Optimal PI-Control & Verification of the SIMC Tuning Rule

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Outline

1. Motivation: Ziegler-Nichols open-loop method
2. SIMC PI(D)-rule & derivation
3. Definition of optimality (performance & robustness)
4. Optimal PI control of first-order plus delay processes
5. Comparison of SIMC with optimal PI
6. Improved SIMC-PI for time-delay process
7. Further work and conclusion

Trans. ASME, **64**, 759-768 (Nov. 1942).

Optimum Settings for Automatic Controllers

By J.G. ZIEGLER[1] and N. B. NICHOLS[2] • ROCHESTER, N. Y.

In this paper, the three principle control effects found in present controllers are examined and practical names and units of measurement are proposed for each effect.

varying its output air pressure, repositions a diaphragm-operated valve. The controller may be measuring temperature, pressure, level, or any other variable, but we will completely divorce the

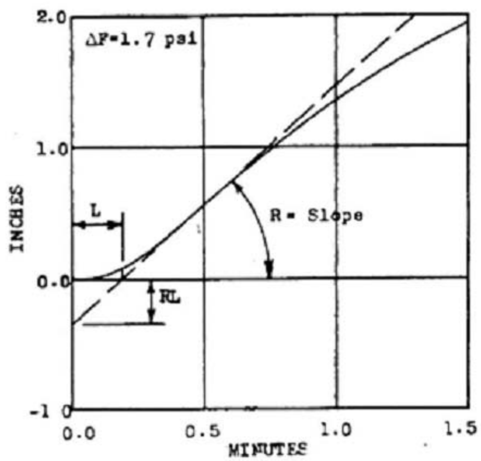

FIG. 8 REACTION CURVE

Reset-Rate Determination From Reaction Curve. Since the period of oscillation at the ultimate sensitivity proves to be 4 times the lag. A substitution of 4 L for P_u in previous equations for optimum reset rate gives an equation expressing this reset rate in terms of lag. For a controller with proportional and auto-matic-reset responses, the optimum settings become

$$\textit{Sensitivity} = \frac{0.9}{R_1 L}\ \textit{psi per in.}$$

$$\text{Reset Rate} = \frac{0.3}{L}\ \text{per min}$$

At these settings the period will be about 5.7L, having been in-creased, by both the lowering of sensitivity and the addition of automatic reset.

My notation:

$$k' = R,\ \theta = L$$

$$K_c = \frac{0.9}{k'}\frac{1}{\theta},\ \tau_I = 3.3\theta$$

Disadvantages Ziegler-Nichols:

1. Rather aggressive settings & No tuning parameter
2. Uses only two pieces of information (k', θ)
3. Poor for processes with large time delay (θ)

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252 *Ind. Eng. Chem. Process Des. Dev.* **1986**, *25*, 252–265

Internal Model Control. 4. PID Controller Design

Daniel E. Rivera, Manfred Morari,* and Sigurd Skogestad

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For a large number of single input–single output (SISO) models typically used in the process industries, the Internal Model Control (IMC) design procedure is shown to lead to PID controllers, occasionally augmented with a first-order lag. These PID controllers have as their only tuning parameter the closed-loop time constant or, equivalently, the
...al PID controllers. As a
...ı dead time are derived
...ɔustness is demonstrated.

Table I. IMC-Based PID Controller Parameters[a]

	model	$y/y_s = \tilde{g}_+ f$	controller	$k_c k$	τ_I	τ_D	τ_F	comments
A	$\frac{k}{\tau s + 1}$	$\frac{1}{\epsilon s + 1}$	$\frac{1}{k}\frac{\tau s + 1}{\epsilon s}$	$\frac{\tau}{\epsilon}$	τ	–	–	–
B	$\frac{k}{(\tau_1 s + 1)(\tau_2 s + 1)}$	$\frac{1}{\epsilon s + 1}$	$\frac{(\tau_1 s + 1)(\tau_2 s + 1)}{k\epsilon s}$	$\frac{\tau_1 + \tau_2}{\epsilon}$	$\tau_1 + \tau_2$	$\frac{\tau_1\tau_2}{\tau_1 + \tau_2}$		
C	$\frac{k}{\tau^2 s^2 + 2\zeta\tau s + 1}$	$\frac{1}{\epsilon s + 1}$	$\frac{\tau^2 s^2 + 2\zeta\tau s + 1}{k\epsilon s}$	$\frac{2\zeta\tau}{\epsilon}$	$2\zeta\tau$	$\frac{\tau}{2\zeta}$		
D	$k\frac{-\beta s + 1}{\tau s + 1}$	$\frac{-\beta s + 1}{\epsilon s + 1}$	$\frac{\tau s + 1}{k(\beta + \epsilon)s}$	$\frac{\tau}{\beta + \epsilon}$	τ	–		
E	$k\frac{-\beta s + 1}{\tau s + 1}$	$\frac{-\beta s + 1}{(\beta s + 1)(\epsilon s + 1)}$	$\frac{\tau s + 1}{ks(\beta\epsilon s + 2\beta + \epsilon)}$	$\frac{\tau}{2\beta + \epsilon}$	τ	–		
F	$k\frac{-\beta s + 1}{\tau^2 s^2 + 2\zeta\tau + 1}$	$\frac{-\beta s + 1}{\epsilon s + 1}$	$\frac{\tau^2 s^2 + 2\zeta\tau s + 1}{k(\beta + \epsilon)s}$	$\frac{2\zeta\tau}{\beta + \epsilon}$	$2\zeta\tau$	$\frac{\tau}{2\zeta}$		
G	$k\frac{-\beta s + 1}{\tau^2 s^2 + 2\zeta\tau s + 1}$	$\frac{-\beta s + 1}{(\beta s + 1)(\epsilon s + 1)}$	$\frac{\tau^2 s^2 + 2\zeta\tau s + 1}{k(\beta\epsilon s + 2\beta + \epsilon)s}$	$\frac{2\zeta\tau}{2\beta + \epsilon}$	$2\zeta\tau$	$\frac{\tau}{2\zeta}$		
H	$\frac{k}{s}$	$\frac{1}{\epsilon s + 1}$	$\frac{1}{k\epsilon}$	$\frac{1}{\epsilon}$	–	–		
I	$\frac{k}{s}$	$\frac{2\epsilon + 1}{(\epsilon s + 1)^2}$	$\frac{2\epsilon s + 1}{k\epsilon^2 s}$	$\frac{2}{\epsilon}$	2ϵ	–	–	(6)
J	$\frac{k}{s(\tau s + 1)}$	$\frac{1}{\epsilon s + 1}$	$\frac{\tau s + 1}{k\epsilon}$	$\frac{1}{\epsilon}$	–	τ	–	–
K	k ...	$2\epsilon s + 1$ ...	$(\tau s + 1)(2\epsilon s + 1)$ ...	$2\epsilon + \tau$ ...	$2\epsilon + \tau$	$2\epsilon\tau$ ...	–	(6)

Table II. IMC-Based PID Parameters for $g(s) = ke^{-\theta s}/(\tau s + 1)$ and Practical Recommendations for ϵ/θ

controller	kk_c	τ_I	τ_D	recommended ϵ/θ (> $0.1\tau/\theta$ *always*)
PID	$(2\tau + \theta)/(2\epsilon + \theta)$	$\tau + (\theta/2)$	$\tau\theta/(2\tau + \theta)$	>0.8
PI	$\theta/\tau = 0.1$	1.54		>1.7
improved PI	$(2\tau + \theta)/2\epsilon$	$\tau + (\theta/2)$		>1.7

Disadvantages IMC-PID:

1. Many rules
2. Poor disturbance response for «slow»/integrating processes (with large τ_1/θ)

Motivation for developing SIMC PID tuning rules (1998)

For teaching & easy practical use, rules should be:

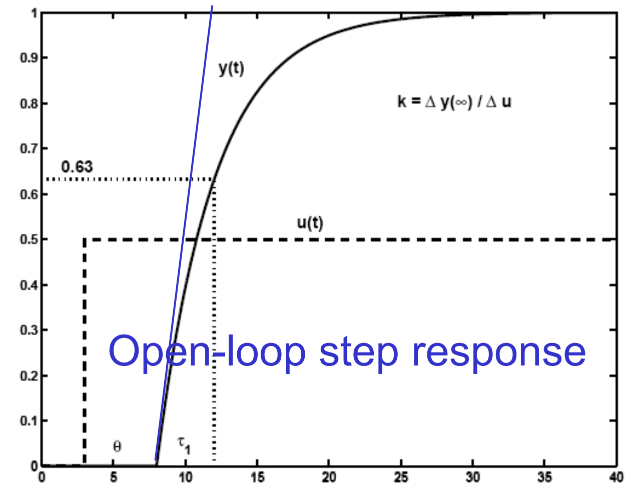
- Model-based
- Analytically derived
- Simple and easy to memorize
- Work well on a wide range of processes

2. SIMC PI tuning rule

1. Approximate process as first-order with delay (e.g., use "half rule")
   - **k = process gain**
   - **τ_1 = process time constant**
   - **θ = process delay**
2. Derive SIMC tuning rule:

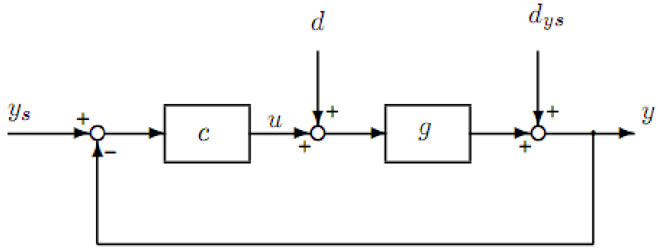
$$K_c = \frac{1}{k} \cdot \frac{\tau_1}{(\tau_c+\theta)}$$
$$\tau_I = \min(\tau_1, 4(\tau_c + \theta))$$

$\tau_c \geq -\theta$: Desired closed-loop response time (tuning parameter)



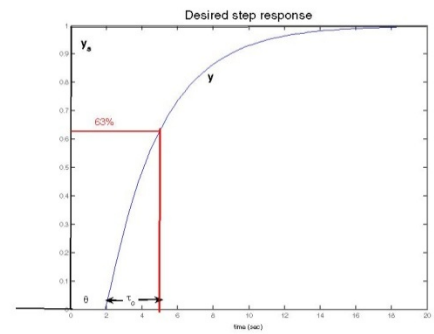
IMC ≈ SIMC for small τ_1 ($\tau_I = \tau_1$)
Ziegler-Nichols ≈SIMC for large τ_1 if we choose $\tau_c = 0$
(aggressive!) $\left(K_c = \frac{0.9}{k}\frac{\tau_1}{\theta},\ \tau_I = 3.3\theta\right)$

Reference: S. Skogestad, "Simple analytic rules for model reduction and PID controller design", *J.Proc.Control*, Vol. 13, 291-309, 2003



$$\frac{y}{y_s} = \frac{gc}{1+gc}$$

Derivation SIMC tuning rule (setpoints)



- Controller: $c(s) = \frac{1}{g(s)} \cdot \frac{1}{\frac{1}{(y/y_s)_{\text{desired}}} - 1}$
- Consider second-order with delay plant: $g(s) = k\frac{e^{-\theta s}}{(\tau_1 s+1)(\tau_2 s+1)}$
- Desired first-order setpoint response: $\left(\frac{y}{y_s}\right)_{\text{desired}} = \frac{1}{\tau_c s+1}e^{-\theta s}$
- Gives a "Smith Predictor" controller: $c(s) = \frac{(\tau_1 s+1)(\tau_2 s+1)}{k}\frac{1}{(\tau_c s+1-e^{-\theta s})}$
- To get a PID-controller use $e^{-\theta s} \approx 1 - \theta s$ and derive

IMC-rule uses Pade: $e^{-\theta s} \approx \frac{1-\frac{\theta}{2}s}{1+\frac{\theta}{2}s}$.
Gives PID even for first-order process

$$c(s) = \frac{(\tau_1 s + 1)(\tau_2 s + 1)}{k}\frac{1}{(\tau_c + \theta)s}$$

which is a cascade form PID-controller with

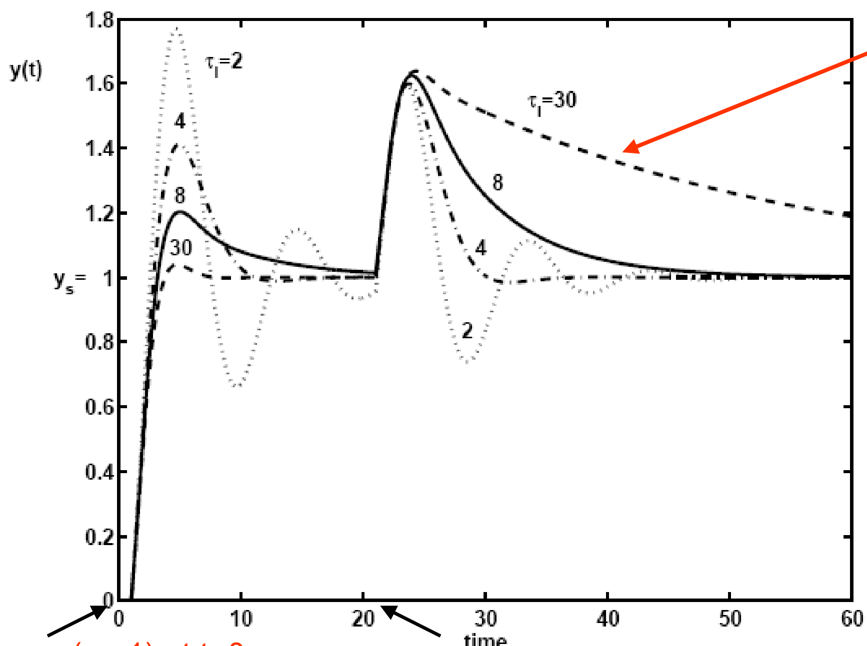
$$K_c = \frac{1}{k}\frac{\tau_1}{\tau_c + \theta}; \quad \tau_I = \tau_1; \quad \tau_D = \tau_2$$

First-order process ($\tau_2 = 0$):
Get PI-controller

- τ_c is the sole tuning parameter

Effect of integral time on closed-loop response

$\tau_I = \tau_1 = 30$



Setpoint change (y_s=1) at t=0

Input disturbance (d=1) at t=20

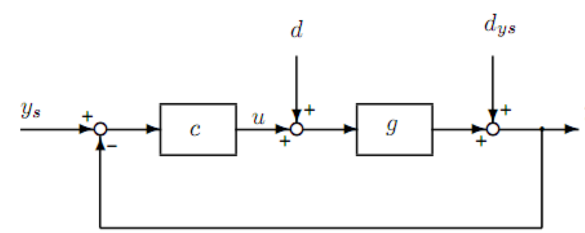


Figure 2: Effect of changing the integral time τ_I for PI-control of "slow" process $g(s) = e^{-s}/(30s+1)$ with $K_c = 15$. Load disturbance of magnitude 10 occurs at $t = 20$.

Too large integral time: Poor disturbance rejection

Too small integral time: Slow oscillations

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SIMC: Integral time correction

- Setpoints: $\tau_I = \tau_1$ ("IMC-rule"). Want smaller integral time for disturbance rejection for "slow" processes (with large τ_1), but to avoid "slow oscillations" must require:

$$\tau_I \geq 4(\tau_C + \theta)$$

- Derivation: $G(s) = k\frac{e^{-\theta s}}{\tau_1 s+1} \approx \frac{k'}{s}$ where $k' = \frac{k}{\tau_1}$; $C(s) = K_c\left(1 + \frac{1}{\tau_I s}\right)$

Closed-loop poles:

$$1 + GC = 0 \Rightarrow 1 + \frac{k'}{s}K_c\left(1 + \frac{1}{\tau_I s}\right) = 0 \Rightarrow \tau_I s^2 + k' K_c \tau_I s + k' K_c = 0$$

To avoid oscillations we must not have complex poles s:

$$B^2 - 4AC \geq 0 \Rightarrow k'^2 K_c^2 \tau_I^2 - 4k' K_c \tau_I \geq 0 \Rightarrow k' K_c \tau_I \geq 4 \Rightarrow \tau_I \geq \frac{4}{k' K_c}$$

Inserted SIMC-rule for $K_c = \frac{1}{k'}\frac{1}{\tau_c+\theta}$ then gives

$$\tau_I \geq 4(\tau_c + \theta)$$

- Conclusion SIMC:

$$K_c = \frac{1}{k'} \cdot \frac{1}{(\tau_c + \theta)} \qquad (k' = k/\tau_1)$$

$$\tau_I = \min(\tau_1, 4(\tau_c + \theta))$$

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SIMC PI tuning rule

$$K_c = \frac{1}{k} \cdot \frac{\tau_1}{(\tau_c + \theta)}$$
$$\tau_I = \min(\tau_1, 4(\tau_c + \theta))$$

$\tau_c \geq -\theta$: Desired closed-loop response time (tuning parameter)

- For robustness select: $\tau_c \geq \theta$

Two questions:

- How good is really the SIMC rule?
- Can it be improved?

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S. Skogestad, "Simple analytic rules for model reduction and PID controller design", *J.Proc.Control*, Vol. 13, 291-309, 2003

"Probably the best simple PID tuning rule in the world"

How good is really the SIMC PI-rule?

Want to compare with:

- Optimal PI-controller

  for class of first-order with delay processes

Optimal ant

versus

SIMC ant

3. Optimal controller

- **Multiobjective.** Tradeoff between
  - Output performance — High controller gain ("tight control")
  - Robustness
  - Input usage
  - Noise sensitivity

  } Low controller gain ("smooth control")

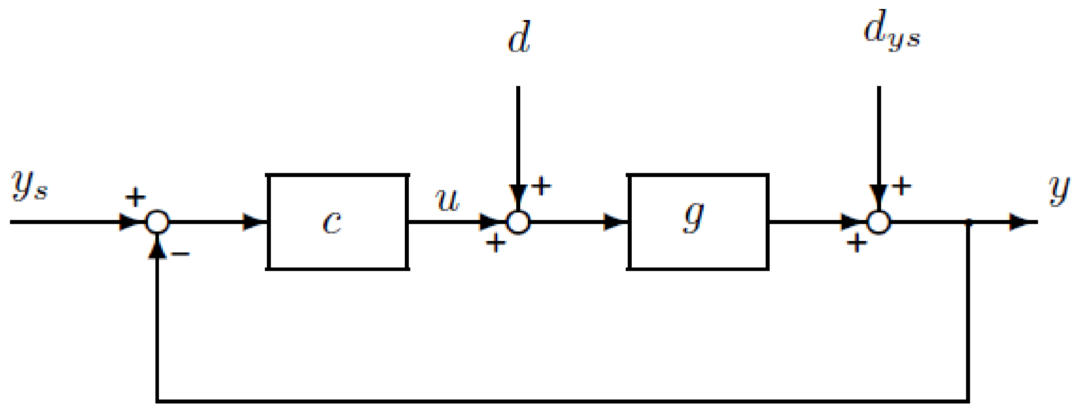
- Quantification
  - Output performance:
    - Frequency domain: weighted sensitivity $||W_pS||$
    - Time domain: IAE or ISE for setpoint/disturbance
  - Robustness: M_s, M_t, GM, PM, Delay margin, …
  - Input usage: $||KSG_d||$, TV(u) for step response
  - Noise sensitivity: $||KS||$, etc.

Our choice:

J = avg. IAE for Setpoint & disturbance

M_s = peak sensitivity

IAE output performance (J)



IAE = Integrated absolute error = $\int |y - y_s| dt$, for step change in y_s or d

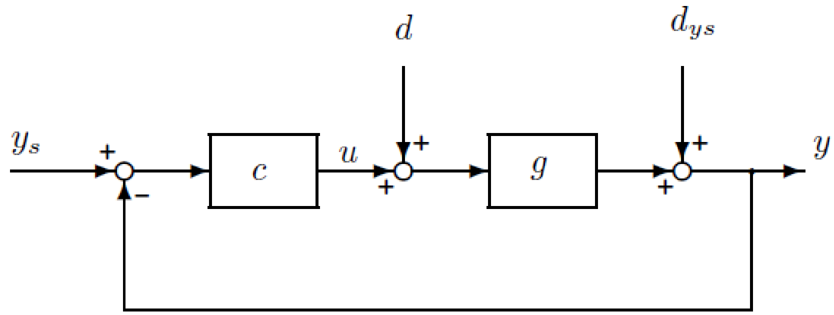
$$J(c) = 0.5\frac{\text{IAE}_{ys}(c)}{\text{IAE}^o_{ys}} + 0.5\frac{\text{IAE}_d(c)}{\text{IAE}^o_d}$$

weight IAE^o_{ys}: PI-optimal for setpoint y_s (or d_{ys}) ($M_s = 1.59$)
weight IAE^o_d: PI-optimal for disturbance $d (M_s = 1.59)$

Cost J is independent of:

1. process gain (k)
2. setpoint (y_s or d_{ys}) and disturbance (d) magnitude
3. unit for time

4. Optimal PI-controller: Minimize J for given M_s



$$J(c) = 0.5\frac{\text{IAE}_{ys}(c)}{\text{IAE}^o_{ys}} + 0.5\frac{\text{IAE}_d(c)}{\text{IAE}^o_d}$$

$\min_c J(c)|_{M_s=m}$

PI-controller: $c(s) = K_c\left(1 + \frac{1}{\tau_I s}\right)$

First-order with delay processes: $g(s) = \frac{k}{\tau_1 s+1}e^{-\theta s}$

$\theta = 1, \tau_1/\theta = [0, \infty]$

$m = [..., 1.2, 1.59, 1.7, 2....]$

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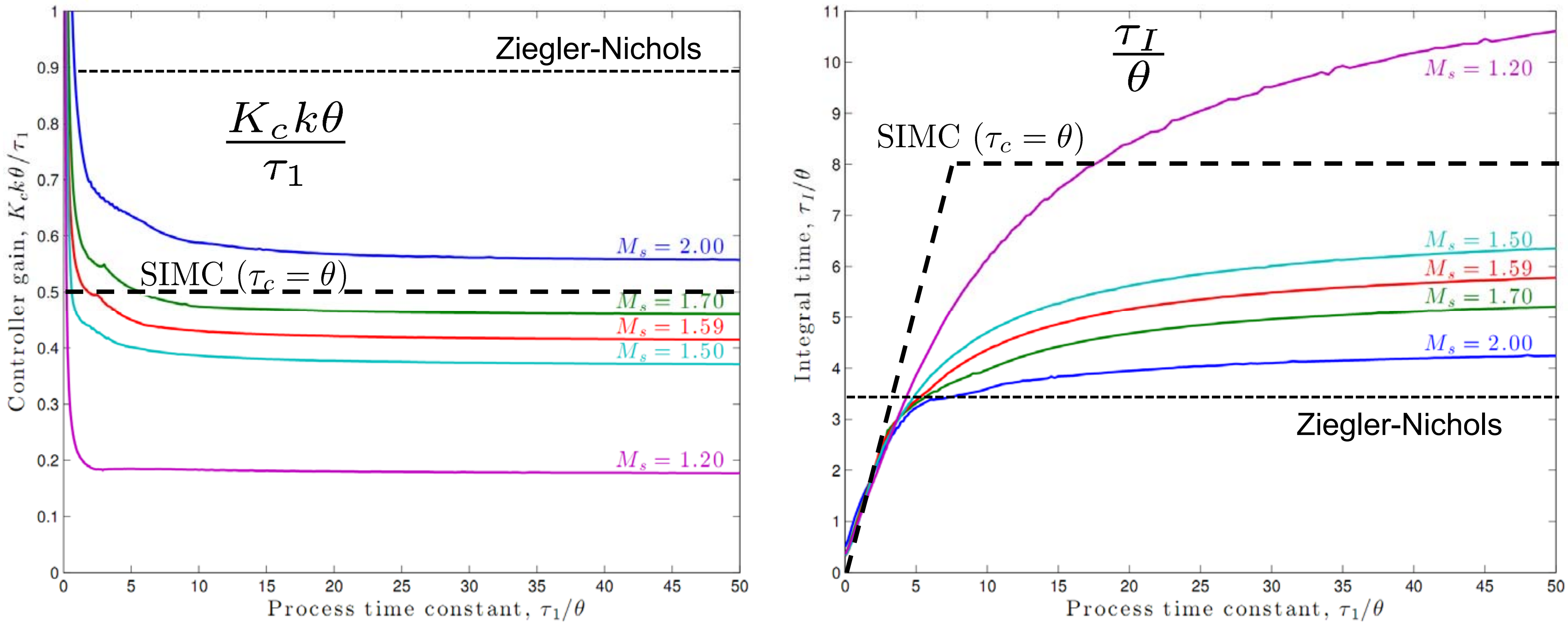
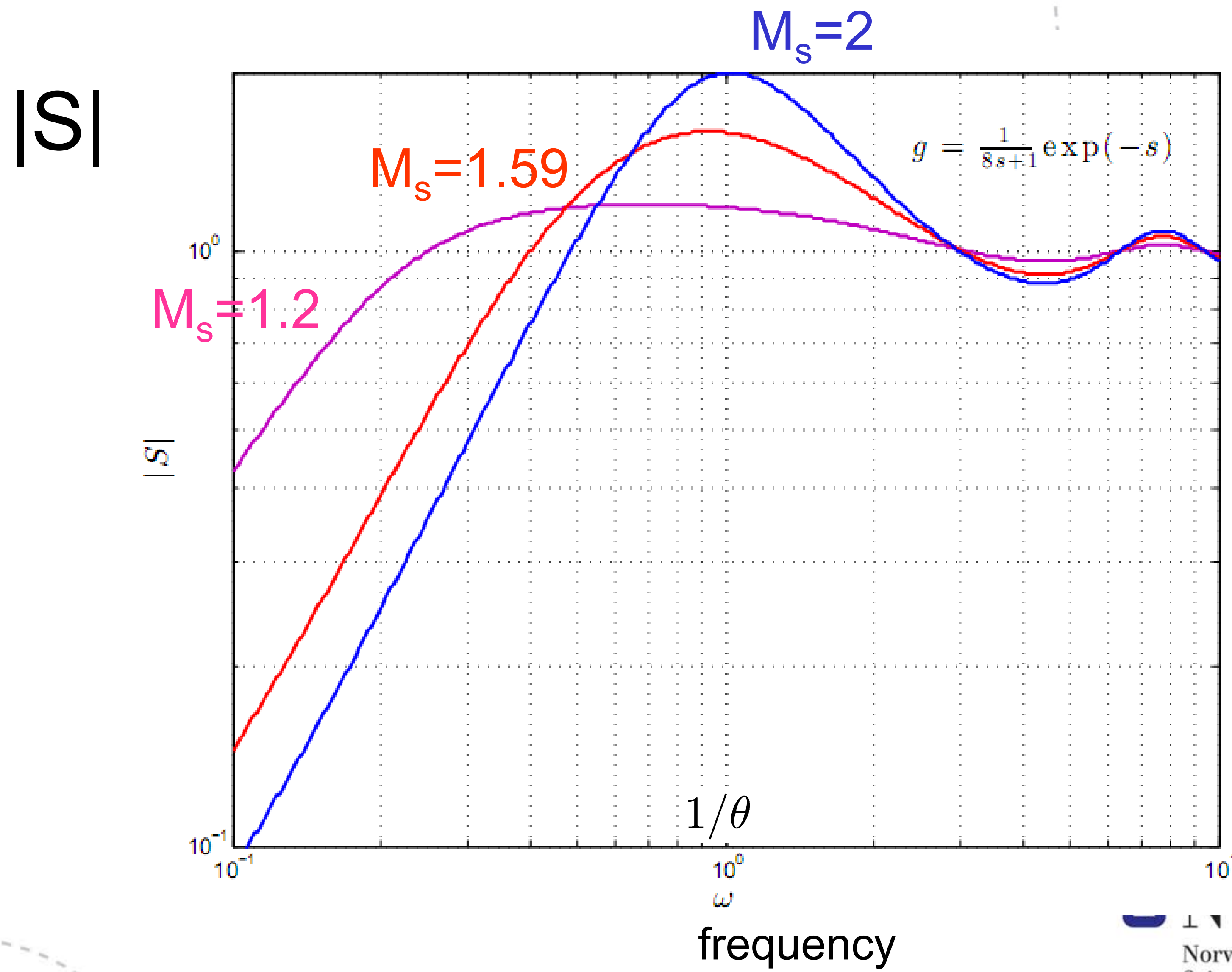
Optimal PI-settings vs. process time constant (τ_1/θ)



$$\text{SIMC:} \quad \frac{K_c k\theta}{\tau_1} = \frac{\theta}{(\theta+\tau_c)}, \quad \tau_I = \min(\tau_1, 4(\tau_c+\theta))$$

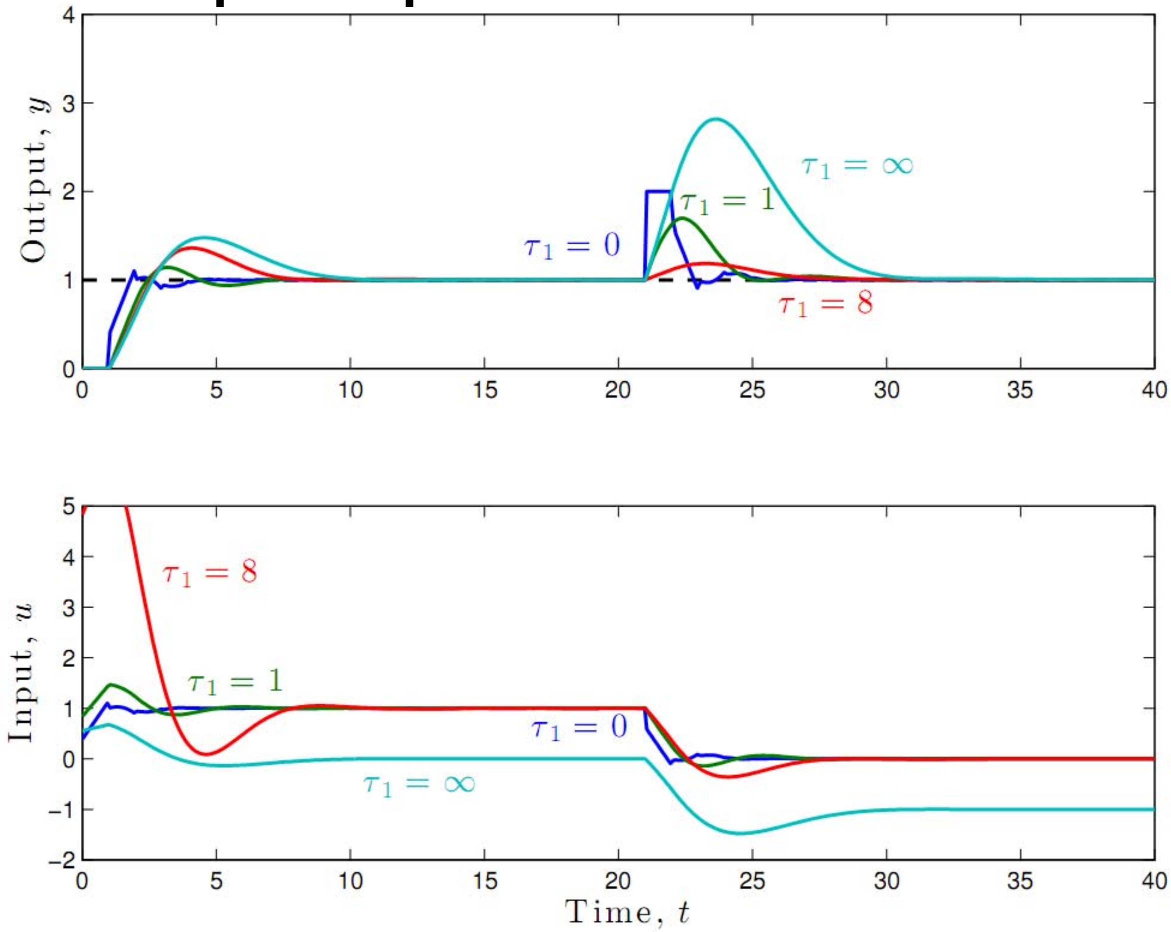
$$\text{Ziegler-Nichols:} \quad \frac{K_c k\theta}{\tau_1} = 0.9, \ \tau_I = 3.3\theta$$

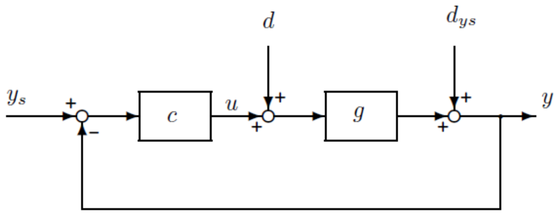
Optimal sensitivity function, S = 1/(gc+1)



Optimal closed-loop response

M_s=2





4 processes, $g(s)=k\, e^{-\theta s}/(\tau_1 s+1)$, Time delay θ=1.
Setpoint change at t=0, Input disturbance at t=20,

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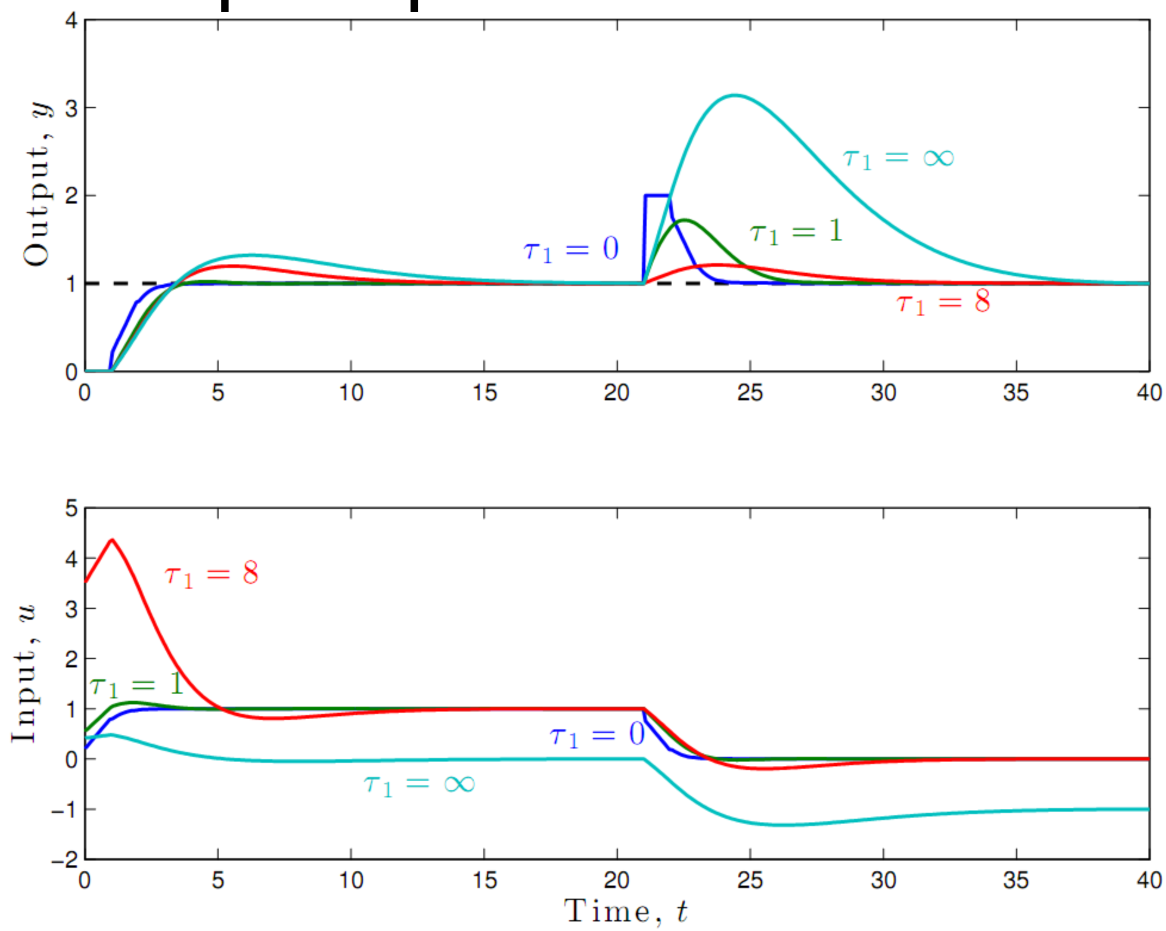
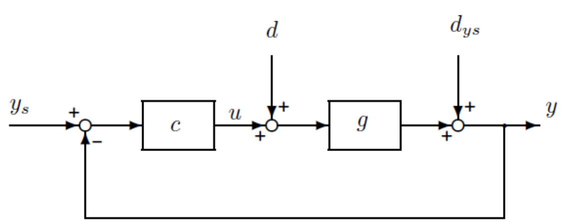
Optimal closed-loop response

M_s=1.59





Setpoint change at t=0, Input disturbance at t=20,
g(s)=k $e^{-\theta s}/(\tau_1 s+1)$, Time delay θ=1

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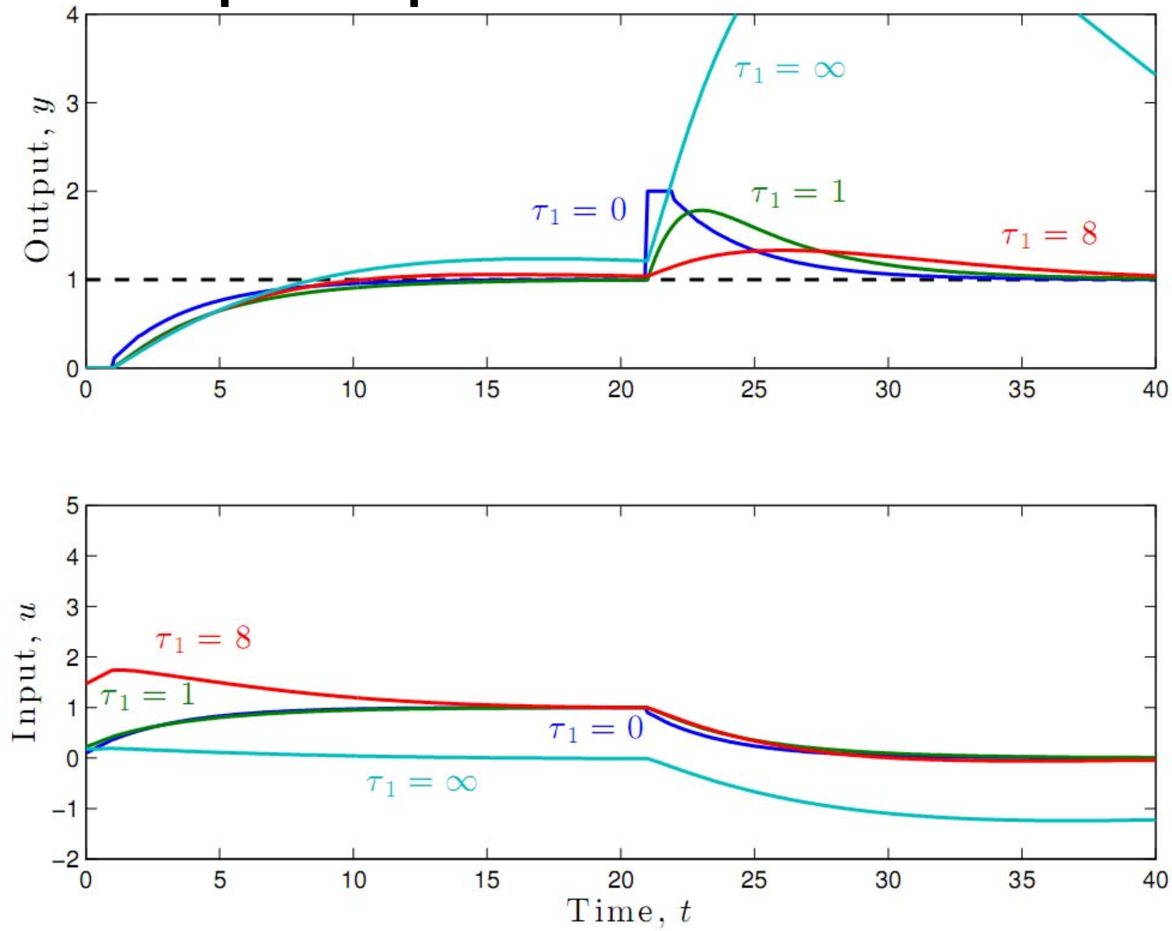
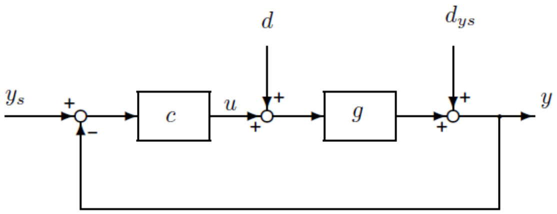
Optimal closed-loop response

M_s=1.2





Setpoint change at t=0, Input disturbance at t=20,
$g(s)=k\ e^{-\theta s}/(\tau_1 s+1)$, Time delay θ=1

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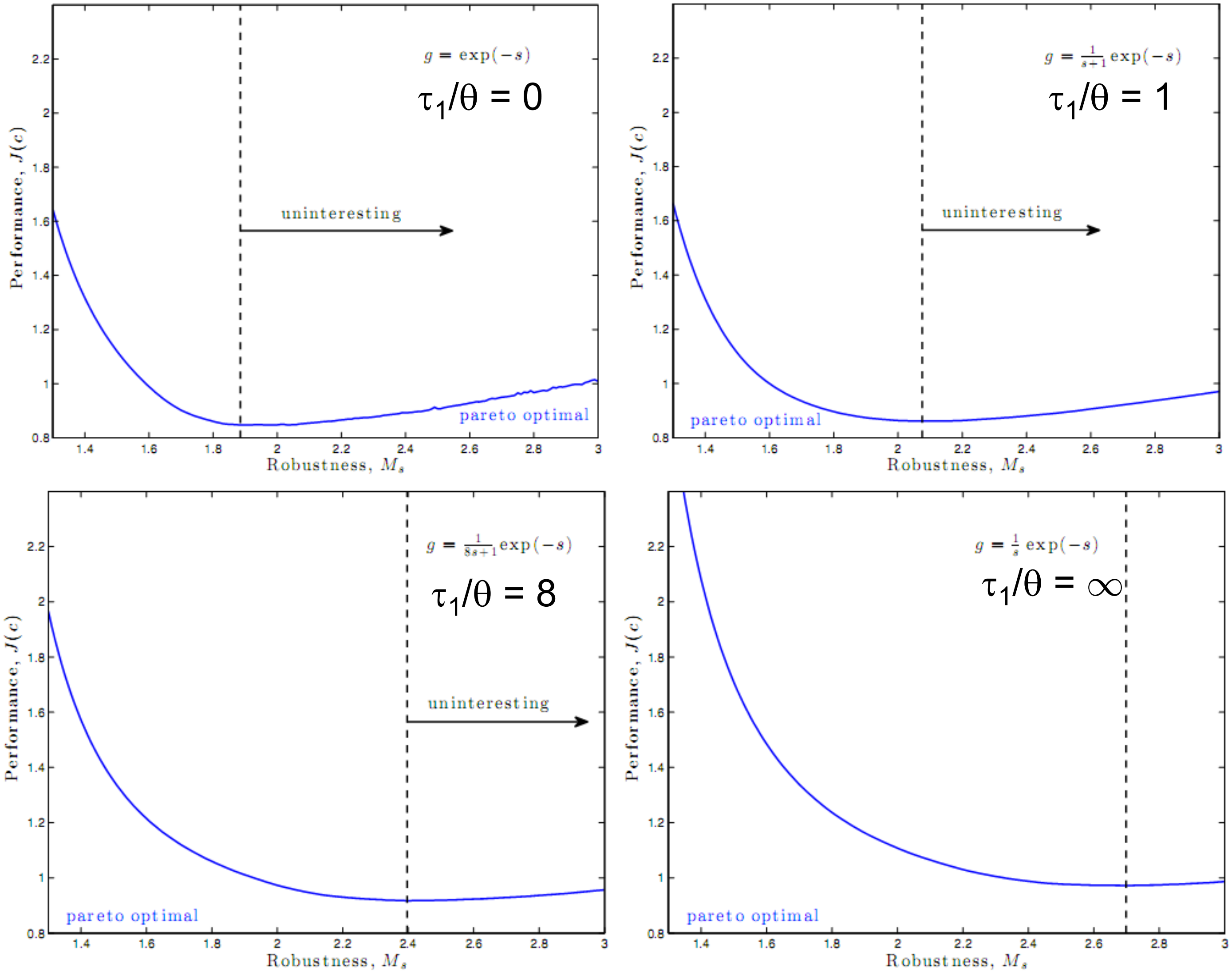
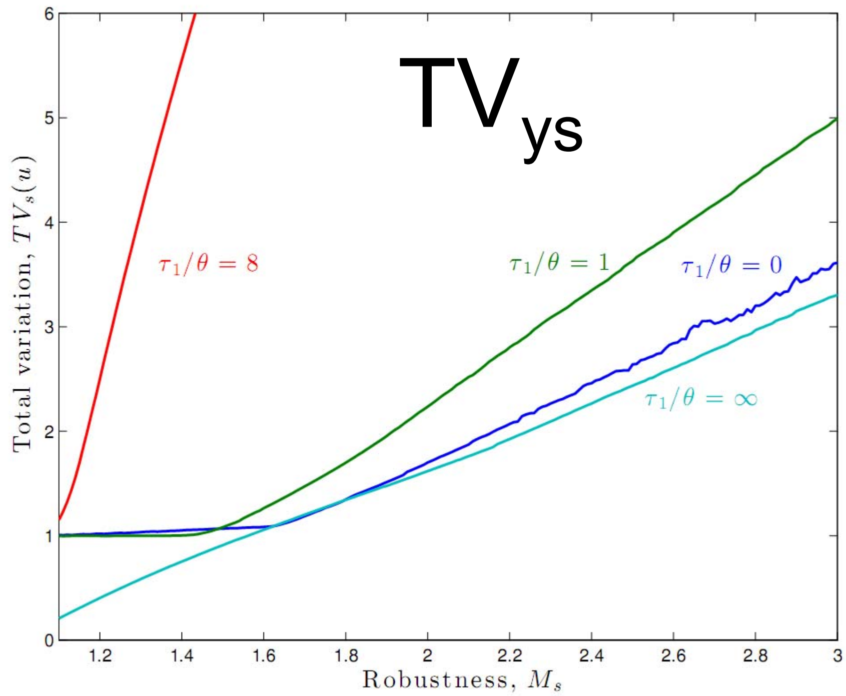
Optimal IAE-performance (J) vs. M_s

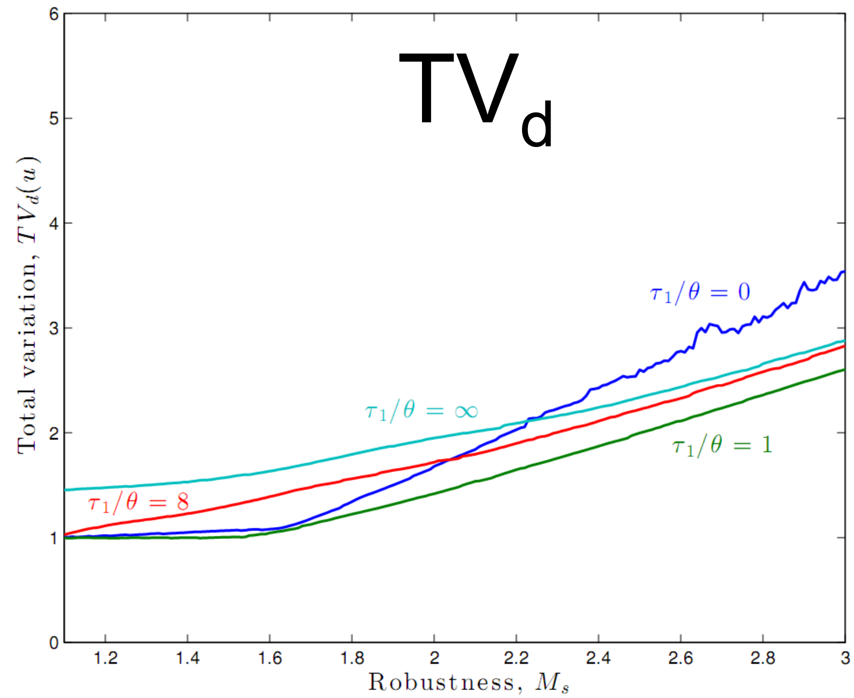


Optimal ant

$$J(c) = 0.5\frac{\text{IAE}_{ys}(c)}{\text{IAE}_{ys}^{o}} + 0.5\frac{\text{IAE}_{d}(c)}{\text{IAE}_{d}^{o}}$$

Input usage (TV) increases with M_s





$$\mathrm{TV}(u) = \int_0^\infty \left|\frac{du}{dt}\right| dt = \Sigma_{i=1}^{\infty} |u_i - u_{i-1}|$$

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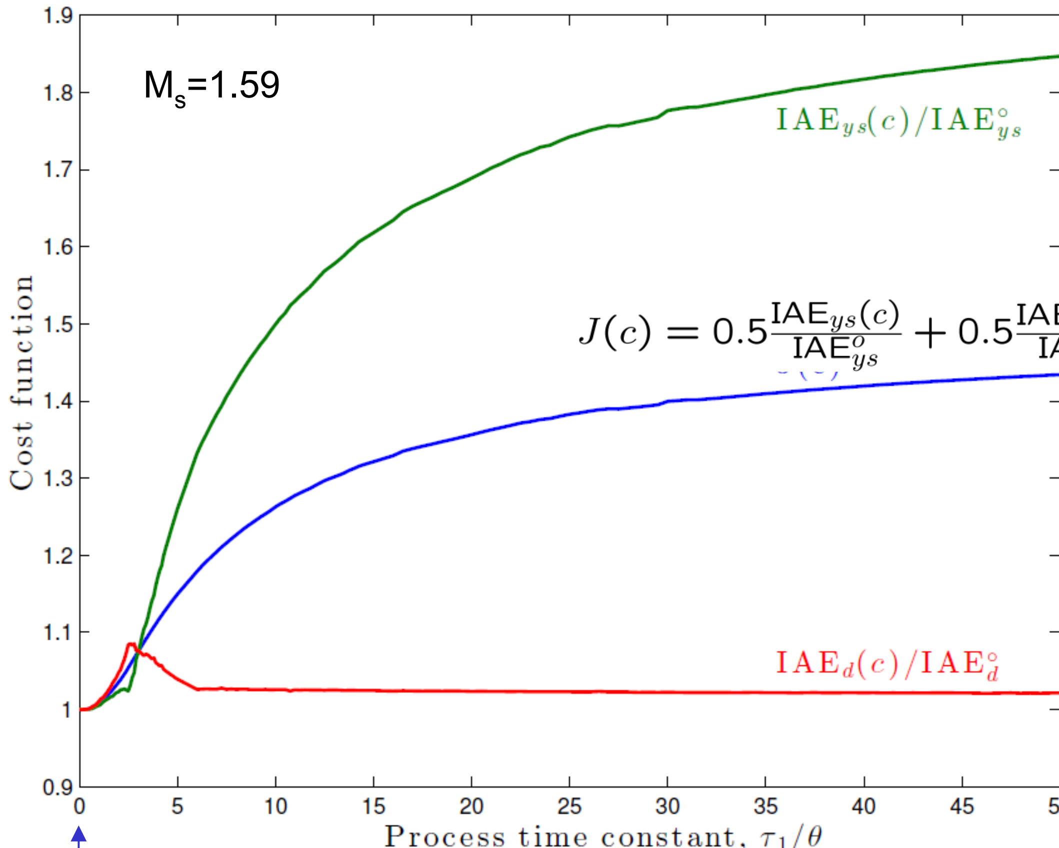
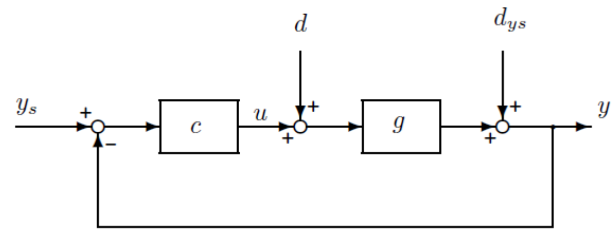
Setpoint / disturbance tradeoff



$$J(c) = 0.5\frac{\mathrm{IAE}_{ys}(c)}{\mathrm{IAE}_{ys}^{o}} + 0.5\frac{\mathrm{IAE}_{d}(c)}{\mathrm{IAE}_{d}^{o}}$$



Optimal controller:
Emphasis on disturbance d

Pure time delay process: J=1, No tradeoff
(since setpoint and disturbance the same)

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Setpoint / disturbance tradeoff

Table 1. Optimal PI-controllers ($M_s = 1.59$) and corresponding IAE-values for four processes.

	Setpoint			Input disturbance			Optimal combined (minimize J)					
Process	K_c	τ_I	IAE°_{ys}	K_c	τ_I	IAE°_d	K_c	τ_I	IAE_{ys}	IAE_d	J	M_s
e^{-s}	0.20	0.32	1.609	0.20	0.32	1.609	0.20	0.32	1.607	1.607	1	1.59
$\frac{e^{-s}}{s+1}$	0.54	1.10	2.073	0.50	1.0	2.016	0.54	1.10	2.087	2.038	1.00	1.59
$\frac{e^{-s}}{8s+1}$	4.0	8	2.171	3.34	3.7	1.134	3.46	4.0	3.096	1.164	1.23	1.59
$\frac{e^{-s}}{s}$	0.50	∞	2.174	0.40	5.8	15.09	0.41	6.3	4.318	15.38	1.50	1.59

IAE_{ys} is for a unit setpoint change. IAE_d is for a unit input disturbance.

Optimal for setpoint: $\tau_I=\tau_1$ (except time delay process)
Integrating process ($\tau_1=\infty$): No integral action

5. What about SIMC-PI?

SIMC ant

$$K_c = \frac{1}{k'} \cdot \frac{1}{(\theta+\tau_c)}$$
$$\tau_I = \min(\tau_1, 4(\tau_c + \theta))$$

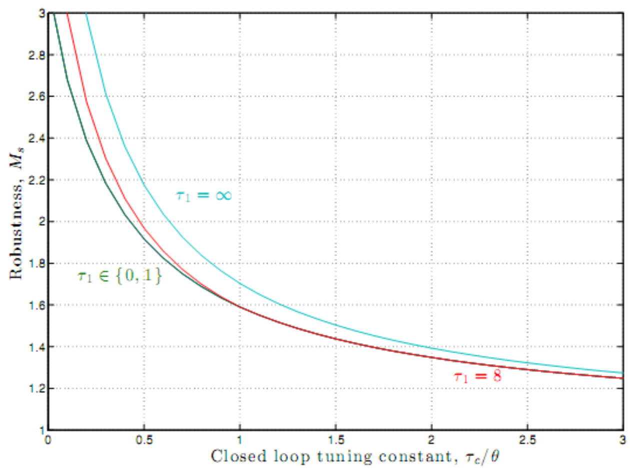
Tuning parameter: τ_c

Tight control with good robustness: Select $\tau_c = \theta$ (effective delay)

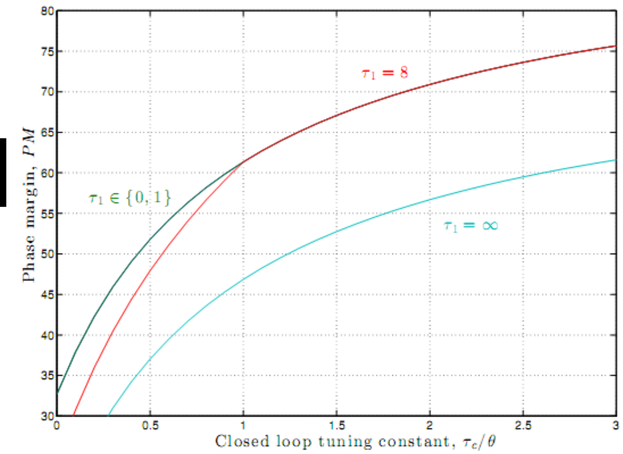
- Gives M_s between 1.59 and 1.7

SIMC: Tuning parameter (τ_c) correlates nicely with robustness measures

M_s

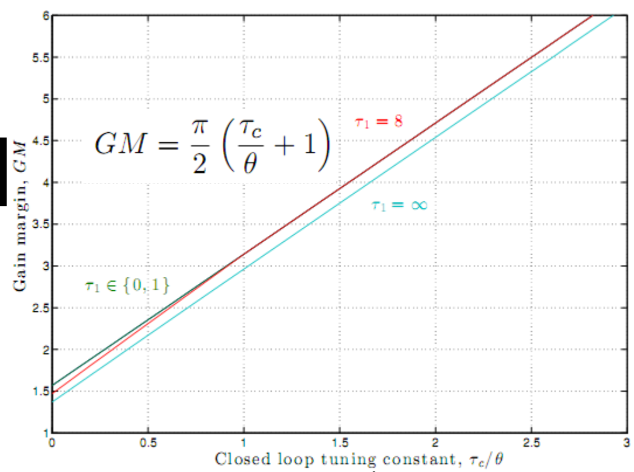


PM

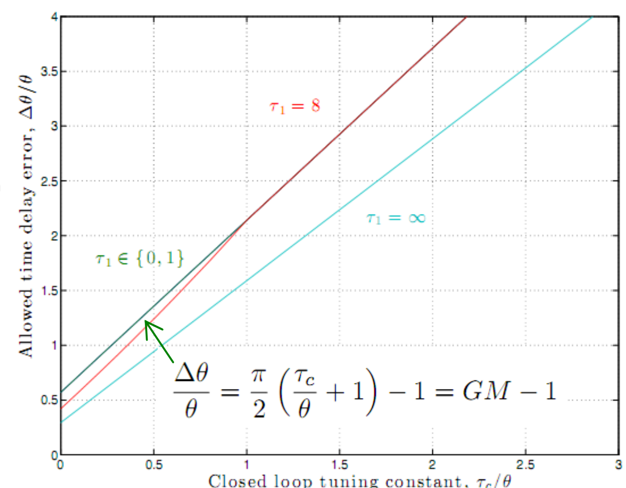


SIMC a

GM



$$\frac{\Delta\theta}{\theta}$$



τ_c/θ τ_c/θ

What about SIMC-PI performance?

$$K_c = \frac{1}{k'} \cdot \frac{1}{(\theta+\tau_c)}$$
$$\tau_I = \min(\tau_1, 4(\tau_c + \theta))$$

SIMC ant

Evaluate performance (J) as a function of M_s (by varying τ_c) and compare with optimal

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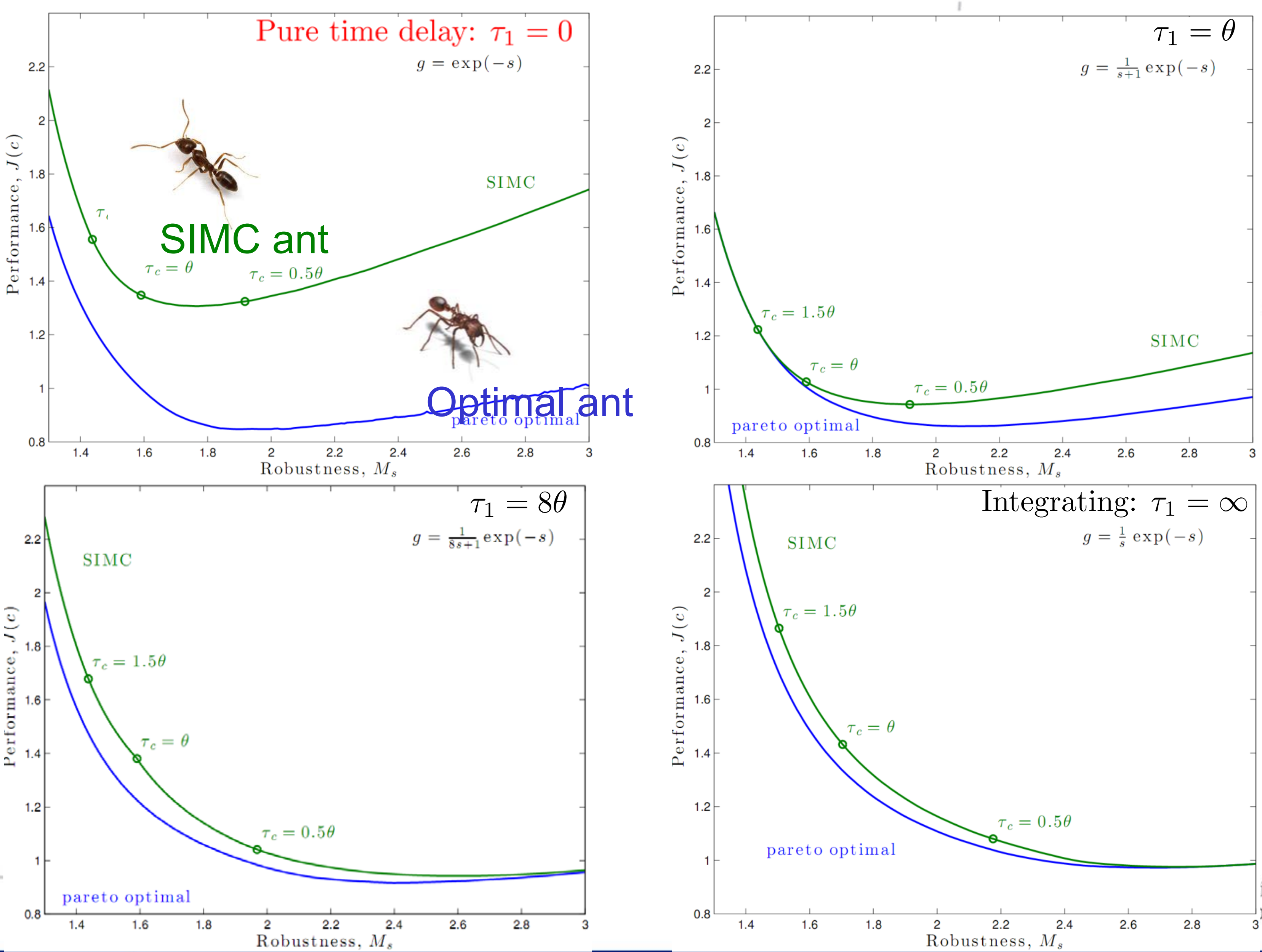
Comparison of J vs. M_s for optimal and SIMC for 4 processes



Conclusion (so far):
How good is really the SIMC rule?

- Varying τ_C gives (almost) Pareto-optimal tradeoff between performance (J) and robustness (M_s)
- $\tau_C = \theta$ is a good "default" choice
- Not possible to do much better with any other PI-controller!

- Exception: Time delay process

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6. Can the SIMC-rule be improved?

Yes, possibly for time delay process

$$K_c = \frac{1}{k} \cdot \frac{\tau_1}{(\tau_c + \theta)}$$
$$\tau_I = \min(\tau_1, 4(\tau_c + \theta))$$

Time delay process, $g = ke^{-\theta s}$ $(\tau_1 = 0)$:
SIMC-rule gives integrating controller:

$$K_c = 0, \ \tau_I = 0, \ K_I = \frac{K_c}{\tau_I} = \frac{1}{k(\tau_c + \theta)}$$

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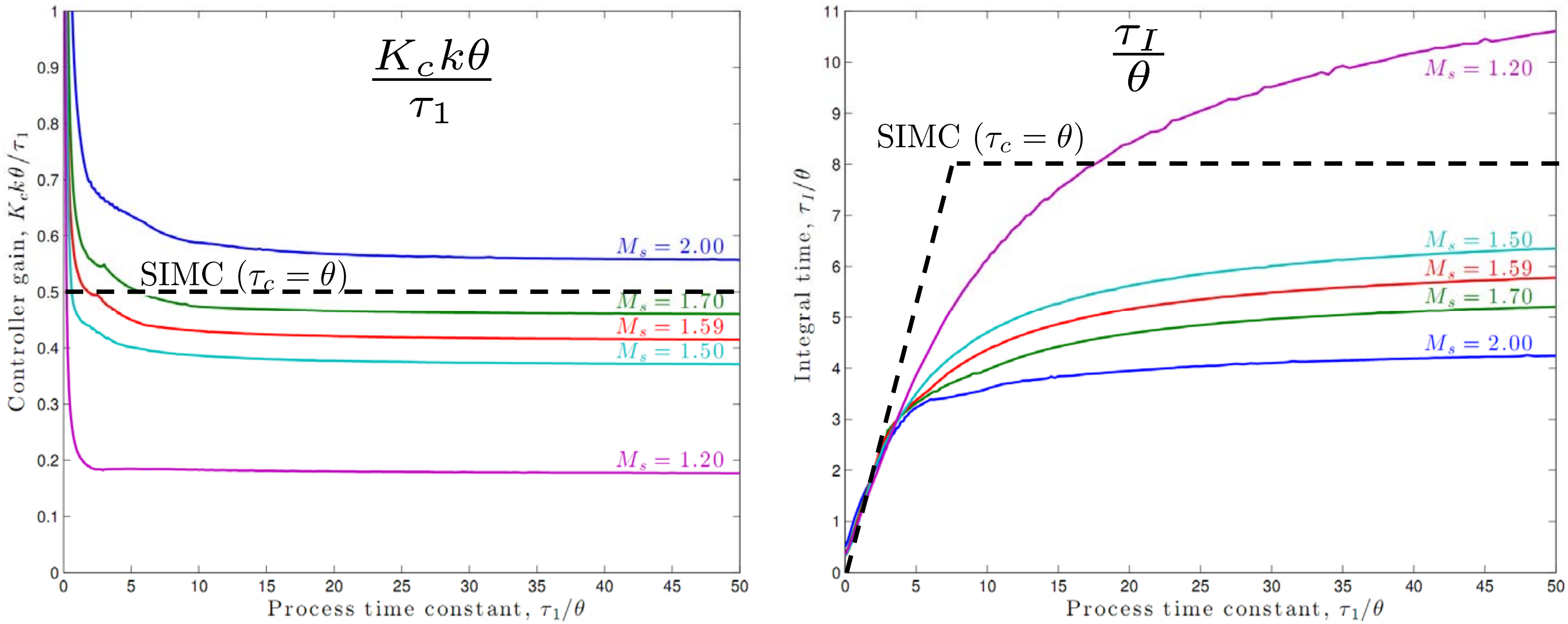
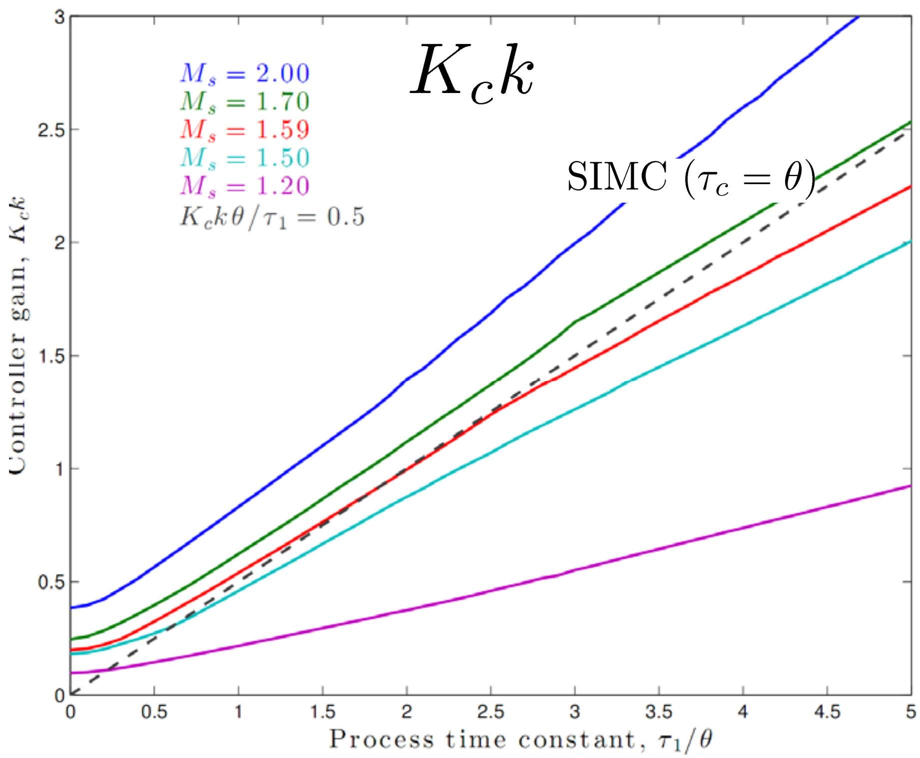
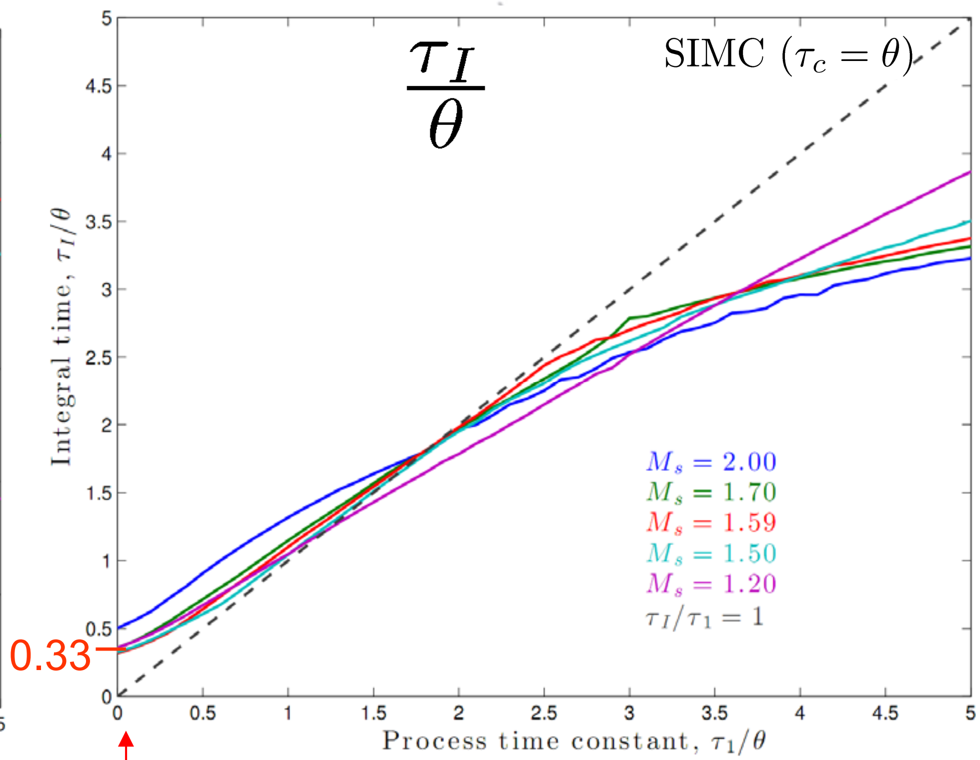
Optimal PI-settings vs. process time constant (τ_1/θ)



SIMC: $\frac{K_c k\theta}{\tau_1} = \frac{\theta}{\tau_c+\theta}, \quad \tau_I = \min(\tau_1, 4(\tau_c+\theta))$

Optimal PI-settings (small τ_1)





Time-delay process

SIMC: $\tau_I=\tau_1=0$

$$\text{SIMC:} \quad K_c k = \frac{\tau_1}{\tau_c+\theta}, \quad \tau_I = \min(\tau_1, 4(\tau_c+\theta))$$

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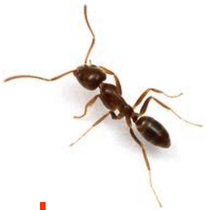
Improved SIMC-rule: Replace τ_1 by $\tau_1+\theta/3$

$$K_c = \frac{1}{k} \cdot \frac{\tau_1 + \frac{\theta}{3}}{(\theta + \tau_c)}$$

$$\tau_I = \min(\tau_1 + \frac{\theta}{3}, 4(\tau_c + \theta))$$

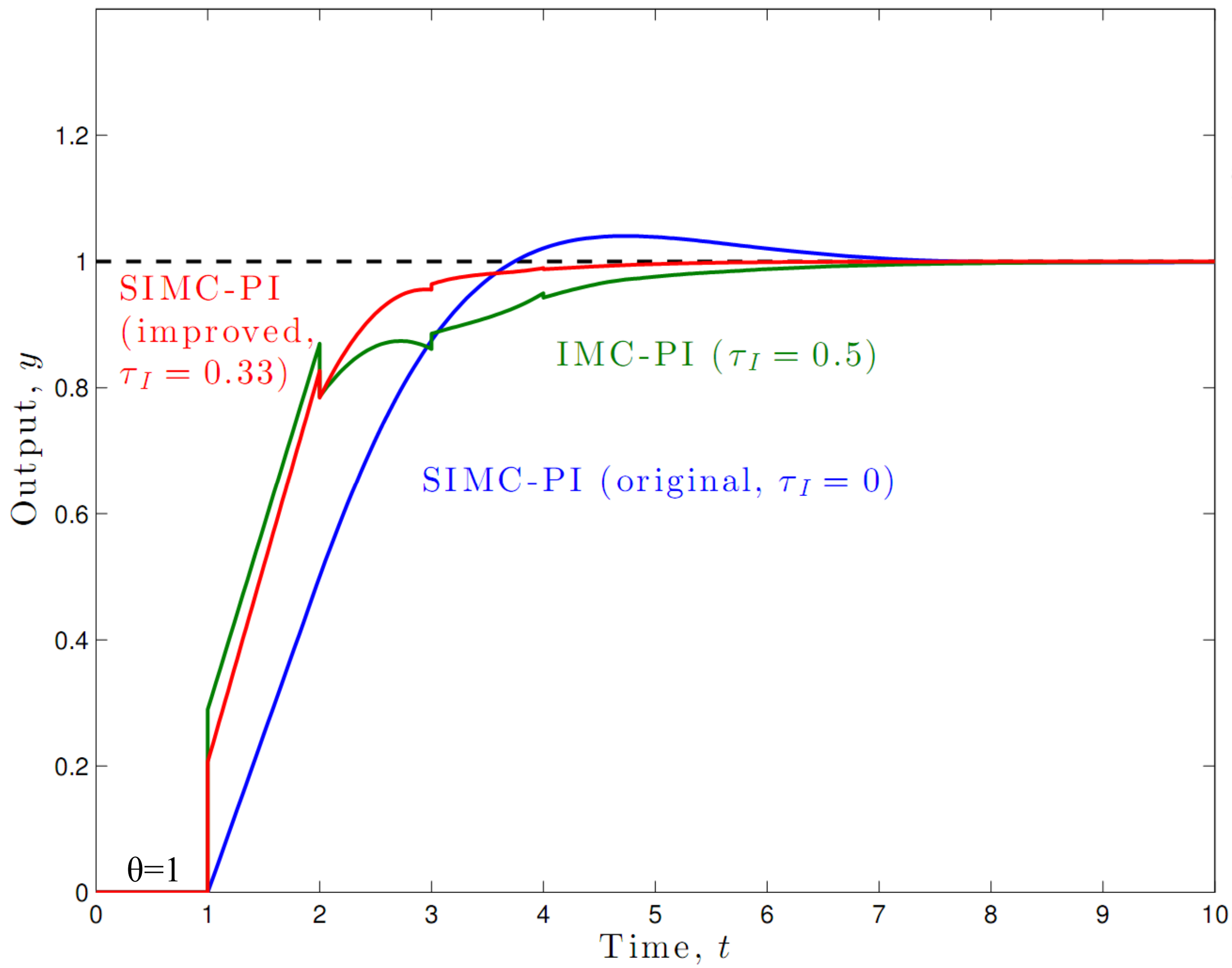
Tuning parameter: τ_c

Time delay process ($\tau_1 = 0$): $\tau_I = \frac{\theta}{3}$



Improved SIMC ant

Step response for time delay process



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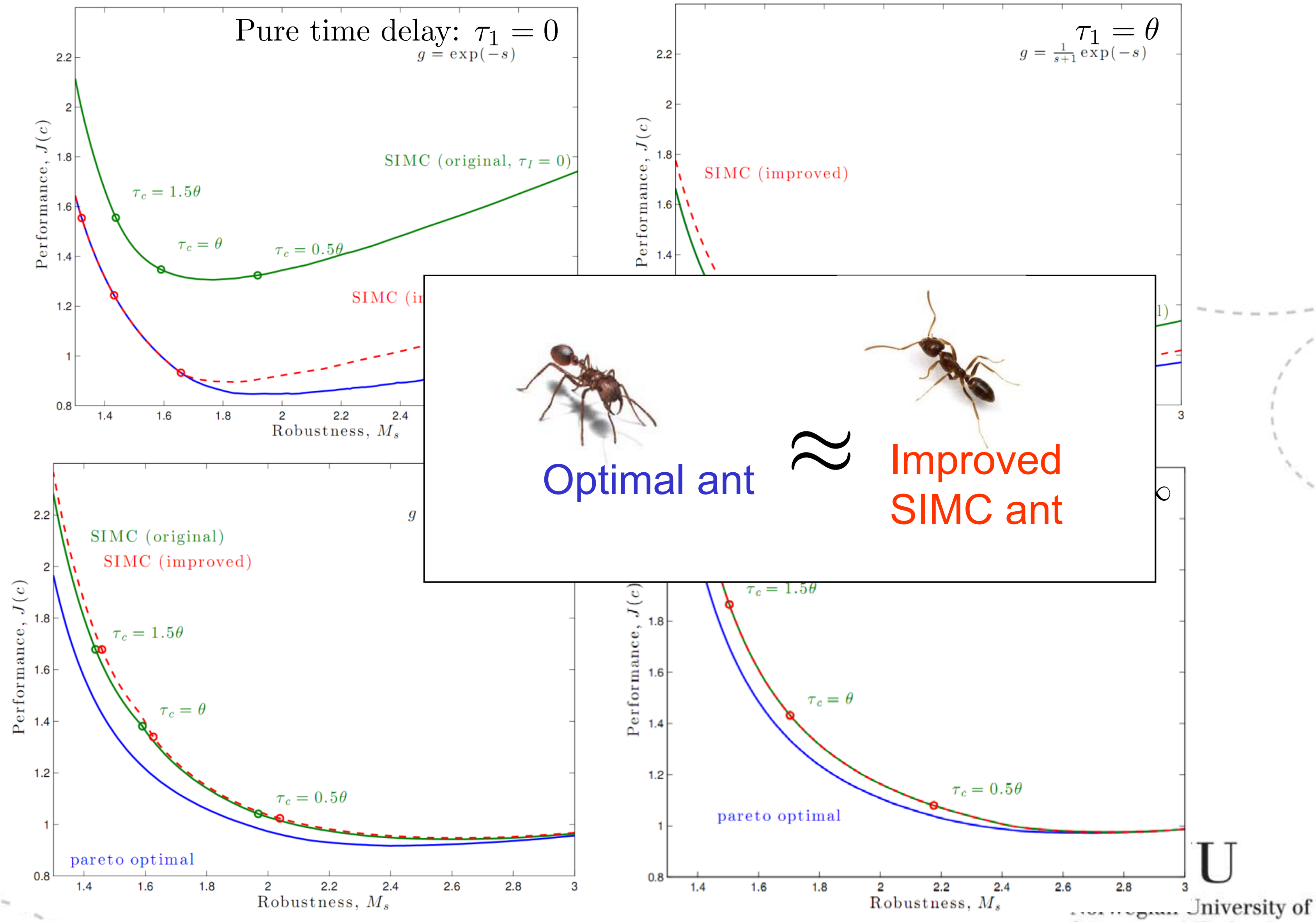
Time delay process: Setpoint and disturbance responses same + input response same

Comparison of J vs. Ms for optimal and SIMC-improved



CONCLUSION: SIMC-improved almost «Pareto-optimal»

7. Further work

- More complex controllers than PI:
  - Definition of problem becomes more difficult
  - Not sufficient with only IAE (J) and M_s
    - input usage
    - noise sensitivity
    - robustness
- Optimal PID
  - And comparison with SIMC-PID rule
- Comparison with truly optimal controller
  - Including Smith Predictor controllers

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8. Conclusion

Questions:

1. How good is really the SIMC-rule?
   – Answer: Pretty close to optimal, except for time delay process
2. Can it be improved?
   – Yes, to improve for time delay process: Replace τ_1 by $\tau_1+\theta/3$ in rule to get ”Improved-SIMC”

- “Probably the best simple PID tuning rule in the world”

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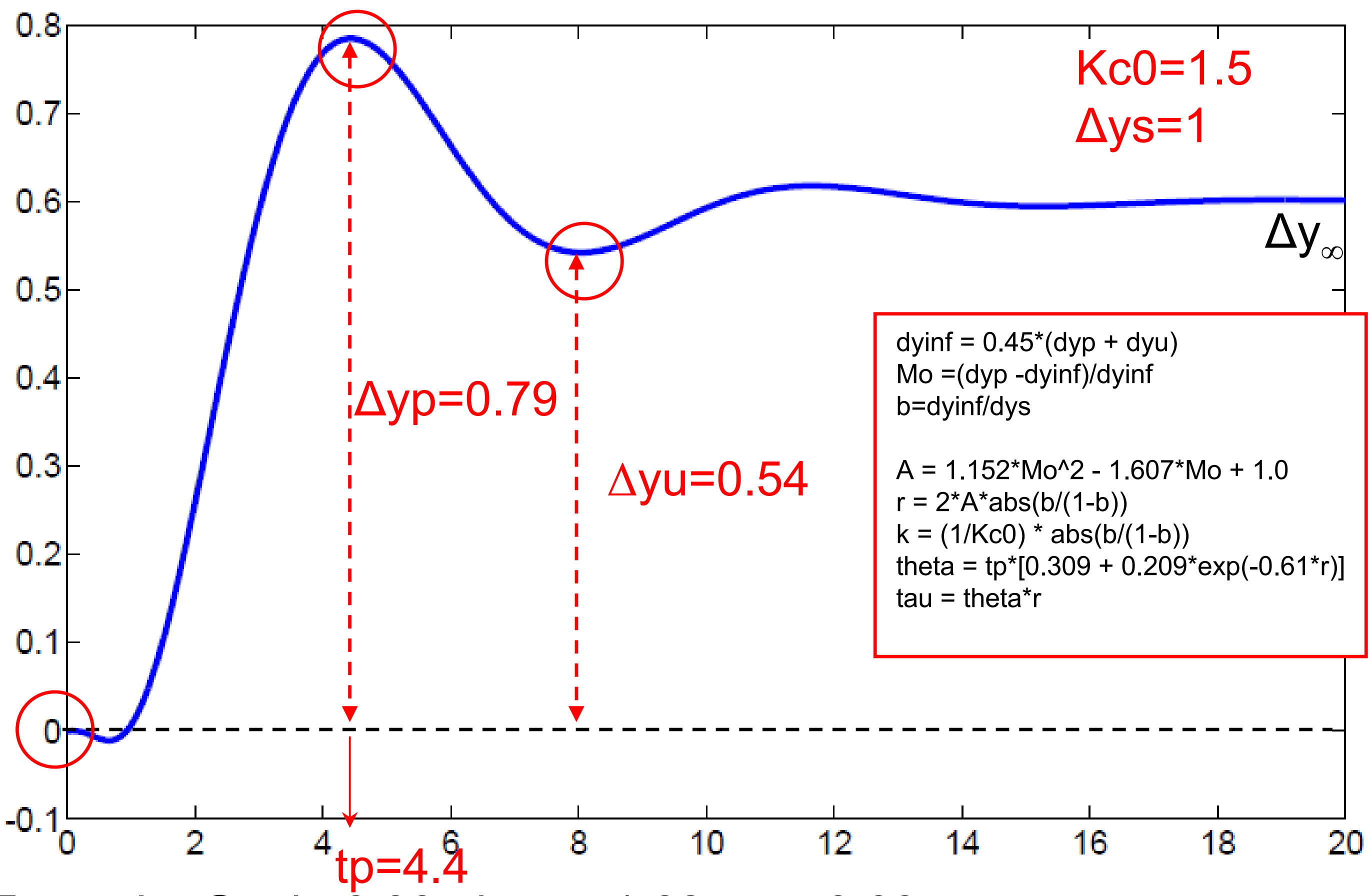
extra

Model from closed-loop response with P-controller



Example: Get k=0.99, theta =1.68, tau=3.03

Ref: Shamssuzzoha and Skogestad (JPC, 2010)
+ modification by C. Grimholt (Project, NTNU, 2010; see also PID12r paper + new PID-book 2012)