

Controlled Variables from Optimal Operation Data

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Abstract

In this paper we show how optimal operation data and concepts of self-optimizing control can be used for finding controlled variables which give optimal operation for the disturbances included in the data set. The method extracts the operation strategy which is hidden in the optimal data and may help to analyze and improve operation in the common case where it is difficult or very expensive to obtain a good model.

Keywords: Controlled variable selection, data based methods, self-optimizing control

1. Introduction

For many processes, obtaining a good mathematical process model is important for successful operation. However, obtaining a good model is often inhibited by several factors, such as a tight budget and limited knowledge or time. Thus, obtaining a good process model and keeping the model up to date is one of the major bottlenecks for the application of advanced process control in industrial applications [1]. It is therefore desirable to minimize the modeling effort, while still achieving good process performance. In this work we present a method which is based on logged process data, which is readily available for many processes in industry. This data is used to find self-optimizing controlled variables whose optimal setpoint does not change with varying disturbances. Previously, self-optimizing control structure design has been based on a process model. The contribution of this paper is to show how past process data can be analyzed to determine good controlled variables.

2. Motivation and problem formulation

An example of a system which is hard to model is a marathon runner. However, it is easy to collect data from runners, such as e.g. heart rate, stride frequency, temperature, blood oxygen content and breathing frequency. The data from the best runs of the runners subject to expected disturbances such as hilly terrain and wind is collected in an optimal data matrix Y . This data is used to determine a linear combination of measurements, which is (almost) constant for all the best runs. By running such as to keep this linear combination of variables at their optimal values, an optimal running strategy can be implemented.

Similarly, in a process plant, some operators may be able to operate the process more profitably than others. Analyzing the "optimal operation data" of these operators can reveal linear combinations of variables, which other operators can use as a guidance when operating the plant. Alternatively, these variables can be used for feedback control.

We assume that optimal operation corresponds to minimizing a cost J , and that the optimization problem can be approximated in deviation variables around the optimal point

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as

$$\min_u J = \begin{bmatrix} u^T & d^T \end{bmatrix} \begin{bmatrix} J_{uu} & J_{ud} \\ J_{du} & J_{dd} \end{bmatrix} \begin{bmatrix} u \\ d \end{bmatrix} \quad (1)$$

where $u \in \mathbb{R}^{n_u}$ and $d \in \mathbb{R}^{n_d}$ are the inputs and the disturbances, respectively. In order for a minimum to be unique, we require that J_{uu} is positive definite. For each degree of freedom u we search for a controlled variable c which is a linear combinations of measurements, $c = Hy$. If the variables give acceptable performance when controlled at constant setpoints, they are called self-optimizing, as defined in [2]: *Self-optimizing control is when we can achieve an acceptable loss with constant setpoint values for the controlled variables (without the need to reoptimize when disturbances occur)*. The loss is defined as $L = J(u, d) - J(u^{opt}, d)$, where u is the input generated by the current operating policy, for example adjusting u such that $c = Hy$ is kept constant.

3. Data method

The new method for finding these measurement combinations is directly inspired by the null-space method [3] which we present in the following.

3.1. Null space Method

This method is based on the quadratic approximation of the cost function (1). In addition it is assumed that a linear noise free measurement model is available, $y = G^y u + G_d^y d$. Here, $y \in \mathbb{R}^{n_y}$ is the vector of linear independent measurements and G^y, G_d^y are the gain matrices of the system.

Theorem 1 (Null space method). *Given a sufficient number of noise-free linear independent measurements, $n_y \geq n_u + n_d$, select H such that $HF = 0$, where $F = \partial y^{opt} / \partial d$ is the optimal sensitivity matrix. Then controlling $c = Hy$ to zero gives optimal operation with zero loss.*

Proof: Close to d^{nom} , by definition of F we have $y^{opt}(d) - y^{opt}(d^{nom}) = F(d - d^{nom})$. The optimal change in the controlled variables is: $c^{opt}(d) - c^{opt}(d^{nom}) = HF(d - d^{nom})$. Since H is selected such that $HF = 0$ optimal variation $c^{opt} - c_{nom}^{opt}$ is zero, too. Hence, controlling $c = Hy$ to zero leads to optimal operation. \square

The optimal sensitivity matrix F is usually obtained numerically, by optimizing a model or by linearizing at the nominal point, and evaluating $F = G_d^y - G^y J_{uu}^{-1} J_{ud}$ [3].

3.2. Using optimal operation data

In practice, we do not have F , but we have good data, and obtaining the optimal sensitivity matrix from a model is often a major challenge. Therefore it is desirable to avoid using a model. Considering the optimal sensitivity matrix $F = \partial y^{opt} / \partial d = [f_1, f_2, \dots, f_{n_d}]$, we see that if the matrix is augmented by any (combination) of its columns, e.g. if $Y = [F, f_1]$, the left null space remains unchanged. Since we require the data Y to come from optimal operation, all disturbances in it have been rejected optimally. Therefore all columns in Y are linear combination of the columns of F . This proves the following result:

Theorem 2 (Optimal data method - No noise). *Given sufficient measurements $n_y \geq n_u + n_d$, and optimal measurement data Y , where we for each distinct disturbance d there is at least one column in Y . Then the optimal measurement combination can be determined by selecting H such that $HY = 0$.*

The H matrix may also give valuable insight into the operation policy. After scaling and centering of the data, the elements in the left singular vector of Y can be used to analyze the operation strategy. We will demonstrate this in an example from economy below.

In practice, the data matrix Y will not be consistent such that a null space $HY = 0$ exists, either because of too many disturbances, or more likely because of measurement noise. One approach to handle this is to do a singular value decomposition $Y = U\Sigma V^T$, and select the transpose of the n_u columns in U which correspond to the smallest singular values in Σ . This is equivalent to approximating Y by the closest matrix with rank n_u .

More generally, the minimum loss method (exact local method) of [4] may be used, to handle cases with measurement noise, but this requires that we also have some “non-optimal” data:

Theorem 3 (Optimal data method with noise [4]). *Given noisy optimal measurement data Y and given “nonoptimal” data for the effect of the inputs (degrees of freedom) u on the measurements Y , so $\Delta y = G^y \Delta u$, the optimal measurement combination can be determined by finding the H which minimizes $\|(HG^y)^{-1}HY\|_F$.*

Note that we want HG^y to be large, that is we want to use “sensitive” measurements. With the sensitivities small and with little measurement noise, the contribution from the term HG^y is small, and then Theorem 2 is sufficient.

4. Case studies

4.1. Optimal operation of a chemical reactor (use of Theorem 2)

We consider a CSTR with a reaction $A \rightleftharpoons B$, Fig 1. The feed contains mainly component A , and the objective is to maximize the profit which is calculated as the difference between the income from selling the product B and the cost of cooling: $P = p_B C_B - p_{cool} T_i^2$. T_i is the cooling temperature which can be manipulated to optimize performance. The feed concentrations are the main disturbances, and the concentrations reactor temperature are measured, so $y = [C_A, C_B, T]$. The optimal operation data is obtained by applying the NCO tracking procedure as described in [5] in combination with finite difference gradient estimates, where the input is perturbed to obtain a gradient estimate, and based on this estimate, it is adjusted to iteratively force the gradient to zero. The optimal data is collected into the data matrix Y , and a singular value decomposition $Y = U\Sigma V^T$ gives $(\sigma_1, \sigma_2, \sigma_3) = (86.5, 4.8, 0.28)$. Since there is one input, T_i , we select the column in U corresponding to $\sigma_3 = 0.28$. This gives a controlled variable $c = Hy$ with $H = [-0.77 \ 0.63 \ 0.005]$. In Fig. 2 the simulated disturbance scenario is given and Fig. 3 shows the input usage when applying NCO tracking (to generate the optimal data) and when using a PI controller to control $c = Hy = -0.77C_A + 0.63C_B + 0.005T$ to zero. Due to the continuous feedback control, controlling $c = Hy$ gives much smoother input action than we have in the “optimal” data. Comparing the final profit in Fig. 4, shows that controlling the obtained invariant gives practically the same performance.

4.2. Economy example (use of Theorem 2)

We consider economic indicators from 1991 to 2006 for France, Germany, Italy, Norway, UK, USA. The data is taken from [6]. The “measurements” y for each country are interest rate (y_1), unemployment (y_2), the industrial production index (IPI, y_3), the consumer price index (CPI, y_4), tax revenue (% of GDP, y_5) and exchange rate to SDR (special drawing

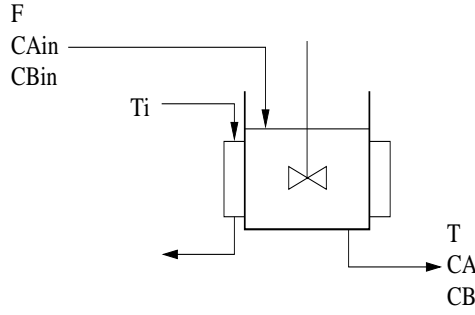


Figure 1. CSTR

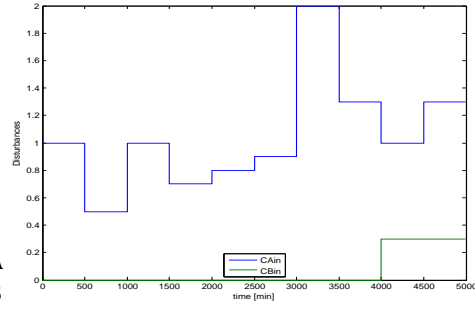


Figure 2. Disturbances $C_{A,in}$, $C_{B,in}$

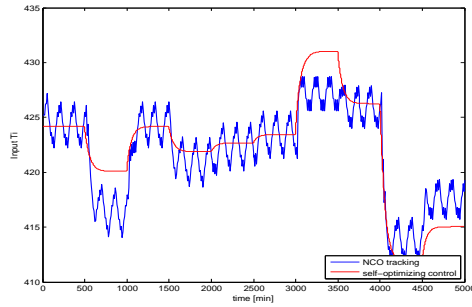


Figure 3. Inputs SOC and NCO tracking

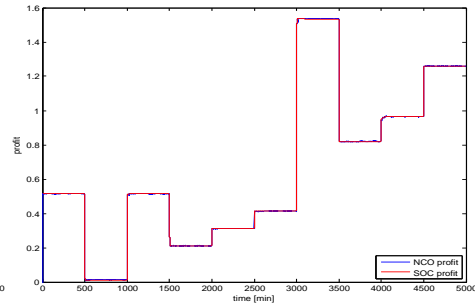


Figure 4. Profit Comparison

rights, a “lumped” currency derived from the Yen, US Dollars, British Pounds and Euros, y_6). The GDP growth, Fig 5, is the criterion for optimality. The measurements of year prior to the three years with highest GDP growth are used for Y . This results in $H = [-0.67 \ -0.02 \ 0.22 \ 0.62 \ 0.32 \ 0.10]$, Fig. 6. The most influential factors are the interest rate (-0.67) and the inflation rate (0.62). This is not unexpected, because the interest rate is used as a handle to control inflation. Of course the economics of countries is too complex to be described accurately by our selected variables, but we have shown that applying our method to economic data can reveal some of the operation strategy behind the data.

5. Discussion and conclusion

The proposed “null space data method” picks out the weak directions in the data Y , whereas other “chemometric” regression methods concentrate on the strong directions in the data. An important reason for this is that we assume that the data is optimal, and we look for hidden combinations in this data that characterize the optimum. On the other hand, in regression methods one looks for relationships between variables X and Y . To show that the methods are different, assume our data contains two data sets, $Y = [Y_1 \ X]^T$ and we want to find how the relationship between Y_1 and X . We assume that $\dim(Y_1) = \dim(u) = n_u$. From our method, the problem becomes $\min_H \| [H[Y_1 \ X]^T] \|_F$. Here, H is not unique, so we have that if H is an optimal solution, so is DH , where D is an invertible matrix [4]. This degree of freedom may be used to set $H = [I \ H_x]$, and we optimize the problem $\min_{H_x} \| Y + H_x X \|_F$, which has the least squares solution $H_x =$

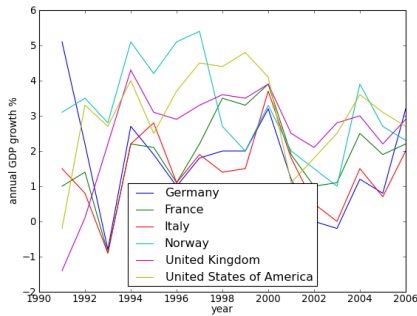


Figure 5. Annual GDP growth

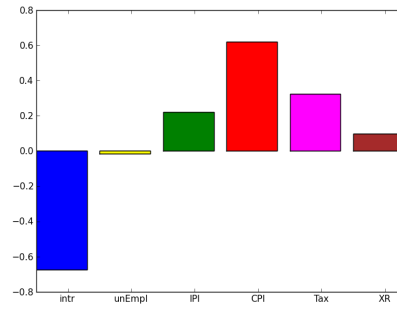


Figure 6. Magnitude of elements in H

$-YX^\dagger$. Thus our method is equivalent to the normal regression methods for problems where the norm of $\|HY\|_F$ is small, such that the contribution from the term $J_{uu}(HG^y)^{-1}$ can be neglected, that is, for the noise free case.

However, a significant difference to standard regression methods for the case when we simply minimize $\|HY\|_F$, is that we do not distinguish between Y_1 and X data and try to find a relationship between these, but instead focus on finding invariant variable combinations $c = Hy = H_y y_1 + H_x x = 0$.

Our method has the advantage that it only uses data and does not rely on a model. Thus it is applicable to systems where it is very expensive or impossible to obtain an accurate model. Not even the cost function has to be known as long as the data is optimal. However, it is important that the data is consistent in the sense that the data gives the correct optimal sensitivity $F = dy^{opt}/dd$ and contains little measurement noise.

The main drawback is that we rely on optimal data, and performance cannot be improved beyond the learning data. However, one could obtain the optimal data using some expensive method, and then analyze it to find a cheap method which gives similar performance, as is done in the CSTR example above. Other applications could be to find the “secret” of good operators or the “control strategy” of a marathon runner or of some economy.

6. References

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