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Generalized Internal Model Control for Balancing Input/Output Disturbance Response

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ABSTRACT: Based on internal model control (IMC), we present a design method to take into account both input and output disturbances. The proposed design provides generalized IMC filters that can be used to obtain good results in terms of output sensitivity (favoring output disturbances), or in terms of input sensitivity (therefore placing the emphasis on load disturbances). If both input and output disturbances are expected, the design offers the possibility of obtaining a balance that improves the overall disturbance rejection response.

1. INTRODUCTION

The objective of a control system is to make the output y behave in a desired way by manipulating the plant input u. There are basically two different problems:[1,2] the *Servo* problem, which concerns the tracking of the reference signal r, and the *Regulator* problem, which aims at rejecting the disturbances d entering the control loop. In both cases, the controller K is designed to make the control error ($e = y - r$) small.

This work exclusively addresses the *Regulator* problem. Note that, if the resulting tracking performance was not suitable, this could be fixed in a second step by introducing a reference prefilter.[2,3] More generally, the *Servo* and the *Regulator* problems can be solved independently, using a two-degree-of-freedom (2DOF) topology.[1,4,5] In what follows, we will assume that disturbances cannot be measured and can enter both at the input and at the output of the plant P. Therefore, a feedforward strategy[6,7] is not advantageous in the considered scenario, where the feedback controller completely determines the disturbance response.

To cope with the input/output *Regulator* problem, here, we rely on the internal model control (IMC) paradigm.[3] Historically, the inherent shortcomings of the IMC method have resulted in the search for new filters and/or alternative procedures: for minimum-phase (MP) unstable plants, Campi et al.[8] suggested a filter that allows easy adjustment of the closed-loop bandwidth, as well as robustness improvement. For stable plants, Horn et al.[9] modified the conventional filter for enhanced input disturbance attenuation. From a broader viewpoint, a simple IMC-based procedure applicable to both stable and unstable plants and aimed at input disturbances was presented by Lee et al.[10] Some years later, Dehghani et al.[11] reported the difficulties with the IMC procedure in an exhaustive manner and, in order to undergo them, devised a numerical design blending IMC and $\mathcal{H}_\infty$ ideas. Although the latter design offers great versatility, it requires judicious choices for some frequency weights and for the desired closed-loop response, which may lead to design pitfalls, as noted in ref 12. Along these lines, a simpler IMC-like $\mathcal{H}_\infty$ design overcoming basic limitations of IMC has been reported by Alcántara et al.[13]

The analytical solution presented here can be seen as the $\mathcal{H}_2$ counterpart of that described in ref 13. With respect to ref 13, some assumptions have been removed: i.e., the plant model is not restricted to be purely rational nor to contain at least one Right Half-Plane (RHP) zero. In addition, plants with complex poles have been included in the discussion. An interesting aspect of the herefore-adopted $\mathcal{H}_2$ approach is that it unifies the previous designs,[8−10] resulting into a more general structure for the IMC filter. The distinguishing feature of the new filter is that it allows one to balance the input/output regulatory performance in a simple manner. This is a fundamental tradeoff, disregarded in refs 8−10, that cannot be overcome using a 2DOF control configuration or a related approach as done in the literature.[14−16]

An outline for the rest of the article is given as follows. Section 2 states the problem formally and reviews basic material about $\mathcal{H}_2$ optimization and IMC. The proposed design is introduced in Section 3, and then it is illustrated by example in Section 4, to obtain different balances of input/output disturbance attenuation. Finally, Section 5 summarizes the main ideas and makes some concluding remarks.

2. PROBLEM STATEMENT AND BACKGROUND MATERIAL

Integrating (and close to integrating) processes are very common in industry (e.g., level systems and pulp and paper plants). For illustration purposes, let us consider a pure integrator process controlled by a proportional−integral (PI) compensator. The situation is depicted in Figure 1a. As it can be seen in Figure 1b, a proportional controller ($K_p = 5$, $K_i = 0$) yields excellent results when output disturbances are the main concern. However, the corresponding response to load disturbances is not satisfactory. In order to suppress the steady-state error due to input disturbances, integral action is necessary. The response to

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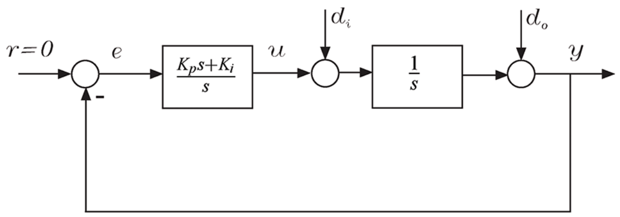
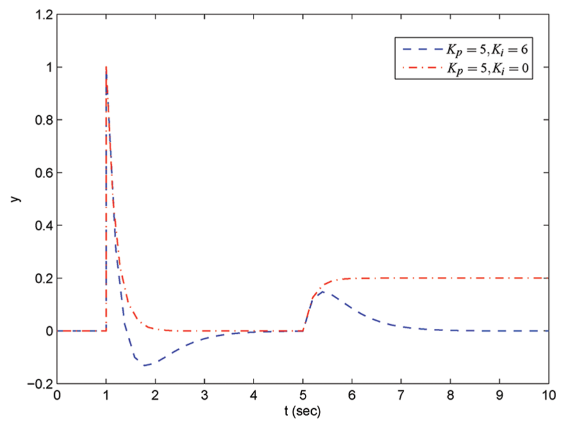


(a) Integrator process with PI controller.



(b) Time responses to unity step disturbances at the output ($t = 1$) and at the input ($t = 5$) of the plant.

Figure 1. Motivating example: (a) integrator process with PI controller and (b) time responses to unity step disturbances at the output ($t = 1$) and at the input ($t = 5$) of the plant.

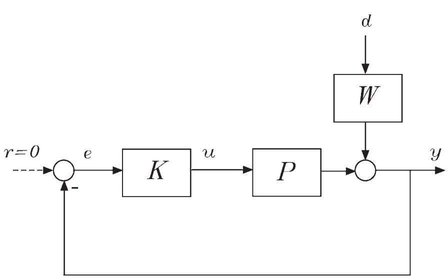


Figure 2. Basic setup for the input/output *Regulator* problem.

the alternative settings $K_p = 5$, $K_i = 6$ confirms this point. Figure 1b clearly shows that a fundamental tradeoff exists between the input and output disturbances. Therefore, if both types of disturbances are present, a tradeoff tuning methodology would help improve the overall disturbance response. These issues concerning input/output disturbances are the theme of this article. To set the problem, we make use of single-input—single-output (SISO) linear models of the form

$$y = Pu + Wd \tag{2.1}$$

for which the corresponding feedback setup is depicted in Figure 2. In (2.1), W (which is not a physical component as P or K) represents a frequency weight that will be designed to make it easy to balance the disturbance response at the input and at the output of the plant. By also absorbing the input type (e.g., step-like disturbances) into the weight W, we will assume hereafter that d in Figure 2 is an impulse (i.e., $d(s) = 1$). To derive the feedback controller K, we will examine the structure of $\mathscr{H}_2$-optimal controllers. By using an $\mathscr{H}_2$-optimal controller, here, we understand one such that the integrated square error,

$$\|e\|_2^2 = \int_0^\infty e^2(t)\, dt \tag{2.2}$$

is minimized for a particular input. Bearing in mind that $e = -SWd = -SW$, where S is the sensitivity function (defined as $S \doteq 1/(1 + PK)$), we can state problem (2.2) in the frequency domain as

$$\min_{\mathscr{C}} \|e\|_2^2 = \min_{\mathscr{C}} \frac{1}{2\pi}\int_{-\infty}^{\infty} |S(j\omega)W(j\omega)|^2\, d\omega \tag{2.3}$$

where $\mathscr{C}$ denotes the set of *internally stabilizing* controllers. Internal stability is the requirement that all the closed-loop transfer functions be stable, implying that unstable pole/zero cancellation between the plant P and the controller K is not allowed. It is well-known that the IMC parametrization of the feedback controller,[3]

$$K = \frac{Q}{1 - PQ} \tag{2.4}$$

allows one to write all the closed-loop relations affinely in Q (e.g., $S = 1 - PQ$, $T = PQ$). Then, in terms of Q, the following fundamental result solves (2.3).

Theorem 2.1 (Morari and Zafiriou[3]). Let us factor both the plant P and the weight W into an all-pass and a MP portion, so that $P = P_a P_m$ and $W = W_a W_m$. Use the parameters l and k to denote the number of integrators and unstable poles of P, respectively. Now, assume that the weight W contains $l' \geq l$ integrators and the first $0 \leq k' \leq k$ unstable poles of P, and define b_P and b_W as

$$b_P = \prod_{i=1}^{k} \frac{-s + \pi_i}{s + \bar{\pi}_i} \quad \text{and} \quad b_W = \prod_{i=1}^{k'} \frac{-s + \pi_i}{s + \bar{\pi}_i} \tag{2.5}$$

with $\pi_1, ..., \pi_k$ being the unstable poles of P. Then, the $\mathscr{H}_2$-optimal (internally stabilizing) Q is given by

$$Q = b_P (P_m b_W W_m)^{-1} \{(b_P P_a)^{-1} b_W W_m\}_* \tag{2.6}$$

where the operator $\{\cdot\}_*$ denotes that, after a partial fraction expansion (PFE) of the operand, all terms involving the poles of P_a^{-1} are omitted.

Note that it is straightforward how to select W for the extreme cases at hand. For example, if only step output disturbances are considered, the weight should be $W = 1/s$, whereas $W = P/s$ for the case of step disturbances entering at the input of the plant. A more difficult problem is how to select W systematically for balanced operation. In addition, W should be such that it allows to adjust the robustness/performance tradeoff. The selection of W will be fully addressed in Section 3. We end this section by observing the following facts.

Remark 2.1. The optimal solution in (2.6) is only dependent on the MP part of W. Consequently, W can be restricted to be MP without a loss of generality (i.e., $W = W_m$).

Remark 2.2. For MP (possibly unstable) plants ($P_a = 1$), the optimal solution in (2.6) becomes $Q = P_m^{-1}$, independent of W.

3. PROPOSED INPUT/OUTPUT REGULATOR DESIGN

This section first addresses the selection of a suitable weight W for the problem at hand (Section 3.1). After selecting W, an analytical solution for Q is given based on the $\mathscr{H}_2$ minimization criterion (see Section 3.2). Finally, in Section 3.3, we examine the nominal performance, robust stability, and robust performance properties of the derived controller.

3.1. Selection of *W*. Let us take $P = P_aP_m$, as in Section 2, and use d_d to denote the generating polynomial of the disturbance (i.e., $d_d = s$ for steps, $d_d = s^2$ for ramps, etc.). For the sake of clarity, it is temporarily assumed that $P_a \neq 1$ and that P does not have any complex poles or zeros, nor any pole at the origin. We also assume that P has slow/unstable poles at $s = -1/\tau_1, ..., -1/\tau_k$. Then, we make the following choice of the weight:

$$W = \frac{(\lambda s + 1)^n}{d_d}\prod_{i=1}^{k}\frac{\gamma_i s + 1}{\tau_i s + 1} \tag{3.1}$$

with

$$n = \max\{1, \delta(d_d) + \delta(P) - 1\} \tag{3.2}$$

where $\delta(d_d)$ and $\delta(P)$ denote the degree of d_d and the relative degree of P, respectively. For the common case of step disturbances ($d_d = s$), (3.2) simplifies to $n = \max\{1, \delta(P)\}$. Finally, λ and $\gamma_1, ..., \gamma_k$ in (3.1) are tuning parameters verifying that

$$\lambda > 0, \quad \gamma_i \in [\lambda, |\tau_i|] \tag{3.3}$$

Here, we recall that the main objective of our design is to consider disturbances entering both at the input and at the output of the plant. In addition, the design must account for model uncertainty. The rationale behind the selection of W in (3.1) is explained below:

- In order to explain the role of λ and γ_i separately, let us start considering that $\lambda = 0$. Then, we have that $W = (1/d_d)\Pi_{i=1}^{k}[(\gamma_i s + 1)/(\tau_i s + 1)]$. Now, by making $\gamma_i = \lambda = 0$, $i = 1, ..., k$, the weight is $W = (1/d_d)\Pi_{i=1}^{k}[1/(\tau_i s + 1)]$. For this choice of γ_i, the design will provide good results, in terms of input disturbance attenuation, since we are including the slow/unstables poles of P in W. In other words, the disturbance passes through the conflicting poles of the plant (note that fast stable poles do not impose a tradeoff between input/output regulatory performance).
- At this point, we can improve the output disturbance response by increasing the value of each γ_i. To see this, let us consider that γ_i is set to the upper bound of the interval described by (3.3) (i.e., we take $\gamma_i = |\tau_i|$). It is then clear that $|W| = 1/|d_d|$, for which (2.3) optimizes the ISE for output disturbances.
- So far, we have assumed that $\lambda = 0$. Let us suppose now that each γ_i has been fixed to a particular value. As we increase the value of λ, the minimization in (2.3) will penalize the magnitude of S at higher frequencies, resulting in a slower closed loop. Therefore, λ can be used to adjust the robustness/performance tradeoff. Regarding n, the value in (3.2) will ensure the properness of the final controller (this point will be clarified later).

Remark 3.1. For simplicity, the γ_i parameters could be determined from a single parameter $\gamma \in [0,1]$, as indicated below:

$$(\gamma_1, ..., \gamma_k)^{\mathrm{T}} = (1 - \gamma)(\lambda, ..., \lambda)^{\mathrm{T}} + \gamma(|\tau_1|, ..., |\tau_k|)^{\mathrm{T}} \tag{3.4}$$

3.2. Analytical Solution. The next step toward obtaining the IMC controller is to use Theorem 2.1. Because $W = W_m$ in (3.1) contains the unstable poles of P, we have $b_P = b_W$, and (2.6) simplifies to $Q = (P_mW)^{-1}\{P_a^{-1}W\}_*$. This is a valid controller when P is non-minimum phase (NMP), in the sense that it is internally stabilizing and proper. However, recalling Remark 2.2, for MP plants (i.e., $P_a = 1$) the solution is $Q = P_m^{-1}$, regardless of the value of W. As a consequence, Q may be improper, and it would be necessary to extend Q by cascading a filter, as in the conventional IMC procedure. We want to avoid this approach and obtain a proper solution directly from the specified weight W. Toward this objective, we finally propose the following solution:

$$Q = (P_mW)^{-1}\{P_a^{-1}W\}_\bigstar \tag{3.5}$$

where $\{\cdot\}_\bigstar$ acts like $\{\cdot\}_*$, but also removes the possibly nonstrictly proper terms after the PFE. The $\{\cdot\}_\bigstar$ operator gives the same result as $\{\cdot\}_*$ when the plant is NMP. When P is MP, the actuation of $\{\cdot\}_\bigstar$ can be understood in terms of $\{\cdot\}_*$ as follows:

$$\{P_a^{-1}W\}_\star = \left.\left\{\left(\underbrace{P_a e^{-sh}}_{P_a'}\right)^{-1} W\right\}_*\right|_{h=0}$$

That is to say, we consider a fictitious delay h, apply $\{\cdot\}_*$, and then evaluate at $h = 0$. The following example illustrates how to calculate (3.5).

Example 3.1. Let us consider the (possibly unstable) First Order Plus Time Delay (FOPTD) model, $P = K_g[e^{-sh}/(\tau s + 1)]$, for which $P_m = (K_g/(\tau s + 1)$, $P_a = e^{-sh}$. We assume that $|\tau| \gg h > 0$ ($k = 1$). In addition, we assume step-like disturbances, i.e., $d_d = s$, and take $n = 1$. By substitution into (3.1), we get $W = (\lambda s + 1)(\gamma s + 1)/[s(\tau s + 1)]$, with $\lambda > 0$, $\gamma \in [\lambda, |\tau|]$. If $h > 0$, the proposed controller described by (3.5) is identical to the $\mathscr{H}_2$-optimal one:

$$\begin{aligned} Q &= \frac{(\tau s + 1)^2 s}{K_g(\lambda s + 1)(\gamma s + 1)}\left\{e^{sh}\frac{(\lambda s + 1)(\gamma s + 1)}{s(\tau s + 1)}\right\}_\bigstar \\ &= \frac{(\tau s + 1)^2 s}{K_g(\lambda s + 1)(\gamma s + 1)}\left\{e^{sh}\frac{(\lambda s + 1)(\gamma s + 1)}{s(\tau s + 1)}\right\}_* \\ &= \frac{(\tau s + 1)^2 s}{K_g(\lambda s + 1)(\gamma s + 1)}\left(\frac{1}{s} - \frac{\tau e^{-h/\tau}\tau[1 - (\lambda/\tau)][1 - (\gamma/\tau)]}{\tau s + 1}\right) \\ &= \frac{(\tau s + 1)(\{\tau - \tau e^{-h/\tau}[1 - (\lambda/\tau)][1 - (\gamma/\tau)]\}s + 1)}{K_g(\lambda s + 1)(\gamma s + 1)} \end{aligned} \tag{3.6}$$

If $h = 0$, P becomes MP ($P_a = 1$). In this case, we make $h = 0$ in (3.6) and we arrive at

$$Q = \frac{(\tau s + 1)(\{\tau - \tau[1 - (\lambda/\tau)][(1 - (\gamma/\tau)]\}s + 1)}{K_g(\lambda s + 1)(\gamma s + 1)} \tag{3.7}$$

In particular, note that $Q \to P_m^{-1} = (\tau s + 1)/K_g$ as $\lambda \to 0$, implying that the $\mathscr{H}_2$-optimal solution is approached for small values of λ.

The following proposition summarizes the most basic properties of the proposed controller. [Although we are assuming that W is given by (3.1), property (P3) holds generally for any MP weight.]

Proposition 3.1. The IMC controller Q in eq 3.5 is such that

(P1) Q is $\mathscr{H}_2$-optimal if P is NMP. In the MP case, Q tends to be $\mathscr{H}_2$-optimal when $\lambda \to 0$, provided that $\delta(d_d) \geq 1$.

(P2) Q is proper and stable.

(P3) $S = 1 - PQ = 0$ at the poles of W.

Proof. Consult Appendix A.1. ■

Property (P1) can be interpreted as the combination of the two steps of the IMC procedure into a single one. Properties (P2) and (P3) imply that Q is realizable and internally stabilizing (because W contains the poles of P). In addition, property (P3) means asymptotic rejection of the disturbances (because the denominator of W contains the generating polynomial d_d, recall the *Internal Model Principle*[2,3]).

Remark 3.2. From (3.5) and property (P3), Q and $1 - PQ$ have zeros at the k slow/unstable poles of P. These zeros get canceled when forming the equivalent unity feedback controller $K = Q/(1 - PQ)$. This means that adjusting λ, γ_i in W does not change the structure of the final controller, but rather, only its parameters.

Remark 3.3. Strictly speaking, properties (P2) and (P3) are not sufficient conditions for internal stability when P is a delayed unstable system. As explained in ref 17, in this case, there are irremovable RHP pole/zero cancellations in K that do not allow a direct implementation. In general, Q can be approximated by a practical controller K (e.g., proportional−integral−derivative (PID) type) by following different methodologies.[15,16,18]

3.2.1. Analytical Solution in Terms of Alternative IMC Filters. The proposed controller described by (3.5) can be expressed as

$$Q = P_m^{-1} f \tag{3.8}$$

with $f = W^{-1}\{P_a^{-1}W\}_\star$. Let us take $W = n_w/d_w$. Now, considering how $\{\cdot\}_\star$ acts and taking into account property (P3) in Proposition 3.1, we can alternatively express f as

$$f = \frac{\chi}{n_w} = \frac{\sum_{i=0}^{\delta(d_w)-1} a_i s^i}{(\lambda s + 1)^n \prod_{i=1}^{k} (\gamma_i s + 1)} \tag{3.9}$$

where $a_0, ..., a_{\delta(dw)-1}$ are determined from the following system of linear equations:

$$T|_{s=\pi_i} = P_a f|_{s=\pi_i} = 1 \qquad i = 1,...,\delta(d_w) \tag{3.10}$$

with π_i ($i = 1, ..., \delta(d_w)$ being the poles of W ($\pi_i = -1/\tau_i$). From (3.1), $\delta(d_w) = k + \delta(d_d)$ in general, except when P is stable and we take $\gamma_i = \tau_i$ for all i. In the latter case, the weight (3.1) simplifies to $W = (\lambda s + 1)^n/d_d$, and $\delta(d_w) = \delta(d_d)$. Note that, as long as the a_i coefficients satisfy (3.10), the filter time constants λ and γ_i can be selected freely without any concern for nominal stability. In more detail, (3.10) corresponds to

$$\begin{pmatrix} \pi_1^{\delta(d_w)-1} & \cdots & \pi_1 & 1 \\ \vdots & \ddots & \vdots & \vdots \\ \pi_{\delta(d_w)}^{\delta(d_w)-1} & \cdots & \pi_{\delta(d_w)} & 1 \end{pmatrix} \begin{pmatrix} a_{\delta(d_w)-1} \\ \vdots \\ a_0 \end{pmatrix} = \begin{pmatrix} P_a^{-1} n_w|_{s=\pi_1} \\ \vdots \\ P_a^{-1} n_w|_{s=\pi_{\delta(d_w)}} \end{pmatrix} \tag{3.11}$$

In the context of step-like inputs, the filter (3.9) generalizes some previously reported filters in the following way:

- For stable plants, by taking $\gamma_i = \tau_i$, the conventional IMC filter[3] is obtained. However, if we take $\gamma_i = \lambda$, then the filter that has been described in ref 9 results.
- Essentially, the filter suggested in ref 8 for MP unstable plants corresponds to taking $\gamma_i \to \infty$ in (3.9). In the general unstable plant case, the filter that has been described in ref 10 is recovered by choosing $\gamma_i = \lambda$.

Finally, using Lagrange-type interpolation theory,[3] it is possible to develop an expression for (3.9) explicitly:

$$f = \frac{1}{n_w} \sum_{j=1}^{\delta(d_w)} (P_a^{-1} n_w)|_{s=\pi_j} \prod_{\substack{i=1 \\ i \neq j}}^{\delta(d_w)} \frac{s - \pi_i}{\pi_j - \pi_i} \tag{3.12}$$

[This formula is not valid for repeated poles.]

3.3. Nominal Performance, Robust Stability, and Robust Performance. In any practical design method, robust performance is the ultimate goal: we want the controller to work well under uncertain circumstances. Assuming that a condition for robust stability is met, the next subsection gives an upper bound for the performance degradation with respect to the nominal case. How the robustness/performance compromise is influenced by the tuning parameters λ and $\{\gamma_i\}_i$ is addressed later in Section 3.3.2.

3.3.1. General Relations. From Section 2, the ISE for an output disturbance d ($d = 1/d_d$) is given by

$$\mathrm{ISE}_o = \int_0^\infty e^2(t)\, dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |S d_d^{-1}(j\omega)|^2\, d\omega \tag{3.13}$$

Similarly, when d enters at the input of the plant, the corresponding ISE is

$$\mathrm{ISE}_i = \int_0^\infty e^2(t)\, dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |PS d_d^{-1}(j\omega)|^2\, d\omega \tag{3.14}$$

Equations (3.13) and (3.14) indicate the nominal performance achieved by the final design in terms of input/ouput disturbance attenuation. Robust stability can be assessed by the well-known condition[2,3]

$$\|\Delta T\|_\infty = \sup_\omega |\Delta(\omega) T(j\omega)| < 1 \tag{3.15}$$

where $\Delta(\omega) \geq 0$ is a bound for the plant multiplicative uncertainty. In practice, nominal performance and robust stability alone are not enough, because some plants in the uncertain set may be on the verge of instability, yielding very poor performance. Therefore, it is necessary to guarantee some degree of robust performance. To this aim, it is useful to have an upper

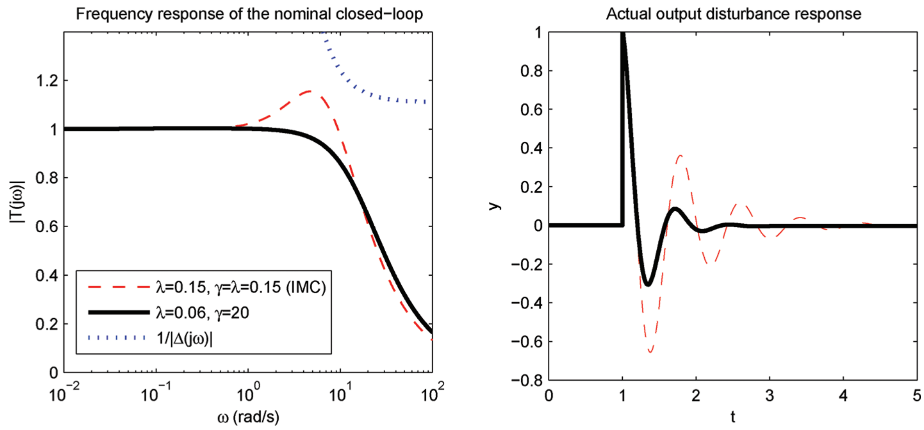


Figure 3. Frequency and time responses (Example 1), $\Delta = (\tilde{P} - P)/P$.

bound for both ISE_i and ISE_o. The worst error is generated by the worst plant, which can be expressed as $P(1 + \delta(s)\Delta(\omega))$ for some $\delta(s)$ such that $|\delta(j\omega)| \leq 1$. Using the inequality $|1 + P(1 + \delta\Delta)K| \geq |1 + PK| - |PK|\Delta$, the actual sensitivity function $\mathscr{S}$ can be bounded as

$$|\mathscr{S}| = \left|\frac{1}{1 + P(1 + \delta\Delta)K}\right| \leq \left|\frac{1}{1 - |\Delta T|}\right||S| \qquad (3.16)$$

From (3.13), (3.14), and (3.16), the following upper bounds for the actual errors result:

$$\mathrm{ISE}_o \leq \overline{\mathrm{ISE}_o} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left|\frac{1}{1 - |\Delta T|}\right|^2 |Sd_d^{-1}|^2 \, \mathrm{d}\omega \qquad (3.17)$$

$$\mathrm{ISE}_i \leq \overline{\mathrm{ISE}_i} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left|\frac{1}{1 - |\Delta T|}\right|^2 |PSd_d^{-1}|^2 d\omega \qquad (3.18)$$

As it is logical, the modeling error increases the (finite) gap between ISE_i (ISE_o) and $\overline{\mathrm{ISE}_i}$ ($\overline{\mathrm{ISE}_o}$) as the stability boundary in (3.15) is approached, exhibiting the typical tradeoff between nominal performance and performance degradation.[2,3,19]

3.3.2. Tuning Guidelines for the Parameters λ and γ_i. In view of eqs (3.13), (3.14), (3.17), and (3.18), nominal performance is captured in terms of S, whereas robust performance is expressed using T (which also determines robust stability (3.15)). The shape of these transfer functions depends on the values of the tuning parameters. The role of λ is the same as in the conventional IMC: basically, for a given value of each γ_i, increasing λ makes the system slower, to the detriment of ISE_o and ISE_i, but favoring the robust stability condition (3.15) by reducing the closed-loop bandwidth. Let us consider now that λ, γ_j ($j = 1, ..., k$, $j \neq i$) have been fixed, and see which is the influence of γ_i. From earlier discussion, when $\gamma_i = \lambda < |\tau_i|$, W is asking for good load disturbance rejection by forcing $S = 0$ at $s = -1/\tau_i$, which may be responsible for a large peak on $|S|$ and $|T|$ and a somewhat aggressive response.[13] As we increase γ_i, W specifies lower gains for $|S|$ at middle-high frequencies, which, via a *waterbed effect* argument[2,13] is achieved augmenting $|S|$ at low frequencies. Consequently, augmenting γ_i has an *smoothing* effect. In particular, this means that improving the response to output disturbances will also make the system slower. As it will be shown in Section 4, after increasing γ_i, λ can be decreased to compensate for the reduction of the closed-loop bandwidth. In summary, tuning γ_i also has an effect on robustness, but it should be clear that the way of affecting the robustness properties is different: λ is more related to the closed-loop bandwidth, which, by the robust stability condition (3.15), is responsible for robustness in the high-frequency region (model uncertainty). On the other hand, the γ_i parameters affect the mid-frequency robustness properties, altering the peaks of the sensitivity functions. More precisely, augmenting γ_i contributes to flattening the frequency response.

4. SIMULATION EXAMPLES

In this section, we consider three simulation examples to illustrate the features of the proposed procedure. For evaluating robustness, we use the peak of the sensitivity function

$$M_S \doteq \|S\|_\infty = \sup_\omega \left|\frac{1}{1 + PK(j\omega)}\right| \qquad (4.1)$$

Because M_S is the inverse of the shortest distance from the Nyquist curve of $L = PK$ to the critical point $-1 + 0j$, small values of M_S indicate good robustness. For a reasonably robust system, an upper bound for the M_S value can be fixed at ~ 2.[2] Another robustness indicator used throughout the examples is given by $M_T \doteq \|T\|_\infty \doteq \sup_\omega |T(j\omega)|$. The robustness interpretation for M_T (the peak of $|T|$) comes from the robust stability condition (3.15). To quantify the input usage, we compute the total variation (TV) of the input u:

$$\mathrm{TV} \approx \sum_{i=1}^{\infty} |u_{i+1} - u_i| \qquad (4.2)$$

where $\{u_i\}_{i=1}^{\infty}$ denotes a discretization sequence of u. In the examples that follow, we restrict our attention to (unity) step disturbances ($d_d = s$), as it is commonly done in the literature.

Example 1. The purpose of this preliminary example is to illustrate the different effect of the λ and γ parameters, completing the discussion in Section 3. We will consider the process $\tilde{P} = -100(10s + 1)(0.02s + 1)/[(-100s + 1)(s + 1)(0.2s + 1)]$, modeled as $P = -100(10s + 1)/[(-100s + 1)(s + 1)]$. The proposed design yields $Q = P_m^{-1}f = P^{-1}f$, where

$$f = \frac{a_1 s + 1}{(\lambda s + 1)(\gamma s + 1)} \qquad (4.3)$$

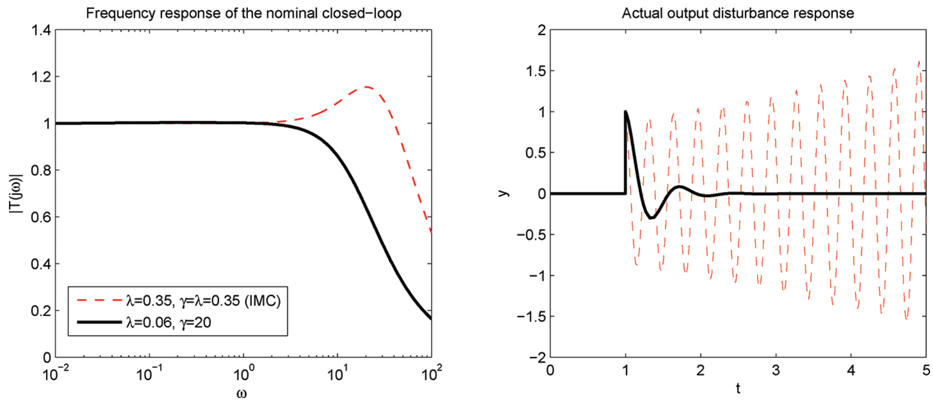


Figure 4. Frequency and time responses (Example 1).

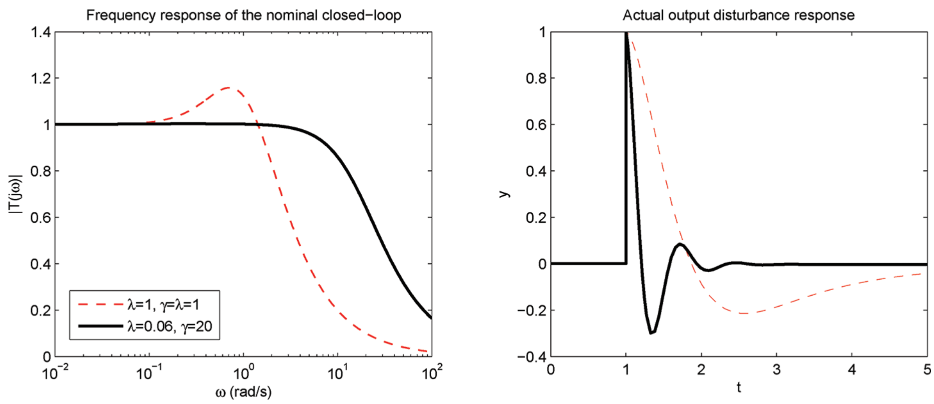
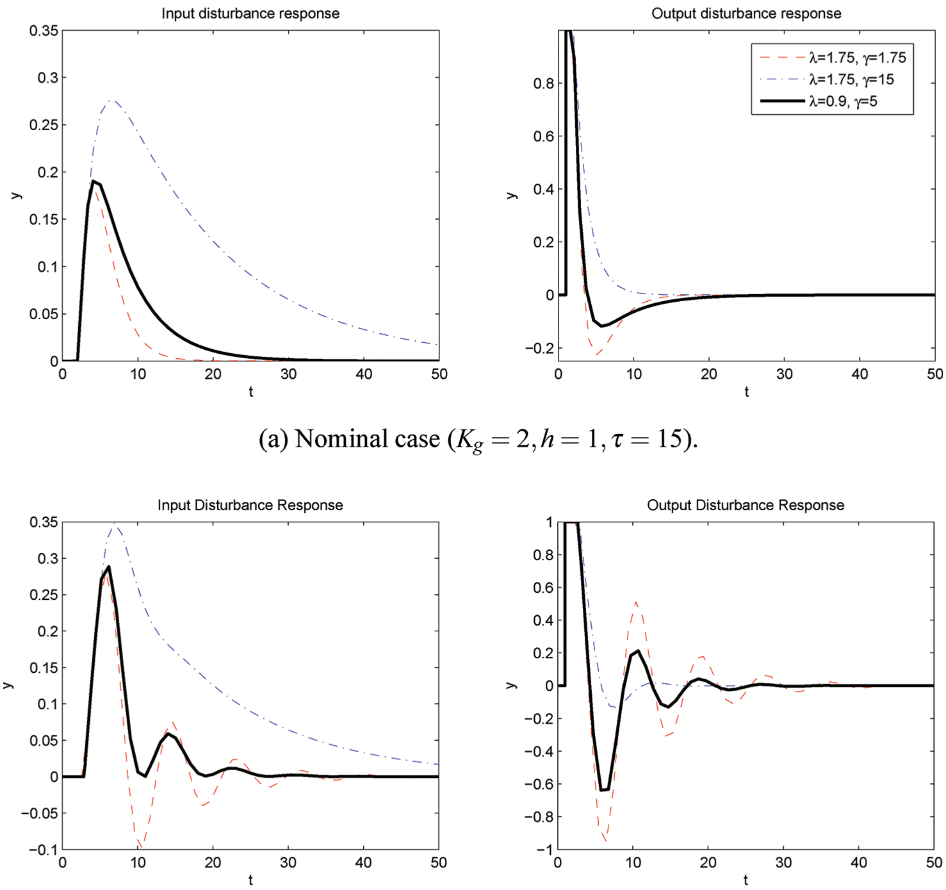


Figure 5. Frequency and time responses (Example 1).

with $a_1 = 100\{[1 + (\lambda/100)][1 + (\gamma/100) - 1]\}$ and $\gamma \in [\lambda, 100]$. For $\gamma = \lambda$, (4.3) coincides with the conventional IMC filter used in refs 3 and 10, which, in this case, favors input disturbances. As a consequence, the response for output disturbances may be undesirable. Figure 3 displays the time/frequency responses for $\lambda = \gamma = 0.15$. As it can be seen, the peak in $|T|$ ($M_T = 1.16$) degrades robust stability, read as $|T(j\omega)| < 1/|\Delta(j\omega)|\ \forall\omega$, and it is responsible for the large oscillations in the response to the output disturbance. We know that, by increasing γ (we take $\gamma = 20$), it is possible to improve this response. As stated in Section 3, this also tends to make the system slower. In order to preserve the original closed-loop bandwidth, the parameter λ can be decreased (we finally take $\lambda = 0.06$). As shown in Figure 3, this retuning allows to keep the original closed-loop bandwidth while avoiding the peak in $|T|$ (now, $M_T = 1$). The resulting outcome is better robustness and smoother response. It is remarkable that it is not possible to press down the peak of $|T|$ by using the classical filter (for which $\gamma = \lambda$), as illustrated in Figures 4 and 5. Clearly, if one uses the standard filter structure, the only reasonable option is to detune the controller, moving the peak of $|T|$ to lower frequencies (see Figure 5). This will improve robustness at the expense of nominal performance. In summary, even if there is an interaction between λ and γ, their roles are significantly different.

Example 2. As pointed out in the early work,[20] an optimal controller designed for a specific type of disturbance (e.g., a step acting at the input of the plant) may result in very poor performance if the actual disturbance (e.g., a step acting at the output) is different from the one considered at the design stage. In this example, we examine how a balance between the response of input and output disturbances can be achieved, focusing on the FOPTD model, $P = K_g(e^{-sh})/(\tau s + 1)$ ($K_g = 2, h = 1, \tau = 15$). The controller was already calculated in (3.7), and the corresponding filter has the same form as (4.3), taking now $a_1 = \tau - \tau[1 - (\lambda/\tau)][1 - (\gamma/\tau)]$. Let us start by selecting $\lambda = \gamma = 1.75$, which provides $M_S = 1.69$, $M_T = 1.28$, $\mathrm{TV}_i = 2.98$, $\mathrm{TV}_o = 39.78$ (TV_i and TV_o denote the total variation with respect to the input and output disturbances, respectively). As shown by Figure 6a, good attenuation of load disturbances is obtained. However, a somewhat large undershoot occurs for output disturbances. In the uncertain case ($h = 1.9$), we can see that the system becomes quite oscillatory ($\mathrm{TV}_i = 10.72, \mathrm{TV}_o = 82$); see Figure 6b. By choosing $\gamma = \tau = 15$, we can avoid the undershoot in the output disturbance response and improve the robustness margins, now $M_S = 1.33$, $M_T = 1$, $\mathrm{TV}_i = 2$, $\mathrm{TV}_o = 16.14$. As a result, the responses are smoother in the uncertain case ($\mathrm{TV}_i = 2.52$, $\mathrm{TV}_o = 18.6$), experiencing less performance degradation. However, the performance for load disturbances is poor, showing a sluggish return to steady state (this fact, which is sometimes called *loss of integral action*, is specially relevant for very lag-dominant plants with high gain[20]). To reach a compromise, we finally retune the controller taking $\lambda = 0.9$, $\gamma = 5$. The latter values give $M_S = 1.6$, $M_T = 1.13$, $\mathrm{TV}_i = 2.55$, $\mathrm{TV}_o = 40.67$. In the uncertain case, $\mathrm{TV}_i = 6.73$, $\mathrm{TV}_o = 67.63$. The data concerning this example have been collected in Table 1.

Example 3. Lastly, we consider a stable, second-order system with a pair of poorly damped poles (this case is studied in



(a) Nominal case ($K_g = 2, h = 1, \tau = 15$).

(b) Uncertain case ($K_g = 2, h = 1.9, \tau = 15$).

Figure 6. Input/output step responses for Example 2: (a) nominal case ($K_g = 2$, $h = 1$, $\tau = 15$) and (b) uncertain case ($K_g = 2$, $h = 1.9$, $\tau = 15$).

Table 1. Data Summary for Example 2

			Nominal Case $K_g = 2$, $h = 1$, $\tau = 15$				Uncertain Case $K_g = 2$, $h = 1.9$, $\tau = 15$			
Model : $K_g\left(\frac{e^{-sh}}{\tau s + 1}\right)$			Input dist.		Output dist.		Input dist.		Output dist.	
Tuning of (4.3)	M_S	M_T	TV	ISE	TV	ISE	TV	ISE	TV	ISE
$\lambda = 1.75, \gamma = 1.75$	1.69	1.28	2.98	0.17	39.78	2.39	10.72	0.31	82	6.3
$\lambda = 1.75, \gamma = 15$	1.33	1	2	0.66	16.14	2.34	2.52	0.69	18.6	3.06
$\lambda = 0.9, \gamma = 5$	1.6	1.13	2.55	0.21	40.67	2.71	6.73	0.29	67.63	5.92

Appendix A.2). The model is given by

$$P = K_g \frac{e^{-sh}}{(s/\omega_n)^2 + 2(\xi/\omega_n)s + 1}$$

$$(K_g = 4, h = 1, \omega_n = 0.5, \xi = 0.25)$$

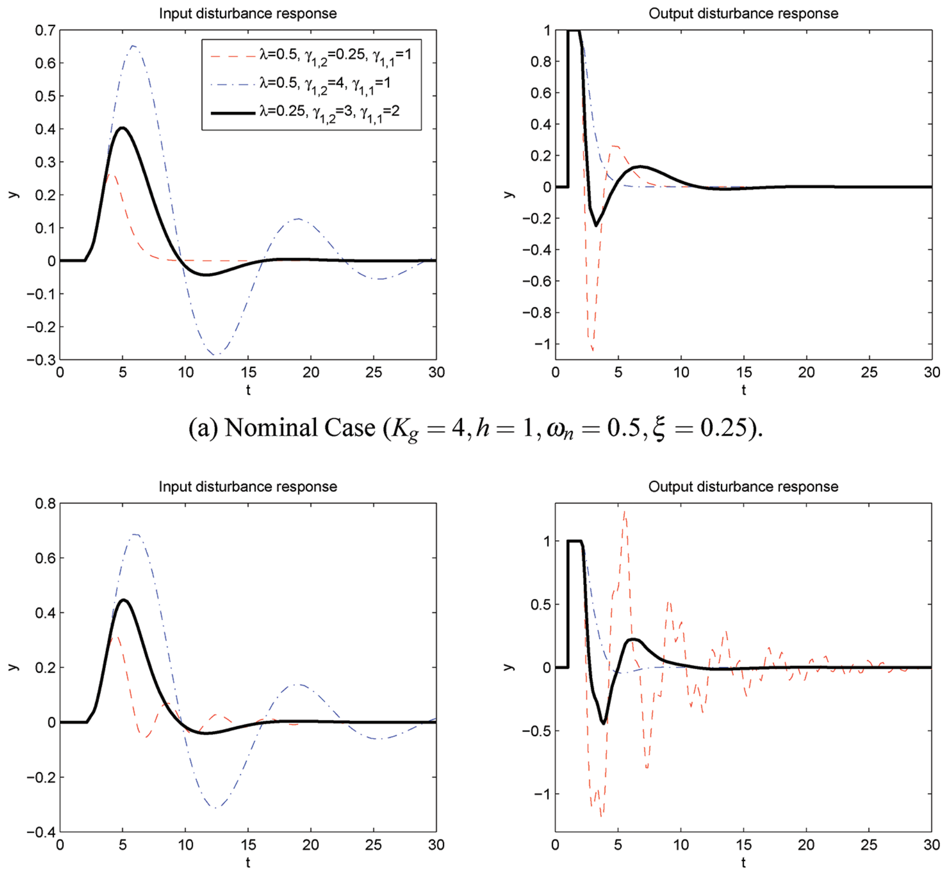
Our design suggests the controller $Q = P_m^{-1}f = [(s/\omega_n)^2 + 2(\xi/\omega_n)s + 1]f$, where f has the following structure:

$$f = \frac{a_2 s^2 + a_1 s + a_0}{(\lambda s + 1)^2(\gamma_{1,2}s^2 + \gamma_{1,1}s + 1)} \tag{4.4}$$

where the a_i coefficients satisfy the condition that $P_a f = e^{-s}f = 1$ at the poles of the weight:

$$W = \frac{(\lambda s + 1)^2\left(\gamma_{1,2}s^2 + \gamma_{1,1}s + 1\right)}{s\left[(s/\omega_n)^2 + 2(\xi/\omega_n)s + 1\right]}$$

First, we select $\lambda = 0.5$. For output disturbances, we then take $\gamma_{1,2} = (1/\omega_n)^2 = 4$, $\gamma_{1,1} = 2\xi/\omega_n = 1$, which results in $f = 1/(0.5s + 1)^2$. The associated responses can be seen in Figure 7 for both the nominal and uncertain cases. The tuning $\lambda = 0.5$, $\gamma_{1,2} = 4$, $\gamma_{1,1} = 1$ gives $M_S = 1.62$, $M_T = 1$, $TV_i = 4.91$, $TV_o = 140$ (nominal case), and $TV_i = 6.5$, $TV_o = 142$ (uncertain case). This design gives



(a) Nominal Case ($K_g = 4, h = 1, \omega_n = 0.5, \xi = 0.25$).

(b) Uncertain Case ($K_g = 4, h = 1.15, \omega_n = 0.5, \xi = 0.25$).

Figure 7. Input/output step responses for Example 3: (a) nominal case ($K_g = 4$, $h = 1$, $\omega_n = 0.5$, $\xi = 0.25$) and (b) uncertain case ($K_g = 4$, $h = 1.15$, $\omega_n = 0.5$, $\xi = 0.25$).

good results for output disturbances, because the slightly damped poles are canceled by the feedback controller. However, because no additional damping is really provided, these modes appear when excited from the input of the plant. Consequently, the response to load disturbances is quite oscillatory. To obtain much better performance for load disturbances, we select $\lambda = 0.5$, $\gamma_{1,2} = \lambda^2 = 0.25$, $\gamma_{1,1} = 2\lambda = 1$. For these settings, the filter f is defined as

$$f = \frac{2.9658s^2 + 2.3389s + 1}{(0.5s + 1)^3}$$

As desired, the response to load disturbances has been improved noticeably. However, a great undershoot appears for output disturbances, indicating that robustness has been seriously degraded: $M_S = 3.88$, $M_T = 2.96$. The corresponding input usage is given by $\mathrm{TV}_i = 41.44$, $\mathrm{TV}_o = 1840$ (nominal case) and $\mathrm{TV}_i = 236.8$, $\mathrm{TV}_o = 8365$ (uncertain case, $h = 1.15$). A tradeoff between the two designs considered so far can be obtained by selecting $\gamma = 0.25$, $\gamma_{1,2} = 3$, $\gamma_{1,1} = 2$, which corresponds to the filter

$$f = \frac{4.682s^2 + 2.06s + 1}{(0.25s + 1)(3s^2 + 2s + 1)}$$

With this retuning, we ultimately get $M_S = 2.21$, $M_T = 1.35$, $\mathrm{TV}_i = 17.21$, $\mathrm{TV}_o = 903$ (nominal case), and $\mathrm{TV}_i = 24.4$, $\mathrm{TV}_o = 1121$ (uncertain case). The idea for selecting $\gamma_{1,2}$, $\gamma_{1,1}$ is to place the complex poles of f to the left of those of the plant P, and with increased damping factor. A summary of the results obtained can be consulted in Table 2.

To conclude this example, we will consider the simplified structure for f given by (A.6) in the Appendix A.2, which, in the case at hand, has the form

$$f = \frac{a_2s^2 + a_1s + a_0}{(\lambda s + 1)^2(\gamma s + 1)^2} \tag{4.5}$$

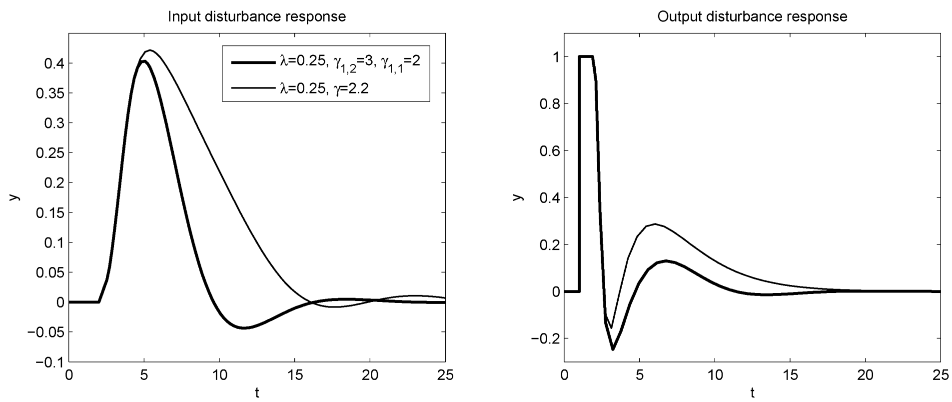
where the a_i coefficients satisfy the condition that $P_a f = e^{-s} f = 1$ at the poles of the weight:

$$W = \frac{(\lambda s + 1)^2(\gamma s + 1)^2}{s\left[(s/\omega_n)^2 + 2(\xi/\omega_n)s + 1\right]}$$

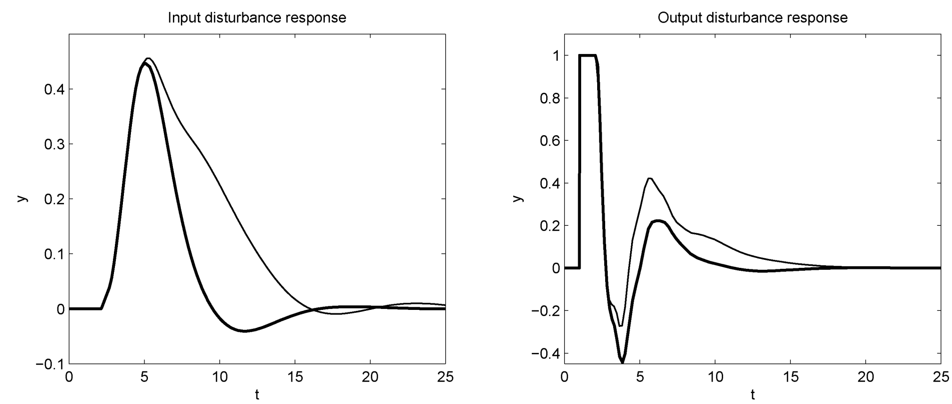
This filter has the same structure that was suggested by Campi et al.[8] for MP unstable plants. If we choose $\gamma = \lambda$, we recover the design for input disturbances. The purpose now is to show that, although this filter can also be used to improve robustness with respect to the design for input disturbances, the robustness enhancement generally requires one to sacrifice more nominal performance than when using the full-structure

Table 2. Data Summary for Example 3

Model : $K_g\left[\frac{e^{-sh}}{(s/\omega_n)^2 + 2(\xi/\omega_n)s + 1}\right]$			Nominal Case $K_g = 4, h = 1, \omega_n = 0.5, \xi = 0.25$				Uncertain Case $K_g = 4, h = 1.15, \omega_n = 0.5, \xi = 0.25$			
			Input dist.		Output dist.		Input dist.		Output dist.	
Tuning of (4.4)	M_S	M_T	TV	ISE	TV	ISE	TV	ISE	TV	ISE
$\lambda = 0.5, \gamma_{12} = 0.25, \gamma_{11} = 1$	3.88	2.96	41.44	0.59	1840	11.55	236.8	0.85	8365	33.82
$\lambda = 0.5, \gamma_{12} = 4, \gamma_{11} = 1$	1.62	1	4.91	3.78	140	8.19	6.5	4.73	142	8.34
$\lambda = 0.25, \gamma_{12} = 3, \gamma_{11} = 2$	2.21	1.35	17.21	2.14	903	9.78	24.4	2.62	1121	12.5



(a) Nominal Case ($K_g = 4, h = 1, \omega_n = 0.5, \xi = 0.25$).



(b) Uncertain Case ($K_g = 4, h = 1.15, \omega_n = 0.5, \xi = 0.25$).

Figure 8. Time responses for Example 3 with simplified filter structure : (a) nominal case ($K_g = 4$, $h = 1$, $\omega_n = 0.5$, $\xi = 0.25$) and (b) uncertain case ($K_g = 4$, $h = 1.15$, $\omega_n = 0.5$, $\xi = 0.25$).

filter with $\gamma_{1,2}$, $\gamma_{1,1}$. Taking $\lambda = 0.25$, $\gamma = 2.2$, the concrete filter

$$f = \frac{8.354s^2 + 3s + 1}{(0.25s + 1)(2.2s + 1)^2}$$

results, for which $M_S = 2.24$, $M_T = 1.4$. These robustness indicators are only slightly worse than those obtained for the previous tradeoff tuning ($M_S = 2.21$, $M_T = 1.35$). However, the overall performance is considerably worse, as can be appreciated from Figure 8. This shows the necessity of considering complex conjugate poles in the filter f for the best tradeoff design.

5. CONCLUSIONS

We have presented a regulatory design method that considers whether the disturbances enter at the input or at the output of the plant. When both types of disturbances are expected, the design allows one to reach a balance. This is achieved by means of considering a weighted sensitivity problem, where the weight is mostly guided by the input type and the conflictive poles of the plant. The final solution can be interpreted in terms of alternative IMC filters, which allow one to adjust both the robustness/performance and the input/output disturbance tradeoffs. Simulation examples have shown that improving the rejection of input disturbances inherently requires larger peaks of the sensitivity functions, resulting into more aggressive responses. Further work will focus on the application to PID control.

A. APPENDIX

A.1. Proof of Proposition 3.1. *Proof.*

(P1) The $\mathcal{H}_2$-optimal solution is given by $Q = (P_m W)^{-1}\{P_a^{-1}W\}_*$. The difference, with respect to the proposed solution amounts to the $\{\cdot\}_*$ operator. If $P_a \neq 1$, $\{\cdot\}_*$ and $\{\cdot\}_\star$ coincide, because $\{P_a^{-1}W\}_*$ is strictly proper. Thus, in this case, (3.5) is optimal (with respect to the selected W). When $P_a = 1$, $\{P_a^{-1}W\}_\star \rightarrow \{P_a^{-1}W\}_*$ when $\lambda \rightarrow 0$. This is because $P_a^{-1}W = W$ tends to become strictly proper (the n zeros at $s = -1/\lambda$ of W move to infinity). Based on the definition of $\{\cdot\}_*$,$\{\cdot\}_\star$, both yield the same result when applied to strictly proper operands. Thus, the proposed Q tends to $Q = (P_m W)^{-1}\{P_a^{-1}W\}_* = P_m^{-1}$ when $\lambda \rightarrow 0$.

(P2) From (3.1) and (3.5), straightforward algebra shows that the structure of Q is given by

$$Q = \frac{P_m^{-1}\chi}{(\lambda s + 1)^n(\gamma_1 s + 1)\cdots(\gamma_k s + 1)} \tag{A.1}$$

where χ is a polynomial of degree $\delta_d(d_d) + k - 1$. Therefore, $\delta(Q) = n - \delta(P) - \delta(d_d) + 1$. Selecting n as in (3.2) provides $\delta(Q) \geq 0$, implying that Q is proper. Stability is also easy to check: the poles of Q are the left half-plane (LHP) zeros of P, collected in P_m, and the zeros of W, which are also in the LHP.

(P3) Equivalently, we will show that $T = 1 - S = 1$ at the poles of W. The complementary sensitivity function is

$$T = PQ = (P_a^{-1}W)^{-1}\{P_a^{-1}W\}_\star \tag{A.2}$$

If W has a pole at $s = p$ of multiplicity m, then we can write $P_a^{-1}W = (\phi(s))/(s-p)^m$, and (A.2) can be expressed as

$$T = \frac{(s-p)^m}{\phi(s)}\left(\cdots + \sum_{i=1}^{m-1}\frac{\alpha_i}{(s-p)^i} + \frac{\alpha_m}{(s-p)^m} + \cdots\right) \tag{A.3}$$

where $\alpha_m = \phi(p)$. Then, it is clear that $T|_{s=p} = [\alpha_m/(\phi(s))]|_{s=p} = 1$.

■

A.2. Extension to Plants with Integrators or Complex Poles. It has been shown in Section 3.2.1 that the proposed design amounts to the selection of a convenient filter f, so that $Q = P_m^{-1}f$. Here, we detail the structure of such a filter (passing over W for brevity) when P has integrators and/or complex conjugate poles. To keep it simple, we address each situation one at a time.

(*i*) *P has (exclusively) l poles at the origin*:
Let P have, exclusively, l poles at the origin. Then, the corresponding filter is

$$f = \frac{\sum_{i=0}^{\delta(d_W)-1} a_i s^i}{(\lambda s + 1)^n \prod_{i=1}^{l}(\gamma_i s + 1)} \tag{A.4}$$

where $\delta(d_W) = l + \delta(d_d)$. The only difference, with respect to (3.9), is that, now, $\gamma_i \in [\lambda,\infty)$, whereas for slow/unstable poles, we had $\gamma_i \in [\lambda, |\tau_i|]$. This can be easily understood, because an integrator corresponds to a pole with an infinitely large time constant.

(*ii*) *P has (exclusively) m complex conjugate poles*:
Let us suppose that the m complex conjugate poles are located at $-\xi_i\omega_{n_i} \pm j\omega_{n_i}(1-\xi_i^2)^{1/2}$. Then, the structure of the filter is

$$f = \frac{\sum_{i=0}^{\delta(d_W)-1} a_i s^i}{(\lambda s + 1)^n \prod_{i=1}^{m}(\gamma_{i,2}s^2 + \gamma_{i,1}s + 1)} \tag{A.5}$$

where $\delta(d_W) = 2m + \delta(d_d)$. For input disturbances, $\gamma_{i,2} = \lambda^2$,$\gamma_{i,1} = 2\lambda$, so that $\Pi_{i=1}^m(\gamma_{i,2}s^2 + \gamma_{i,1}s + 1) = (\lambda s + 1)^{2m}$. For output disturbances, we want $\Pi_{i=1}^m(\gamma_{i,2}s^2 + \gamma_{i,1}s + 1)$ to be equal to $(1/\omega_{n_i}^2)\Pi_{i=1}^m(s^2 + 2|\xi|\omega_{n_i} + \omega_{n_i}^2)$, which is achieved for $\gamma_{i,2} = (1/\omega_{n_i})^2$, $\gamma_{i,1} = 2|\xi|\omega_{n_i}$ (in this extreme case, only when P is stable, $\delta(d_W) = \delta(d_d)$, as explained in Section 3.2.1). It is not so simple now to determine an interval for $\gamma_{i,1}$, $\gamma_{i,2}$ as in the real poles case (this point is illustrated in Section 4). An exception occurs if the complex poles are well-damped ($|\xi_i|$ close to one), in this case, we can disregard the imaginary part and treat the complex conjugate pairs as double real poles at $s = \omega_{n_i}\xi_i$. This allows one to simplify the filter structure to be

$$f = \frac{\sum_{i=0}^{\delta(d_W)-1} a_i s^i}{(\lambda s + 1)^n \prod_{i=1}^{m}(\gamma_i s + 1)^2} \tag{A.6}$$

with $\gamma_i \in [\lambda, 1/|\omega_{n_i}\xi_i|]$.

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