^y} n^{y'}, \quad (7)$$

such that

$$\left|\left| [d' \, n^{y'}]^T \right|\right|_2 \leq 1. \quad (8)$$

The local average loss resulting from a given control structure represented by H can be evaluated using ([6], [7])[1]

$$L = \frac{1}{2} \left|\left| J_{uu}^{1/2} (HG^y)^{-1} HY \right|\right|_F^2 \quad (9)$$

Remark 1. *The matrices $F = \frac{\partial y^{opt}}{\partial d}$ and G^y can be obtained by re-optimizing the process and a finite difference approximation, or as done in this work, by using the efficient sensitivity calculation routines in sIPOPT [9]. The software provides the sensitivity of the KKT solution with respect to parameters (disturbances d). As sIPOPT re-uses the matrix factorizations from the NLP solver IPOPT [10], the sensitivities are obtained by a simple backsolve. Similarily, the inverse of the reduced Hessian J_{uu} can be easily obtained from sIPOPT.*

[1] Our definition of the average loss is slightly different from [7], where it is defined as $L = \frac{1}{6(n_y+n_d)} \left|\left| J_{uu}^{1/2} (HG^y)^{-1} HY \right|\right|_F^2$. Although this definition is formally correct, one may reduce the loss arbitrarily by adding (unused) measurements, $n_y \to \infty$. To avoid this, we define the average loss as in (9).

B. Null-space methods

If there is no noise, and we have sufficient independent measurements ($n_y \geq n_u + n_d$), then H may be chosen in the left null space of F. In this case we have $HF = 0$, and since $W_{n^y} = 0$, we also have that $HY = 0$. Hence the local loss from (9) is zero. In practice, however, there will always be measurement noise ($W_{n^y} \neq 0$), and the CVs obtained from the null space method will cause a loss, which can be unacceptably high. Alstad et al. [6] present the extended null-space method, which selects H in the null-space of F (optimal disturbance rejection), and uses the remaining degrees of freedom to minimize the effect of noise. We, however, aim at minimizing the combined effect of disturbances and noise. Therefore, we use Eq. (5) for determining candidates for controlled variables.

C. Selecting subsets of measurements

The above expressions for optimal measurement combinations are valid for controlled variables c which contain a given set of measurements y. However in practice it is often desirable to use subsets of the available measurements, because the performance does not improve significantly beyond including a certain number of measurements, while installation and maintenance costs increase.

For finding the best subsets of measurements, branch-and-bound methods have been developed [11], [12], [13]. Recently [14] proposed a method to implement certain structural constraints on H.

In this paper we will not dwell on the available methods, but rather investigate some sets of controlled variables for a HEN with a stream split. For finding sets of measurements, we use the freely available software `b3av.m` [13] for MATLAB™.

III. HEAT EXCHANGER NETWORK

A. Process description

We consider the HEN in Fig. 1, where a feed stream with temperature T_0 and heat capacity w_0 is split into 6 lines, which are heated independently before they are merged again. The structure of this HEN is the same as the structure of the HEN for feed pre-heating of the crude oil distillation unit at the Mongstad refinery in Norway. The objective is to adjust the feed split such that the end temperature is maximized,

$$\min J = -T_{end}. \quad (10)$$

This corresponds to minimizing the energy required for the crude oil distillation unit. The heat exchanger network has 30 measurable temperatures and 14 flows. Thus, the potentially

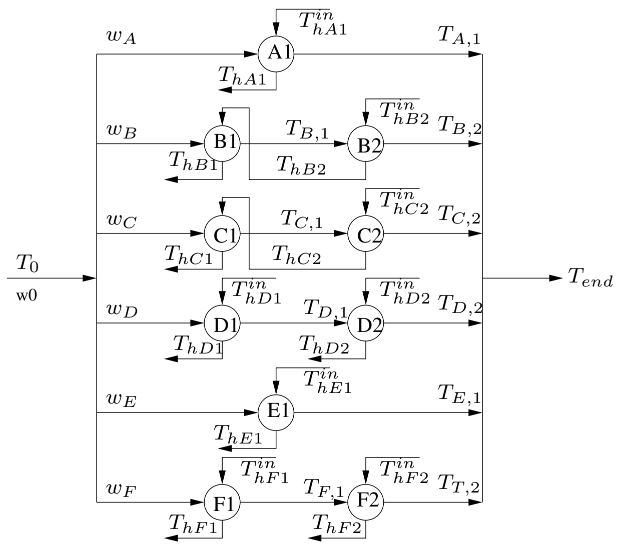

Fig. 1. Heat exchanger network

available measurement vector is

$$\begin{aligned} y = \big[&T_0,\, T_{end},\, T_{1A},\, T_{h1A},\, T_{h1A}^{in}, \\ &T_{2B},\, T_{1B},\, T_{h1B},\, T_{h2B},\, T_{h2B}^{in}, \\ &T_{2C},\, T_{1C},\, T_{h1C},\, T_{h2C},\, T_{h2C}^{in}, \\ &T_{2D},\, T_{1D},\, T_{h1D},\, T_{h2D},\, T_{12D}^{in},\, T_{h2D}^{in}, \\ &T_{1E},\, T_{h1E},\, T_{h1E}^{in}, \\ &T_{2F},\, T_{1F},\, T_{h1F},\, T_{h2F},\, T_{h1F}^{in},\, T_{h2F}^{in}, \\ &w_A,\, w_{h1A},\, w_B,\, w_{h2B},\, w_C,\, w_{h2C}, \\ &w_D,\, w_{h1D},\, w_{h2D},\, w_E,\, w_{h1E},\, w_F,\, w_{h1F},\, w_{h2F}\big]\,. \end{aligned} \quad (11)$$

We assume that all temperature measurements have an uncertainty (noise) of +/-1°C, that is $W_{n^{y_T}} = 1°C\ \mathrm{I}_{30\times30}$, where $\mathrm{I}_{30\times30}$ denotes the identity matrix of dimension 30×30.

The flow measurements are assumed to have an uncertainty of 2% of the nominal value, resulting in $W_{n^{y_w}} = diag(0.64, 0.96, 0.92, 1.85, 0.48, 0.72, 0.59, 2.37, 0.68, 0.77, 1.60, 1.11, 0.94, 3.50)$ kW/K. The resulting overall noise weighting matrix is

$$W_{n^y} = \begin{bmatrix} W_{n^{y_T}} & 0 \\ 0 & W_{n^{y_w}} \end{bmatrix}. \quad (12)$$

B. Heat exchanger network model

The heat exchangers are modelled with simple energy balances. We present the model of the heat exchanger in the first line (A) in detail here; the heat exchangers on the other lines are modelled analogously. The main assumptions are incompressible fluids and constant specific heat capacities in all flows.

The energy balance around the cold and hot sides give

$$Q_{A1} = w_A(T_{A,1} - T_0) = w_{hA1}(T_{hA1}^{in} - T_{hA1}), \quad (13)$$

where Q_{A1} denotes the heat, $w_A = m_A c_P$ is the heat capacity flow rate (product of the mass flow rate $m_{A,1}$ and the specific heat capacity c_P) of stream A, w_{hA1} is the heat capacity flow rate of the hot stream $hA1$, and T_X^{in} and T_X denote the temperatures of stream X at the inlet and outlet of the heat exchanger, respectively.

The transfered heat is calculated as

$$Q_{A1} = UA_{A1}\Delta T_{lm}, \quad (14)$$

where the logarithmic temperature difference is calculated using the Underwood approximation [15]:

$$(\Delta T_{lm})^{1/3} = \frac{1}{2}[(\Delta T_1)^{1/3} + (\Delta T_2)^{1/3}] \quad (15)$$

where $\Delta T_1 = T_{hA1}^{in} - T_{A,1}$ and $\Delta T_2 = T_{h,A1} - T_0$ denote the temperature differences on the two sides of the heat exchanger. This approximation is very close to the logarithmic mean temperature, while it has better numerical properties. In particular, it is still defined when $\Delta T_1 = \Delta T_2$, while the exact logarithmic mean temperature is not, which can cause problems during the optimization.

An energy balance around the mixer gives

$$T_{end} = \frac{1}{w_0} \sum_{i=A,B,\ldots,F} w_i T_i, \quad (16)$$

and a mass balance around the splitter:

$$w_0 = \sum_{i=A,B,\ldots,F} w_i. \quad (17)$$

C. Degrees of freedom analysis and nominal optimum

Under operation, the hot flow rates and temperatures are given, as well as the feed temperature T_0. Since the total feed flow rate is fixed, the flow in 5 of 6 lines can be set freely. The sixth flow is given from the law of mass conservation. Thus, there are 5 degrees of freedom, and we need to find $n_c = 5$ controlled variables. The nominal parameter values are listed in Table I, and the corresponding optimal measurement values are given in Table II.

IV. Simulation Scenarios

We consider two scenarios: In Scenario 1, the varying disturbances are assumed to be only the flow rates of the feed and the hot streams, and the heat transfer UA in all heat exchangers. In Scenario 2 the temperatures of incoming streams are considered as additional disturbances.

For both scenarios, we consider different choices of CVs, including single measurements and selected measurement combinations. Their performance is compared in terms of the average loss L calculated from equation (9).

A. Scenario 1. Flow rate and heat transfer disturbances

The flow and heat transfer disturbances for Scenario 1 are given together with their numerical values in Table III. In the next sections, we consider the performance of some selected control structures.

TABLE I

NOMINAL PARAMETER VALUES

Symbol	Value	Unit	Description
T_0	133	°C	Feed temperature
w_0	225	kW/K	Feed heat capacity
T_{h1A}^{in}	300	°C	Hot stream 1A temperature
w_{h1A}	48	kW/K	Hot stream 1A heat capacity
UA_{A1}	131	kW/K	Heat transfer coefficient times area A1
$wh1B$	92.5	kW/K	Hot stream 1B heat capacity
UA_{B1}	102.7	kW/K	Heat transfer coefficient times area B1
T_{h2B}^{in}	270.3	°C	Hot stream 2B temperature
UA_{B2}	88.64	kW/K	Heat transfer coefficient times area B2
$wh1C$	35.8	kW/K	Heat capacity stream 1C
UA_{C1}	84	kW/K	Heat transfer coefficient times area C1
T_{h2C}^{in}	245	°C	Hot stream 2C temperature
UA_{C2}	133.6	kW/K	Heat transfer coefficient times area C2
T_{12D}^{in}	226	°C	Hot stream 1D temperature
w_{h1D}	118.5	kW/K	Heat capacity stream 1D
UA_{D1}	132.8	kW/K	Heat transfer coefficient times area D1
T_{h2D}^{in}	273.8	°C	Hot stream 2D temperature
w_{h2D}	33.9	kW/K	Heat capacity stream 2D
UA_{D2}	41.6	kW/K	Heat transfer coefficient times area D2
T_{h1E}^{in}	256.4	°C	Hot stream 1E temperature
w_{h1E}	79.9	kW/K	Heat capacity stream 1E
UA_{E1}	190.9	kW/K	Heat transfer coefficient times area E1
T_{h1F}^{in}	203	°C	Hot stream 1F temperature
w_{h1F}	47.2	kW/K	Heat capacity stream 1F
UA_{F1}	49.4	kW/K	Heat transfer coefficient times area 1F
T_{h2F}^{in}	248	°C	Hot stream 2F temperature
w_{h2F}	175.1	kW/K	Heat capacity stream 2F
UA_{F2}	224.1	kW/K	Heat transfer coefficient times area 2F

TABLE II

NOMINAL OPTIMAL MEASUREMENT VALUES

Variable	Value	Variable	Value
T_{end}	255.676 °C	T_{h2D}	247.350 °C
T_{1A}	283.102 °C	T_{1E}	251.430 °C
T_{h1A}	200.358 °C	T_{h1E}	199.429 °C
T_{2B}	261.281 °C	T_{2F}	244.393 °C
T_{1B}	231.824 °C	T_{1F}	164.622 °C
T_{h1B}	206.632 °C	T_{h1F}	165.818 °C
T_{h2B}	255.680 °C	T_{h2F}	222.716 °C
T_{2C}	243.184 °C	w_A	31.8637 kW/K
T_{1C}	213.075 °C	w_B	45.9091 kW/K
T_{h1C}	171.672 °C	w_C	23.8251 kW/K
T_{h2C}	224.962 °C	w_D	29.4667 kW/K
T_{2D}	254.175 °C	w_E	38.4364 kW/K
T_{1D}	223.745 °C	w_F	55.4990 kW/K
T_{h1D}	203.435 °C		

1) Open loop operation: Simply leaving the split at the nominal optimal values results in an average loss of

$$L_{OL1} = 0.3147°C \tag{18}$$

Although this value is quite low, it may be reduced further by controlling the right variables.

2) Controlling a combination of all measurements:

a) Exact local method: Using a combination $c = Hy$ of all measurements y, for our scenario the average loss is

$$L_{All1} = 0.0284°C \tag{19}$$

This is the locally best loss that can be achieved in this scenario using a linear measurement combination.

TABLE III

FLOW AND HEAT TRANSFER DISTURBANCES WITH MAGNITUDES FOR SCENARIO 1

Variable	Weighting in W_d	Description
w_0	11.25 kW/K	Feed heat capacity
w_{h1A}	2.40 kW/K	Hot stream 1A heat capacity
UA_{A1}	13.10 kW/K	Heat transfer coefficient times area A1
UA_{B1}	10.20 kW/K	Heat transfer coefficient times area B1
w_{h2B}	4.60 kW/K	Hot stream 2B heat capacity
UA_{B2}	8.90 kW/K	Heat transfer coefficient times area B2
UA_{C1}	8.40 kW/K	Heat transfer coefficient times area C1
w_{h2C}	1.80 kW/K	Hot stream 2C heat capacity
UA_{C2}	13.40 kW/K	Heat transfer coefficient times area C2
w_{h1D}	5.90 kW/K	Heat capacity stream 1D
UA_{D1}	13.30 kW/K	Heat transfer coefficient times area D1
w_{h2D}	1.70 kW/K	Heat capacity stream 2D
UA_{D2}	4.20 kW/K	Heat transfer coefficient times area D2
w_{h1E}	4.00 kW/K	Heat capacity stream 1E
UA_{E1}	19.10 kW/K	Heat transfer coefficient times area E1
w_{h1F}	2.40 kW/K	Heat capacity stream 1F
UA_{F1}	4.90 kW/K	Heat transfer coefficient times area 1F
w_{h2F}	8.80 kW/K	Heat capacity stream 2F
UA_{F2}	22.40 kW/K	Heat transfer coefficient times area 2F

b) Null space method: If we neglect the noise and select H in the left null space of F, the performance of the plant with noise is dependent on which basis vectors of the null space are chosen for H. For two different sets of basis vectors H_1 and H_2 we have

$$\begin{aligned} L_{All1}^{NullSpace}(H_1) &= 19.1711°C \\ L_{All1}^{NullSpace}(H_2) &= 2.0702°C \end{aligned} \tag{20}$$

The loss is very different for different choices of basis vectors of the left null space of F. If there was no noise, $W_d = 0$, we would have $L_{All1}^{NullSpace}(H_1) = L_{All1}^{NullSpace}(H_2) = 0$. We do not further consider the issue of selecting the best basis vectors using the extended null-space method [6], because this is beyond the scope of this paper, and because the exact local method (Eq. (5)) selects the best (optimal) H in presence of noise and disturbances.

3) Using subsets of measurements: The average loss when using different subsets of measurements in linear variable combinations is shown in blue in Fig. 2. The largest reduction in the loss comes when increasing the number of measurements from 5 to 6. The reason for this is that the disturbances may enter the plant in all 6 lines. When using only 5 measurements from 5 lines, the disturbance in the 6th line is not detected. Adding a 6th measurement from the remaining line makes it possible to detect disturbances affecting this line, too, and therefore reduces the loss significantly.

Above a certain number of measurements the loss does not significantly decrease further. From Fig. 2 it is evident that the loss decreases very little for more measurements than 15, and above 20 measurements the decrease in loss is hardly observable[2]. The loss is very low for this scenario, and in

[2]It decreases slightly for some more measurements, but for the last 9 measurements it is completely flat. These last 9 measurements are the inlet temperatures of the feed and the hot streams, which are assumed to be constant. Thus they do not provide any further information for disturbance rejection in Scenario 1.

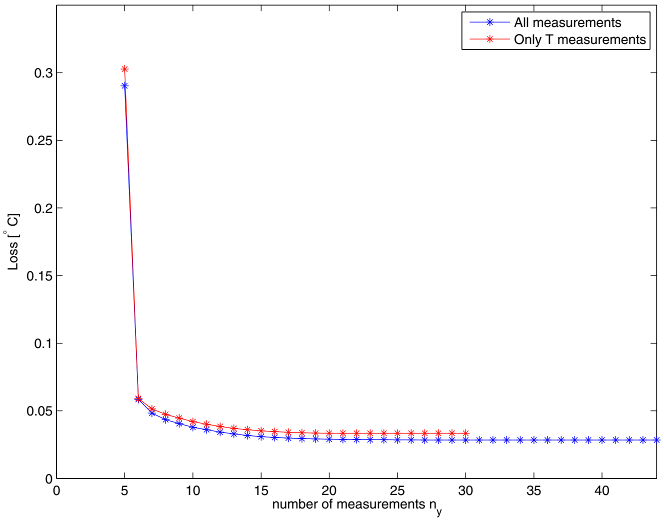

Fig. 2. Scenario 1 (Flow rate and UA disturbances): Loss for best combinations of different number of measurements n_y

most practical cases, open loop operation will be sufficient. However, if one wants to optimize performance further, it does not seem reasonable invest in measurement equipment for more than 10-15 measurements.

a) Only temperature measurements: Typically, not all measurements are equally capital intensive. E.g. temperature measurements are generally much cheaper than flow measurements. Therefore we next consider cases with different sets of temperature measurements. The relationship between the average loss and the number of temperature measurements included in the CVs is given in red in Fig. 2. The trend is very similar to when flow measurements are used, too. After a sharp initial decrease, the curve flattens out quickly. The average loss resulting from using all temperature measurements is

$$L_{AllT1} = 0.0335°C. \quad (21)$$

This is only little higher than when using all measurements, including flows. Note that also here, the constant inlet temperatures do not contribute to minimizing the loss.

b) Only specific temperature measurements: In many practical cases we may have that the outlet temperatures of the cold streams are measured, i.e.

$$y = [T_{1A}, T_{1B}, T_{2B}, T_{1C}, T_{2C}, T_{1D}, T_{2D}, T_{1E}, T_{1F}, T_{2F}]^T \quad (22)$$

In this case the average loss using a linear combination is

$$L_{outTemp1} = 0.0639°C \quad (23)$$

c) Single temperature measurements: The simplest strategy (beside open loop operation) is to control as many single temperatures as there are degrees of freedom. The best set gives a loss of

$$L_{Single\,T1} = 0.3027°C \quad (24)$$

TABLE IV

TEMPERATURE DISTURBANCES WITH MAGNITUDES FOR SCENARIO 2

Variable	Weighting in W_d	Description
$T0$	$13.0°C$	Feed temperature
T^{in}_{h1A}	$30.0°C$	Hot stream 1A temperature
T^{in}_{h2B}	$27.0°C$	Hot stream 2B temperature
T^{in}_{h2C}	$24.5°C$	Hot stream 2C temperature
T^{in}_{12D}	$22.6°C$	Hot stream 1D temperature
T^{in}_{h2D}	$27.4°C$	Hot stream 2D temperature
T^{in}_{h1E}	$25.6°C$	Hot stream 1E temperature
T^{in}_{h1F}	$20.3°C$	Hot stream 1F temperature
T^{in}_{h2F}	$24.8°C$	Hot stream 2F temperature

with the measurements

$$y = [T_{1A}, T_{2B}, T_{h2C}, T_{2D}, T_{1E}]^T. \quad (25)$$

B. Scenario 2. Flow, heat transfer, and temperature disturbances

In this more general scenario, in addition to varying flow rates and heat transfer coefficients, as in the previous section, we assume that the stream temperatures are also varying in the ranges given in Table IV. We proceed as in the previous section with considering the different cases with different measurements involved.

1) Open loop operation: Operating the heat exchanger in open loop with the nominal values for the splits, results in a quite high average loss of

$$L_{OL2} = 2.2836°C. \quad (26)$$

2) Controlling a combination of all measurements:

a) Exact local method: Using all available measurements, the exact local method gives an average loss of

$$L_{All2} = 0.0318°C. \quad (27)$$

which is just a little bit higher than the loss in Scenario 1.

b) Null space method: If we neglect the noise and select H in the left null space of F, such that $HF = 0$, the actual loss with noise is again dependent on the choice of basis vectors for H. Two examples are

$$L^{NullSpace}_{All1}(H_1) = 19.1711°C$$
$$L^{NullSpace}_{All1}(H_2) = 2.0702°C \quad (28)$$

The loss is the same as in Scenario 1, since the same columns of the null space have been selected, and the left null space for Scenario 1 is contained in the null space for Scenario 2.

3) Using subsets of measurements: Plotting the number of measurements used in the controlled variable versus the loss, Figure 3 in blue, indicates that it is not necessary to include all measurements, since the loss is not reduced significantly when including more than 10 measurements. Moreover, we find that above 10 measurements including flow measurements does not give any significant advantage in terms of loss over using only temperature measurements.

a) Only temperature measurements: Using only temperatures, the best combination results in a very small a loss of

$$L_{AllT2} = 0.0438°C. \quad (29)$$

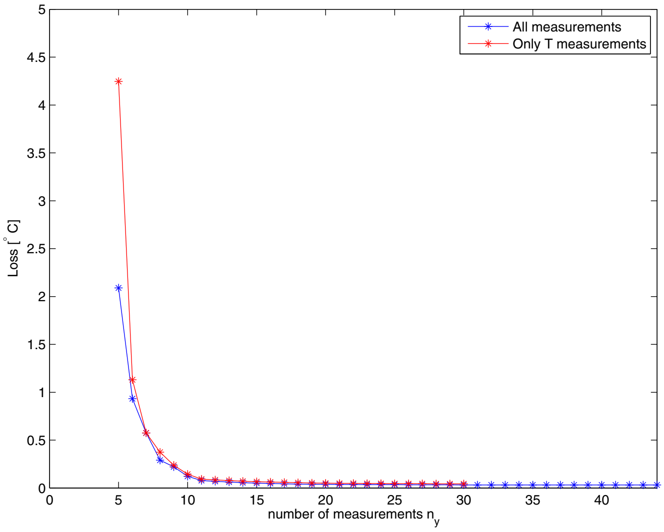

Fig. 3. Scenario 2 (Flow rate, Temperature and UA disturbances): Loss for best combinations of different number of measurements n_y

b) Only specific temperatures measurements: Using only the outlet temperatures of the cold streams, as in Eq. (22) gives a loss of

$$L_{outTemp2} = 16.9079°C, \tag{30}$$

which is very high. However, if we add some disturbance measurements, namely the stream inlet temperatures,

$$\begin{aligned} y =&[T_0, T_{1A}, T^{in}_{h1A}, T_{2B}, T^{in}_{h2B}, T_{2C}, T^{in}_{h2C}, T_{2D}, T_{1D}, \\ &T^{in}_{h1D}, T^{in}_{h2D}, T_{1E}, T^{in}_{h1E}, T_{2F}, T_{1F}, T^{in}_{h1F}, T^{in}_{h2F}] \end{aligned} \tag{31}$$

the loss is reduced to

$$L_{outTemp2a} = 0.0961°C \tag{32}$$

Thus, measuring the disturbances (the stream inlet temperatures) reduces the loss orders of magnitudes.

c) Single temperature measurements: Controlling single temperature measurements, leads to a loss of

$$L_{SingleT2} = 4.2459°C \tag{33}$$

with the measurement set

$$y = [T_{h1A}, T_{1B}, T_{1C}, T_{2D}, T_{h1E}] \tag{34}$$

V. Discussion and Conclusions

This HEN case study shows clearly that the control structure design has a strong impact on the performance. Especially in cryogenic processes and systems which process large quantities, like refineries, improving the average end temperature just 0.5-1°C leads to significant economics savings.

Although the disturbances in Scenario 1 are included in Scenario 2, we treated Scenario 1 separately, because it shows that the stream inlet temperatures are the disturbances, which have the strongest effect on the loss. If it could be guaranteed that the inlet temperatures remain constant (Scenario 1), then keeping the splits constant would most likely be sufficient.

The situation changes dramatically if the stream inlet temperatures are varying (Scenario 2). Here an open loop policy causes high loss, and controlling single measurements performs even worse. In this case, however, controlling good temperature measurement combinations can reduce the loss up to around 2 orders of magnitude.

The relationship between the number of measurements used and the loss, Fig. 2 and 3, shows that including flow measurements does not reduce the loss significantly, provided enough temperature measurements are available. Controlling combinations of about 10 or more temperature measurements results in a very small loss.

The null space method, which gives zero loss without noise, has been shown to give very poor performance, sometimes even worse than open loop operation, because the loss is dependent on which set of null space basis vectors is chosen as H. Thus, this case study clearly demonstrates how necessary it is to take noise into account when finding H.

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